Fly-around Motion Control Based on Exact Linearization with Adaptive Law

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1. Introduction

Formation flight is a new key technology that will help to extend space activities such as space debris elimination, communication with satellites having malfunctioning attitude control systems, and observation of a specific surface of a rotating asteroid in deep space. These applications can be realized by a chaser satellite flying around a target satellite and tracking a specific surface of the target satellite. In the present Note, this motion is referred to as fly-around motion. In order to achieve this motion, the chaser satellite requires not only attitude tracking but also position tracking. The attitude and position tracking control problem has been widely studied for formation flight.[1,2,3] A group of small satellites that require small sensor devices, such as CCD cameras, is expected to be the desired scheme for formation flight. The use of only a small on-board CCD camera to measure the line of sight (LOS), which is the angle between the sight direction of the on-board camera of the chaser satellite and the direction to the target satellite, requires a tracking control method based on LOS.

Therefore, in the present Note, a control scheme by which to achieve fly-around motion using LOS information is proposed based on the exact linearization method.[4] The performance of the fly-around motion depends on the inertia property of the target because the momentum of inertia dominates rotational motion. However, determining exactly the inertia of a malfunctioning target satellite or an asteroid in advance is almost impossible. In order to overcome this problem, an adaptive method for estimating the inertia ratios of a target is presented in the present Note, provided that the principal axes of the target are known. This assumption may be overly restrictive, but is needed in designing the adaptive method to work with an unknown target satellite. In addition, if a target is rotating fast, the large amount of force and the fuel relative to the mass of the chaser satellite will be needed in order to make the chaser satellite artificially orbit the target at a rate equal to the spin rate of the target. Therefore, in the present Note, only a case in which a target is rotating slowly is considered. A numerical example is given in order to verify the validity of the proposed control scheme.

2. Model Description

2.1. Equation of motion

The target is assumed to be a malfunctioning satellite, which may be uncontrolled, or an asteroid in deep space. When gravitational and orbital effects such as the Coriolis force are neglected, the equations of motion and attitude kinematics for the chaser satellite and the target satellite can be represented, respectively, as follows:

$$m_i \dot{\mathbf{v}}_i + m_i \boldsymbol{\omega}_i \times \mathbf{v}_i = \mathbf{f}_i \tag{1}$$

$$\mathrm{J}_i \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times \mathrm{J}_i \boldsymbol{\omega}_i = \mathbf{t}_i \quad (i = t, c) \tag{2}$$

$$\dot{\mathbf{q}}_i = \frac{1}{2} \begin{bmatrix} -\boldsymbol{\omega}_i^{\times} & \boldsymbol{\omega}_i \\ -\boldsymbol{\omega}_i^T & 0 \end{bmatrix} \mathbf{q}_i \tag{3}$$

where the subscripts c and t denote the chaser and target satellite, respectively, $\mathbf{f}_c$ and $\mathbf{t}_c$ are the control force and torque, respectively, given to the chaser satellite, $\mathbf{r}$, $\mathbf{v}$, $\boldsymbol{\omega}$ and $\mathbf{q}(=[\tilde{\mathbf{q}}^T q_4]^T)$ are the position, velocity, angular velocity and quaternion, respectively, of the satellites, m and J are the mass and the inertia tensor, respectively, of the satellites, and the notation $\mathbf{z}^{\times}$ denotes a skew-symmetric matrix. Note that the velocity, angular velocity, force and torque are indicated in the satellite body-fixed frames of each satellite. In the present study, differences in position, velocity, and angular velocity between the chaser and the target are defined in the chaser satellite body-fixed frame as $\mathbf{r}_e = \mathbf{r}_c - C_t^c \mathbf{r}_t$, $\mathbf{v}_e = \mathbf{v}_c - C_t^c \mathbf{v}_t$, and $\boldsymbol{\omega}_e = \boldsymbol{\omega}_c - C_t^c \boldsymbol{\omega}_t$, respectively, where C_t^c is the direct cosine matrix from the chaser satellite body-fixed frame to that of the target satellite.

