RELIABILITY OF SOIL POROSITY ESTIMATION FROM SEISMIC WAVE VELOCITIES

Geotechnical and Geophysical Site Characterisation 5 – Lehane, Acosta-Martínez & Kelly (Eds)

Reliability of soil porosity estimation from seismic wave velocities

S. Foti & F. Passeri
Politecnico di Torino, Turin, Italy

ABSTRACT: Soil porosity is a state parameter of fundamental importance for several geotechnical problems. Geophysical testing provides appealing strategies for the determination of soil porosity, as several geophysical parameters are directly related to soil porosity. In particular the theory of wave propagation in saturated porous media, developed by Biot in the 1950s, allows the determination of soil porosity from the measured velocity of propagation of compressional and shear waves. A formal assessment of the reliability of the estimated porosity values is of primary importance to evaluate the applicability of this approach to solve practical geotechnical problems. In this paper the propagation of measurement uncertainties on the estimated values of soil porosity is theoretically evaluated. Moreover, experimental data of multiple acquisitions of cross-hole tests are considered. Data collected by different operators are also used to assess the confidence interval associated to different equipment, acquisition practices and testing methodology.

1 INTRODUCTION

Porosity is widely recognised as a key parameter as it affects the mechanical and hydraulic response of soils. A crucial issue in its determination is typically represented by the difficulties in collecting high quality undisturbed samples for coarse-grained soils (Jamiolkowski 2012) or for deep formations (Musso et al. 2015). Usually in geotechnical engineering, the relative density of sands is obtained through empirical correlations with in situ tests, whose reliability is not easily assessed.

In geomechanics of reservoirs and rock physics, porosity is commonly derived through analytical and/or (semi-)empirical formulations from geophysical parameters measured in wells (e.g. Mavko et al. 1998).

Foti et al. (2002) proposed an approach for porosity assessment on the basis of measured seismic wave velocity by adopting the formulation by Biot (1956a, 1956b) for linear elastic, isotropic and fully saturated porous media. In the Biot theory linear poroelasticity is applied with simultaneous superposition of fluid and solid phases in the same region of space.

Under the assumption of undeformable solid grains, the non-linear functional relationship to evaluate soil porosity n can be written as (Foti et al. 2002):

$$n = \frac{\rho^s - \sqrt{(\rho^s)^2 - \frac{4(\rho^s - \rho^w)K^w}{V_p{}^2 - 2\left(\frac{1-\nu_{sk}}{1-2\nu_{sk}}\right)V_s{}^2}}}{2(\rho^s - \rho^w)} \qquad (1)$$

where: ρ^s and ρ^w are respectively the mass densities of the soil particles and pore water; K^w is the bulk modulus of the pore water; V_p and V_s are the velocities of propagation of the dilatational and shear waves, respectively; ν_{sk} is the Poisson's ratio of the (evacuated) soil skeleton.

The equation (1) was also validated in several case studies (Foti et al. 2002, Foti & Lancellotta 2004, Lai & Crempien 2012, Jamiolkowski 2012, Callerio et al. 2013). The stability of the inversion procedure and the problem well-posedness were established by exploring the connection with ν_{sk} at different V_p-V_s couples (Lai & Crempien 2012). Results proved a general low-dependence on the parameter, except for high velocities (stiff soils), whereas the study offers a useful guide in adopting the simplified formulation.

In the present paper an analysis of the uncertainties is conducted, making use of the basic tools of the error propagation theory (Taylor 1997). The final aim is to reveal the influence of the parameters involved in the porosity equation (1). A particular attention is paid to the role of the Poisson's ratio of the evacuated soil skeleton and to the velocity of compressional waves in water, which appear to be the most influent a-priori parameters on the final estimate.

Finally, two case studies are reported, with experimental data from Zelazny Most (Poland) and Mirandola (Italy). The first example regards an ad-hoc geophysical survey in which repeated travel times and inclinations measures were carried out for a cross-hole experiment, aiming at reaching a well-defined statistical population. The latter includes data from different companies, operators and field techniques to investigate the soil using cross-hole and suspension loggings tests.

