



POLITECNICO DI TORINO
Repository ISTITUZIONALE

Temporal dynamics of small perturbations for a two-dimensional growing wake

Original

Temporal dynamics of small perturbations for a two-dimensional growing wake / Scarsoglio, Stefania; Tordella, Daniela; W. O., Criminale. - ELETTRONICO. - XI(2007), pp. 221-223. ((Intervento presentato al convegno 11th European Turbulence Conference tenutosi a Porto, Portugal nel June, 25-29 2007.

Availability:

This version is available at: 11583/1543298 since: 2018-02-26T18:53:32Z

Publisher:

CIMNE

Published

DOI:10.1007/978-3-540-72604-3_70

Terms of use:

openAccess

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)

Temporal dynamics of small perturbations for a 2D growing wake

Scarsoglio S.¹, Tordella D.¹, and Criminale W.O.²

¹ Dipartimento di Ingegneria Aeronautica e Spaziale, Politecnico di Torino, Torino 10129, Italy stefania.scarsoglio@polito.it, daniela.tordella@polito.it

² Department of Applied Mathematics, University of Washington, Box 352420, Seattle, WA 98195-2420, USA lascalea@amath.washington.edu

1 Introduction

A general three-dimensional initial-value perturbation problem is presented to study the linear stability of a two-dimensional growing wake. The base flow has been obtained by approximating it with an expansion solution for the longitudinal velocity component that considers the lateral entrainment process [1]. By imposing arbitrary three-dimensional perturbations in terms of the vorticity, the temporal behaviour, including both the early time transient as well as the long time asymptotics, is considered [2], [3], [4]. The approach has been to first perform a Laplace-Fourier transform of the governing viscous disturbance equations and then resolve them numerically by the method of lines. The base model is combined with a change of coordinate [5]. Base flow configurations corresponding to a R of 35, 50, 100 and various physical inputs are examined. In the case of longitudinal disturbances, a comparison with recent spatio-temporal multiscale Orr-Sommerfeld analysis [6], [7] is presented.

2 The initial-value problem

The base flow is viscous and incompressible. To define it, the longitudinal component of an approximated Navier-Stokes expansion for the two-dimensional steady bluff body wake [1], [8] has been used. The x coordinate is parallel to the free stream velocity, the y coordinate is normal. The coordinate x_0 plays the role of parameter of the system together with the Reynolds number. The analytical expression for the wake profile is $U(y; x_0, R) = 1 - a(R)x_0^{-1/2}e^{-(Ry^2)/(4x_0)}$, where $a(R)$ depends on the Reynolds number [8]. By changing x_0 , the base flow profile locally approximates the behaviour of the actual wake generated by the body. The equations are

$$\nabla^2 \tilde{v} = \tilde{\Gamma} \quad (1)$$

$$\frac{\partial \tilde{\Gamma}}{\partial t} + U \frac{\partial \tilde{\Gamma}}{\partial x} - \frac{\partial \tilde{v}}{\partial x} \frac{d^2 U}{dy^2} = \frac{1}{R} \nabla^2 \tilde{\Gamma} \quad (2)$$

$$\frac{\partial \tilde{\omega}_y}{\partial t} + U \frac{\partial \tilde{\omega}_y}{\partial x} + \frac{\partial \tilde{v}}{\partial z} \frac{dU}{dy} = \frac{1}{R} \nabla^2 \tilde{\omega}_y \quad (3)$$

where $\tilde{\omega}_y$ is the transversal component of the perturbation vorticity, while $\tilde{\Gamma}$ is defined as $\tilde{\Gamma} = \frac{\partial \tilde{\omega}_z}{\partial x} - \frac{\partial \tilde{\omega}_x}{\partial z}$. All physical quantities are normalized with respect to the free stream velocity, the spatial scale of the flow D and the density. By introducing the moving coordinate transform $\xi = x - U_0 t$ [5] and performing a combined Laplace-Fourier decomposition of the dependent variables in terms of ξ and z , the governing equations become

