

Tutorial lecture on Characterization and Macromodeling of 3D Interconnects

*Original*

Tutorial lecture on Characterization and Macromodeling of 3D Interconnects / GRIVET TALOCIA, Stefano. - ELETTRONICO. - (2004). ((Intervento presentato al convegno 8th IEEE Workshop on Signal Propagation on Interconnects (SPI) tenutosi a Heidelberg (Germany) nel May 9-12, 2004.

*Availability:*

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# Characterization and Macromodeling of 3D Interconnects

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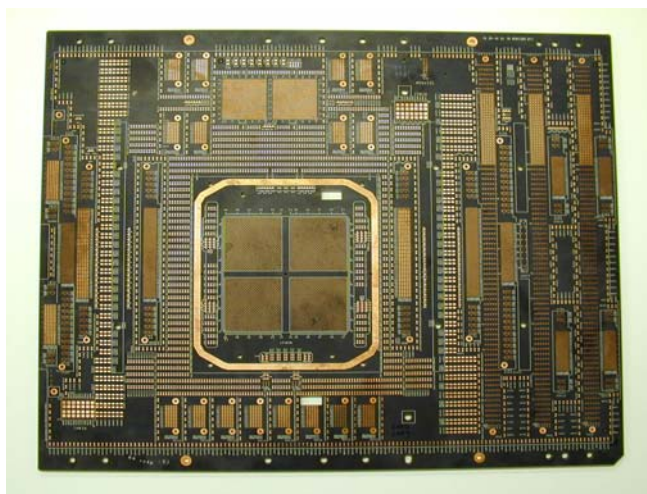
<http://www.eln.polito.it/research/emc>



S. Grivet-Talocia, SPI tutorial, 9 May 2004



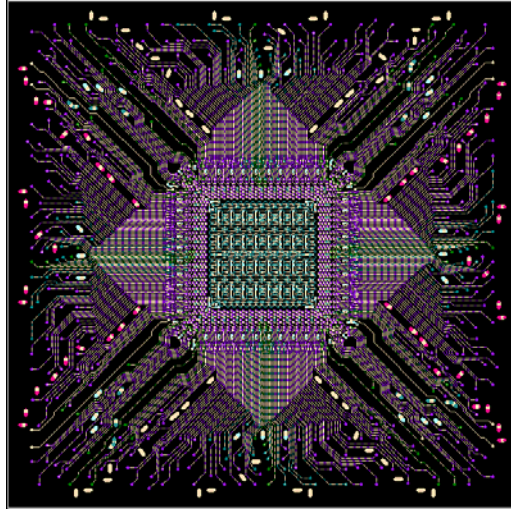
## Introduction



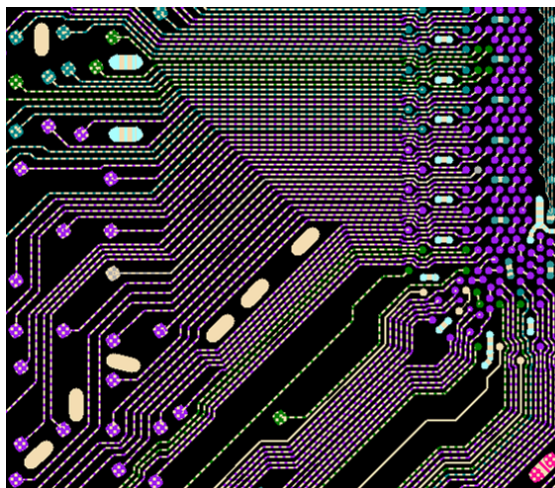
S. Grivet-Talocia, SPI tutorial, 9 May 2004