2 PARAMETRIC UNCERTANTIES ANALYSES

In physics, there are two kinds of measurements: direct and indirect. Uncertainties are related to experimental errors (systematic or random) during the measuring stage or introduced in the subsequent interpretation process that provides a derivative physical quantity.

The error propagation theory can be used to characterise the influence of each parameter that appears in the porosity formulation in (1), in this example specialized for a cross-hole test configuration.

The starting point is to consider all uncertainties involved in the direct measures as randomly distributed and independent. These hypotheses enable us to assume each parameter as normally distributed (following a Gaussian probability distribution). We consider $n = n\ (\rho^s, \rho^w, K^w, V_p = d/t_p, V_s = d/t_s, \nu_{sk})$ as a several variable function, where d is the travel distance and t_i the travel times for each seismic wave (with $i = p, s$). Supposing that ρ^s, …, ν_{sk} are measured with fractional uncertainties $\varepsilon_{\rho s}$, …, $\varepsilon_{\nu sk}$ and the measured values are used to compute the function n, the uncertainties in porosity are never larger than the ordinary sum:

$$\varepsilon_n n = \left|\frac{\partial n}{\partial V_p}\right| V_p(\varepsilon_{t_p} + \varepsilon_d) + \left|\frac{\partial n}{\partial V_s}\right| V_s(\varepsilon_{t_s} + \varepsilon_d) + \\ + \left|\frac{\partial n}{\partial K^w}\right| \left(2\rho^w {V_w}^2 \varepsilon_{V_w}\right) + \left|\frac{\partial n}{\partial \nu_{sk}}\right| \nu_{sk}\varepsilon_{\nu_{sk}} + \\ + \left|\frac{\partial n}{\partial \rho_w}\right| \rho_w \varepsilon_{\rho_w} + \left|\frac{\partial n}{\partial \rho_s}\right| \rho_s \varepsilon_{\rho_s} \qquad (2)$$

In (2) $V_p = d/t_p$ and $V_s = d/t_s$ are associated to the fractional uncertainties on distance and travel-times, as they are the actual measured quantities in cross-hole tests. Moreover the bulk modulus of the fluid is defined as $K^w = \rho^w {V_w}^2$, whereas $\partial/\partial n$ are partial derivatives.

It is essential to emphasise that quantities ν_{sk} and V_w are not typically evaluated through an experimental procedure on site (as for d or t_i). In the following they have been defined by a presumptive best value and an associated own uncertainty.

Many authors examined the typical Poisson's ratio for sands and its dependence on other parameters of the soil. For example, Nakagawa et al. (1997) and Bates (1989) illustrated the connection between ν_{sk} and effective confining pressure for different example of sands. Wichtmann & Triantafyllidis (2010) analysed the link between ν_{sk} and size distribution characteristics, whereas they are in accordance with Xiaoquiang et al. (2013), which stated the dependence on confining pressure and void ratio. An additional study is reported by Kumar & Madhusadhan (2010), where the Poisson ratio is also analysed changing the relative density of the sand. Taking into account this literature, it has been assumed $\nu_{sk} = 0.25 \pm 0.1$.

As for the velocity of compressional waves in water, in accordance with Lubbers & Graaff (1998), it is calculated by a formula considering temperature-dependence in the 10°C - 20°C range, determining the reference value and dispersion as $V_w = 1464.8 \pm 17.4$ m/s. In this context our aim was to identify a realistic situation of underground fluctuating temperature, for the illustration of the proposed approach. However it is evident that a more precise evaluation of the water temperature allows minimising the contribution of this physic parameter to the uncertainty on the porosity evaluation.

In this study other ancillary parameters are always assumed as $\rho^s = 2.7$ g/cm^3 and $\rho^w = 1$ g/cm^3.

