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*Original*

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# Velocity derivative statistics in shearless turbulence

*Euromech colloquium 501*

Daniela Tordella, Michele Iovieno

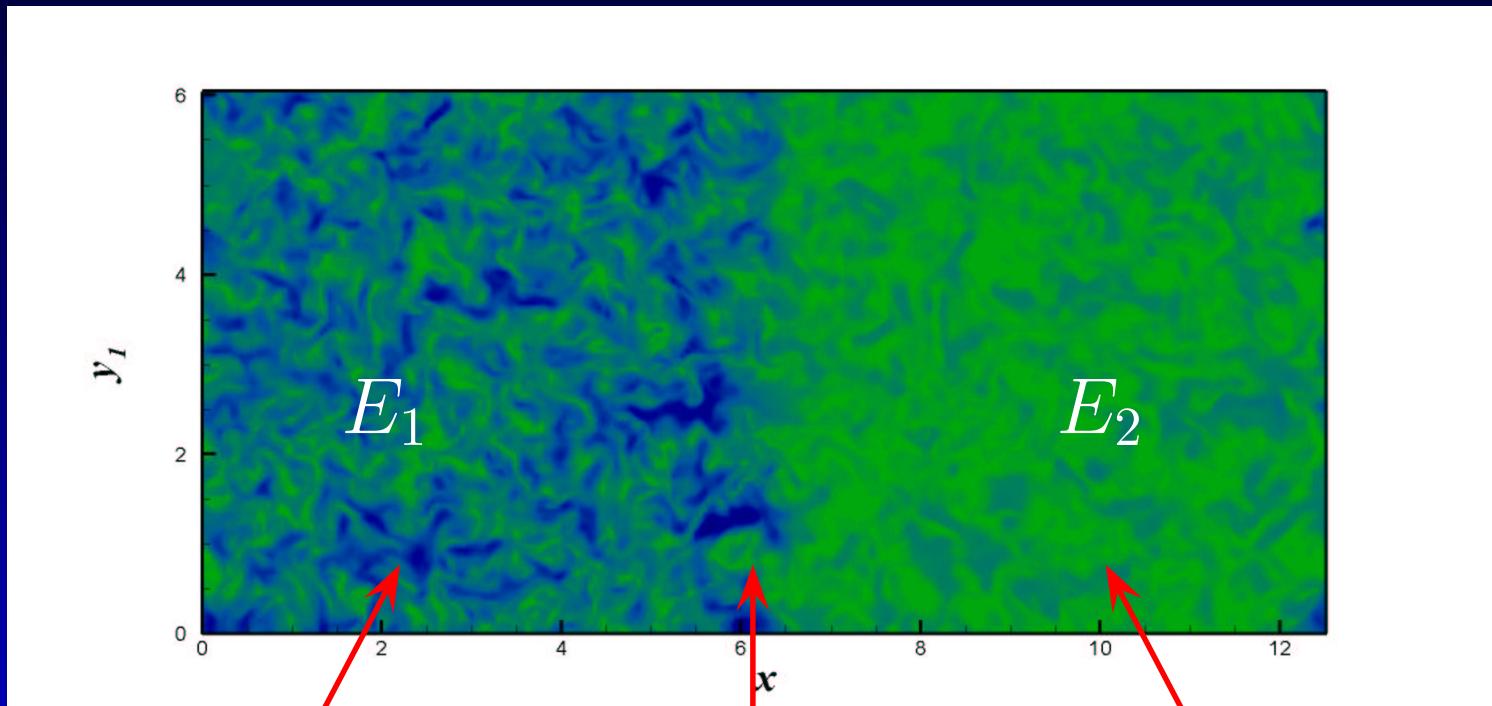
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# Turbulent shearless mixing

Ref: *J. Fluid Mech.* **549**, 441-451, (2006).



[Run Movie 1-2](#)

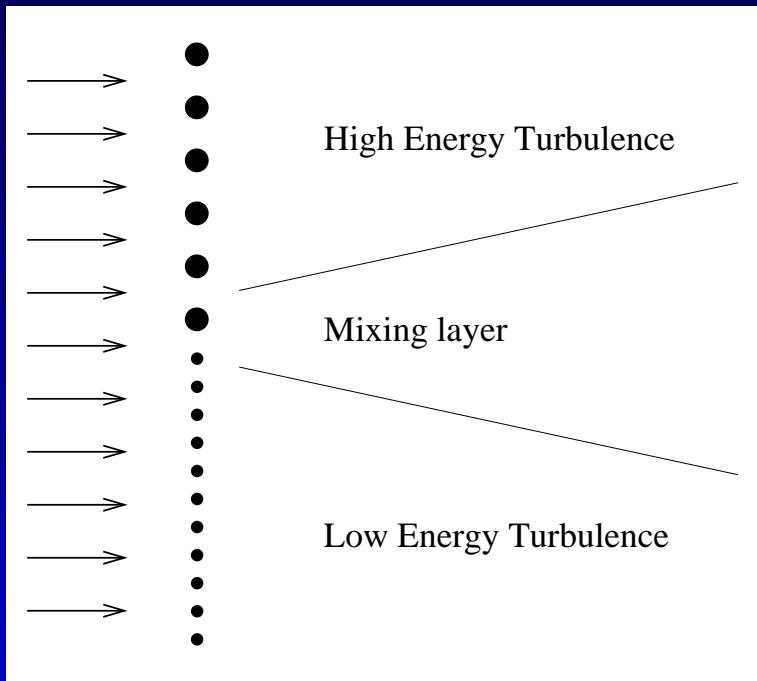
1-High energy turbulence

Mixing layer

2-Low energy turbulence



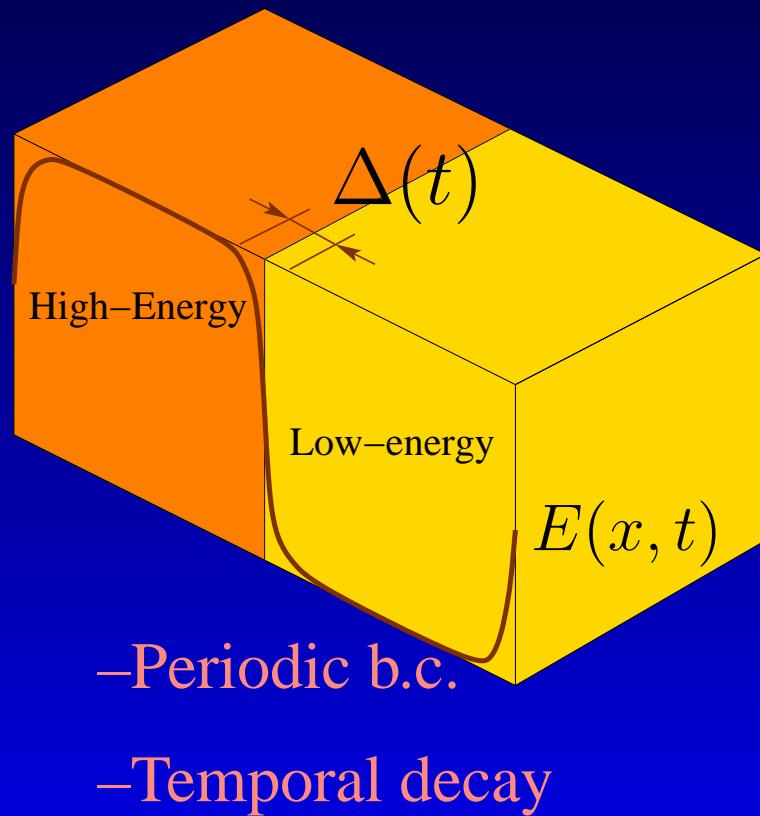
# State of the art



- Grid turbulence experiments:
  - ▶ Gilbert *JFM* 1980
  - ▶ Veeravalli-Warhaft *JFM* 1989



# State of the art



- Grid turbulence experiments:
  - ▶ Gilbert *JFM* 1980
  - ▶ Veeravalli-Warhaft *JFM* 1989
- Numerical experiments:
  - ▶ Briggs *et al.* *JFM* 1996
  - ▶ Knaepen *et al.* *JFM* 2004
  - ▶ Tordella-Iovieno *JFM* 2006
  - ▶ Iovieno-Tordella-Bailey *PRE* 2008)

# Main features

- Self-similar stage of decay
- $\overline{u_1^2} \approx \overline{u_2^2} \approx \overline{u_3^2}$
- High intermittency, function of:
  - ▶ gradient of turbulent kinetic energy  
 $\Rightarrow \mathcal{E} = E_1/E_2$
  - ▶ gradient of integral scale  $\Rightarrow \mathcal{L} = \ell_1/\ell_2$