## Introduction

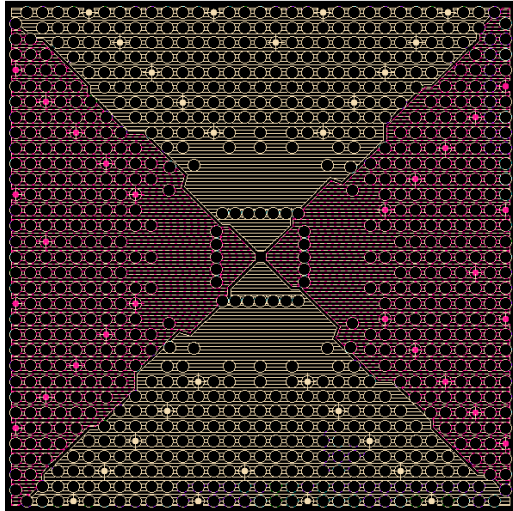


## Introduction



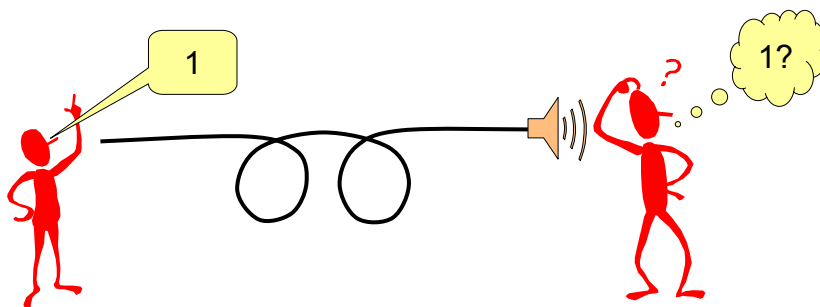


## Introduction



## Introduction

**High-speed Data transmission** requires  
**integrity of the signals**  
thru lines, bends, vias, connectors, ...

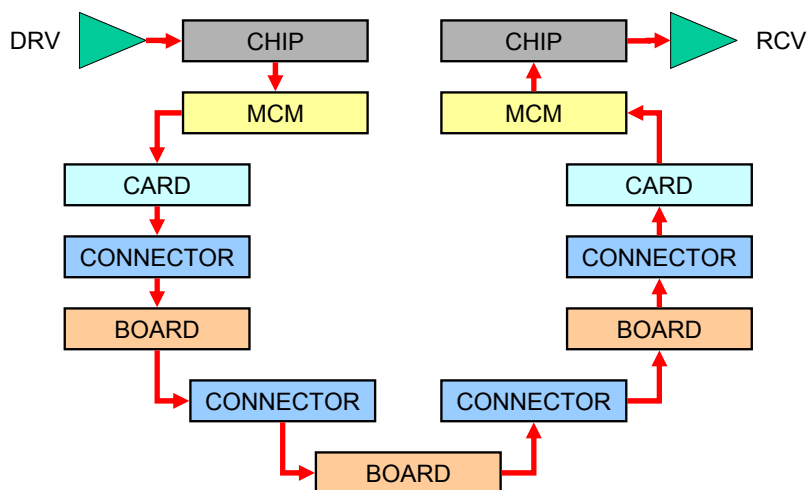




## An example

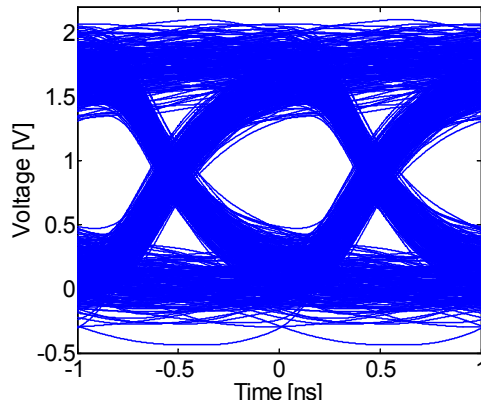


## An example





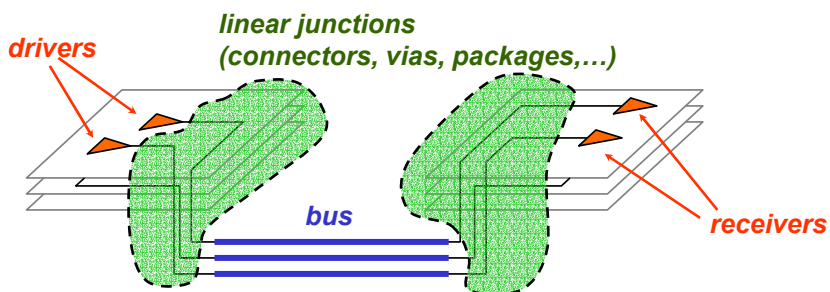
## An example

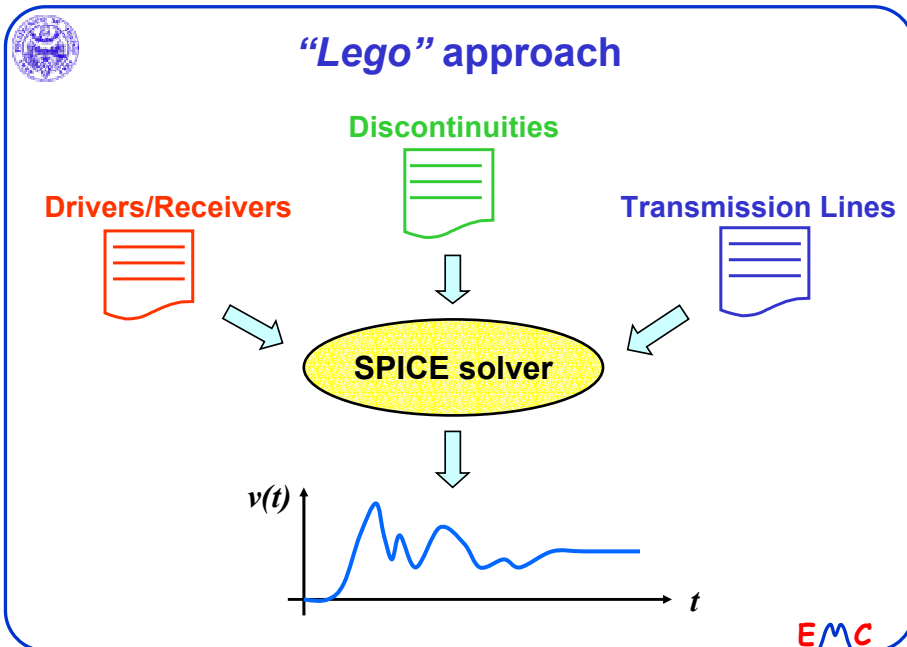
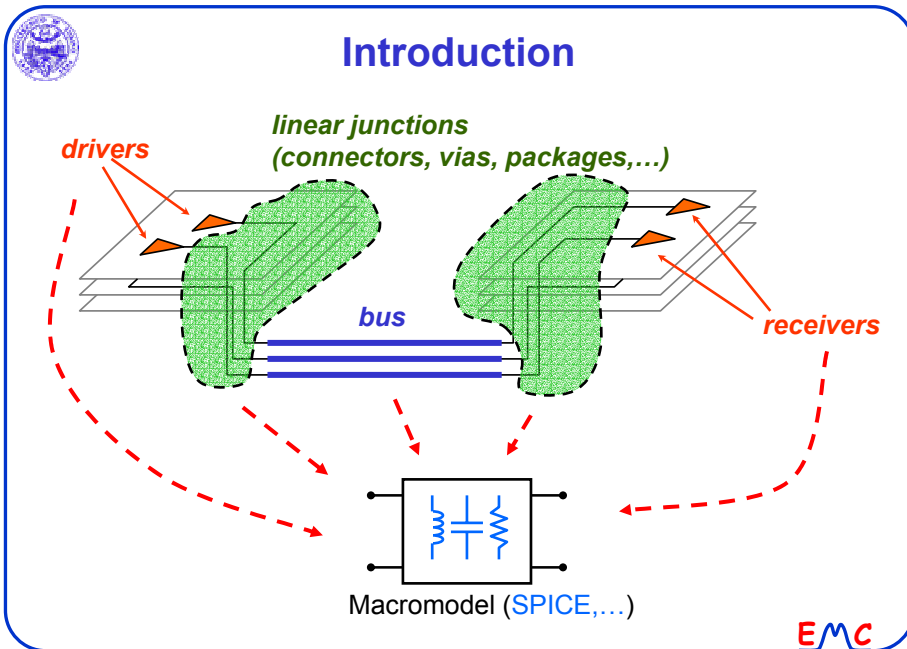


## Introduction

Signal Integrity issues in high-speed digital systems

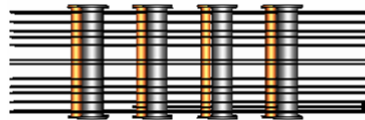
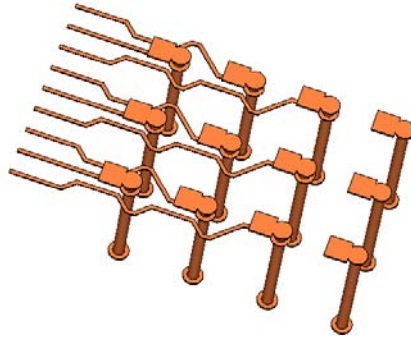
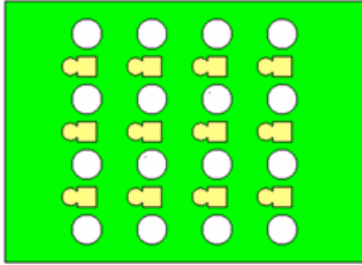
Crosstalk, couplings, reflections, losses, dispersion, attenuation, resonances, ground noise, nonlinear effects, radiation, EMI, ...



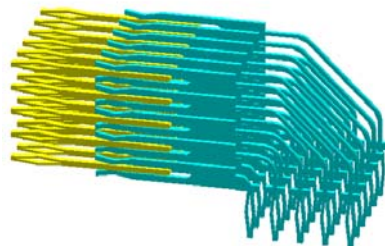
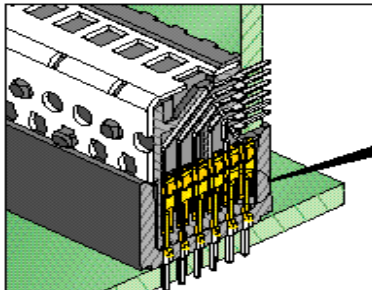




## 3D Interconnects



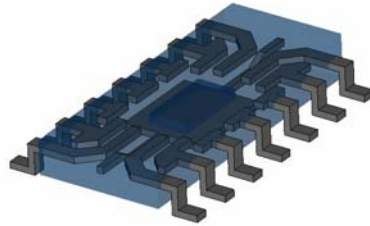
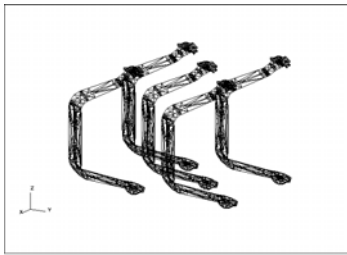
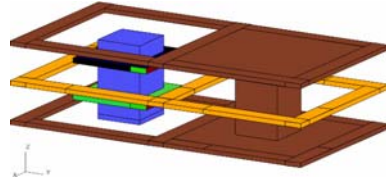
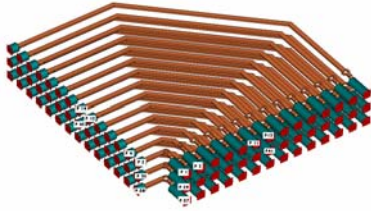
## 3D Interconnects







