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# Analytical Stress Intensity Factors for cracks at blunted V-notches

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## Abstract

An analytical expression for the Stress Intensity Factor (SIF) related to a crack emanating from a blunted V-notch root is put forward. Different notch amplitudes, ranging from  $0^\circ$  to  $180^\circ$ , and different crack length to root radius ratios, ranging from 0 to 10, are taken into account. The analysis is limited to mode I loading conditions, as long as the crack length is sufficiently small with respect to the notch depth. The proposed formula improves significantly the predictions of the relationships available in the Literature, by considering a notch amplitude dependent parameter: its values are estimated through a finite element analysis (FEA).

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*Keywords:* Blunted V-notches, brittle fracture, mode I, SIF

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## 1. Introduction

Different criteria have been introduced for the investigation of brittle fracture at blunted V-notches under mode I loading (Leguillon and Yosibash 2003, Gomez and Elices 2004, Taylor 2004, Pugno and Ruoff 2004, Gomez et al. 2006, Picard et al. 2006, Ayatollahi and Torabi 2010, Carpinteri et al. 2011,2012).

These approaches, all involving a material length, differ from each other by the failure condition: it can be a stress requirement (Taylor 2004, Pugno et al. 2004, Ayatollahi and Torabi 2010), an energy balance (Lazzarin and Berto 2005, Gomez et al. 2006) or it can be expressed by coupling both the considerations (Leguillon and Yosibash 2003, Picard et al. 2006, , Carpinteri et al. 2011,2012).

As concerns energy based approaches the evaluation of the crack driving force still represents a drawback. The analysis reduces, according to Irwin's relationship, to the evaluation of the SIF related to a crack at the notch tip, but

no general analytical expressions are available in the Literature. The solution is thus obtained numerically by means of a FEA (Leguillon and Yosibash 2003, Picard et al. 2006).

Indeed, important results have been derived for U-notches (i.e., the rounded crack case,  $\omega=0^\circ$ ) since the beginning of eighties (Schijve 1982, Lukas 1987). Lukas (1987), for instance, considered the generic geometry of an elliptical hole and proposed an approximating function for the SIF at the notch root. This relationship has been widely applied to U-notches (which can be thought as limit cases to an elliptical hole, when the ratio between the length of the minor axis and that of the major axis tends to zero) by the Scientific Community (Gomez et al. 2006, Carpinteri et al. 2012), despite its range of validity is very restricted.

On the other hand, for what concerns generic notch amplitudes  $\omega \neq 0^\circ$ , an important analytical contribution was presented in a previous work (Carpinteri et al. 2011). The expression was proposed to fulfil the asymptotic limits of very short and very long cracks. It was numerically verified for  $\omega=90^\circ, 120^\circ$  and  $150^\circ$  (the three notch angles considered in the paper), showing the maximum deviation for  $\omega=90^\circ$  (nearly 7%). The error was expected to slightly increase for lower notch amplitudes.

In this work, an improved relationship is proposed, by introducing a novel parameter  $m$ , function of the notch amplitude, for  $0^\circ \leq \omega < 180^\circ$ . Its values are obtained via FEAs carried out by means of FRAN2D<sup>®</sup> code (Wawrzynek and Ingrassia 1991).

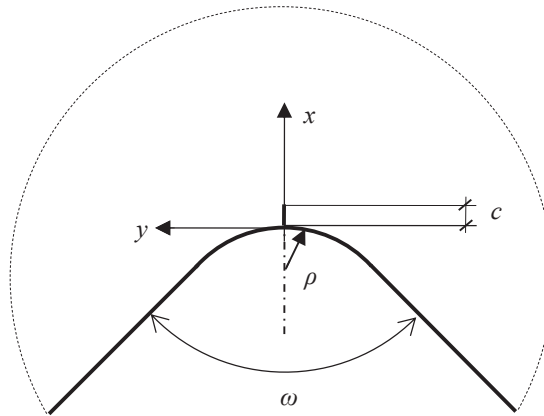


Fig. 1 Rounded V-notch with a crack stemming from the notch root.

