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A MULTIOBJECTIVE MAXIMAL COVERING LOCATION PROBLEM INCORPORATING INTER-CITY TRAFFIC NETWORK

Kenji ISHIZAKI*

Abstract Location-allocation models provide a valuable tool for operationalizing the central place theory. This paper attempts to integrate the marketing principle and the traffic principle in Christaller's central place theory using multiobjective model. The model is formulated with two objectives: (a) minimize the total population uncovered by each service, and (b) minimize the total cost of constructing arcs between centers. Some computational results indicate that the model is possible to derive the flexible system of central place hierarchy.

Key words: maximal covering location problem, shortest path problem, multiobjective programming, Christaller's traffic principle

1. Introduction

Location-allocation models for seeking the optimal solution of facility location problem are stated using mathematical programming formulation. It has been pointed out that such mathematical optimization techniques are suitable for operationalizing the concepts of classical location theories, especially central place theory (Beaumont 1987). In modelling central place theory as the optimization problem, it is important to verify the objectives to derive the theoretical central place systems. For instance, in the studies using location-allocation models, some of works defined as p-median problem (e.g. Dökmeci 1973; Puryear 1975), and others emphasized the similarity to covering problem (e.g. Storbeck 1988, 1990; Stratiff and Cromley 2010).

Especially, with regard to Christaller's central place theory (Christaller 1966), as Saey (1973) and Beaumont (1987) have pointed out, one possible interpretation is that of the set covering problem that seeks fewest locations while ensuring coverage over a region within the upper limit of the range of a good. Ishizaki (1992) demonstrated that the marketing principle of Christaller's central place theory was able to be formulated as hierarchical set-covering problem, and applied the model to a hypothetical lattice network. However, the solution of the model using top-down method brought up the question of how to generate the hexagonal network of theoretical central place system. As a result of reinterpreting the locational principle to reply to this question, the

* Nara Women's University

marketing principle was able to be modelled as a part of the generalized maximal covering location problem (Ishizaki 1995). Then, how can other principles of Christaller's central place theory be defined as mathematical programming formulation? The administrative principle that lower-level cities are entirely within the territory of the higher-level city can be formulated by adding the coherent constraints (Serra and ReVelle 1993; Şahin and Süral 2007). On the other hand, the traffic principle that aims to align small towns with traffic networks (e.g. highways or railways) between major cities is not able to be formulated easily. The reason is because it is difficult to consider simultaneously two problems of where to locate cities for supplying the goods or services and which to connect between cities as traffic network.

There are some studies concerned with the optimal network design of facility location problem. O'Kelly (1986) proposed the hub location problem that facilities act as switching points in network connecting a set of interacting nodes, and various models to solve hub-and-spoke systems have been developed (Farahani *et al*. 2013). While the hub location problem aims to decrease the number of links between nodes by minimizing the demand-weighted total travel cost, Melkote and Daskin (2001a, b) demonstrated the models that simultaneously optimize facility locations and the design of the underlying transportation network. These models clearly distinguished the total travel cost from demand nodes to facilities and the total link construction cost among the objective function, and Bigotte *et al*. (2010) expanded this model to hierarchical facility location problem. However, it is necessary to take into account a trade-off relationship between the facility location problem and the traffic network problem, because two objectives of these problems are likely to be competing.

This paper presents a new generalized model to solve simultaneously both problems of locating the city center and constructing the inter-city traffic network using multiobjective programming. The former problem can be formulated as the maximal covering location problem that is substantially regarded as the marketing principle of Christaller's central place theory. Then, the traffic network problem in the latter is defined as the shortest path problem between cities, and it is shown that a central place system based on the traffic principle is derivable by adding a new constraint to the model of marketing principle.