## 3D Interconnects



## Outline

- Introduction
- **Macromodeling approaches for 3D Interconnects**
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Macromodeling approaches

Macromodeling of 3D interconnects for Signal Integrity

1. Capture physical effects leading to signal degradation
  - Must take into account **3D electromagnetic fields**
    - **Simulation or measurement**
    - Many different characterizations are possible!
2. Use this information to build a macromodel
  - **Many macromodeling approaches available!**



## Macromodeling approaches

### Characterization via equations

Discretization of Maxwell full-wave equations

**Model Order Reduction** methods: build a simplified model from an existing (large) one

### Characterization via port responses (Black Box)

Time or frequency domain

Simulated or measured

**Reduced-Order Model Identification** methods: build a model from samples of the port responses



## Macromodeling approaches

Main goal of all (lumped) macromodeling methods:

**produce a rational approximation**



$$\mathbf{H}_q(s) = \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

### Lumped circuits

- **have rational transfer functions**
- **are governed by Ordinary Differential Equations**

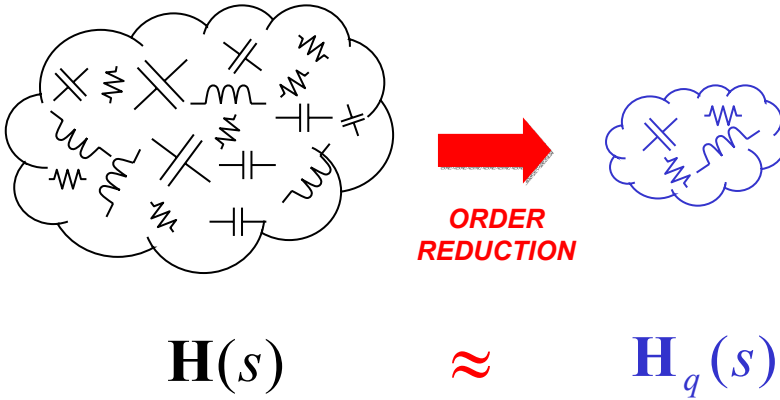


## Outline

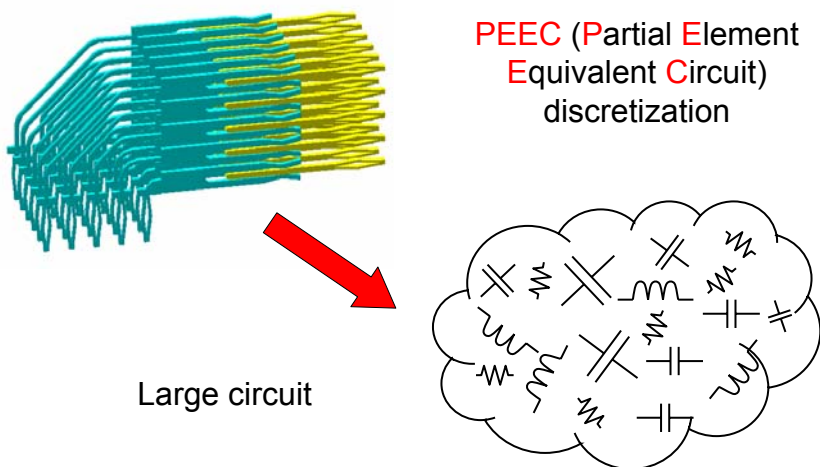
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## Moder Order Reduction

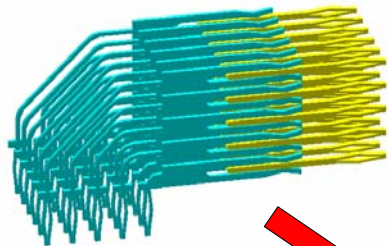


## Possible scenarios



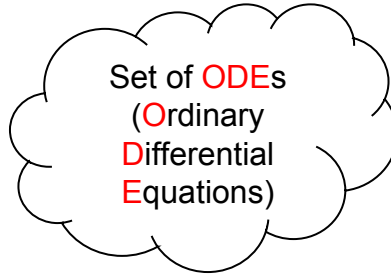


## Possible scenarios

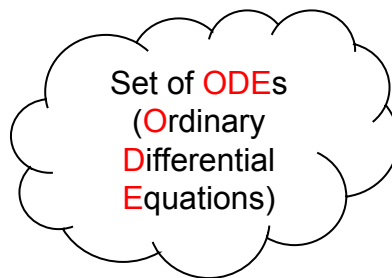
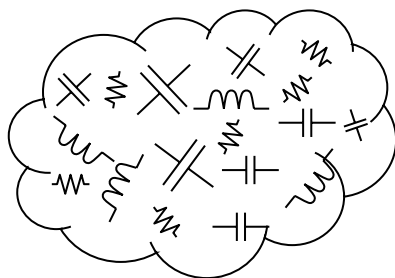


Large system

Spatial discretization of  
Maxwell equations  
(FDTD, FEM, MoM, ...)



## Possible scenarios

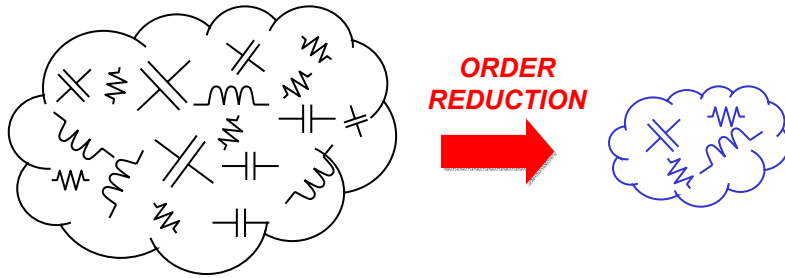


MNA  $\rightarrow$  
$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{G} + s\mathbf{C})^{-1}\mathbf{B}$$



## Approximation via moment matching

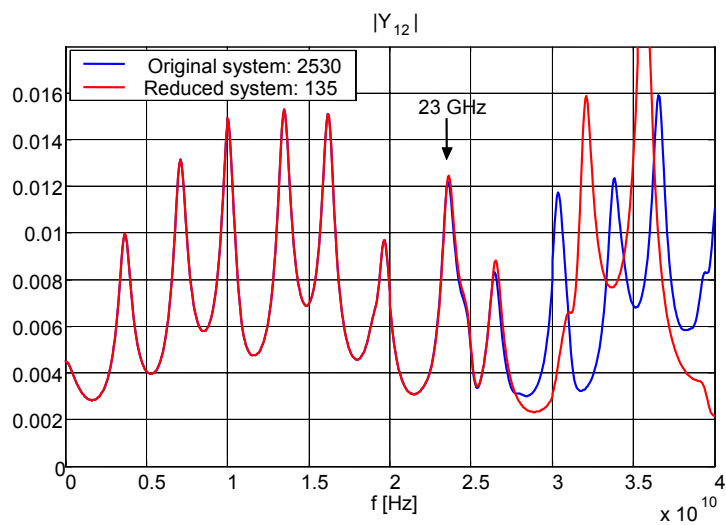


$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1s + \mathbf{M}_2s^2 + \dots + \mathbf{M}_Ns^N + \dots$$

$$\mathbf{H}_q(s) = \mathbf{M}_0 + \mathbf{M}_1s + \mathbf{M}_2s^2 + \dots + \mathbf{M}_qs^q + \dots$$