2.2. Definition of Line of Slight and Attitude Error

In order to simplify analysis, the following assumptions are made: (1) a CCD camera, the screen of which is made up of matrix-structured pixels, is used to sense the direction to the target, (2) the direction of the camera sight is coincident with the $-x$ direction of the chaser satellite body-fixed frame, (3) the relative distance to the target in the x direction and the velocity of the target satellite on the on-board camera screen of the chaser satellite can be sensed.

Under these assumptions, the LOS parameters are defined as follows:

$$\mathbf{l}_{os} = \begin{bmatrix} r_{e_x} & d_y & d_z \end{bmatrix}^T = \begin{bmatrix} r_{e_x} & s(r_{e_y}/r_{e_x}) & s(r_{e_z}/r_{e_x}) \end{bmatrix}^T \tag{4}$$

where s is the focus length of the camera, and d_y and d_z are the y and z coordinates, respectively, indicating the position of the target on the chaser satellite's on-board camera screen, and r_{e_x} indicates the relative distance of the target satellite in the x direction of the chaser satellite body-fixed frame. Note that without loss of generality, s can be set as 1, and hereafter $s = 1$. The attitude error is defined as follows:

$$\mathbf{q}_e = \begin{bmatrix} \tilde{\mathbf{q}}_e \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{t4}\tilde{\mathbf{q}}_c - \tilde{\mathbf{q}}_t^{\times}\tilde{\mathbf{q}}_c - \tilde{\mathbf{q}}_t q_{c4} \\ \tilde{\mathbf{q}}_t^T\tilde{\mathbf{q}}_c + q_{t4}q_{c4} \end{bmatrix} \tag{5}$$

Let parameter $\mathbf{x}$ consist of the angular velocity of the chaser satellite, the position and attitude of the chaser satellite relative to the target satellite, and the velocity and angular velocity of the chaser satellite relative to the target satellite as measured by the chaser satellite, as $\mathbf{x} = \begin{bmatrix} \boldsymbol{\omega}_c^T & \mathbf{r}_e^T & \mathbf{q}_e^T & \mathbf{v}_e^T & \boldsymbol{\omega}_e^T \end{bmatrix}^T$. In order to derive the nonlinear controller based on the LOS parameters, let the vector $\hat{\mathbf{x}}$, which will be exactly linearized in the following section, be $\hat{\mathbf{x}} = \begin{bmatrix} \mathbf{l}_{os}^T & \tilde{\mathbf{q}}_e^T & \dot{\mathbf{l}}_{os}^T & \dot{\tilde{\mathbf{q}}}_e^T \end{bmatrix}^T = \begin{bmatrix} \hat{\mathbf{x}}_1^T & \hat{\mathbf{x}}_2^T & \hat{\mathbf{x}}_3^T & \hat{\mathbf{x}}_4^T \end{bmatrix}^T$.

3. Exact Linearization via Nonlinear Feedback

Let a nonlinear feedback control that includes force and torque be as follows:

$$\begin{bmatrix} \mathbf{f}_c \\ \mathbf{t}_c \end{bmatrix} = \boldsymbol{\alpha}(\mathbf{x}) + \mathrm{B}(\mathbf{x})\hat{\mathbf{u}} \tag{6}$$

where $\boldsymbol{\alpha}(\mathbf{x})$ is a vector $\in R^6$, $\mathrm{B}(\mathbf{x})$ is a matrix $\in R^{6\times 6}$, and $\hat{\mathbf{u}}(=[\hat{\mathbf{u}}_f^T \ \ \hat{\mathbf{u}}_t^T]^T)$ is a fictitious control input $\in R^6$ to control the LOS parameters and the attitude of the chaser satellite relative to the target satellite. Considering the second derivatives of the LOS parameters and quaternion errors and comparing both sides of the equation yields nonlinear functions, $\boldsymbol{\alpha}(\mathbf{x})$ and $\mathrm{B}(\mathbf{x})$ that convert the vector $\hat{\mathbf{x}}$ into an exact-linearized system

$$\dot{\hat{\mathbf{x}}} = \hat{\mathrm{A}}\,\hat{\mathbf{x}} + \hat{\mathrm{B}}\,\hat{\mathbf{u}} \tag{7}$$

$$\hat{\mathrm{A}} = \begin{bmatrix} 0_{6\times 6} & \mathrm{I}_{6\times 6} \\ 0_{6\times 6} & 0_{6\times 6} \end{bmatrix} \tag{8}$$

$$\hat{\mathrm{B}} = \begin{bmatrix} 0_{6\times 6} & \mathrm{I}_{6\times 6} \end{bmatrix}^T \tag{9}$$

where I is the identity matrix.

