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EXTENSION OF THE DISCRETE-ORDINATES TRANSPORT SOLVER IDT TO REGULAR TWO-DIMENSIONAL TRIANGULAR MESHES

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ABSTRACT

In this work, the Integro-Differential Transport solver (IDT), which is one of the transport solvers available in the APOLLO3 lattice code, has been extended to handle unstructured meshes. In particular, the previously implemented method of short characteristics (MoSC) used to solve for the spatial variable in the framework of a S_N approach has been extended to triangular cells, which represent the natural discretization for hexagonal lattice calculations. The coefficients of the collision probability matrices have been evaluated by means of a split-cell algorithm, specialized for dealing with different orientations of the triangle with respect to each discrete ordinate of the S_N sweeping. The correct implementation of the method and its robustness with respect to the skewness and the optical thickness of the triangle has been verified. The method of manufactured solutions has been employed to obtain a numerical estimate of the spatial convergence order of the method. Finally, a benchmark against reference MC calculations for the well known C5G7 benchmark has been succesfully performed.

KEYWORDS: Short characteristics, IDT, Unstructured meshes

1. INTRODUCTION

One among the concepts which is being investigated within the framework of GEN-IV is the fast breeder reactor [1]. Such a reactor is characterized by a hexagonal lattice pattern resulting from the juxtaposition of hundreds of hexagonal assemblies. To handle fast neutron transport simulations of such kind of reactor, a triangular mesh is desirable, e.g. for performing systematic mesh refinement. In this work, IDT [2], which is one of the transport solvers available in the APOLLO3 lattice code [3], has therefore been extended to handle transport calculations in unstructured meshes. In particular, among the implemented methods for the solution of the spatial variable in the framework of a discrete ordinate sweeping, the implemented method of short characteristics (MoSC) has been selected and extended to triangular cells [4]. The MoSC employs the integral transport equation to describe analytically the neutron transport within homogeneous cells. The angular flux within the cells, the angular flux on the interfaces between cells and the angular source are expanded on linear polynomial basis functions. The collision probability matrices are obtained by projecting the integral transport equation on the aforementioned linear basis functions. A routine for computing the coefficients of these matrices has been programmed which takes advantage of the knowledge of the orientation of the triangle with respect to each discrete ordinate to simplify calculations. The correct implementation of the method has carefully been verified and its performances assessed.

2. THE METHOD OF SHORT CHARACTERISTICS FOR TRIANGULAR MESHES

As this work regards a spatial discretization technique, the following discussion will consider the monokinetic transport equation written for a given direction $\mathbf{\Omega}$. In particular, the Method of Characteristics (MOC) is based on the integral form of the transport equation in each triangle T_α in which a domain D is discretized. Written along a characteristic direction the integro-differential equation becomes [5]:

$$\psi\left(x\right) = e^{-\tau(x,0)}\psi\left(0\right) + \int_0^x dx' \cdot e^{-\tau(x,x')} q\left(x'\right) \tag{1}$$

Where the source q includes selfscattering, fission and external source and $\tau\left(x,0\right)$ is the optical distance between 0 and x evaluated along a characteristic direction. Since in a S_N sweeping $\psi\left(0\right)$ and q are known, this represents an explicit formulation for the angular flux ψ at any distance x from the entrance of a characteristic direction within a certain domain. By defining suitable scalar products, this explict form of the transport equation for the angular flux can be projected on polynomial basis functions. By further assuming a constant total cross section for each T_α, an angular flux of the form $\psi\left(\boldsymbol{r}\right) = \boldsymbol{p}\left(\boldsymbol{r}\right)\cdot\boldsymbol{\psi}$ and an angular source of the form $q\left(\boldsymbol{r}\right) = \boldsymbol{p}\left(\boldsymbol{r}\right)\cdot\boldsymbol{q}$, the equations for the MOC are retrieved. In addition to that, in MoSC also the boundary flux is expanded in an analogous way, $\psi_s\left(\boldsymbol{r}\right) = \boldsymbol{p}\left(\boldsymbol{r}\right)\cdot\boldsymbol{\psi}_s$. The result is the following set of equations for the moments of the angular flux:

$$\begin{cases} \left(\boldsymbol{p},\boldsymbol{p}\right)\boldsymbol{\psi} = \sum\limits_{s'}\left(\boldsymbol{p}, e^{-\tau}\boldsymbol{p}_{s'}\right)\boldsymbol{\psi}_{s'} + \left(\boldsymbol{p},\mathrm{K}\boldsymbol{p}\right)\boldsymbol{q} \\ \left\langle\boldsymbol{p}_s,\boldsymbol{p}_s\right\rangle^+\boldsymbol{\psi}_s = \sum\limits_{s'}\left\langle\boldsymbol{p}_s, e^{-\tau}\boldsymbol{p}_{s'}\right\rangle_s^+\boldsymbol{\psi}_{s'} + \left\langle\boldsymbol{p}_s,\mathrm{K}\boldsymbol{p}\right\rangle^+\boldsymbol{q} \end{cases} \tag{2}$$

where the summations over s' represents the sum of contributions from the incoming surfaces to the angular flux within the cell volume and to the angular flux on the outgoing surface, respectively. In matrix form:

$$\begin{cases} \boldsymbol{\psi} = \sum\limits_{s'}\mathrm{I}_{s'}\boldsymbol{\psi}_{s'} + \mathrm{C}\boldsymbol{q} \\ \boldsymbol{\psi}_s = \sum\limits_{s'}\mathrm{T}_{s'\to s}\boldsymbol{\psi}_{s'} + \mathrm{E}_s\boldsymbol{q} \end{cases} \tag{3}$$

where $I_{s'}$ is the Incoming matrix, C is the Collision matrix, $T_{s' \to s}$ is the Transmission matrix and E_s is the Escape matrix. This form, proposed in [4], elegantly conveys the physical meaning of the explicit procedure used in the convolution integral evaluation which is the basis of the MOC in general, and of the MoSC in particular.