# Method

- We consider two kinds of shearless mixings:
  - with a *uniform scale*: the mixing is generated by the gradient of energy only  $\Rightarrow \mathcal{E}$  only parameter ( $\mathcal{L} = 1, \mathcal{E} \neq 1$ )
  - with a *scale* and *energy* gradient:  $\Rightarrow \mathcal{E} \neq 1$  and  $\mathcal{L} \neq 1$
- The behaviour of the mixing in the limit of very high energy ratio  $\mathcal{E}$  is investigated.
- Method: DNS, parallelepiped domain ( $2\pi \times 2\pi \times 2n\pi$ ,  $n = 2$  and  $n = 4$ ),  $Re_\lambda = 45$  and 71, LES, same domain and  $Re_\lambda = 44 - 450$



# Velocity statistics

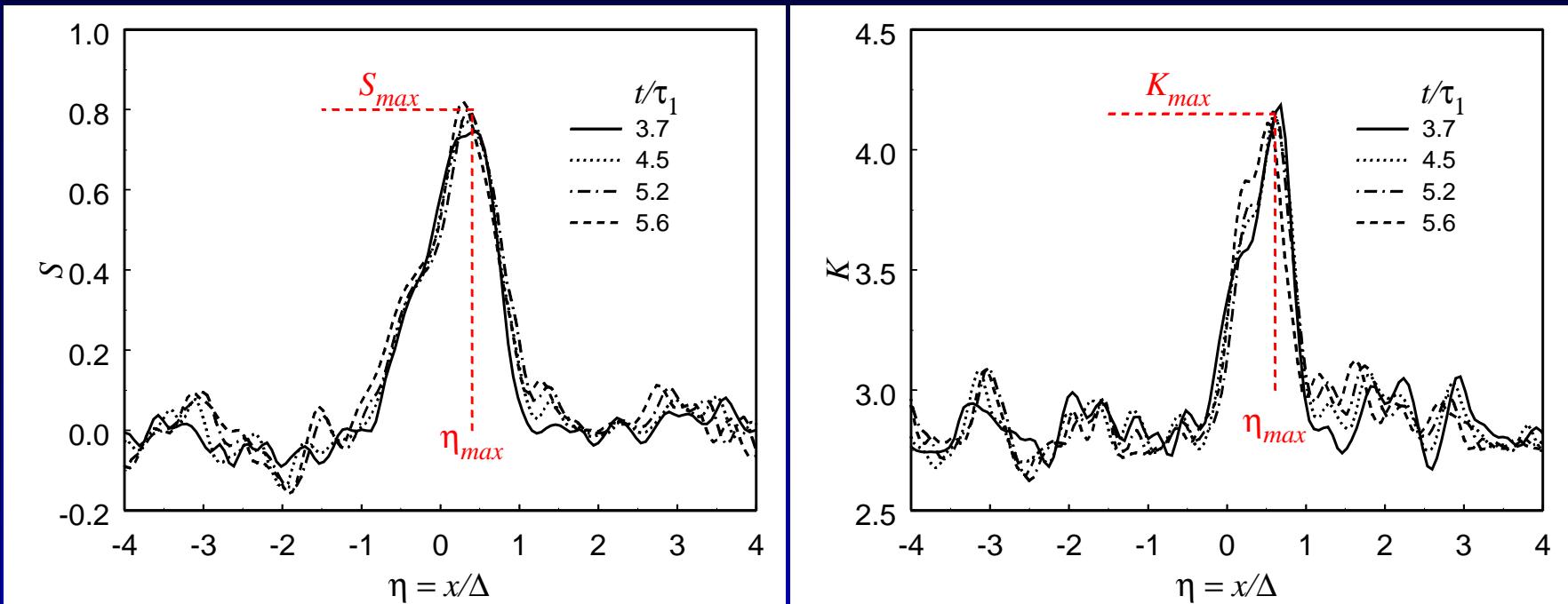
- No shear  $\Rightarrow$  no kinetic energy production,
- The mixing is intermittent even if no gradient of scale is present (*PRE* 2008)
- Intermittency is (*JFM* 2006)
  - ▶ *ENHANCED* if the energy gradient is concurrent with the integral scale gradient ( $\mathcal{L} > 1$ )
  - ▶ *REDUCED* if the energy gradient is opposite to the integral scale gradient ( $\mathcal{L} < 1$ )
- Self-similar stage of evolution



# Intermittency - $\mathcal{E} = 6.7$

$$S = \overline{u^3}/\overline{u^2}^{3/2}$$

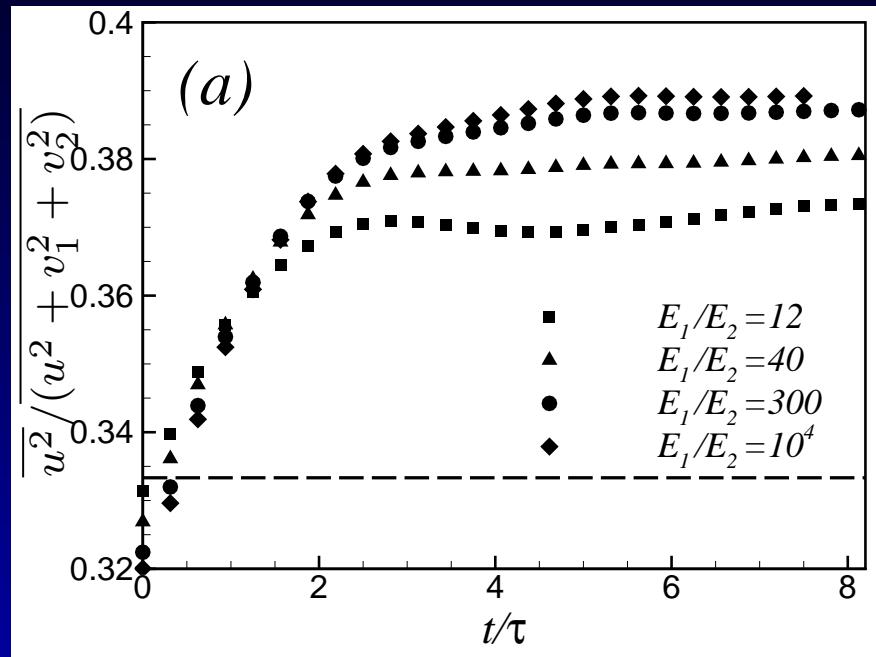
$$K = \overline{u^4}/\overline{u^2}^2$$



$S_{max}, K_{max}$  = maximum of Skewness and Kurtosis  
in the mixing layer  
 $\eta_{max}$  = position of the maximum in the mixing layer