The nonlinear function $\boldsymbol{\alpha}(\mathbf{x})$ includes the angular acceleration of the target satellite. This angular acceleration can be estimated if the inertia tensor of the target is known and the angular velocity of the target can be sensed. The case in which the inertia tensor of the target is known is almost never satisfied in reality, when the target satellite is a non-cooperative satellite, or an asteroid. In order to overcome this problem, an adaptive law to estimate the inertia property of the target satellite will be described for a simple case in the next section.

4. Nonlinear Adaptive Controller

The purpose of the adaptive law derived in this section is to estimate the principal inertia ratios of the target under the assumptions that the principal axis of the target satellite is coincident with its body axis and the inertia tensor of the chaser is known exactly.

A Lyapunov function candidate is selected as:

$$V = \frac{1}{2}\boldsymbol{\eta}^T\boldsymbol{\eta} + \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma} + \frac{1}{2}g\left(1 - r_{ref}/r_{e_x}\right)^2 + \frac{1}{2}h\hat{\mathbf{x}}_2^T\hat{\mathbf{x}}_2/(1 - \hat{\mathbf{x}}_2^T\hat{\mathbf{x}}_2) + \frac{1}{2}\tilde{\mathbf{k}}^T \mathrm{G}^{-1}\tilde{\mathbf{k}} \tag{10}$$

where $\boldsymbol{\eta}$ and $\boldsymbol{\sigma}$ are defined, using constant positive scalars a and b, as $\boldsymbol{\eta} = a\hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_3$ and $\boldsymbol{\sigma} = b\hat{\mathbf{x}}_2 + \hat{\mathbf{x}}_4$, respectively, g and h are constant positive scalars, G is a constant positive definite diagonal matrix, $\tilde{\mathbf{k}}$ is the parameter mismatch between the correct value $\mathbf{k}$ and the estimated value $\hat{\mathbf{k}}$, $\mathbf{k}$ is the inertia ratio vector defined as

$$\mathbf{k} = \left[k_x \;\; k_y \;\; k_z\right]^T = \left[(J_{t22} - J_{t33})/J_{t11} \quad (J_{t33} - J_{t11})/J_{t22} \quad (J_{t11} - J_{t22})/J_{t33}\right]^T \tag{11}$$

and $\hat{\mathbf{x}}_1$ is defined as $\hat{\mathbf{x}}_1 = \left[r_{e_x} - r_{ref} \quad d_y \quad d_z\right]^T$. Note that r_{ref} is a positive, constant reference distance in the x direction of the chaser satellite body-fixed frame so as to avoid collisions between the chaser and target satellites.

By taking the time derivative of Eq.(10), applying control inputs $\hat{\mathbf{u}}_f$ and $\hat{\mathbf{u}}_t$

$$\hat{\mathbf{u}}_f = -a\hat{\mathbf{x}}_3 - c\boldsymbol{\eta} + \hat{\mathbf{u}}'_f \ , \hat{\mathbf{u}}'_f = \begin{bmatrix} -gr_{ref}(1 - r_{ref}/r_{e_x})/r_{e_x}^2 & 0 & 0 \end{bmatrix}^T \tag{12}$$

$$\hat{\mathbf{u}}_t = -b\hat{\mathbf{x}}_4 - d\sigma + \hat{\mathbf{u}}_t{}' \ , \hat{\mathbf{u}}_t{}' = -h\hat{\mathbf{x}}_2/(1 - \hat{\mathbf{x}}_2^T\hat{\mathbf{x}}_2)^2 \tag{13}$$