2. Model Formulation

When constructing an efficient traffic network, "the marketing principle is an awkward arrangement in terms of connecting different levels of the hierarchy" (Dicken and Lloyd 1990: 29). On the other hand, compared with the central place system based on the marketing principle, the traffic principle produces a system that "the central places would thus be lined up on straight traffic routes which fan out the central point" (Christaller 1966: 74). Figure 1 shows the difference of the traffic routes of central place systems derived by two principles: a) marketing principle and b) traffic principle. First of all, a traffic route of level 3 (e.g. major road) is constructed from central place of highest level 4 (e.g. metropolis) to level 3 place when assuming that the traffic routes connect between places of different hierarchical levels. Then, the traffic network is organized by different routes according to where a central place of level 2 is located. In the marketing principle that supplying the goods or services is a prior objective, it is necessary that traffic routes between level 2 place and levels 4 or 3 places are constructed newly because a central place of level 2 is

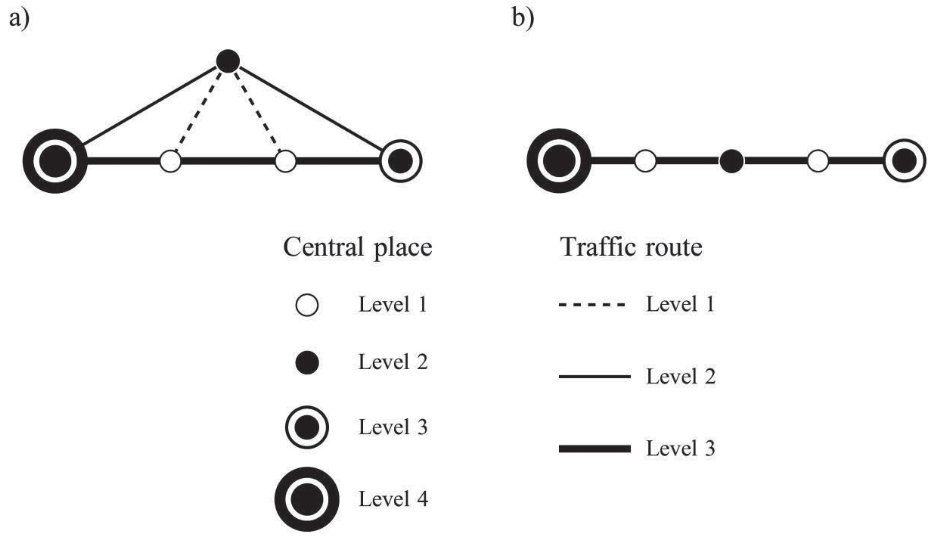

Fig. 1 Traffic routes of the marketing principle and the traffic principle.

located at point away from major cities (Fig. 1a). In contrast, the traffic principle is able to connect different levels of central places by only one traffic route of level 3 because lower cities are located along the route from level 4 place to level 3 place (Fig. 1b). As a result, the traffic route is constructed "as straightly and as cheaply as possible" (Christaller 1966: 74).

Here, we will assume the discrete location problem of centers that supply the goods or services of different types. The hierarchy of central place is determined by the number of centers on the node, and the arcs are simultaneously constructed on the shortest path between central places of different hierarchical levels. Then, the model is able to be formulated as follows as discrete and hierarchical location problem that there are two objectives of supplying the goods or services and reducing the traffic routes:

$$\min \quad w \sum_{m} \sum_{i} a_i X_{im} + (1-w) \sum_{g} c_g Z_g \tag{1}$$

subject to:

$$\sum_{j \in M_{im}} Y_{jm} + X_{im} \geq 1 \quad \forall\, i, m \tag{2}$$

$$Z_g \geq Y_{jm} + Y_{kl} - 1 - \sum_{h \in N_{jk}} Y_{hl} \quad \forall\, j, k, g \in R_{jk}, m, l > m \tag{3}$$

$$Y_{j(m-1)} - Y_{jm} \geq 0 \quad \forall\, j, m \geq 2 \tag{4}$$

$$\sum_{j} Y_{jm} = p_m \quad \forall m \tag{5}$$

$$X_{im} = 0,1 \tag{6}$$

$$Y_{jm} = 0,1 \tag{7}$$

$$Z_g = 0,1 \tag{8}$$

where:

a_i = population of demand node i;
c_g = cost of constructing arc g;
p_m = the number of type m centers;
d_{ij} = distance from node i to node j;
S_m = the maximum service distance of type m center;

$$X_{im} = \begin{cases} 1 \text{ if demand node } i \text{ is not covered by a type } m \text{ center within distance } S_m; \\ 0 \text{ otherwise;} \end{cases}$$

$$Y_{jm} = \begin{cases} 1 \text{ if a type } m \text{ center locates at node } j; \\ 0 \text{ otherwise;} \end{cases}$$

$$Z_g = \begin{cases} 1 \text{ if arc } g \text{ is constructed;} \\ 0 \text{ otherwise;} \end{cases}$$