$$\frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i) \hat{v} = \hat{\Gamma} \quad (4)$$

$$\begin{aligned} \frac{\partial \hat{\Gamma}}{\partial t} = & -ik\cos(\phi)(U - U_0)\hat{\Gamma} + ik\cos(\phi)\frac{d^2 U}{dy^2} \hat{v} \\ & + \alpha_i(U - U_0)\hat{\Gamma} - \alpha_i\frac{d^2 U}{dy^2} \hat{v} + \frac{1}{R}\left[\frac{\partial^2 \hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i)\hat{\Gamma}\right] \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial \hat{\omega}_y}{\partial t} = & -ik\cos(\phi)(U - U_0)\hat{\omega}_y - ik\sin(\phi)\frac{dU}{dy} \hat{v} \\ & + \alpha_i(U - U_0)\hat{\omega}_y + \frac{1}{R}\left[\frac{\partial^2 \hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i)\hat{\omega}_y\right] \end{aligned} \quad (6)$$

where $\hat{f}(y, t; \alpha, \gamma) = \int_{-\infty}^{+\infty} \int_0^{+\infty} \tilde{f}(\xi, y, z, t) e^{i\alpha\xi + i\gamma z} d\xi dz$ is the Laplace-Fourier transform of a general dependent variable, $\phi = \tan^{-1}(\gamma/\alpha_r)$ is the perturbation angle of obliquity, $k = \sqrt{\alpha_r^2 + \gamma^2}$ is the polar wavenumber and $\alpha_r = k\cos(\phi)$, $\gamma = k\sin(\phi)$ are the wavenumbers in ξ and z directions respectively. We choose periodic and bounded initial conditions:

CASE I (symmetric initial condition): $\hat{v}(0, y) = e^{-y^2} \cos(\beta y)$, $\hat{\omega}_y(0, y) = 0$

CASE II (asymmetric initial condition): $\hat{v}(0, y) = e^{-y^2} \sin(\beta y)$, $\hat{\omega}_y(0, y) = 0$

3 Results and Conclusions

The amplification factor G is defined as the normalized energy density [3], namely $G(t; k, \phi) = E(t; k, \phi)/E(t = 0; k, \phi)$. It effectively measures the growth of the energy at time t , for a given initial condition at $t = 0$ (fig. 1). By defining the temporal growth rate [4] as $r = \log|E(t)|/(2t)$ ($E(t)$ is the total perturbation energy) and the angular frequency f as the temporal derivative of disturbance phase, we can evaluate the initial stages of exponential growth and, in the case of 2D disturbances, compare them with the normal mode theory results [6] (fig. 2).

Figure 1 yields three differing examples of early transient periods. Case (a) shows that a growing wave becomes damped, increasing the obliquity angle beyond $\pi/4$. Case (b) corresponds to dispersion relation values far from the saddle point and shows that spatially damped/amplified waves can be temporally amplified/damped. Case (c) demonstrates that perturbations normal to the base flow are stable. Figure 2 presents the comparison between the initial value problem and the Orr-Sommerfeld problem. The results are parameterized with respect to the position x_0 through the polar wavenumber $k = k(x_0)$. Equations are integrated in time beyond the transient until the

temporal growth rate asymptotes to a constant value. We observed a very good agreement with the stability characteristics given by the Orr-Sommerfeld theory for both the symmetric and asymmetric arbitrary disturbances considered.