where c and d are constant positive scalars, and taking the parameter mismatches into account, an adaptive law by which to estimate the inertia ratios of the target can be designed as follows:

$$\dot{\hat{\mathbf{k}}} = -G(\mathrm{Y}_1^T\ \boldsymbol{\eta} + \mathrm{Y}_2^T\ \boldsymbol{\sigma}) \tag{14}$$

where

$$\begin{aligned}
\mathrm{Y}_1 &= \mathbf{r}_e^{\times}\,\mathrm{E}_1^{-1}(\mathbf{q}_e)\,\mathrm{E}_2(\mathbf{q}_e)\mathrm{diag}[\omega_{t2}\omega_{t3}, \omega_{t3}\omega_{t1}, \omega_{t1}\omega_{t2}] \\
\mathrm{Y}_2 &= -\mathrm{E}_2(\mathbf{q}_e)\mathrm{diag}[\omega_{t2}\omega_{t3}, \omega_{t3}\omega_{t1}, \omega_{t1}\omega_{t2}] \\
\mathrm{E}_1(\mathbf{q}_e) &= (q_{e4}\,\mathrm{I}_{3\times3} + \tilde{\mathbf{q}}_e^{\times})/2 \\
\mathrm{E}_2(\mathbf{q}_e) &= (q_{e4}\,\mathrm{I}_{3\times3} - \tilde{\mathbf{q}}_e^{\times})/2
\end{aligned}$$

The estimated inertia ratios of the target satellite, $\hat{\mathbf{k}} = [\hat{k}_x\ \ \hat{k}_y\ \ \hat{k}_z]^T$, are obtained by integrating the adaptive law Eq.(14) with an initial estimated value.

5. Numerical Simulation

The parameters of the numerical simulations are listed in Table 1. The position of the target is assumed to be on the origin in the inertia frame. The performance of the adaptive law is assessed by setting the initial estimated inertia ratios of the target at the erroneous values listed in Table 1. The adaptive law updates the parameters using the tracking errors after the tracking is almost achieved, based on the relationship between the tracking errors and the parameter mismatch.

Figures 1(a), (b) and (c) show the time responses of position errors, quaternion errors, and the inertia ratios for the target as estimated by the adaptive law, respectively. The LOS parameters and attitude errors are successfully controlled so that the chaser satellite flies around the target satellite, tracking a specific surface of the target satellite. The correct values of the inertia ratios for the target are $k_x = -0.5$, $k_y = 0.5$, and $k_z = 0$. The estimated inertia ratios change dramatically at the beginning of maneuvering, and converge to the correct values after approximately 300 s.

6. Conclusions

Fly-around motion control in the absence of a gravitational field or other disturbances has been introduced based on an exact-linearization method. An adaptive law has been provided in order to improve the performance of the controller for the case in which the principal axes of the target are known and the inertia ratios of the target include uncertainty. The large amount of force and the fuel relative to the mass of the chaser satellite is needed in order to make the chaser satellite artificially orbit the target at a rate equal to the spin rate of the target if a target is rotating fast. Therefore, only a case in which the target is rotating slowly has been considered in the present Note. Numerical simulation revealed that even if modeling of the inertia ratios for the target satellite includes uncertainty at the initial time, the proposed adaptive control method can estimate the correct inertia ratios of the target satellite and precisely control the position and attitude of the chaser satellite so that the chaser satellite can fly around the target satellite, tracking a specific surface of the target satellite.

References

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Table 1: Parameters for numerical simulation.