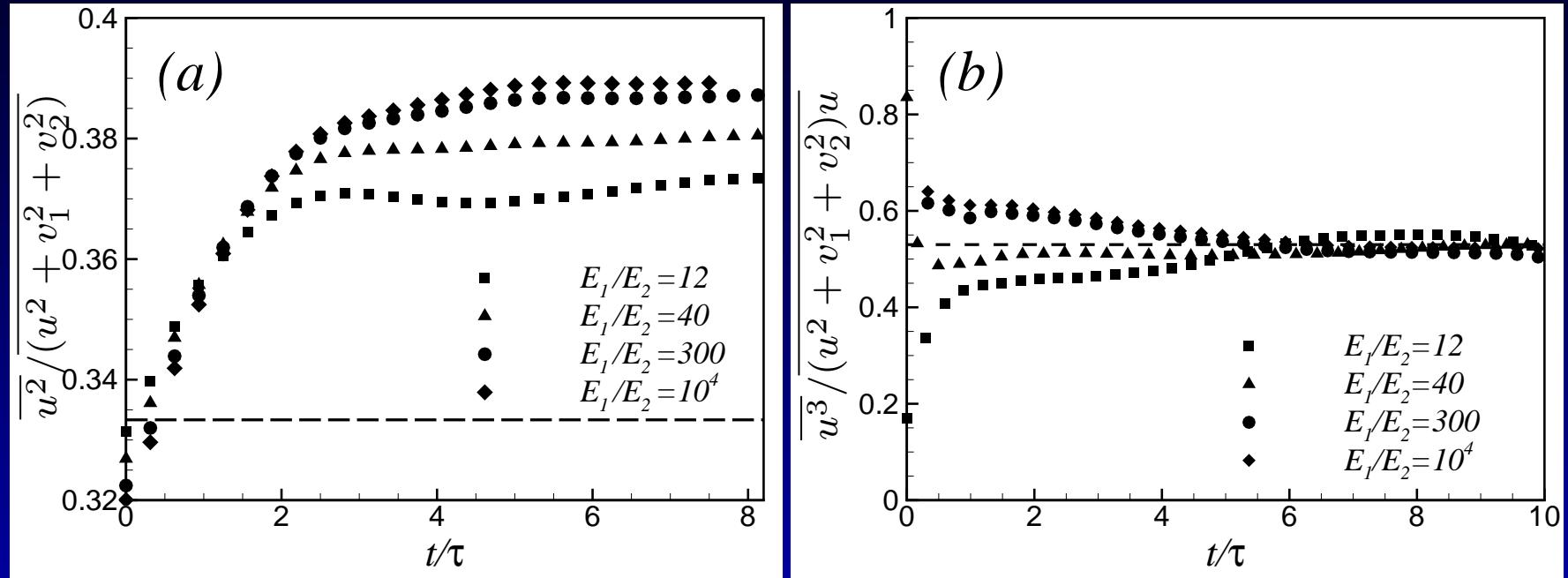
# Anisotropy



Left: second order velocity moment anisotropy



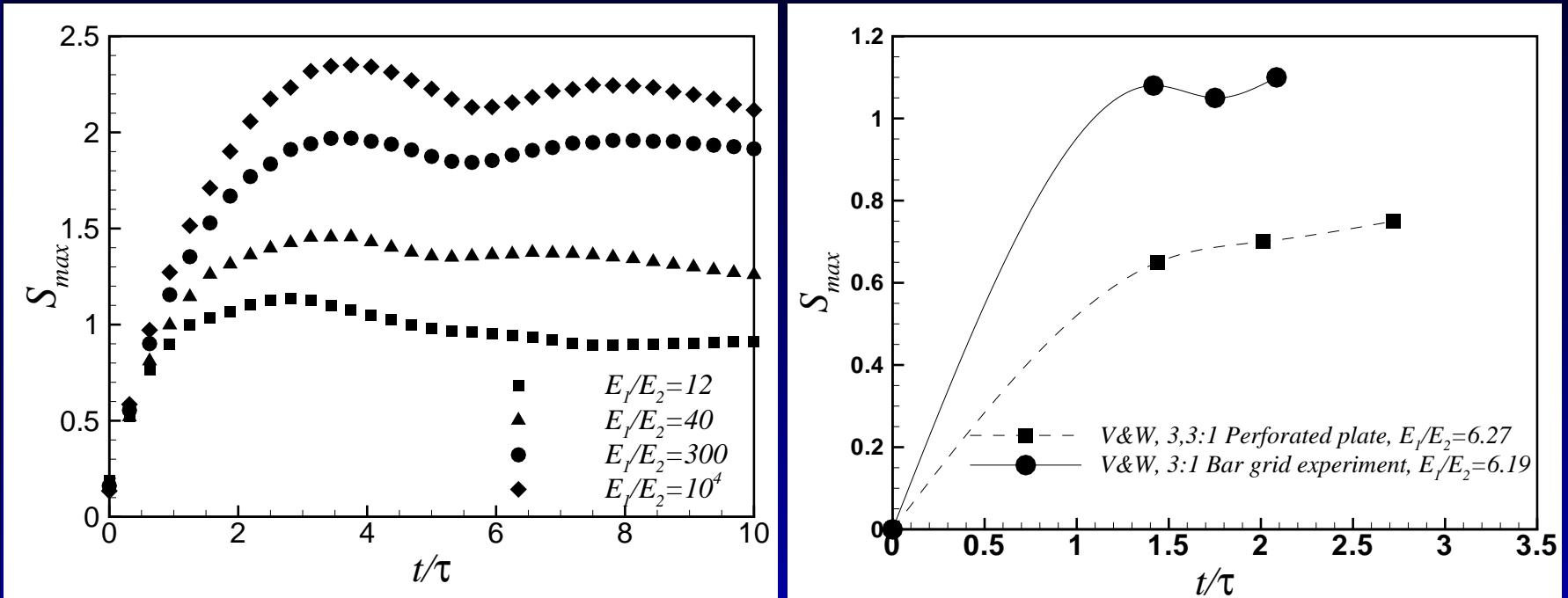
# Anisotropy



Left: second order velocity moment anisotropy  
Right: third order velocity moment anisotropy



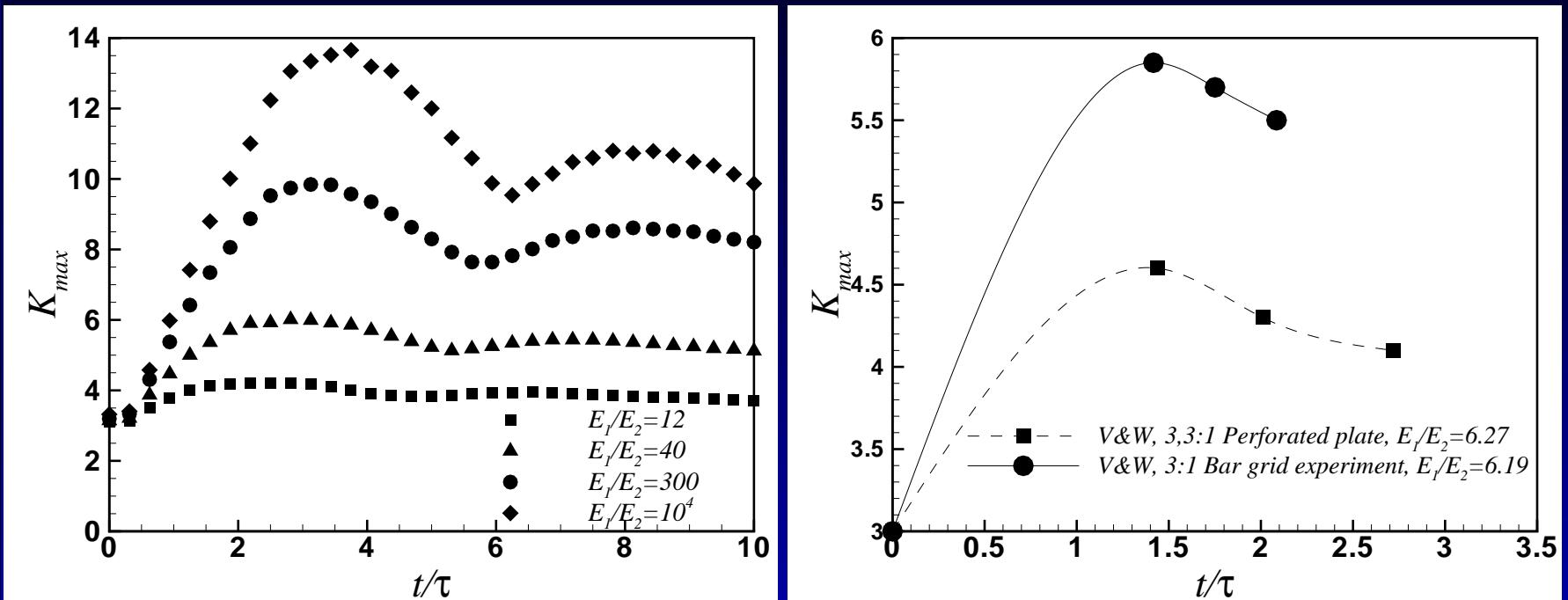
# Inhomogeneity: Skewness evolution



Time evolution of the maximum of skewness within the mixing.



# Inhomogeneity: Kurtosis evolution

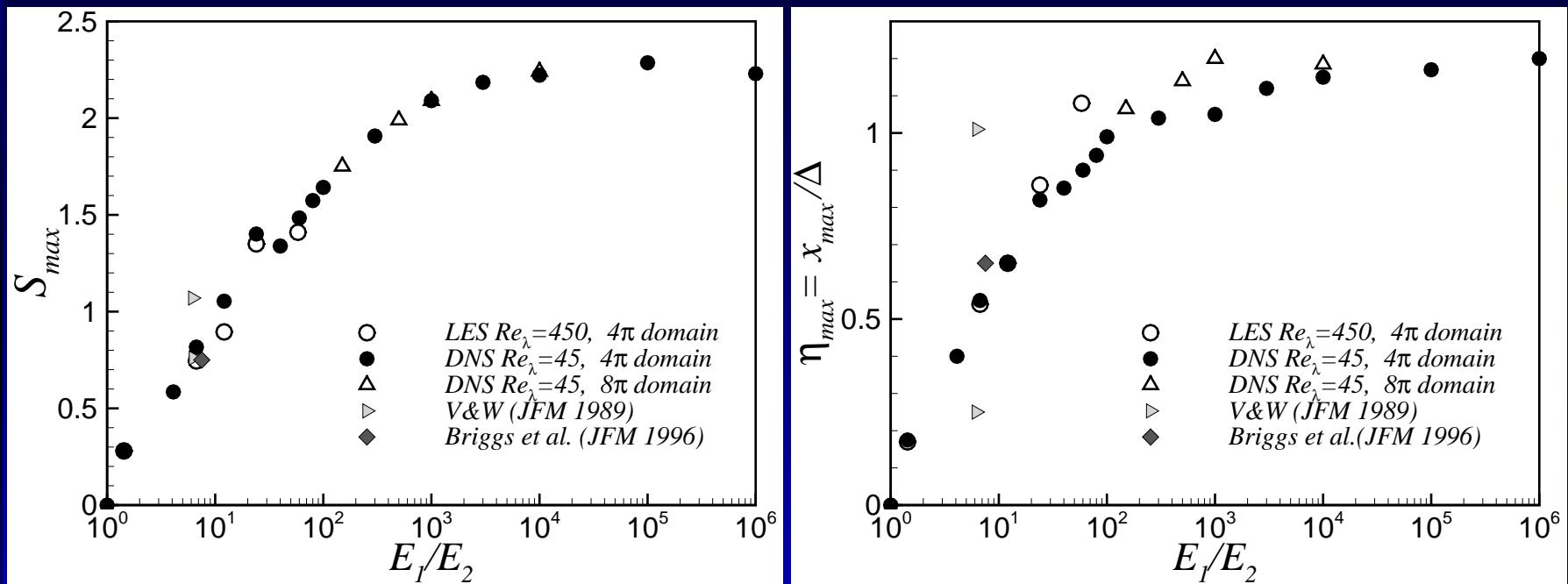


Time evolution of the maximum of kurtosis within the mixing.