## Moment matching: an example





## Moments

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases} \quad \begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases} \quad \begin{cases} \mathbf{A} = -\mathbf{G}^{-1}\mathbf{C} \\ \mathbf{R} = \mathbf{G}^{-1}\mathbf{B} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{L}^T(\mathbf{I} - s\mathbf{A})^{-1}\mathbf{B}\mathbf{u}$$

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1s + \mathbf{M}_2s^2 + \dots$$
$$\text{Moments} \quad \mathbf{L}^T\mathbf{R} \quad \mathbf{L}^T\mathbf{A}\mathbf{R} \quad \mathbf{L}^T\mathbf{A}^2\mathbf{R}$$



## Moment matching techniques

**Explicit**      $\mathbf{M}_i = \mathbf{L}^T\mathbf{A}^i\mathbf{R} \rightarrow \mathbf{H}_q(s)$

**Asymptotic Waveform Evaluation (AWE)**

**Pade` Approximations**

**Complex Frequency Hopping (CFH)**

- Good theoretical properties, convergence
- Bad numerical properties, intrinsic ill-conditioning due to
  - Moment generation
  - Moment matching



## Moment matching techniques



### Implicit

#### Krylov subspace projection methods

- Same information stored in moments
- Much better numerical performance, robustness
- Several versions
  - Arnoldi, PRIMA, Lanczos, ...
- Possibility of preserving stability and passivity by construction!



## Krylov subspaces

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \dots$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{Moments} & \mathbf{L}^T \mathbf{R} & \mathbf{L}^T \mathbf{A} \mathbf{R} & \mathbf{L}^T \mathbf{A}^2 \mathbf{R} \end{array}$$

$$Kr(\mathbf{A}, \mathbf{R}, q) = \text{span}\{\mathbf{R}, \mathbf{A}\mathbf{R}, \mathbf{A}^2\mathbf{R}, \dots, \mathbf{A}^{q-1}\mathbf{R}\}$$

$$\mathbf{V}_q = \text{basis of } Kr(\mathbf{A}, \mathbf{R}, q)$$

Constructed via iterative (stable) algorithms





## Arnoldi (basic) algorithm

$$\begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T \mathbf{x} \end{cases}$$

$$\mathbf{V}_q^T \mathbf{A} \mathbf{V}_q = \mathbf{A}_q$$

$$\mathbf{x} \approx \mathbf{V}_q \mathbf{x}_q$$

$$\begin{cases} \mathbf{x}_q = \mathbf{A}_q \dot{\mathbf{x}}_q + \mathbf{R}_q \mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T \mathbf{x}_q \end{cases}$$



## PRIMA algorithm

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T \mathbf{x} \end{cases}$$

$$\mathbf{V}_q^T \mathbf{C} \mathbf{V}_q = \mathbf{C}_q$$

$$\mathbf{V}_q^T \mathbf{G} \mathbf{V}_q = \mathbf{G}_q$$

$$\mathbf{x} \approx \mathbf{V}_q \mathbf{x}_q$$

$$\begin{cases} \mathbf{G}_q \mathbf{x}_q + \mathbf{C}_q \dot{\mathbf{x}}_q = \mathbf{B}_q \mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T \mathbf{x}_q \end{cases}$$



## Passivity conditions (PRIMA)

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

Can often be enforced  
by construction building  
the original system



$$\begin{cases} \mathbf{G}_q\mathbf{x}_q + \mathbf{C}_q\dot{\mathbf{x}}_q = \mathbf{B}_q\mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T\mathbf{x}_q \end{cases}$$