## 2. Stress Intensity Factor

Let us consider a crack of length  $c$  stemming from a blunted V-notch root (Fig. 1). The root radius is denoted by  $\rho$ . As long as the notch depth  $a$  is sufficiently large with respect to  $c$ ,  $a \gg c$ , the following SIF function was proposed (Carpinteri et al. 2011):

$$K_I(c) = \frac{\beta K_I^{V,\rho} c^{\lambda-1/2}}{\left[ 1 + \frac{q-1}{q} \left( \frac{\beta}{\psi} \right)^{\frac{1}{1-\lambda}} \frac{\rho}{c} \right]^{1-\lambda}} \quad (1)$$

The parameters  $\beta$  (Carpinteri et al. 2010),  $\lambda$  (Williams' eigenvalues),  $q$  and  $\psi$ , depend only on the notch amplitude  $\omega$ . The former two have been reported in Table 1, the third reads

$$q = \frac{2\pi - \omega}{\pi}, \quad (2)$$

while the latter writes

$$\psi = 1.12\sqrt{\pi} \frac{[1 + \eta_0(0)]}{(2\pi)^{1-\lambda}}, \quad (3)$$

$\eta_0(0)$  varying as  $\omega$  varies (Table 1, (Filippi et al. 2002)). Eventually,  $K_I^{V,\rho}$  in Eq. (1) represents the generalized SIF for a null radius  $\rho=0$ .

Equation (1) can be easily written in dimensionless form as:

$$\bar{K}_I(\bar{c}) = \frac{\beta \bar{c}^{\lambda-1/2}}{\left[ 1 + \frac{q-1}{q} \left( \frac{\beta}{\psi} \right)^{\frac{1}{1-\lambda}} \frac{1}{\bar{c}} \right]^{1-\lambda}}, \quad (4)$$

where  $\bar{K}_I(\bar{c}) = \frac{K_I(\bar{c})}{K_I^{V,\rho} \rho^{\lambda-1/2}}$  and  $\bar{c} = c/\rho$ . As already stated, Eq. (4) was found to provide good results for  $\omega=90^\circ$ ,

$120^\circ$  and  $150^\circ$ : its predictions were compared with numerical results (Carpinteri et al. 2011), showing an error ranging from 3% ( $\omega=150^\circ$ ) to nearly 7% ( $\omega=90^\circ$ ).

In order to improve the accuracy of Eq. (4), investigating also lower notch amplitudes, the following generalization is now proposed, by introducing an additional parameter  $m=m(\omega)$ :

$$\bar{K}_I(\bar{c}) = \frac{\beta \bar{c}^{\lambda-1/2}}{\left\{ 1 + \left[ \frac{q-1}{q} \left( \frac{\beta}{\psi} \right)^{\frac{1}{1-\lambda}} \frac{1}{\bar{c}} \right]^m \right\}^{\frac{1-\lambda}{m}}}. \quad (5)$$

Expression (5) reverts to (4) for  $m=1$  and it fulfils the asymptotic limits independently of  $m$ . In fact, for a very long crack,  $\bar{c} \gg 1$ , but still small with respect to the notch depth  $a$ , Eq. (5) yields:

$$\bar{K}_I(\bar{c}) = \beta \bar{c}^{\lambda-1/2}, \quad (6)$$

which provides the SIF related to a crack at the V-notch tip (Hasebe and Iida 1978, Philipps et al. 2008),  $K_I(c) = \beta K_I^{V,\rho} c^{\lambda-1/2}$ . In other words, when  $\bar{c} \gg 1$ , the root radius effect becomes negligible.

On the other hand, for a very short crack,  $\bar{c} \ll 1$ , Eq. (5) leads to:

$$\bar{K}_I(\bar{c}) = \left( \frac{q}{q-1} \right)^{1-\lambda} \psi \sqrt{\bar{c}}, \quad (7)$$

i.e. it reverts to the expression for an edge crack subjected to the local peak stress,  $K_I(c) = 1.122\sigma_{\max} \sqrt{\pi c}$ , where  $\sigma_{\max} = K_I^{V,\rho} [1 + \eta_0(0)] / [2\pi\rho(q-1)/q]^{1-\lambda}$  is the maximum stress attained at the notch tip (Filippi et al. 2002).

The values for the parameter  $m$  to be inserted into Eq. (5) will be evaluated according to the numerical procedure described in the following section.

### 3. Numerical validation

In order to estimate the parameters  $m$  in Eq. (5), a FEA is carried out through FRAN2D<sup>®</sup> code (Wawrzynek and Ingraffea 1991). For each notch amplitude ( $\omega$  ranging from 0° to 150°, with a step of 30°), an element with the following geometric ratios is considered:  $\rho/a=b=0.01$ , being  $b$  the characteristic dimension of the sample. Under such assumptions, the boundary effects can be reasonably disregarded. A unite tensile load is applied, and the stress field ahead of the notch tip is verified to match the analytical expression proposed in the Literature (Filippi et al. 2002). A crack along the bisector is then created: the mesh next to the root is automatically regenerated by the software. The SIFs are evaluated through a displacement correlation technique. Only the significant range  $0 \leq \bar{c} \leq 2$  will be considered in the present analysis.