M_{im} = the set of nodes j within distance S_m, that is $\{j|d_{ij} \leq S_m\}$;

N_{jk}= the set of nodes h closer to node j than node k, that is $\{h|d_{jh} < d_{jk}, h \neq j\}$;

R_{jk}= the set of arcs g on the shortest path from node j to node k;

The objective function (1) contains two objectives; (a) minimize the total population uncovered by each service, and (b) minimize the total cost of constructing arcs between centers. The model is formulated using multiobjective programming that seeks to feasible alternatives to attain two above objectives by adjusting the weight w. The weight w takes a value $0 < w < 1$. Constraint (2) allows X_{im} to equal 1 when the demand node i is not covered by either centers of type m within the maximum distance S_m. Constraint (3) defines the construction of arcs that connect between the nearest centers of different types. For example, suppose that a center of type 1 is located at node j and a center of type 2 is located at node k. If any centers of type 2 are not located at nodes closer to node j than node k, then Z_g is equal to 1 because constraint (3) insures $Z_g \geq 1$, and the arcs that compose the shortest path connecting between node j and node k are constructed. However, if one or more centers of type 2 are located at nodes closer to node j than node k and a center of type 2 is not located at node k, then Z_g is more than 0 or a negative value,

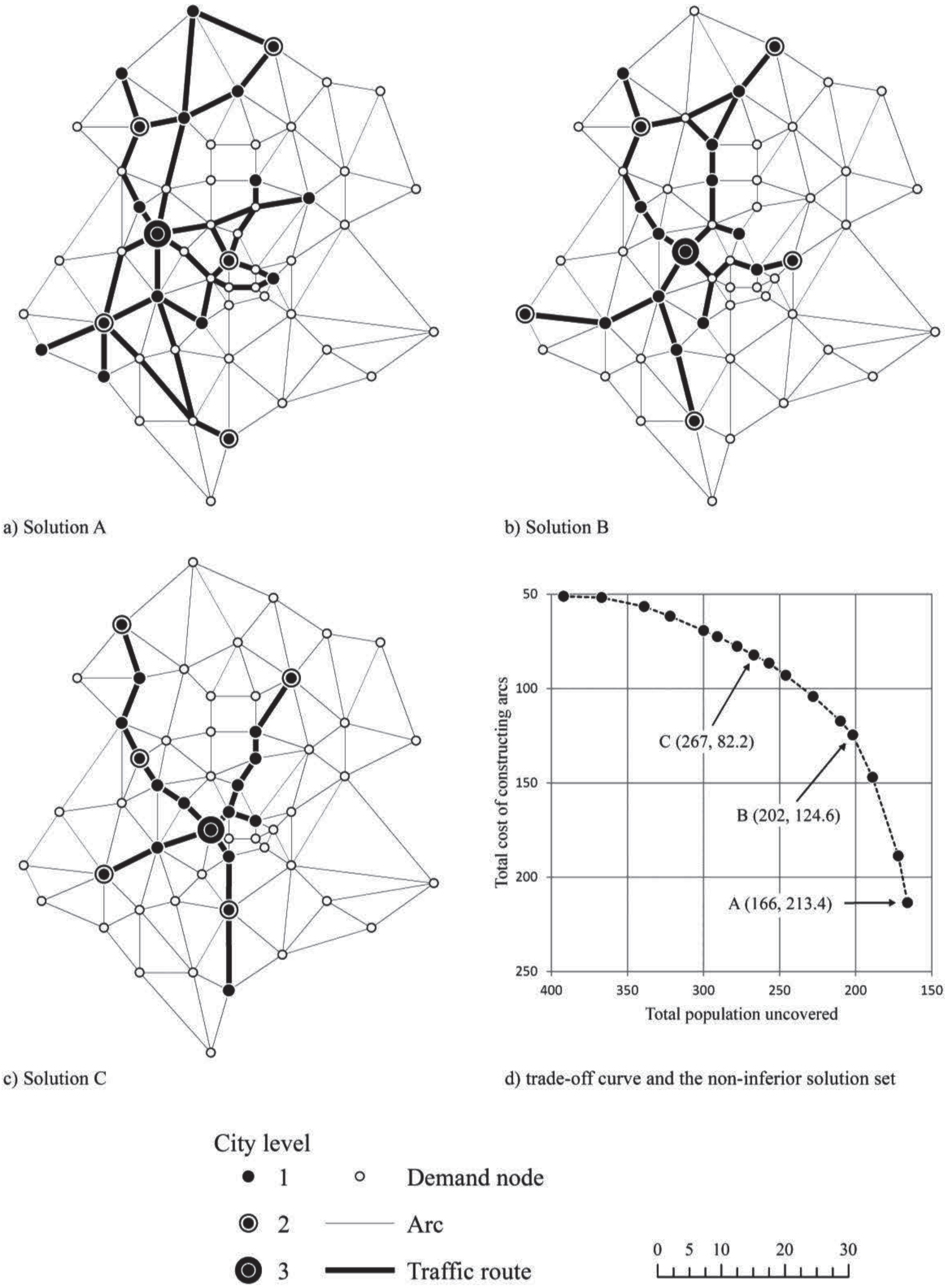

Fig. 2 Non-inferior solutions. In parentheses of Solution A through C among non-inferior solutions, the left numerical value indicates the total population uncovered and the right numerical value indicates the total cost of constructing arcs.

and it becomes $Z_g = 0$ because Z_g is minimized by objective function (1). Constraint (4) shows the successively inclusive hierarchy. It states that, if a center of type m is located at node j, then a lower-type m-1 center must also be located at node j. The number of type m centers is restricted in constraint (5).

3. Computational Experience

The above model was tested on the well-known 55-node Swain network shown in Fig. 2. Each node was considered as a demand node as well as a potential center location, and the arcs connecting between nodes were drawn by referring to Serra *et al*. (1996). The set of arcs constituting each shortest path were setting previously by solving the shortest path problem between two arbitrary nodes. The cost of constructing arc was equal to the length of the arc. Suppose that there were three kinds of center for supplying services of types 1 through 3, and the maximum service distance of each type was assumed to be $S_1 = 5$, $S_2 = 10$, and $S_3 = 20$ respectively. The number of centers from lower-order to higher-order was each $p_1 = 18$, $p_2 = 6$, and $p_3 = 1$. The problem was solved using NUOPT ver. 15.1.0 by NTT DATA Mathematical Systems Inc.