$$\mathbf{G} \geq 0 \quad \mathbf{C} \geq 0$$

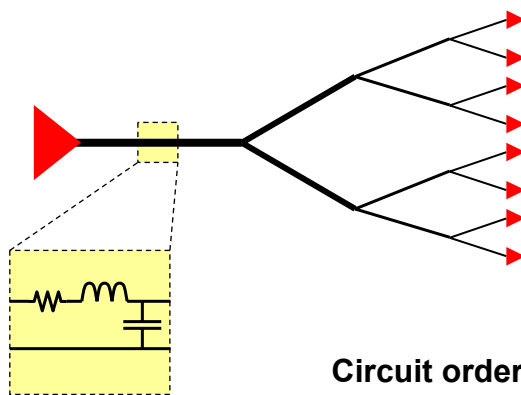
$\mathbf{C}$  symmetric

$$\mathbf{L} = \pm \mathbf{B}$$

$\mathbf{V}_q$  must be full rank



## An example: RLC tree circuit

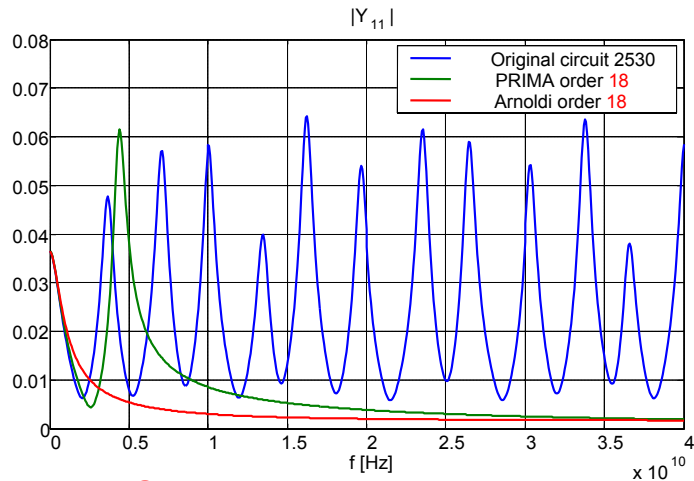


Circuit order: 2530

Ports: 9



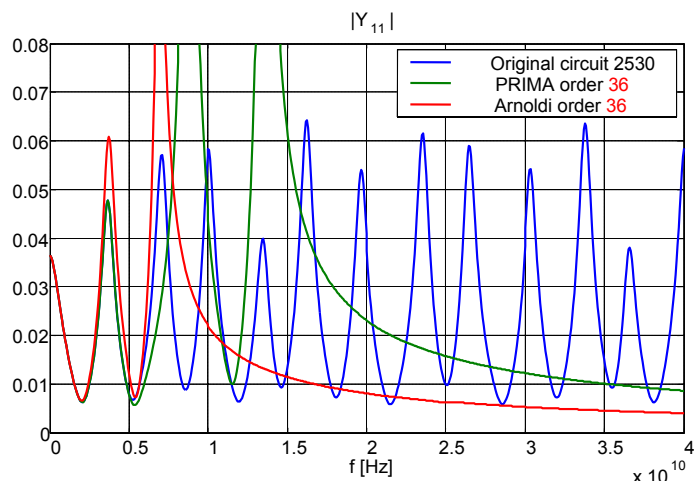
## RLC tree circuit: order reduction



■ 1 GHz



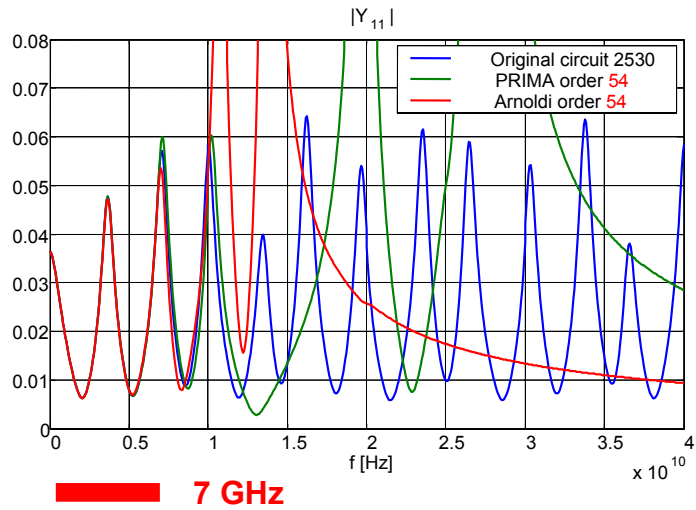
## RLC tree circuit: order reduction



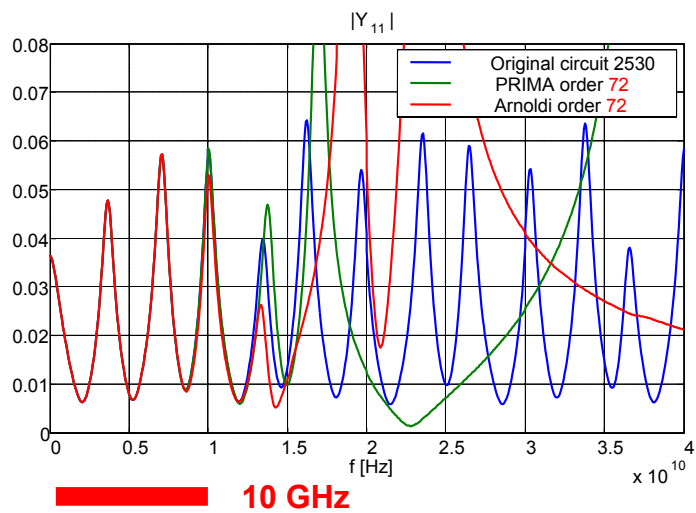
■ 2 GHz



## RLC tree circuit: order reduction

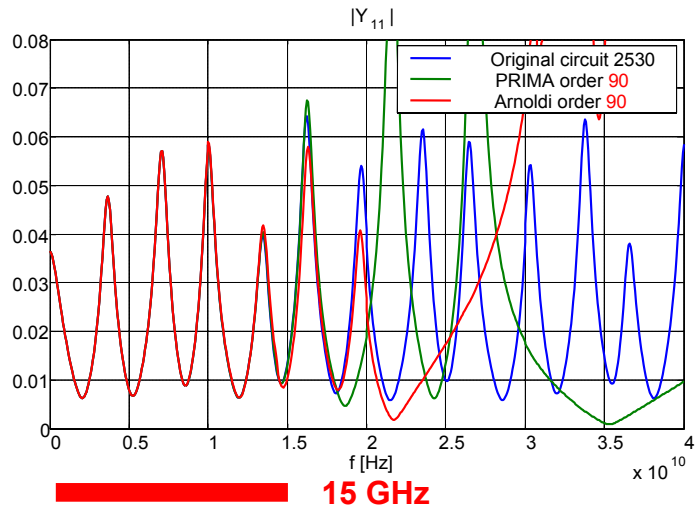


## RLC tree circuit: order reduction

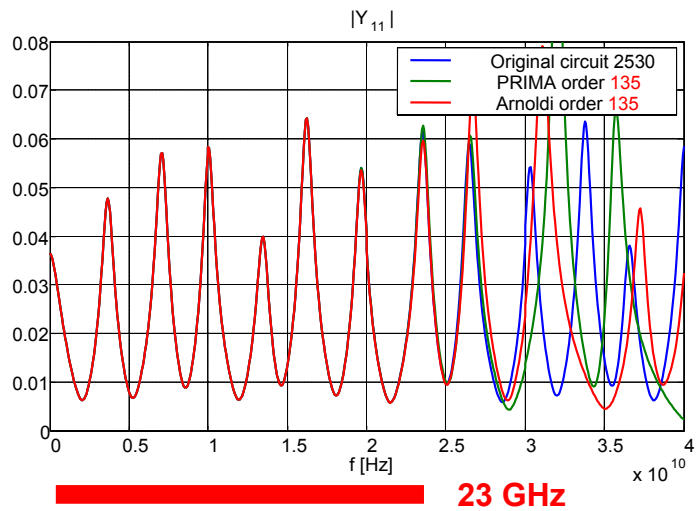




## RLC tree circuit: order reduction

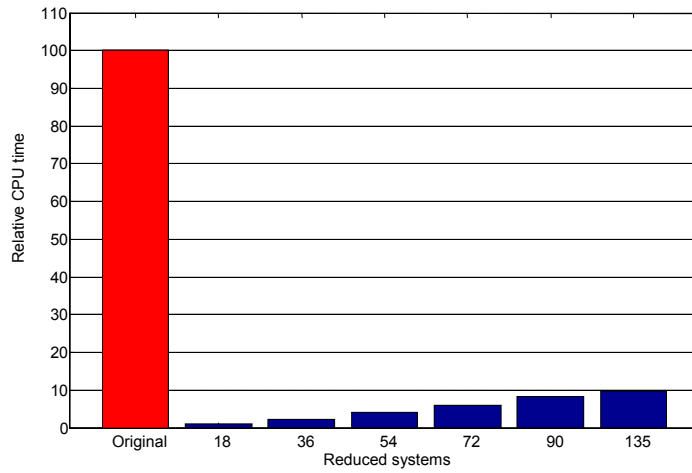


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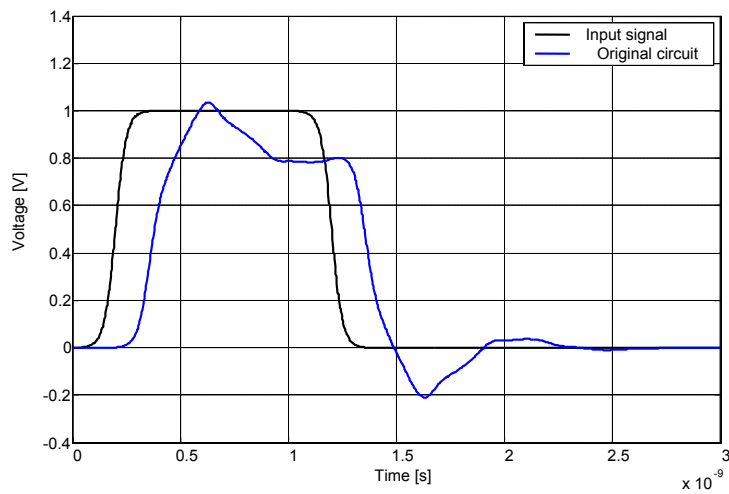