Figure 1 shows the percentage fractional error propagated on n due to 1% error on t_p, d, V_w and ν_{sk}, respectively. It is worth to mention that the percentage fractional error is convenient as a comparison tool, however each parameter has a typical uncertainty range (e.g. 0.1% for d and 40% ν_{sk}). The distance (Fig. 1b) is the most influent parameter involved, with highest percentage fractional errors propagated on n. Following the propagation theory, its influence is the sum of the t_p (Fig. 1a) and t_s contributes, i.e. slightly more than t_p, since t_s has minor importance. A mass density change achieves a small oscillation in the porosity and uncertainties in these two parameters are negligible. The velocity of sound in water shows a marked effect on the calculated porosity (Fig. 1c). Finally, Figure 1d regards the Poisson's ratio of the evacuated soil skeleton. In this case, percentage fractional errors propagated on n are very limited, but the uncertainty associated to this parameter is substantial, so it actually has a large influence on the estimate of porosity.

In Figure 1, the white curves delimitate the most significant area with respect to natural sand deposits. Indeed several couples of V_s-V_p values in Figure 1 are unrealistic for two main causes: the typical porosities of sands and the physical relationships between the variables. In particular high V_p values are not realistic if associated to low V_s values (area above the top white line), whereas couples below the lower white line represent unrealistic values of porosity for a typical coarse grained soil.

Considering the area between the two white curves in Figure 1, it is possible to draw the following conclusions of the parametric analysis for an uncemented sand ($V_s < 500$m/s):

- Uncertainties on the arrival time of P-wave (t_p) are amplified with a factor 3.5 to 5;
- Uncertainties on the distance between the two holes (d) are amplified with a factor 3.5 to 6;
- Uncertainties on compressional wave velocity in the pore fluid (V_w) are amplified with a factor 2.5 to 4;
- Uncertainties on the Poisson's ratio of the (evacuated) solid skeleton are factored with a weight 0.2 to 0.45
- Uncertainties on the other parameters (t_s, ρ^s and ρ^w) are negligible (the corresponding graphs are not reported).

3 CASE STUDIES

3.1 *Zelazny Most*

Repeated measurements of cross-hole experimental data were collected at the site of Zelazny Most tailing dam in two different campaigns in 2011 and 2014 (Callerio et al. 2013, Jamiolkowski 2012-2014, Jamiolkowski & Masella 2015). Measurements of P and S seismic wave propagation were constantly repeated to form a statistical population. Specifically, travel time measurements and deviation surveys were repeated to statistically evaluate average values and related uncertainties.

A statistical assessment of the test repeatability was obtained and 50th, 85th and 95th percentiles have been calculated.