The application of the MOSC to triangular mesh is performed by means of the split-cell algorithm [6], which allows to simplify the evaluation of the coefficients of the four matrices in Eq. (3). The exact implementation for calculating the matrix coefficients depends on the couple $\{\alpha, \mathbf{\Omega}\}$, and in particular on the number of incoming and outgoing sides of T_α for a given $\mathbf{\Omega}$. A schematic for the case of one incoming surface and two outgoing surfaces is provided in Fig. 1. The formalism employed in this work mostly follows [6].

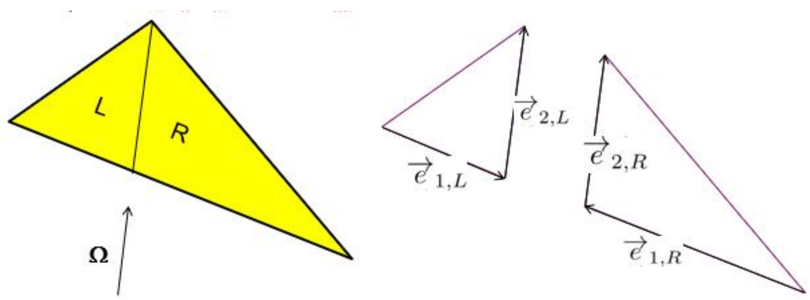

Figure 1: Application of the split-cell algorithm to a cell with one incoming face and two outgoing faces

3. VERIFICATION

First, numerical tests have been performed to verify the robustness of the method and of its implementation in IDT. Differential conservation has been verified up to machine precision for a wide span of triangle skewnesses and cell optical thicknesses. To estimate the convergence order of the method, the method of manufactured solution has instead been employed [7]. If a linear flux is imposed on a cell, the error is again below machine precision. If a parabolic or exponential form of the flux is instead enforced, the error decreases as the characteristic dimension of a cell is reduced, see Fig. 2, following a constant convergence order. Finally, the method has been succesfully benchmarked against MC results for the classical C5G7 benchmark case [8], see again Fig. 2.

4. CONCLUSIONS AND PERSPECTIVE

In this work an extension of the IDT code based on the method of short characteristics has been presented for application to triangular unstructured meshes. The robustness of the method with respect to the triangle skewness and optical thickness has been verified. The convergence order has been numerically estimated by means of the method of manufactured solution, and a classical benchmark, the C5G7, has been succesfully passed. As results appear promising, the extension of the acceleration methods available in IDT to the case of triangular meshes would be a natural prosecution of the present work.

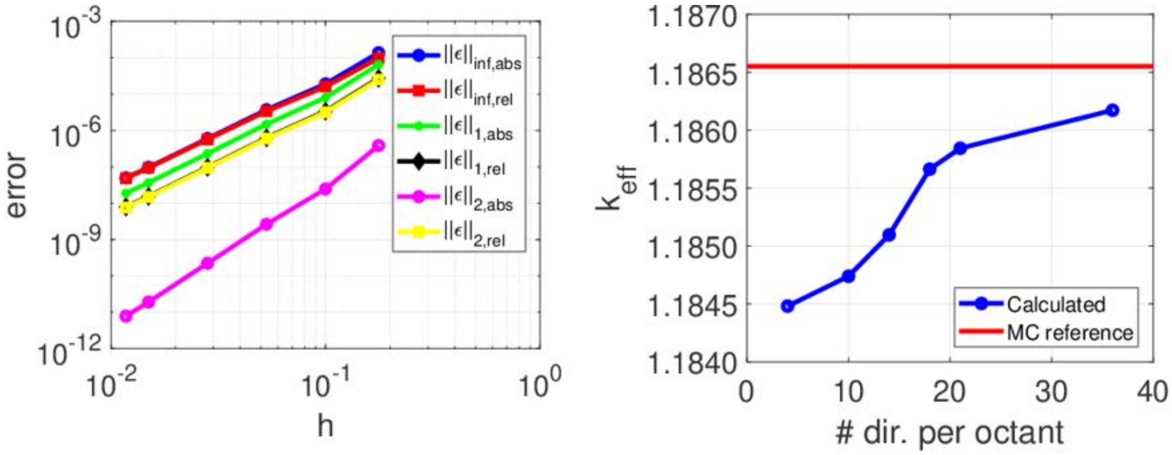

Figure 2: Estimation of convergence order of the method via MMS with various definitions of the error (left) and result of C5G7 benchmark (right)

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