## RLC tree circuit: efficiency

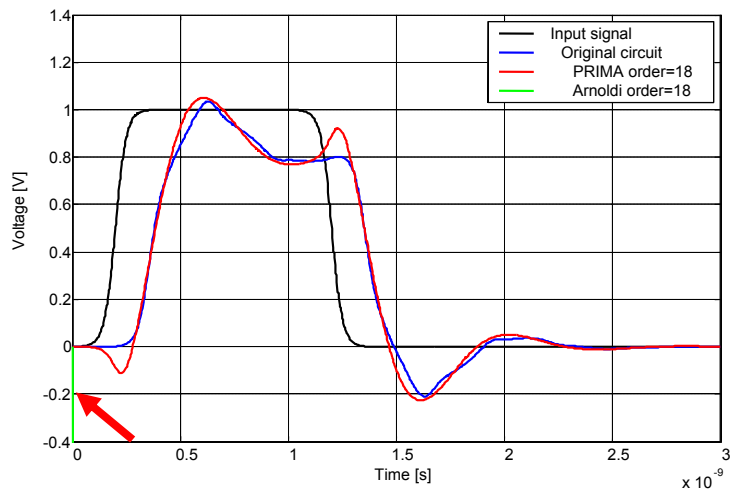


## RLC tree circuit: transient analysis

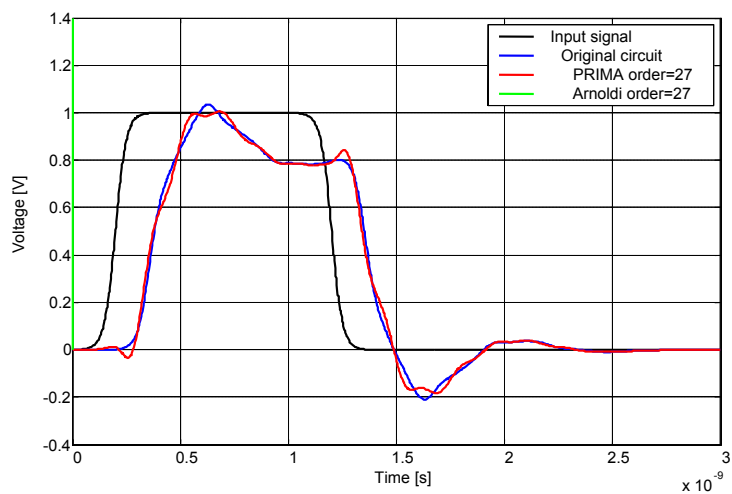




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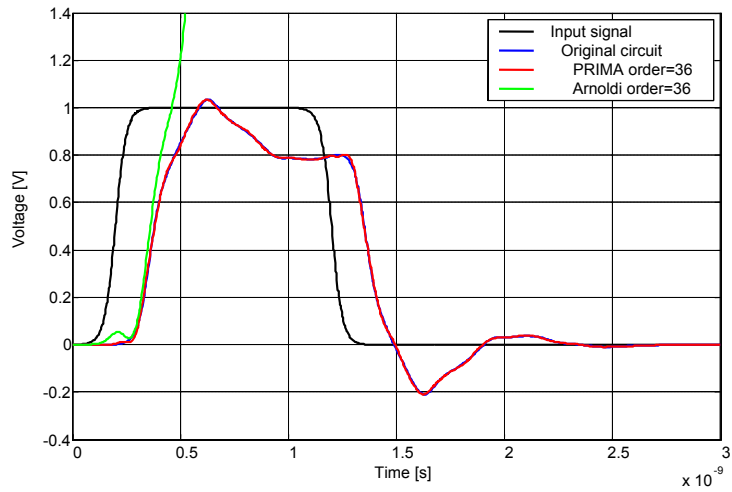


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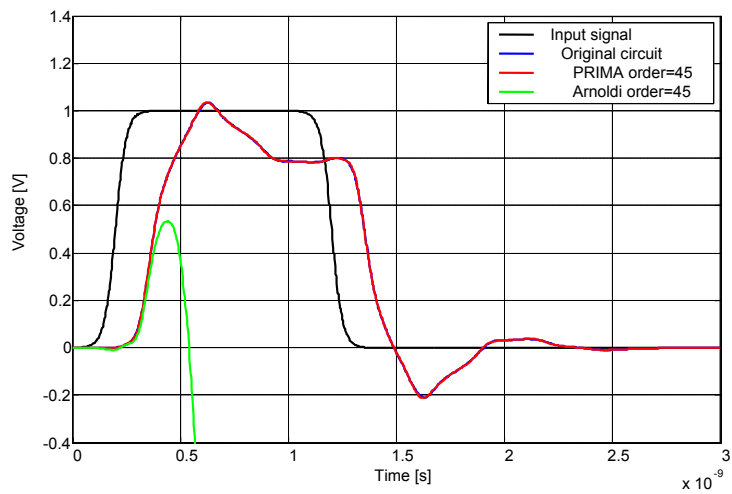




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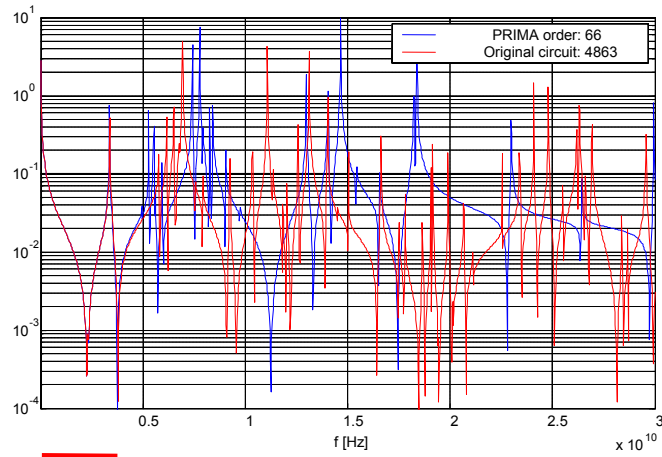






## Example: MNA, 22 ports, order 4863

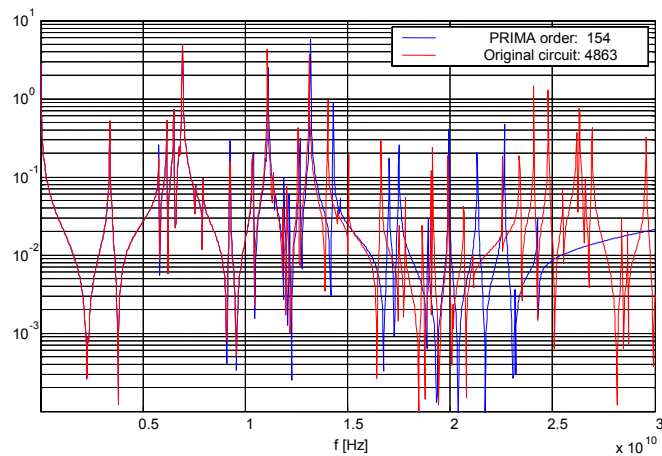
([www.win.tue.nl/niconet/NIC2/benchmodred.html](http://www.win.tue.nl/niconet/NIC2/benchmodred.html))