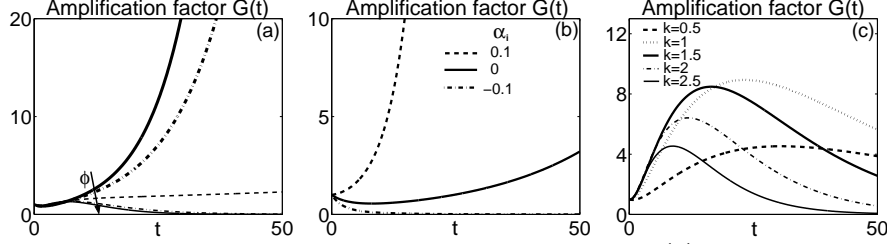


Fig. 1. The amplification factor G as a function of time. (a): $R = 100$, $k = 1.2$, $\alpha_i = -0.1$, $\beta = 1$, $x_0 = 10.15$, $\phi = 0, \pi/8, \pi/4, (3/8)\pi, \pi/2$, symmetric perturbation (case I). (b): $R = 50$, $k = 0.3$, $\beta = 1$, $\phi = 0$, $x_0 = 5.20$, $\alpha_i = -0.1, 0, 0.1$, symmetric perturbation (case I). (c): $R = 100$, $\alpha_i = -0.01$, $\beta = 1$, $\phi = \pi/2$, $x_0 = 7.40$, $k = 0.5, 1, 1.5, 2, 2.5$, symmetric perturbation (case I).

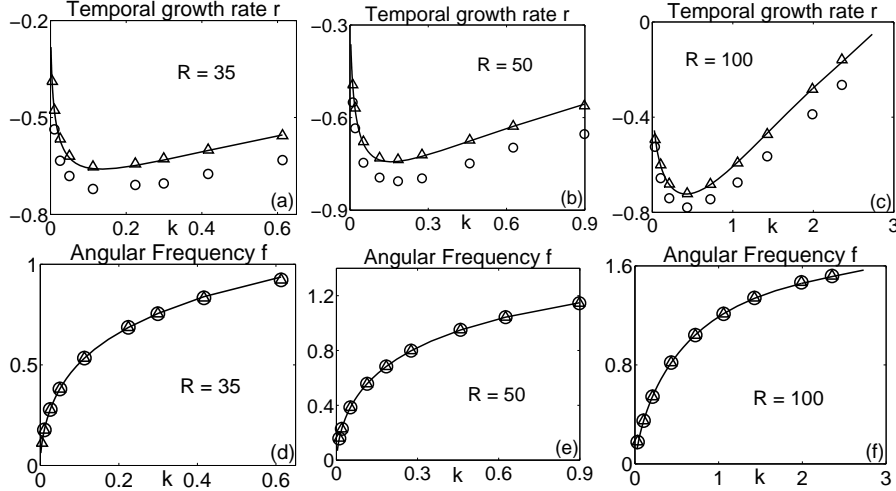


Fig. 2. $\beta = 1$, $\phi = 0$. (a, b, c) Temporal growth rate and (d, e, f) angular frequency. Comparison between present results (triangles: symmetric perturbation, case I; circles: asymmetric perturbation, case II) and normal mode analysis by Tordella, Scarsoglio and Belan, 2006 Phys. Fluids (solid lines). The wavenumber $\alpha = \alpha_r(x_0) + i\alpha_i(x_0)$, $\alpha_r(x_0) = k(x_0)$ is the most unstable wavenumber in any section of the near-parallel wake (dominant saddle point in the local dispersion relation). The wake sections considered are in the interval $3D \leq x_0 \leq 50D$.

References

1. D. Tordella, M. Belan: Phys. Fluids **15**, 7 (2003)
2. P. N. Blossy, W.O. Criminale, L.S. Fisher: J. Fluid Mech. submitted, (2006)
3. D.G. Lasseigne, et al.: J. Fluid Mech. **381**, (1999)
4. W. O. Criminale, et al.: J. Fluid Mech. **339**, (1997)
5. W.O. Criminale, P.G. Drazin: Stud. in Applied Math. **83**, (1990)
6. D. Tordella, S. Scarsoglio and M. Belan: Phys. Fluids **18**, 5 (2006)
7. M. Belan, D. Tordella: J. Fluid Mech. **552** (2006)
8. M. Belan, D. Tordella: Zamm **82**, 4 (2002)