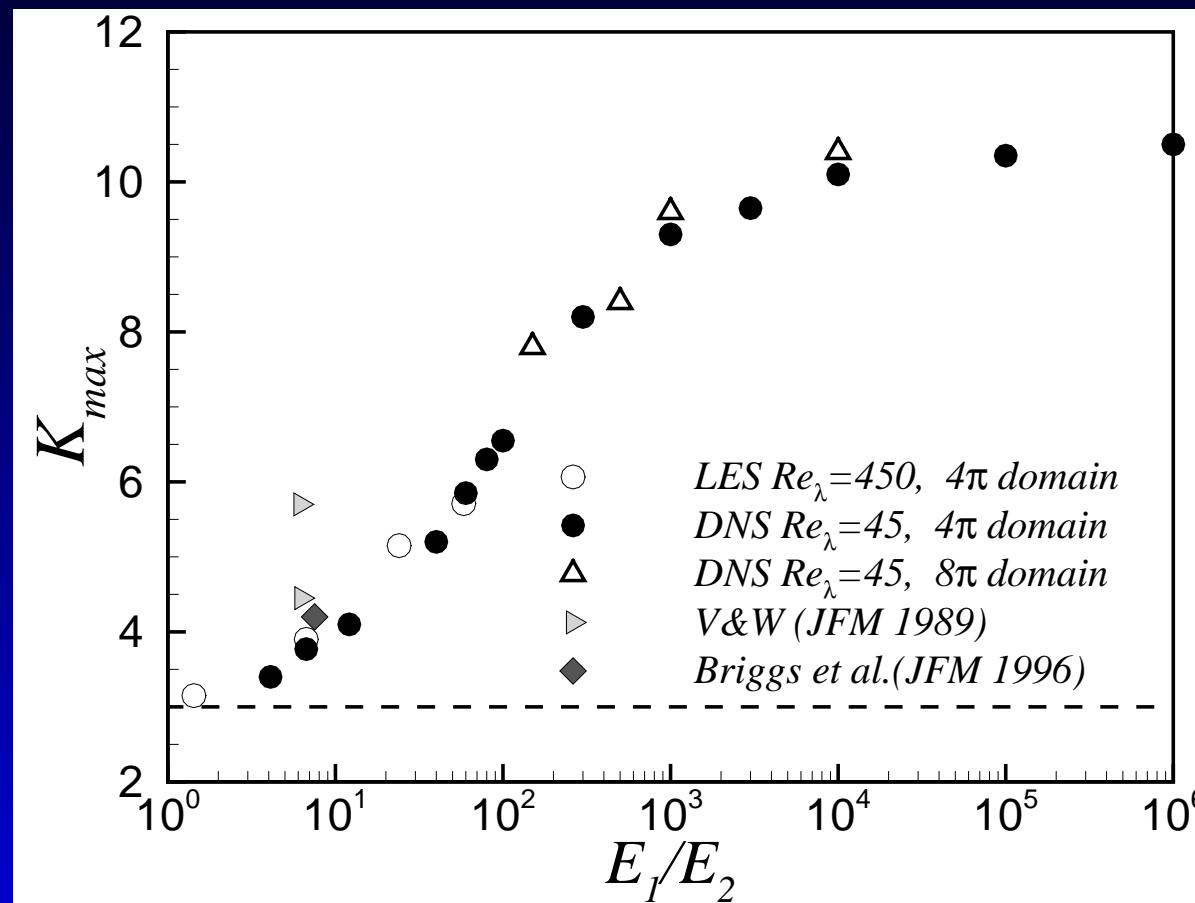
# Asymptote for $E_1/E_2 \rightarrow \infty$

## Skewness and penetration



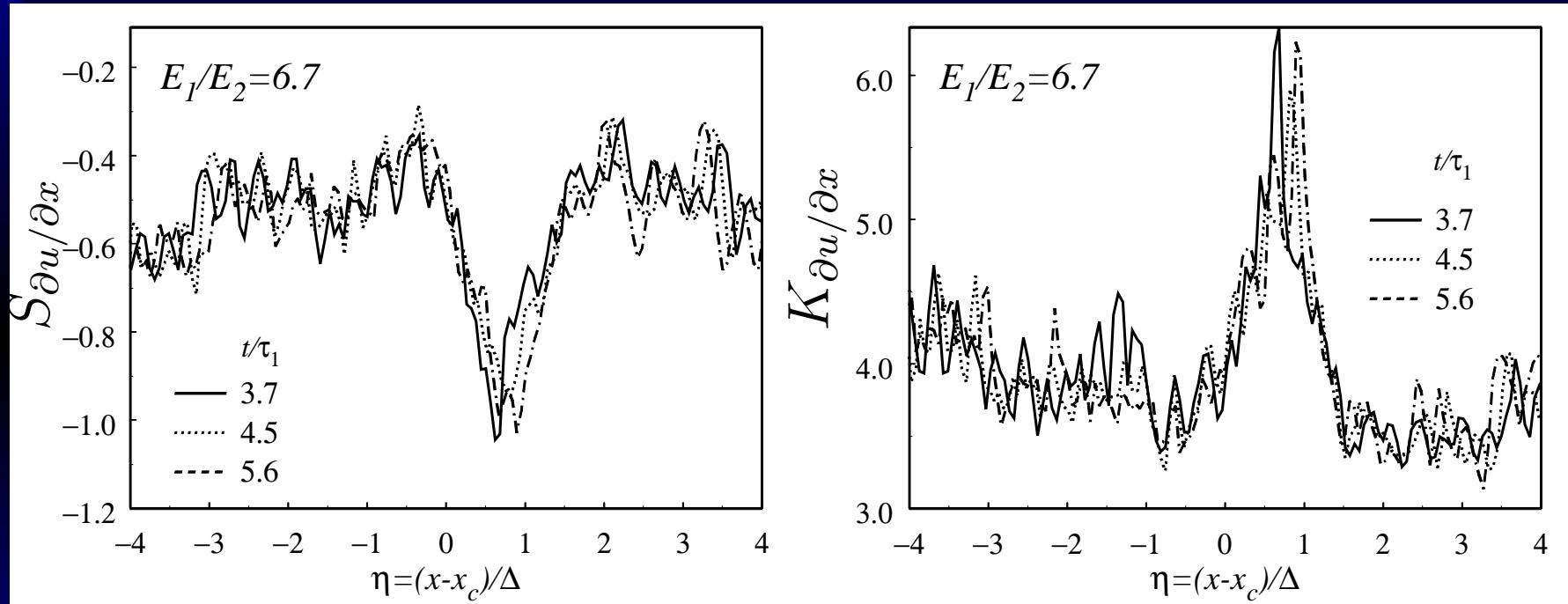
# Asymptote for $E_1/E_2 \rightarrow \infty$