The study is presented in Figs. 2-4: the asymptotic limits provided by Eqs. (6) and (7) are denoted by a dashed line and a dotted line, respectively. Numerical data are represented by circles, while results according to Eq. (4) are described by the continuous thin line. Eventually, predictions related to the proposed formula (5) refer to the continuous thick line: they are obtained by means of the parameters  $m$  reported in Table 1, estimated using an iterative least squares method to improve the fitting with FEA.

Let us start by discussing the results related to the crack case ( $\omega=0^\circ$  and  $K_I^{V,\rho} = K_I^U$ ,  $K_I^U$  being the SIF of the corresponding crack, (Glinka 1985)), according to which Eq. (5) provides

$$\bar{K}_I(\bar{c}) = 2.243 \sqrt{\frac{\bar{c}}{[1 + (5.031\bar{c})^m]^{\frac{1}{m}}}}, \tag{8}$$

while the asymptotic limits (6) and (7) revert to  $\bar{K}_I(\bar{c})=1$  and  $\bar{K}_I(\bar{c})=2.243\sqrt{\bar{c}}$ , respectively. The perfect matching between theoretical predictions related to  $m=1.82$  and numerical data is evident from Fig. 2. Notice that also the estimations according to Lukas' formula (providing errors within 5% as long as  $c/\rho < 0.2$ , Lukas 1987),

$$\bar{K}_I(\bar{c}) = 2.243 \sqrt{\frac{\bar{c}}{1 + 4.5\bar{c}}}, \tag{9}$$

have been drawn (thick dashed line). In Eq. (9), a factor 4.5 instead of 5.031 was considered to improve the fitting for short cracks. On the other hand, for long cracks, Eq. (9) provides  $\bar{K}_I(\bar{c}) \approx 1.056$  with an overestimation of more than 5%.

Table 1. Notch amplitude  $\omega$  dependent parameters

$\omega$ (deg)	$\lambda$	$\beta$	$\eta_\theta(0)$	$m$
0	0.5000	1.000	1.000	1.820
30	0.5015	1.005	1.034	1.473
60	0.5122	1.017	0.9699	1.338
90	0.5445	1.059	0.8101	1.314
120	0.6157	1.161	0.5700	1.255
150	0.7520	1.394	0.2882	1.223
180	1.0000	1.985	-	-

Increasing the notch amplitude (Figs. 3,4), the value for  $m$  to improve the precision of Eq. (5) decreases (Table 1) and, consequently, the discrepancy with estimations according to Eq. (4) decreases as well. Nevertheless, the improvement of Eq. (5) keeps to be significant, reducing the maximum deviation below 1-2%.

Eventually, for the flat edge case ( $\omega=180^\circ$ ), Eq. (5) provides coherently  $\bar{K}_I(\bar{c})=1.122\sqrt{\pi\bar{c}}$ , which leads to  $K_I(c)=1.122\sigma\sqrt{\pi c}$  ( $K_I^{V,\rho} = \sigma$  for  $\omega=180^\circ$ ), i.e. the SIF does not depend on the root radius  $\rho$ . In this case, no numerical simulations are obviously necessary.