When the model is applied by adjusting the weight w in increments of 0.01 from 0.01 to 0.99, sixteen non-inferior solutions shown in Fig. 2d are derived. There is a trade-off relationship between two objectives. Among the non-inferior solutions, Solution A is the same result as the maximal covering location problem because the sum of uncovered population of each service takes a minimum value 166. Figure 2a shows the central place system by non-inferior Solution A. Depending on the result of supplying services for the entire region as much as possible, five cities of level 2 surround the city of level 3 city and twelve cities of level 1 are located away from the higher-level cities. Such dispersed locations of cities are similar to the arrangement of central place system by Christaller's marketing principle. However, many inter-city traffic routes are needed because of connecting between mutually separated cities and the total cost (distance) of constructing arcs becomes maximum value as a result (Fig 2d).

It becomes possible to reduce the total cost of construction arcs on the traffic network by reducing the weight w, and alternative solutions are derived. Solutions B and C are one of such alternative solutions. The central place system by Solution B shown in Fig. 2b indicates that the cities of level 2 are arranged as well as Fig. 2a but the cities of level 1 are located along the shortest path between level 3 and level 2 cities. Furthermore, according to Solution C (Fig. 2c), all levels of cities are aligned approximately on straight lines and a more efficient traffic network is constructed, as if the effect of traffic principle appears.

Thus, the above model is regarded as integrating both the marketing principle and the traffic principle that bring different consequences to the arrangement and the traffic network of central place systems.

4. Summary and Conclusions

This paper proposed to the maximal covering location problem incorporating inter-city traffic network and defined the model to integrate both of the marketing principle and the traffic principle in Christaller's central place theory. By introducing constraint (3), the location problem of center and the connecting problem between cities are able to be solved simultaneously. It was shown that various central place systems are derived by the model using multiobjective programming from the marketing principle to the traffic principle. However, to solve the above problem, greater computational efforts is necessary due to constraint (3) that have the tendency to cause the large size problem as the number of nodes and arcs increase. If we could apply the model toward a more complex problem, it might be also possible to derive a mixed hierarchy of central place system that different principles depending on the levels of cities are interacted. And, it might suggest the similarity with the general hierarchical model by Parr (1978).

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