## Kurtosis



# Longitudinal derivative moments distribution

$$E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$

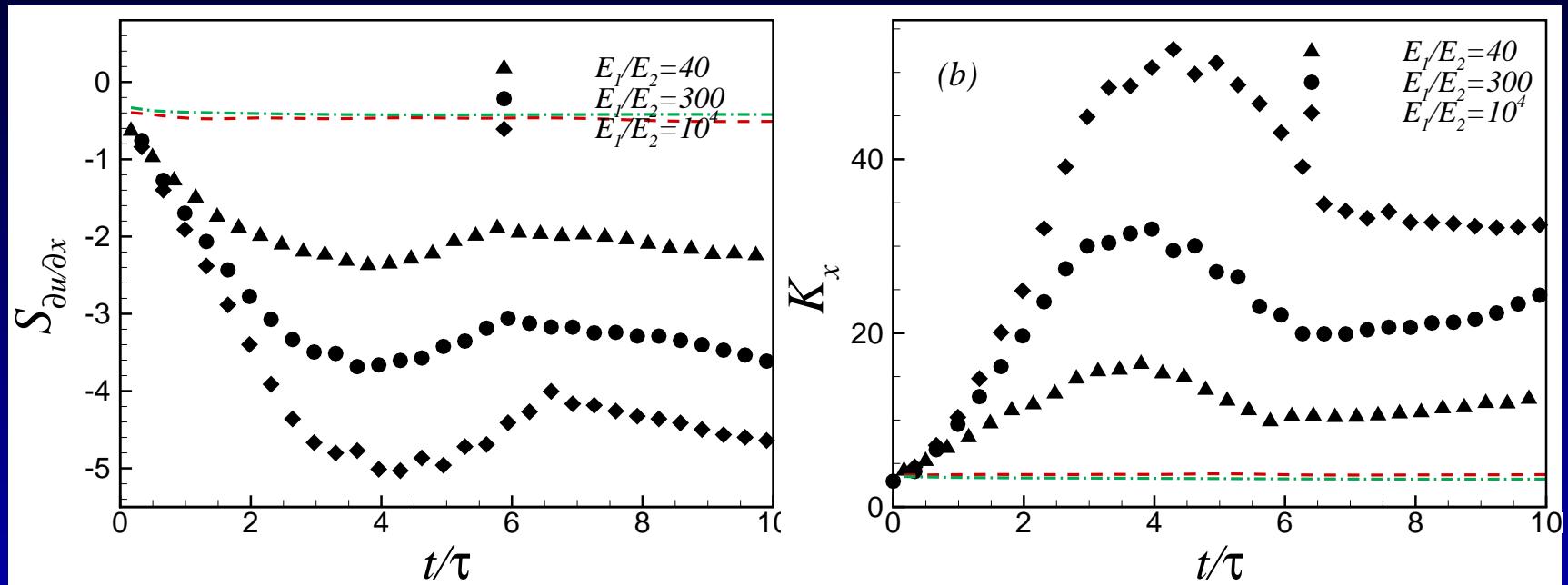


$\eta$  is the dimensionless coordinate along the mixing  
 $\Delta$  is the mixing half-width



# Mixing: Longitudinal derivatives

Time evolution of the peak of the derivative statistics



$u$  = velocity component in the mixing direction

$x$  = direction along the mixing

$\tau$  = initial eddy turnover time

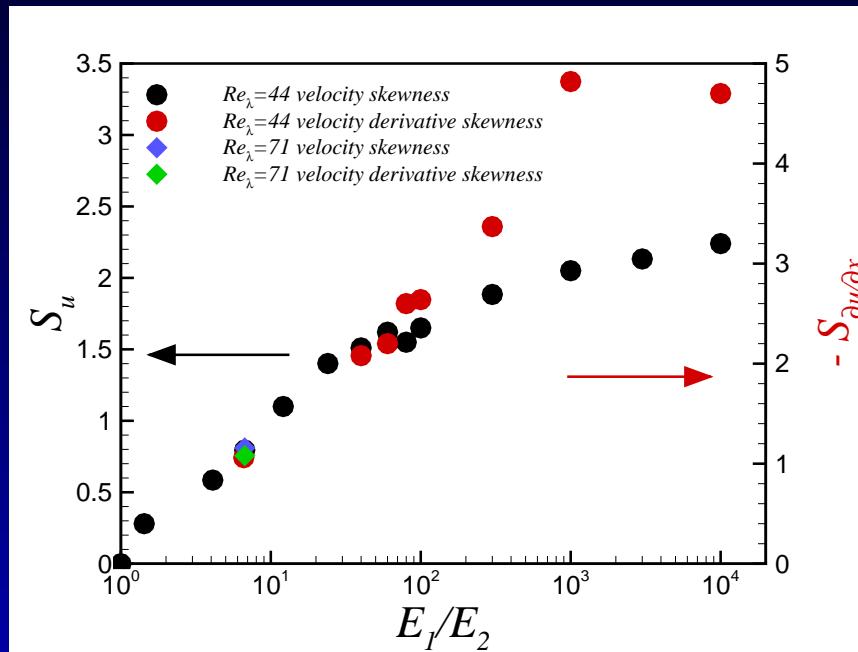
— — — high energy homogeneous region

- · - · - low energy homogeneous region

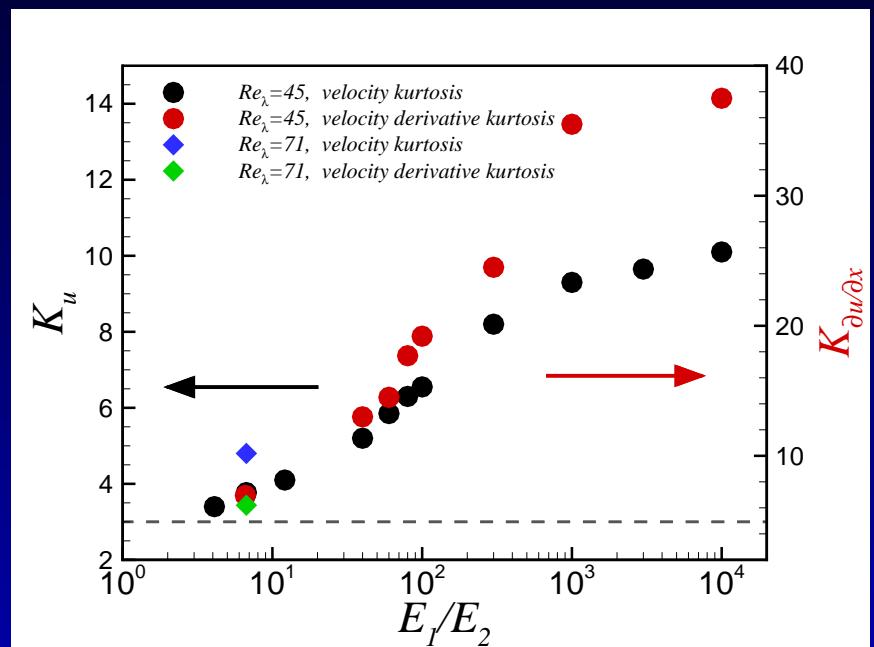


# Asymptote for $E_1/E_2 \rightarrow +\infty$

## Velocity Skewness



## Velocity Kurtosis

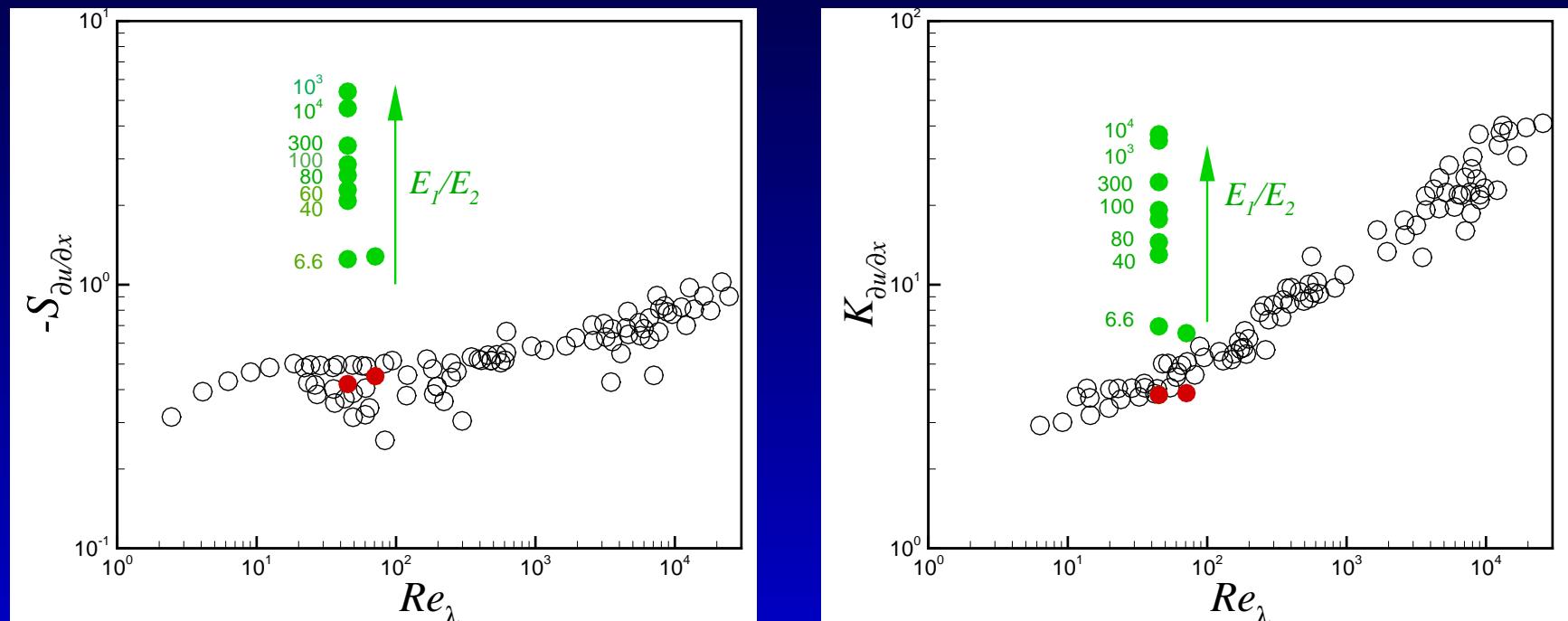


Longitudinal derivative Skewness and Kurtosis of the velocity component along the mixing direction