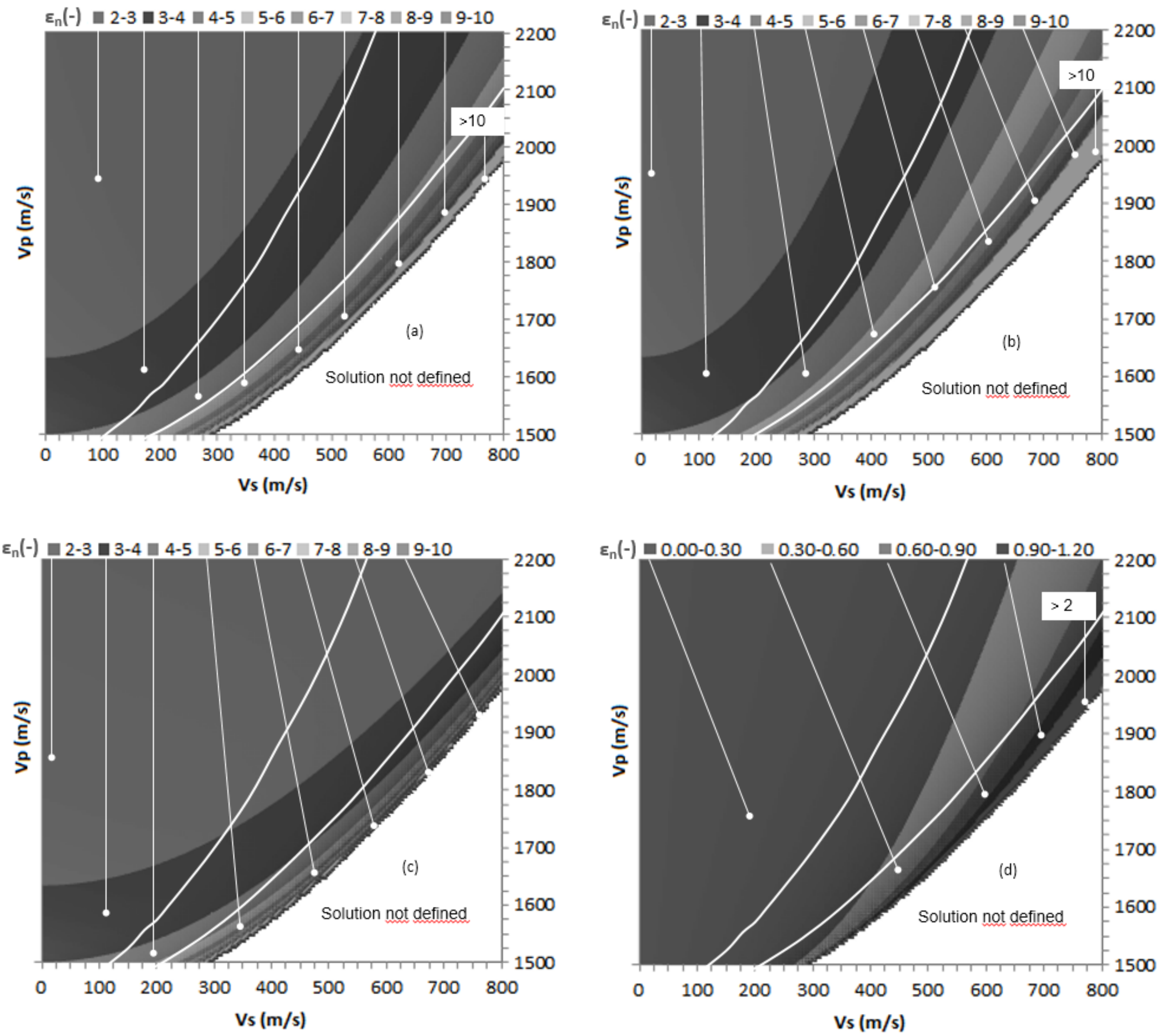


Figure 1. Induced fractional uncertainties on n by: (a) t_p, (b) d, (c) V_w, (d) ν_{sk}.