**3 GHz**



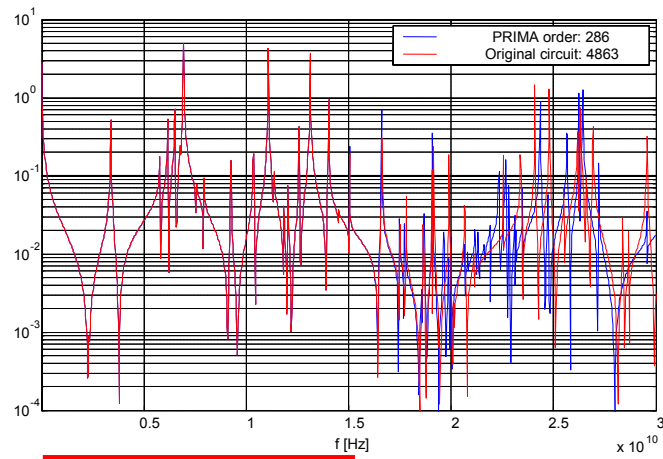
## Example: MNA, 22 ports, order 4863



**6 GHz**



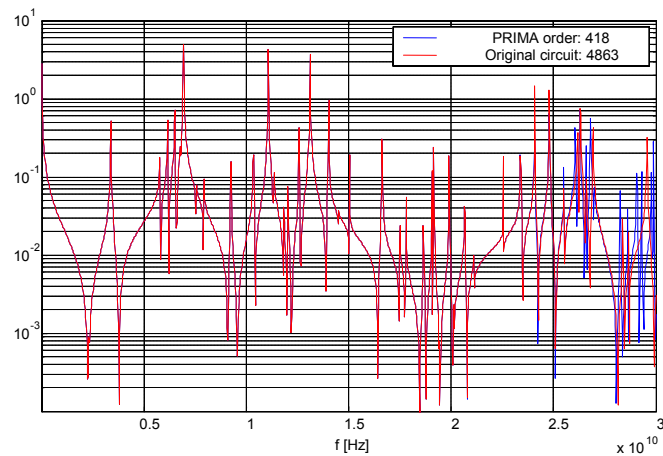
## Example: MNA, 22 ports, order 4863



15 GHz



## Example: MNA, 22 ports, order 4863



24 GHz



## Key references

M.Celik, L.Pileggi, A.Odabasioglu, IC Interconnects Analysis, Kluwer, 2002

*...and references therein*

R.Achar, M.S.Nakhla, Proceedings of the IEEE, Vol.89, 2001, 693-728

*... and references therein*

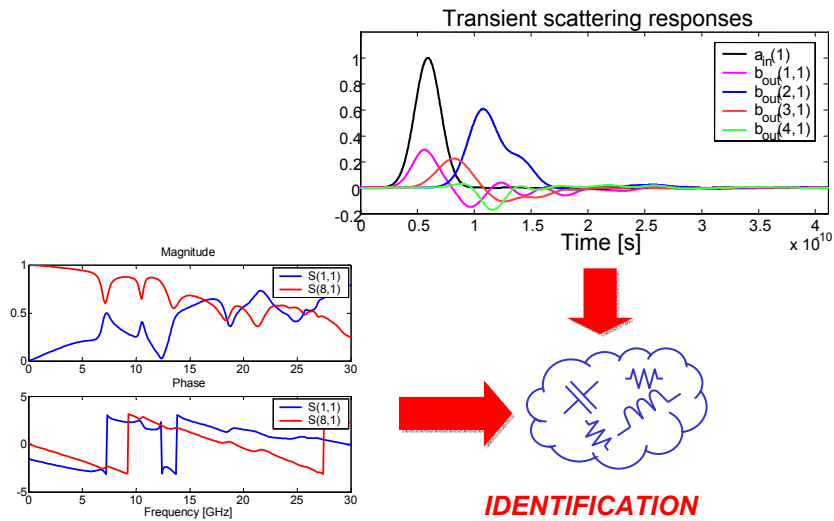


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- Model Order Reduction methods
  - PRIMA
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  - Frequency-Domain Vector Fitting
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## Model Identification



## Model identification

From samples to model: **identification** process

Reduced-order identification: **approximation** process

Several identification methods exist

Characterized by use of different:

- Input data
- Modeling criteria
- Model parameter estimation



## Identification methods

### Block Complex Frequency Hopping (BCFH)

[R.Achar, M.S.Nakhla, *IEEE Proceedings*, Vol.89, 2001]

Rational **Padé** approximation of network functions

**Convergence** property in a neighborhood of the expansion point

**Hopping** along frequency axis to cover the modeling bandwidth

May lead to **ill-conditioned** numerical systems when used for identification from sampled responses



## Identification methods

### Global Rational Approximation

[M.Elzinga, K.L.Virga, J.L.Prince, *IEEE Trans. MTT*, 9/2000]

[W.Beyene, J.Schutt-Aine, *IEEE Trans. CPMT*, Vol.21, 3/1998]

[K.L.Choi, M.Swaminathan, *IEEE Trans. CAS II*, vol.47, 4/2000]

[J.Morsey, A.C.Cangellaris, *Proc. EPEP*, 2001]

[... many, many, many others...]

A matrix of rational functions is **fitted** to the samples of a network function matrix (e.g. the Y matrix)



## Identification methods

### Pencil of Functions

[Y.Hua, T.Sarkar, *IEEE Trans. AP*, vol.37, 2/1989]

Time-domain data

Estimates model poles by **fitting** a sum of exponential functions to the samples of transient port responses

**Poles** obtained as eigenvalues of a generalized eigenvalue problem

**Automatic order estimation**



## Identification methods

### Subspace-based State-Space System Identification methods (4SID)

[M. Viberg, *Automatica*, 12/1995]

[T.McKelvey, H.Akcaay, L.Ljung, *IEEE Trans. AC*, vol.41, 7/1996]

Based on **projections** of data onto orthogonal subspaces, leading to direct state-space estimation

Built-in **automatic order estimation** (based on SVD)

Available in both **time and frequency** domain

Equivalent to Pencil of Functions methods



## Identification methods

### Nevanlinna-Pick Interpolation

[C.P.Coelho, J.R.Phillips, L.M.Silveira, Proc. DATE 2002]

Interpolation of samples of the scattering matrix with a (unitary bounded) matrix rational function

Nice theoretical properties

Very complex

Leads to models with large dynamical order



## Identification methods

### Vector Fitting

[B.Gustavsen, A. Semlyen, IEEE Trans. PD, vol.14, 7/1999]

Performs data fitting with rational functions avoiding nonlinear optimization

Iterative process converging to the dominant poles

Available for both time and frequency domain



## Identification methods

**Identification methods are not expected to work for every possible problem**

**Any method performs well for a certain class of identification problems**

**Vector Fitting** is selected here as one of the most promising methods for a wide range of applications



## Outline

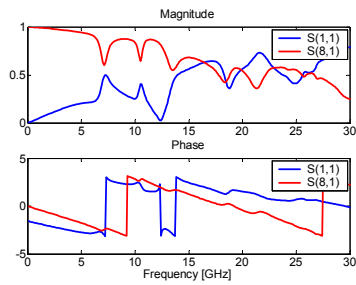
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## Frequency-domain macromodeling

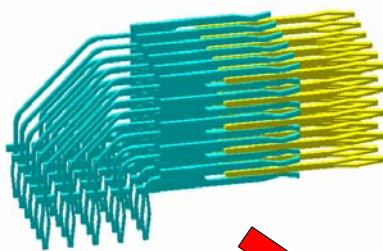
Model identification from frequency-domain responses



**IDENTIFICATION**



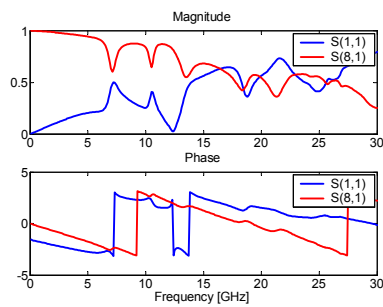
## Possible scenarios



**Frequency-Domain  
full-wave simulation  
(MoM, FEM, ...)**

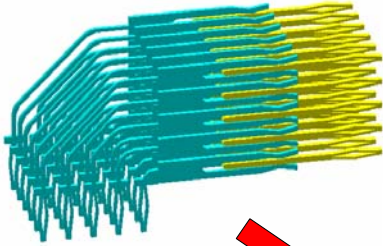


Frequency tables  
of transfer matrix  
(S, Y, Z, ...)