# Comparison with homogeneous turbulence

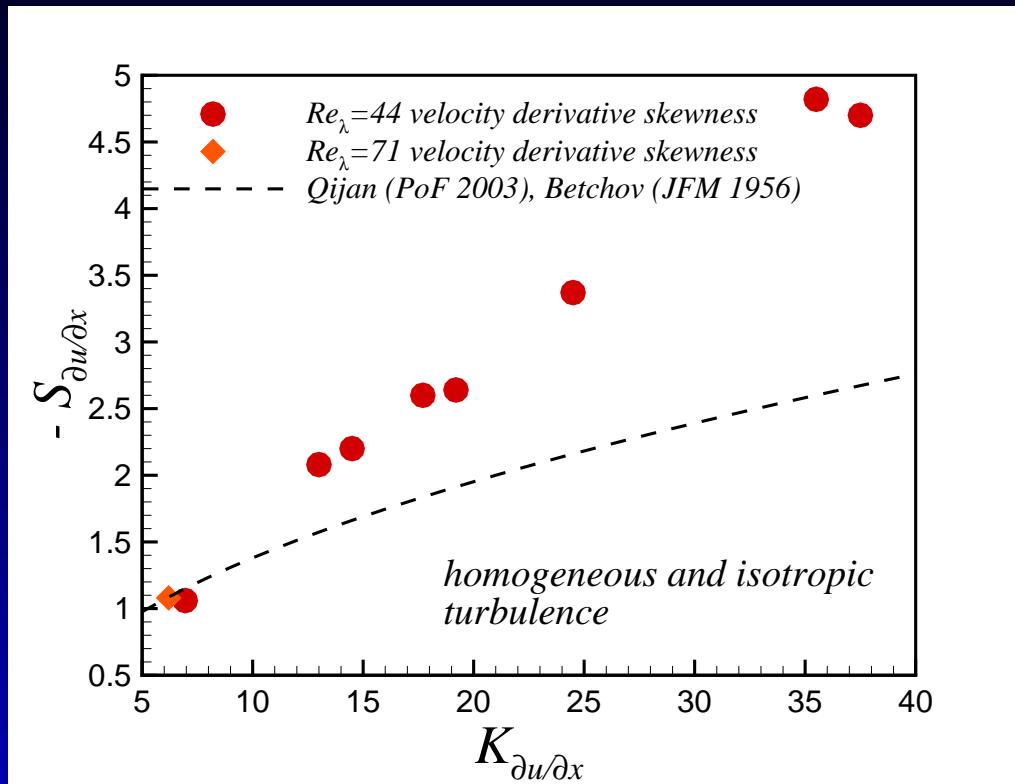
Comparison of longitudinal moments inside the mixing with longitudinal moments in homogeneous and isotropic turbulence  
(data from Sreenivasan and Antonia, *Ann.Rev.Fluid Mech* 1997)



- HIT, present simulations
- Shearless mixings, present simulations
- Homogeneous turbulence



# Comparison with homogeneous turbulence (II)



Comparison with the upper bound

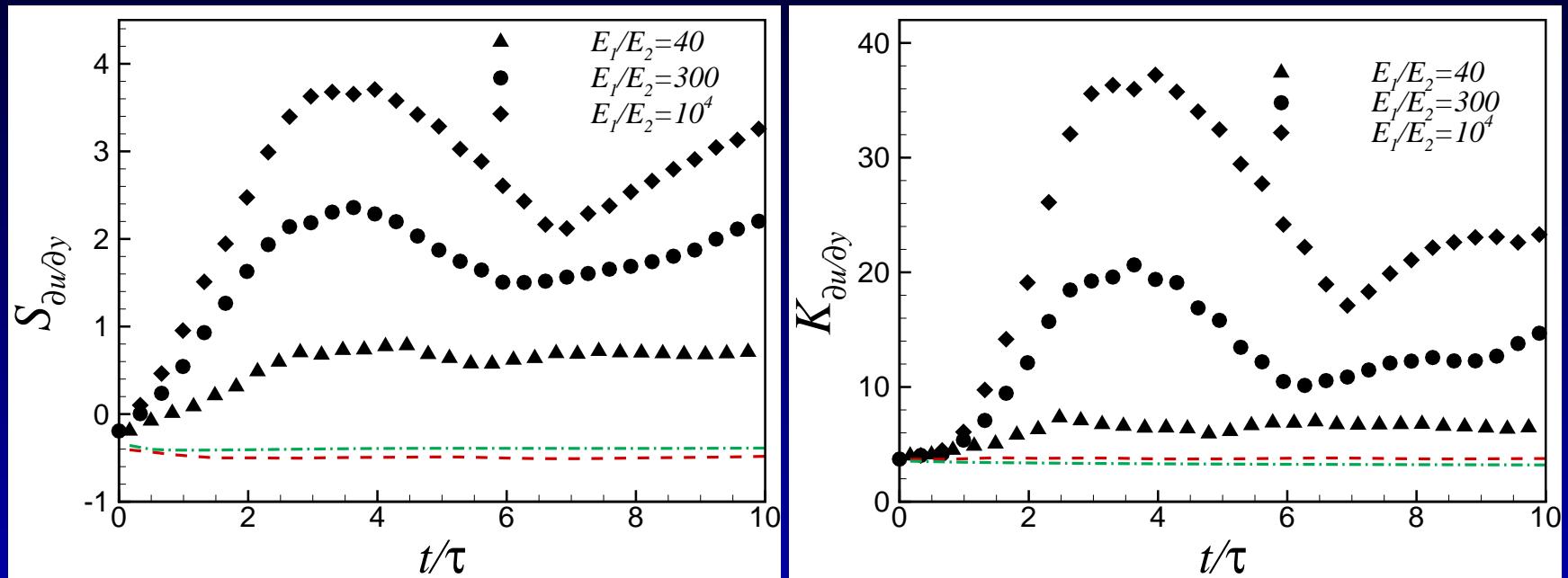
$$-S_{\partial u / \partial x} \leq 2 \left( \frac{K_{\partial u / \partial x}}{21} \right)^{\frac{1}{2}}$$