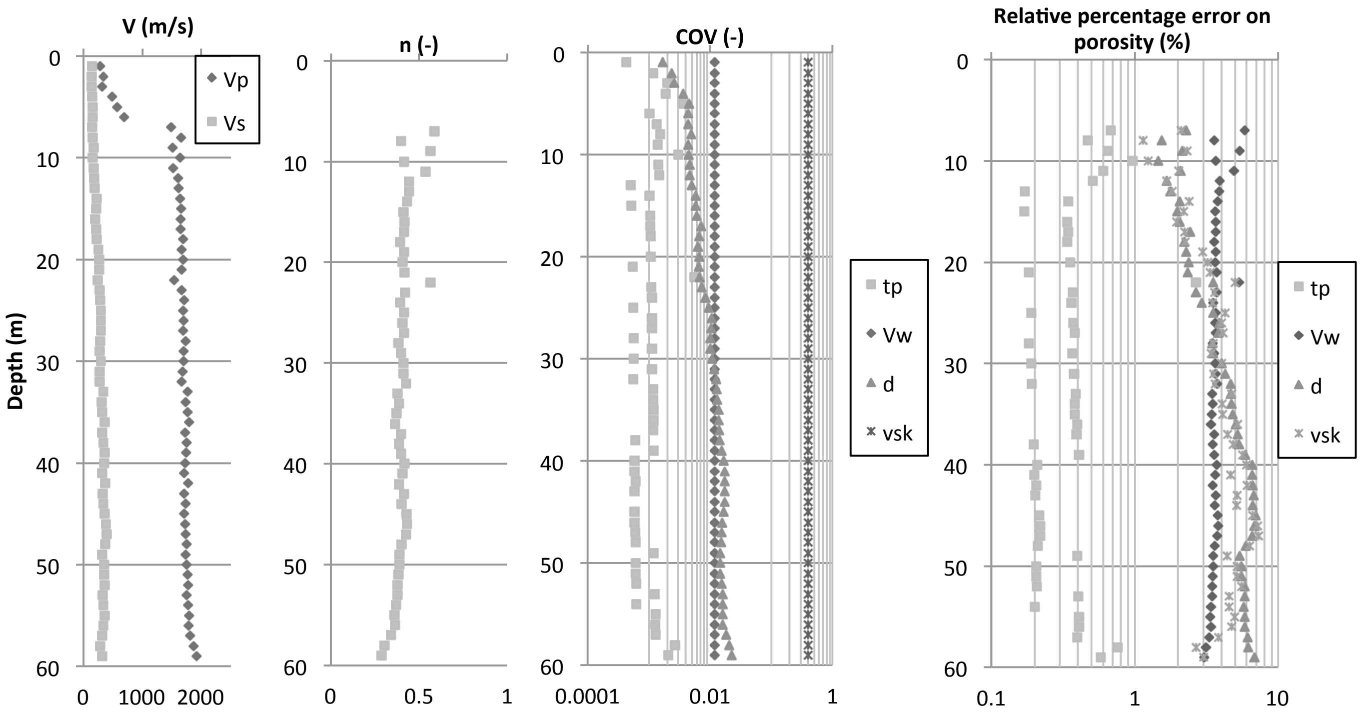


Figure 2. (a) Velocities profiles, (b) estimated porosities, (c) coefficients of variation, (d) relative percentage errors on porosity (XIX 4E-5E).

Evidences for a single borehole (XIX 4E-5E) are illustrated in Figure 2, with the velocity profiles (Figure 2a) and the estimated porosity according to equation 1 (Figure 2b). In Figure 2c and 2d the coefficients of variation and the calculated percentage uncertainty propagated on n are proposed for each parameter involved. Figure 2d shows that the propagated percentage error on *n* due to t_p is one order lower than the uncertainty due to *d*, V_w or ν_{sk}. In any case the uncertainty associated to each parameter is limited to values lower than 7%.

In this case Poisson ratio of the solid skeleton was set to a standard range according to literature ($\nu_{sk} = 0.25 \pm 0.1$).

As no information was available of the actual temperature in the subsoil, the mean value and the associated standard deviation of V_w have been assumed as in Section 2.