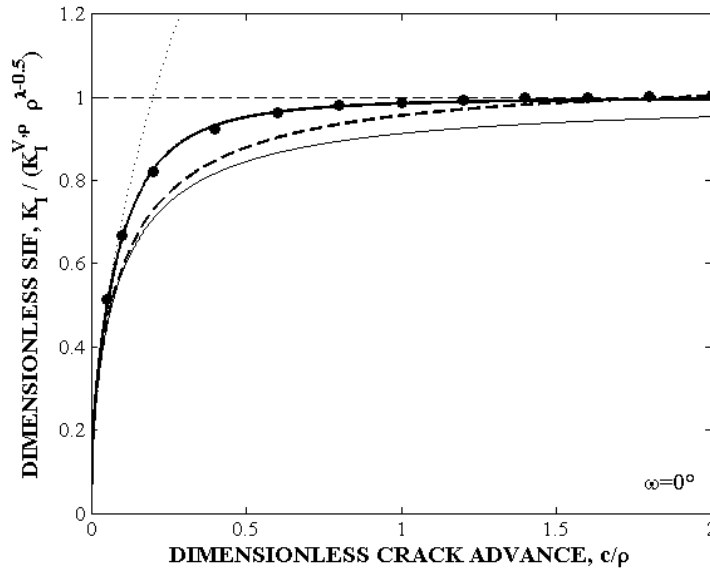


Fig.2 Dimensionless SIF vs. dimensionless crack length for  $\omega=0^\circ$ : asymptotic limit for large cracks (Eq. (6), dashed line); asymptotic limit for small cracks (Eq. (7), dotted line); predictions according to Eq. (4) (continuous thin line); predictions according to Lukas' formula (Eq. (9), thick dashed line); predictions according to the proposed expression (Eq. (5), continuous thick line); numerical data (circles).

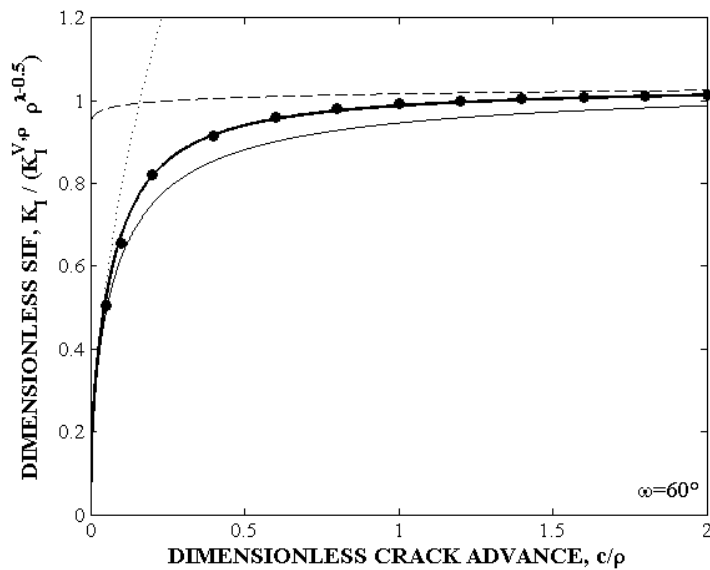


Fig.3 Dimensionless SIF vs. dimensionless crack length for  $\omega=60^\circ$ : asymptotic limit for large cracks (Eq. (6), dashed line); asymptotic limit for small cracks (Eq. (7), dotted line); predictions according to Eq. (4) (continuous thin line); predictions according to the proposed expression (Eq. (5), continuous thick line); numerical data (circles).

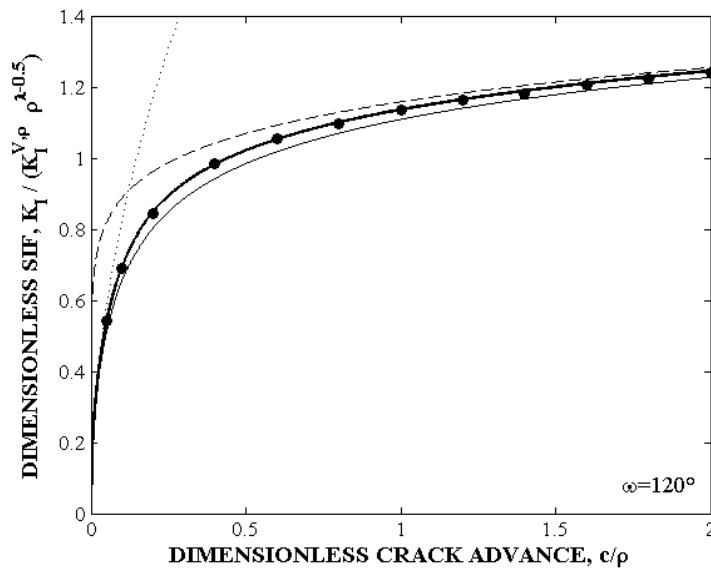


Fig.4 Dimensionless SIF vs. dimensionless crack length for  $\omega=120^\circ$ : asymptotic limit for large cracks (Eq. (6), dashed line); asymptotic limit for small cracks (Eq. (7), dotted line); predictions according to Eq. (4) (continuous thin line); predictions according to the proposed expression (Eq. (5), continuous thick line); numerical data (circles)

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