of the longitudinal skewness in homogeneous turbulence



# Mixing: Transversal derivatives

Time evolution of the peak of the derivative statistics



$u$  = velocity component in the mixing direction

$y$  = direction normal to the mixing

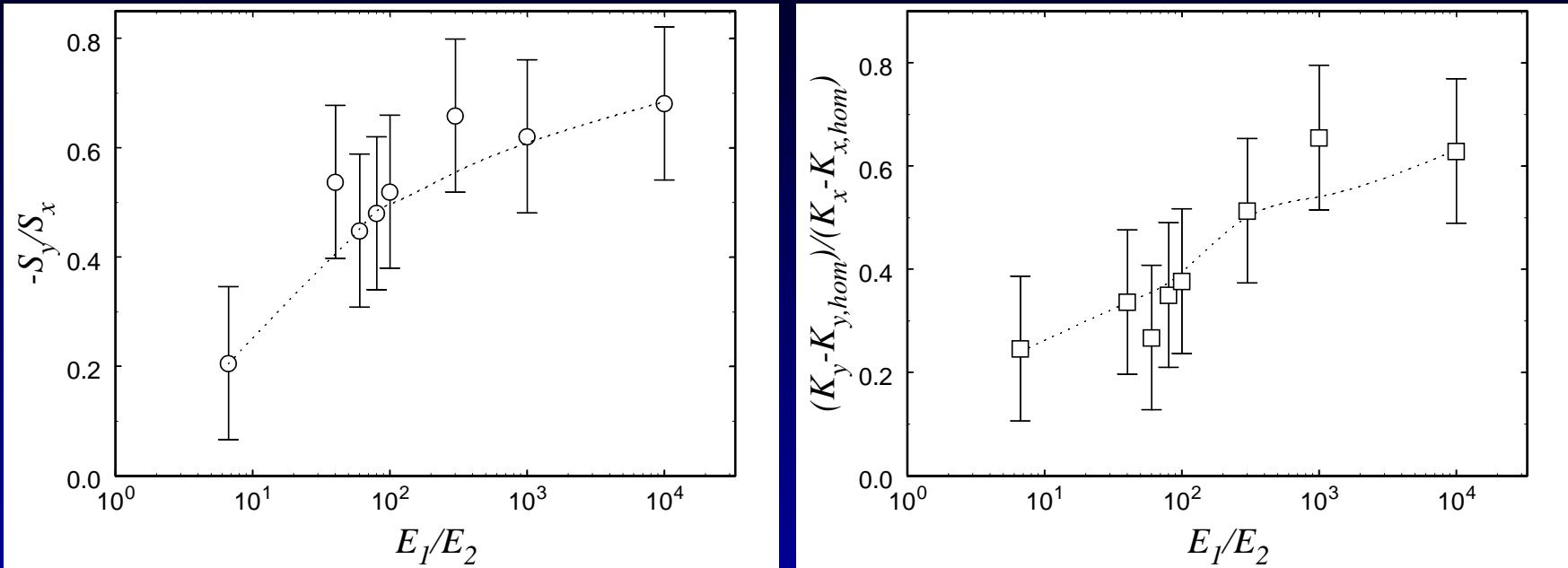
$\tau$  = initial eddy turnover time

— — — high energy homogeneous region

— · — · — low energy homogeneous region



# Small scale anisotropy: uniform integral scale



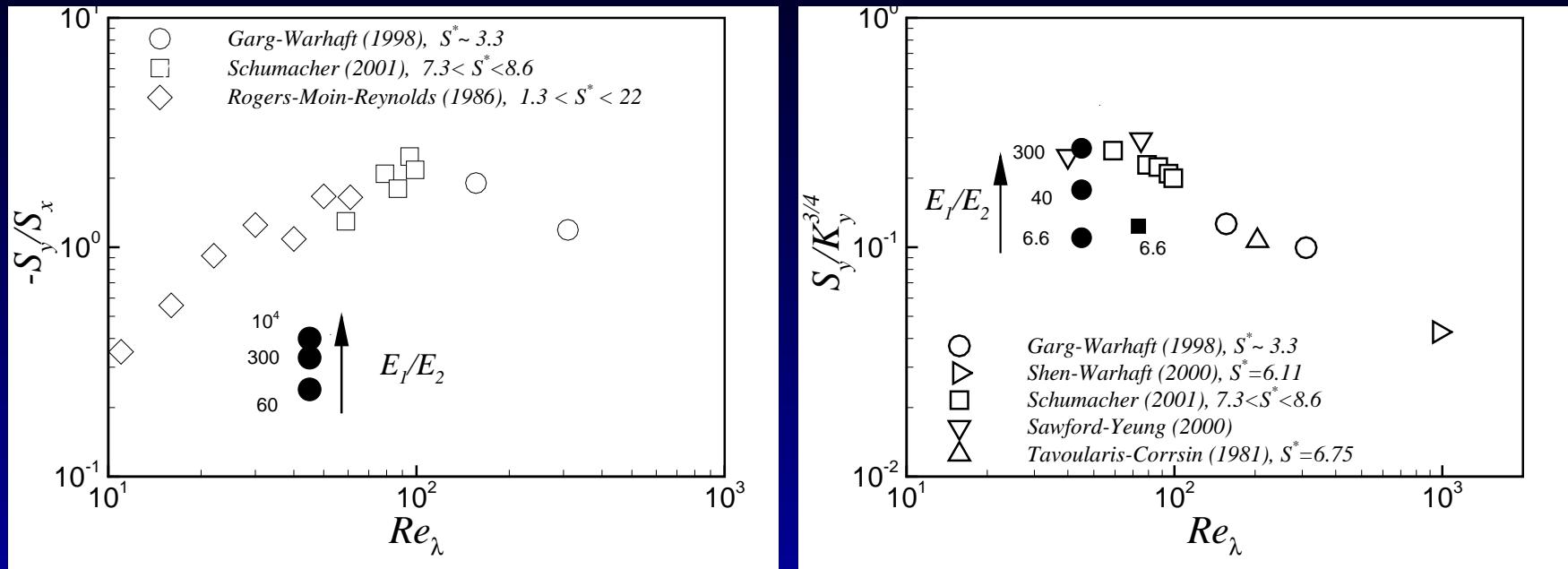
(a) isotropy of the derivative skewness;

(b) isotropy of the derivative kurtosis.

The bars represent the amplitude of the fluctuations of these ratios.



# Comparison with homogenous shear turbulence

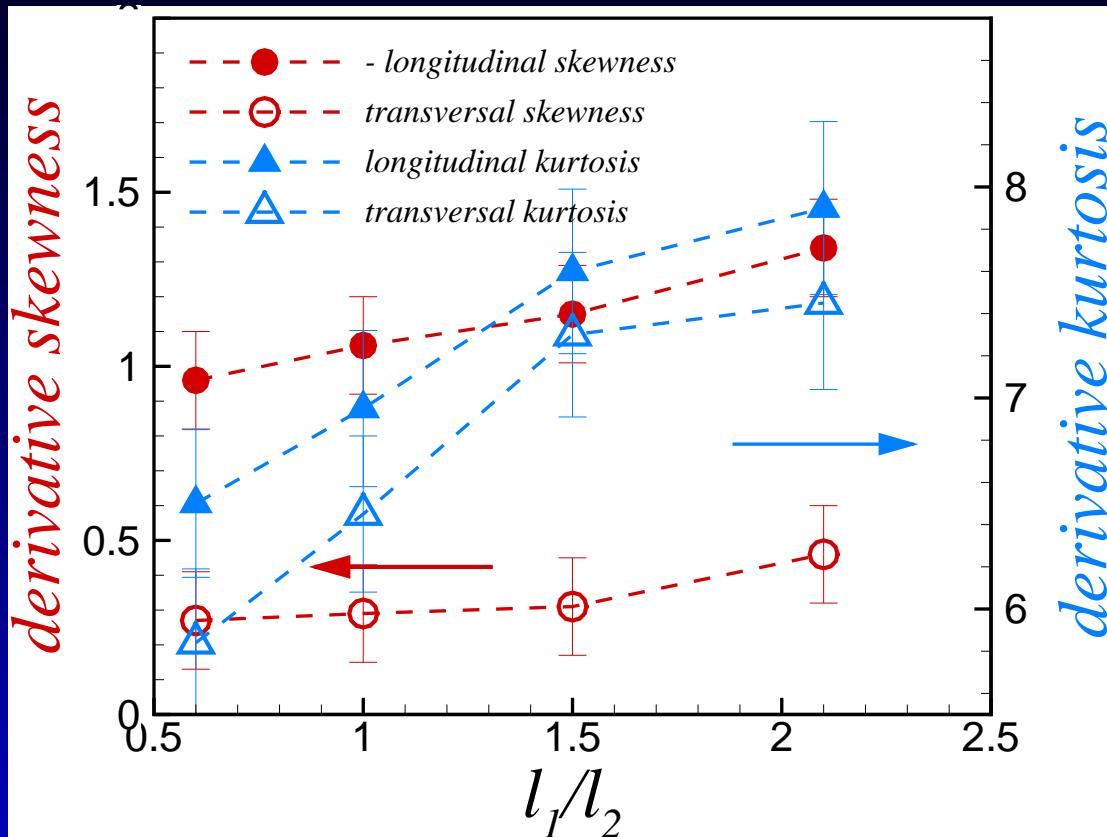


Comparison of anisotropy and intermittency:

- shearless mixing data,  $\bullet Re_\lambda = 45, \square Re_\lambda = 71$
- empty symbols: homogeneous shear flows data.