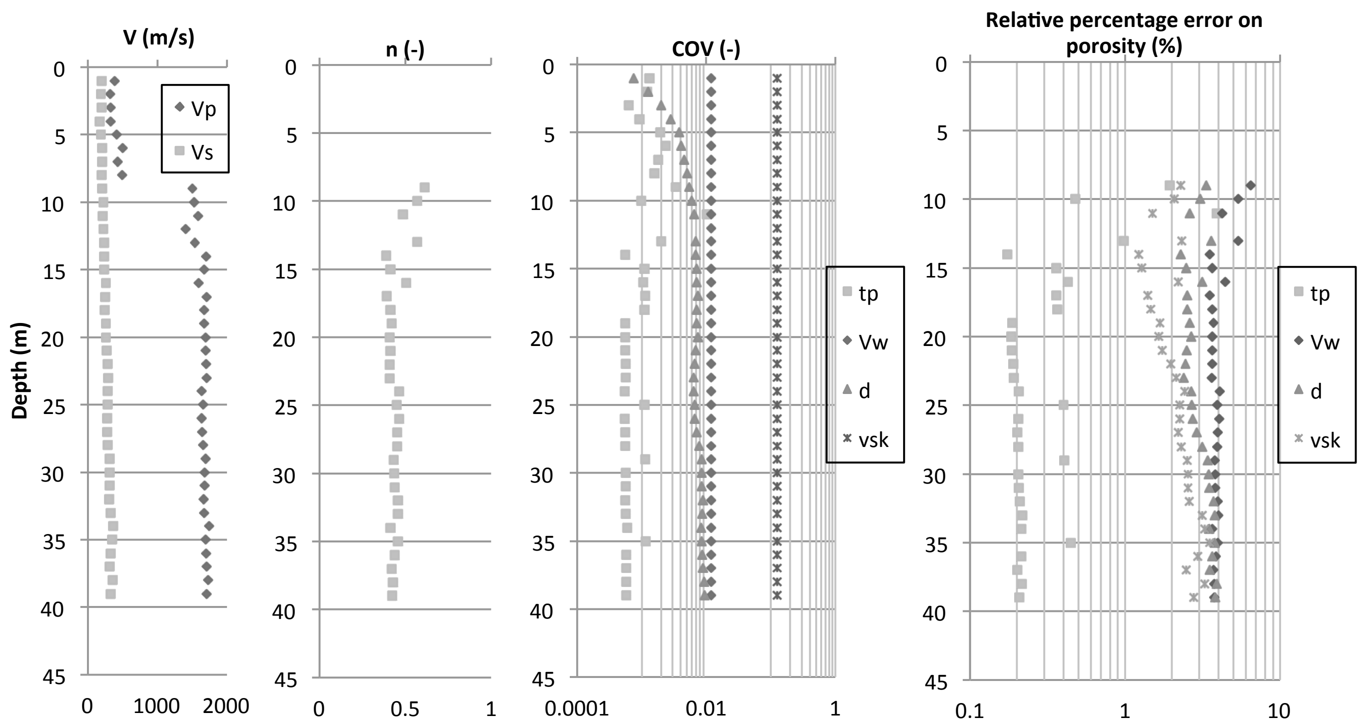


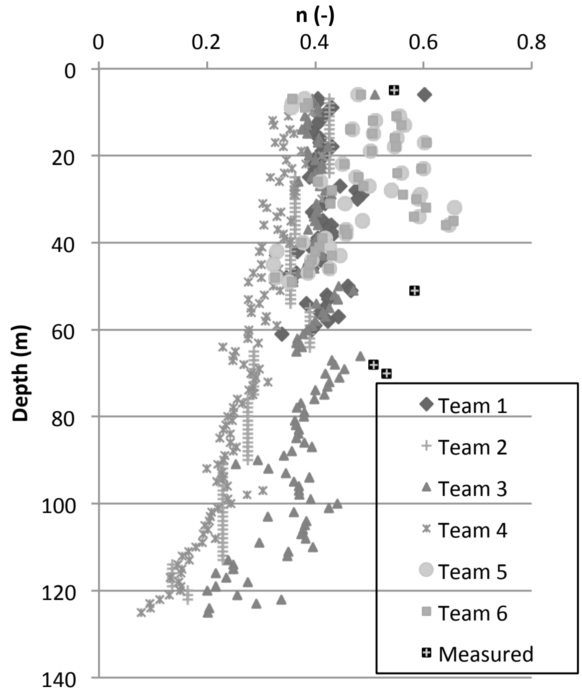
Figure 3. (a) Velocities profiles, (b) estimated porosities, (c) coefficients of variation, (d) relative percentage errors on porosity (VIII 7W-8W).

Figure 3 illustrates another example. For this borehole (VIII 7W-8W) the Poisson's ratio of the evacuated soil skeleton was assessed using data above the water table and the elastic formulations of wave propagation. The mean value of the Poisson's ratio of the evacuated soil skeleton is then 0.29, with a standard deviation of 0.04. Comparing Figure 2d and 3d, it is evident the lower impact of uncertainties from ν_{sk} on porosity assessments. Moreover uncertainties from a single parameter do not exceed 4-5%.

Figures 2 and 3 also show a relevant importance of the velocity of compressional waves in water propagated in the general formula. In both cases, its contribution is always ranging from 3% to 6% and it is often the most influent uncertainty on the calculated porosity. However, these uncertainties could be mitigated by restricting the temperature reference values, if reliable experimental measures are available. Finally, in this particular case, great care was adopted in the travel time and distance measurements, nevertheless clearly the uncertainty on distance evaluation plays a greater role than the uncertainty on travel time estimation (see also Callerio et al. 2013).