Mass and Inertia	target	$\mathrm{J}_t = \mathrm{diag}[10,10,15]$ [kgm^2]
	chaser	$m_c = 500$ [kg], $\mathrm{J}_c = \begin{bmatrix} 300 & -30 & -50 \\ -30 & 400 & -40 \\ -50 & -40 & 300 \end{bmatrix}$ [kgm^2]
Initial state	target	$\mathbf{q}_t = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\boldsymbol{\omega}_t = \begin{bmatrix} 0.01 & 0.01 & 0.01 \end{bmatrix}^T$ [rad/s]
	chaser	$\mathbf{r}_c = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix}^T$ [m], $\mathbf{v}_c = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ [m/s]
		$\mathbf{q}_c = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T$, $\boldsymbol{\omega}_c = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ [rad/s]
Gain		$a = b = c = d = 0.1$ $g = h = 0.001$ $\mathrm{G} = \mathrm{diag}[4, \quad 4, \quad 4] \times 10^5$
Initial estimation for the inertia ratios of the target		$\hat{k}_x(0) = 0, \hat{k}_y(0) = -0.5, \hat{k}_z(0) = 0.5$

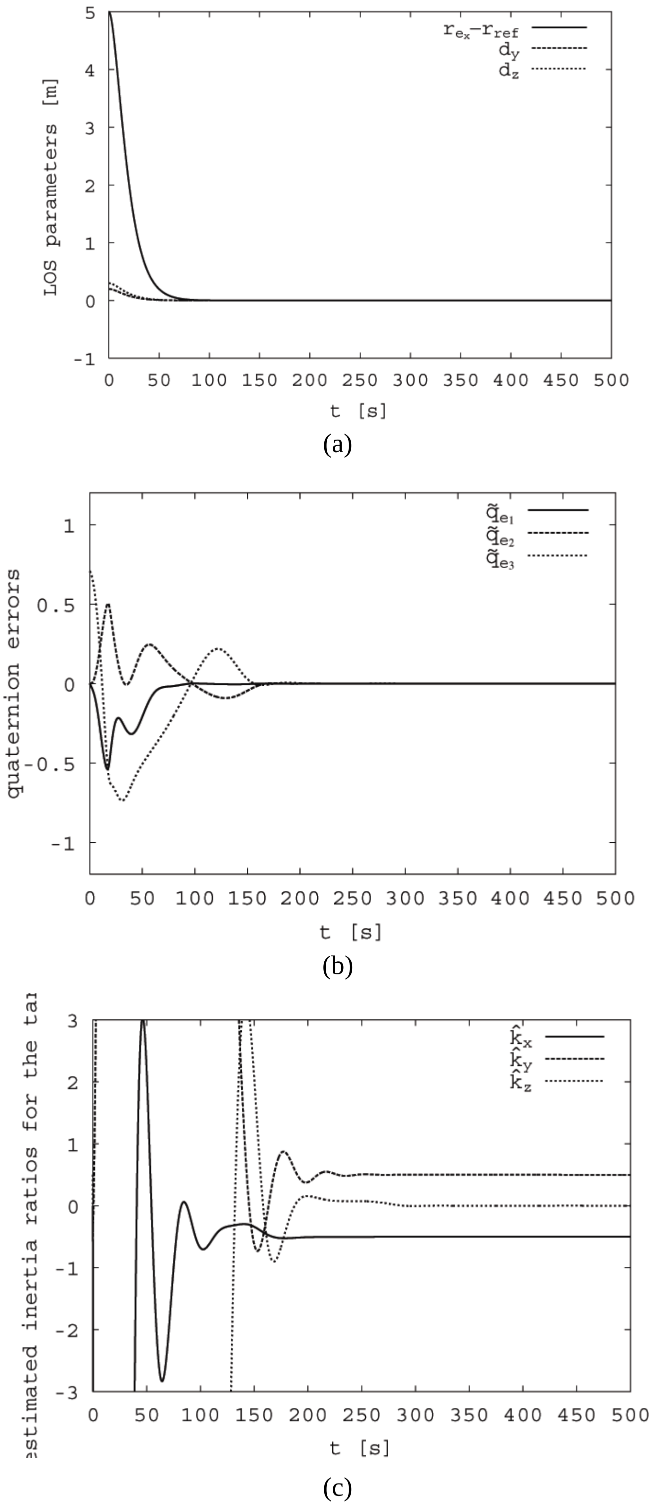

Figure 1: Time responses of LOS parameters (a), quaternion errors (b) , and estimated inertia ratios for the target satellite(c).