# Derivative statistics: non uniform integral scale



# Conclusions

Direct numerical simulations carried over a large range of turbulent energy ratios ( $6 \leq \mathcal{E} \leq 10^4$ ) show:

- a high level of intermittency, comparable with that of shear flows, on longitudinal and transversal derivative moments
- anisotropy of third and fourth order derivative moments increasing with the energy ratio  $E_1/E_2$ .
- a minor influence of the gradient of integral scale on the derivative statistics (small scales) than on the velocity statistics (large and inertial scales)



# Conclusions

Direct numerical simulations carried over a large range of turbulent energy ratios ( $6 \leq \mathcal{E} \leq 10^4$ ) show:

- a high level of intermittency, comparable with that of shear flows, on longitudinal and transversal derivative moments
- anisotropy of third and fourth order derivative moments increasing with the energy ratio  $E_1/E_2$ .
- *Future work*
- higher Reynolds number
- study of the mixing where the initial nonhomogeneity is associated to the integral scale variation only



# Appendix: Shearless mixing statistics

|   | $\bar{u_i u_i u_3} / \bar{u_3^2}^{3/2}$            |      |      | Velocity Kurtosis |           |           | Long. derivative skewness |                      |                      | Long. derivative kurtosis |                      |                      | Trans. Moments       |                      |
|---|--|------|------|-------------------|-----------|-----------|---------------------------|----------------------|----------------------|---------------------------|----------------------|----------------------|----------------------|----------------------|
|   | i=1  | i=2  | i=3  | $K_{u_1}$         | $K_{u_2}$ | $K_{u_3}$ | $S_{\partial_1 u_1}$      | $S_{\partial_2 u_2}$ | $S_{\partial_3 u_3}$ | $K_{\partial_1 u_1}$      | $K_{\partial_2 u_2}$ | $K_{\partial_3 u_3}$ | $S_{\partial_1 u_3}$ | $K_{\partial_1 u_3}$ |
| $E_1/E_2 :$   | Mixings with $\ell_1/\ell_2 = 1, R_\lambda = 45$   |      |      |                   |           |           |                           |                      |                      |                           |                      |                      |                      |                      |
| 6.6   | 0.34   | 0.36 | 0.82 | 3.6               | 3.4       | 4.07      | -0.11                     | -0.10                | -1.06                | 5.0                       | 4.85                 | 6.95                 | 0.29                 | 6.55                 |
| 40  | 0.54   | 0.59 | 1.34 | 5.6               | 6.0       | 5.56      | 0.55                      | 0.80                 | -2.08                | 7.1                       | 6.77                 | 13.0                 | 0.50                 | 8.50                 |
| 80  | 0.63   | 0.69 | 1.57 | 6.4               | 6.7       | 6.67      | 0.95                      | 1.1                  | -2.60                | 8.5                       | 8.6                  | 17.7                 | 0.60                 | 13.0                 |
| 300   | 0.78   | 0.87 | 1.91 | 7.5               | 8.2       | 8.93      | 1.5                       | 2.0                  | -3.37                | 16                        | 14                   | 24.5                 | 1.0                  | 13.6                 |
| 10e4  | 0.92   | 0.95 | 2.20 | 7.8               | 8.2       | 11.6      | 3.4                       | 3.2                  | -4.70                | 20                        | 26                   | 37.3                 | 1.05                 | 23.1                 |
| $\ell_1/\ell_2 :$   | Mixings with $E_1/E_2 \approx 6.6, R_\lambda = 45$ |      |      |                   |           |           |                           |                      |                      |                           |                      |                      |                      |                      |
| 0.6   | 0.32   | 0.33 | 0.77 | 3.45              | 3.4       | 3.85      | -0.14                     | -0.10                | -0.94                | 5.0                       | 4.8                  | 6.50                 | 0.27                 | 5.90                 |
| 6.6   | 0.34   | 0.36 | 0.82 | 3.6               | 3.4       | 4.07      | -0.11                     | -0.10                | -1.06                | 5.0                       | 4.85                 | 6.95                 | 0.29                 | 6.55                 |
| 1.5   | 0.49   | 0.52 | 1.20 | 4.0               | 3.6       | 5.5       | -0.12                     | -0.08                | -1.14                | 5.2                       | 4.9                  | 7.60                 | 0.31                 | 7.70                 |
| 2.1   | 0.55   | 0.58 | 1.40 | 4.9               | 4.4       | 7.5       | -0.05                     | -0.02                | -1.34                | 5.2                       | 5.0                  | 7.80                 | 0.46                 | 8.22                 |
| Mixings with $E_1/E_2 = 6.7 \ell_1/\ell_2 = 1, R_\lambda = 71$                        |  |      |      |                   |           |           |                           |                      |                      |                           |                      |                      |                      |                      |
|   | 0.42   | 0.37 | 0.81 | 3.65              | 3.55      | 4.8       | -0.15                     | -0.19                | -1.08                | 4.45                      | 4.65                 | 6.20                 | 0.30                 | 5.95                 |
| Veeravalli and Warhaft(1989), $E_1/E_2 \approx 7, \ell_1/\ell_2 \approx 1.5 \div 1.7$ |  |      |      |                   |           |           |                           |                      |                      |                           |                      |                      |                      |                      |
| bars  | ==   | ==   | 1.06 | 4.36              | 4.23      | 5.53      | ==                        | ==                   | ==                   | ==                        | ==                   | ==                   | =                    | =                    |
| plate   | ==   | ==   | 0.63 | 3.47              | 3.49      | 4.07      | ==                        | ==                   | ==                   | ==                        | ==                   | ==                   | =                    | =                    |

## Legend:

3 = inhomogeneous direction

1,2 = homogeneous directions

$S_{u_i}, K_{u_i}$  = skewness and kurtosis of  $u_i$

$S_{\partial_j u_i}, K_{\partial_j u_i}$  = skewness and kurtosis of  $\partial u_i / \partial x_j$



# Appendix: homogeneous shear turbulence

| Homogeneous shear turbulence     |       |        |       |       |       | Shearless mixings |           |                 |       |       |       |       |
|----------------------------------|-------|--------|-------|-------|-------|-------------------|-----------|-----------------|-------|-------|-------|-------|
| $Re_\lambda$                     | $S^*$ | Sd-lg  | Kd-lg | Sd-tr | Kd-tr | $Re_\lambda$      | $E_1/E_2$ | $\ell_1/\ell_2$ | Sd-lg | Kd-lg | Sd-tr | Kd-tr |
| <b>Garg-Warhaft 1998</b>         |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 156                              | 3.39  | -0.42  | 7.1   | 0.8   | 11.7  |                   |           |                 |       |       |       |       |
| 310                              | 3.31  | -0.42  | 6.9   | 0.5   | 8.6   |                   |           |                 |       |       |       |       |
| <b>Shen-Warhaft 2000</b>         |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 974                              | 6.11  | =      | =     | 0.24  | *10   |                   |           |                 |       |       |       |       |
| <b>Gylfason-Warhaft 2004</b>     |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 452                              | =     | -0.546 | 9.94  | =     | =     |                   |           |                 |       |       |       |       |
| 877                              | =     | -0.586 | 13.1  | =     | =     |                   |           |                 |       |       |       |       |
| <b>Schumacher 2001</b>           |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 59                               | 7.32  | -0.74  | 6.3   | 0.96  | 5.6   | 45                | 6.6       | 1.0             | -1.06 | 6.95  | 0.29  | 6.55  |
| 79                               | 8.18  | -0.44  | 4.8   | 0.92  | 6.4   | 71                | 6.6       | 1.0             | -1.08 | 6.25  | 0.30  | 5.95  |
| 87                               | 8.58  | -0.50  | 5.3   | 0.90  | 6.4   | 45                | 40        | 1.0             | -2.08 | 13.0  | 0.50  | 8.50  |
| 95                               | 8.22  | -0.37  | 4.3   | 0.92  | 7.2   | 45                | 300       | 1.0             | -2.60 | 17.7  | 0.60  | 13.0  |
| 99                               | 8.37  | -0.40  | 5.0   | 0.87  | 7.1   | 45                | $10^4$    | 1.0             | -3.37 | 24.5  | 1.0   | 13.6  |
| <b>Rogers-Moin-Reynolds 1986</b> |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 11                               | 1.3   | -1.2   | =     | 0.42  | =     | 45                | 6.5       | 0.6             | -0.94 | 6.50  | 0.27  | 5.90  |
| 16                               | 3.3   | -0.95  | =     | 0.53  | =     | 45                | 6.7       | 1.5             | -1.24 | 7.60  | 0.31  | 7.70  |
| 22                               | 6.2   | -0.61  | =     | 0.56  | =     | 45                | 6.7       | 2.1             | -1.34 | 7.80  | 0.46  | 8.22  |
| 30                               | 10    | -0.48  | =     | 0.60  | =     |                   |           |                 |       |       |       |       |
| 40                               | 14    | -0.58  | =     | 0.63  | =     |                   |           |                 |       |       |       |       |
| 50                               | 19    | -0.42  | =     | 0.70  | =     |                   |           |                 |       |       |       |       |
| 61                               | 22    | -0.43  | =     | 0.71  | =     |                   |           |                 |       |       |       |       |
| <b>Sawford-Yeung 2000</b>        |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 40                               | =     | =      | =     | 0.92  | *5.7  |                   |           |                 |       |       |       |       |
| 75                               | =     | =      | =     | 1.20  | *6.5  |                   |           |                 |       |       |       |       |
| <b>Tavoularis-Corrsin 1981</b>   |       |        |       |       |       |                   |           |                 |       |       |       |       |
| 204                              | 6.75  | =      | =     | *0.6  | *10   |                   |           |                 |       |       |       |       |

**Legend:**

$S^* = S \langle u^2 \rangle / \epsilon$   
 dimensionless shear parameter

$Sd$  = derivative skewness,  
 $Kd$  = derivative kurtosis,  
 $tr$  = transversal,  
 $lg$  = longitudinal