3.2 *Mirandola*

The second case study regards a site in the town of Mirandola (Italy), where extensive experimental data were gathered for the InterPACIFIC project (Garofalo et al. 2015), that aimed at assessing the reliability of different geophysical methods for seismic response analyses. In Mirandola several teams used different invasive methods at the same boreholes, carefully collecting information on accuracy (ability to obtain the ideal true value) and repeatability (precision) of each test.



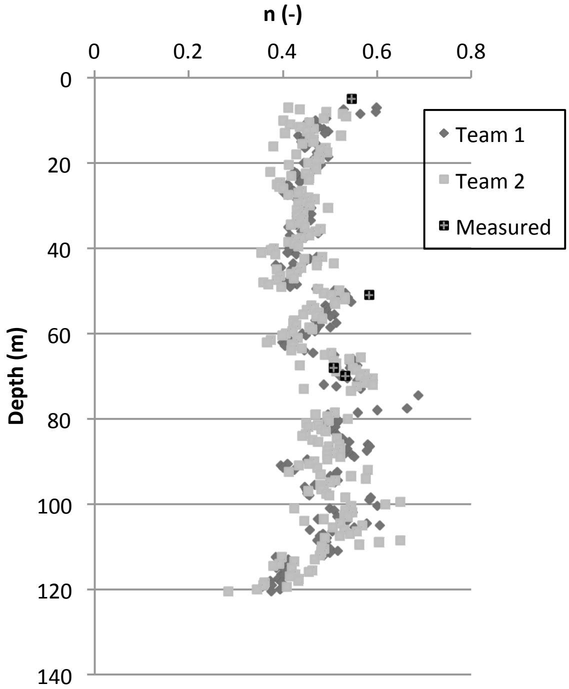


Figure 4. (a) Estimated porosities by different cross-hole tests, (b) estimated porosities by suspension loggings.

In the present study, porosity has been estimated with the seismic velocities measured by each team in cross-hole tests (Fig. 4a) and with P-S suspension logging measurements (Fig. 4b). Values of soil porosities from direct estimates on laboratory samples are also reported as a reference.

Figure 4a shows a very large variability on the results from cross-hole tests. Moreover apparently most of the estimates leads to underestimated values with respect to laboratory values. Apparently, more consistent results are obtained with the values of the P-S suspension logging (Fig. 4b).

4 CONCLUSIONS

In the present study, the uncertainties associated to porosity estimation with the approach proposed by Foti et al. (2002) have been considered. Results of the error propagation procedure show different relevance of the parameters, which appear in the formula. In particular, for a cross-hole test, the care in measuring the distance between the boreholes has huge importance, whereas the travel times revealed minor influence. The S-waves travel times have a very low incidence, especially for realistic V_p-V_s couples, whereas major attention should be paid on P-waves. On the other hand the velocity of compressional waves in water and the Poisson's ratio of the evacuated soil skeleton, which are usually assumed a-priori, are to be estimated with extreme care, since they present relevant effects on the estimate of n. As reported in the case studies these two parameters are the most important together with the distance, and future researches should be conducted in order to evaluate some mitigation possibilities. For example a more accurate investigation of the underground temperature oscillations could lead to narrower V_w uncertainty bounds. Moreover calculation of the Poisson ratio from unsaturated and homogeneous shallow layers can help in the evaluation of a site specific value of ν_{sk}, lowering the uncertainty on this parameter.

In the Mirandola case study the porosities estimations represented a useful tool to verify the results from cross-hole and suspension logging tests. In this case reliable laboratory porosities measurements are compared to estimates from seismic wave velocities to assess the reliability of the latter. Following this approach, the most reliable cross-hole result has been identified. An impressive match between porosities estimated by the suspension logging tests and the laboratory direct evaluation has also been found.

5 ACKNOWLEDGMENTS

The Authors are grateful to prof. Michele Jamiolkowski for making available the experimental data for the Zelasny Most case study and for the highly valuable discussions on the topic of this paper. The Mirandola data were collected for the InterPACIFIC project financed by the Research & Development Program SIGMA funded by EdF, Areva, CEA and ENEL and by CASHIMA project, funded by CEA, ILL and ITER. Regione Emilia Romagna made available additional borehole data and laboratory test results.

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