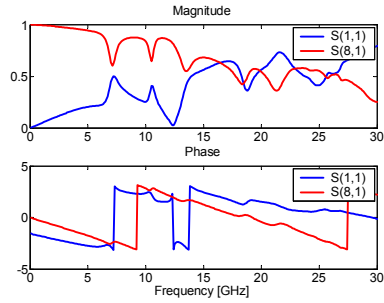
## Possible scenarios



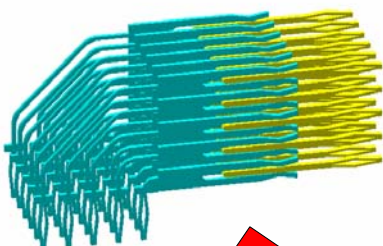
Time-Domain full-wave simulation (FIT, FDTD)

FFT postprocessing

Frequency tables of transfer matrix (S, Y, Z, ...)

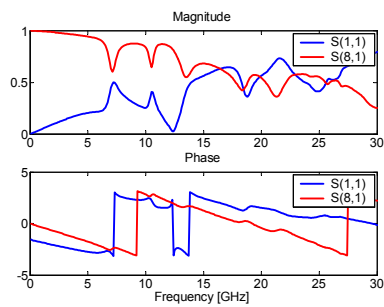


## Possible scenarios



Direct VNA measurement

Frequency tables of transfer matrix (S)





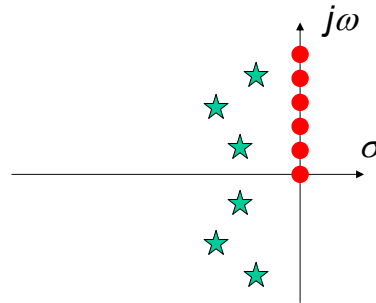
## Frequency-Domain Macromodeling

Input data  
 $\{ \hat{H}(j\omega_k), k = 1, \dots, K \}$

Approximation  

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Fitting condition  
 $H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$



Unknowns:

- Poles  $p_n$
- Residues  $R_n$
- Constant  $H_\infty$



## Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^N \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

- Nonlinear dependence on poles
- Requires nonlinear optimization (e.g. nonlinear least squares)
- Convergence problems (local minima, etc...)





## Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^N \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

**Vector Fitting  
avoids nonlinear optimization**

B. Gustavsen, A. Semlyen, "Rational approximation of frequency responses by **vector fitting**", *IEEE Trans. Power Delivery*, Vol.14, July 1999, pp.1052-1061



## Frequency-Domain Vector Fitting

Input data

$$\left\{ \hat{H}(j\omega_k), \quad k = 1, \dots, K \right\}$$

Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$

- $w(s)$  is unitary for  $s \rightarrow \infty$
- poles  $q_n$  are fixed a priori
- residues  $c_n$  are unknown

Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

The poles of

$w(s) H(s)$   
are  $\{q_n\}$  only!



## Frequency-Domain Vector Fitting

Input data

$$\{\hat{H}(j\omega_k), k=1, \dots, K\}$$

Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$
$$= \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

There are  $N$  zeros  $\{z_n\}$

Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

$$\{p_n\} \cong \{z_n\}$$



## Frequency-Domain Vector Fitting

Input data

$$\{\hat{H}(j\omega_k), k=1, \dots, K\}$$

Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$

$$\left\{ \sum_{n=1}^N \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$



## Frequency-Domain Vector Fitting

Unknowns

$$\{c_n, \tilde{c}_n, \tilde{H}_\infty\}$$

$$\left\{ \sum_{n=1}^N \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

**Linear least squares problem: easy to solve!**

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

Poles of  $H(s)$

EMC  
GROUP



## Frequency-Domain Vector Fitting

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

Poles of  $H(s)$

**Theorem:** the zeros  $\{z_n\}$  are the eigenvalues of

$$\mathbf{Q} = \mathbf{A} - \mathbf{b} \mathbf{c}^T$$

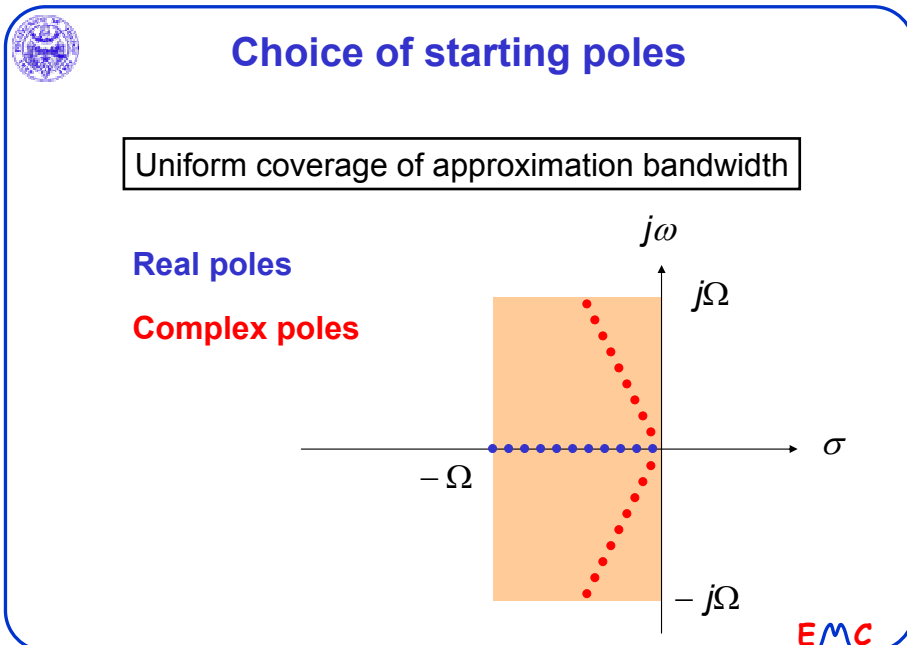
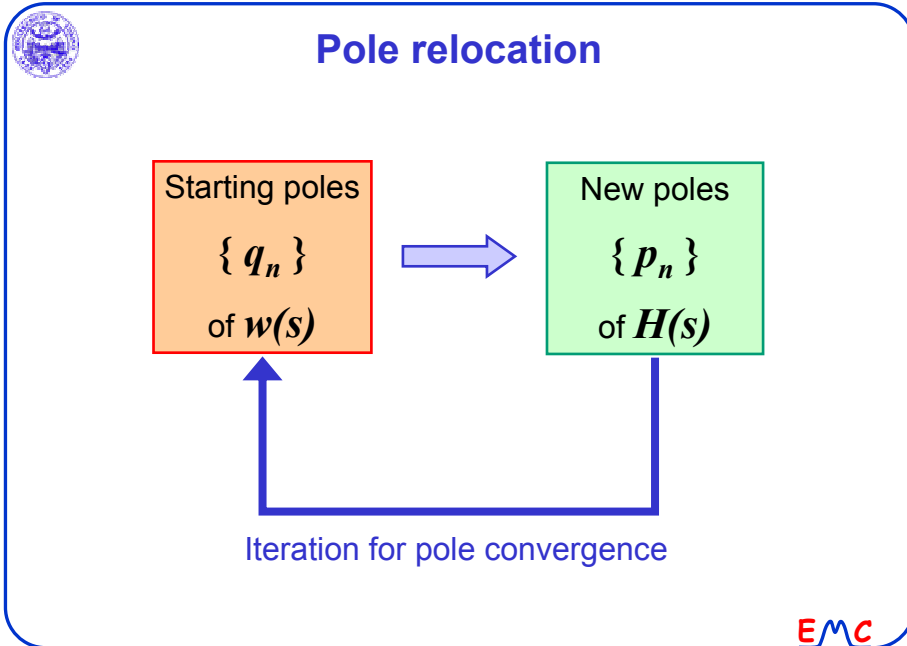
where

$$\mathbf{A} = \text{diag} \{q_n\}$$

$$\mathbf{b} = (1 \quad 1 \quad \dots \quad 1)^T$$

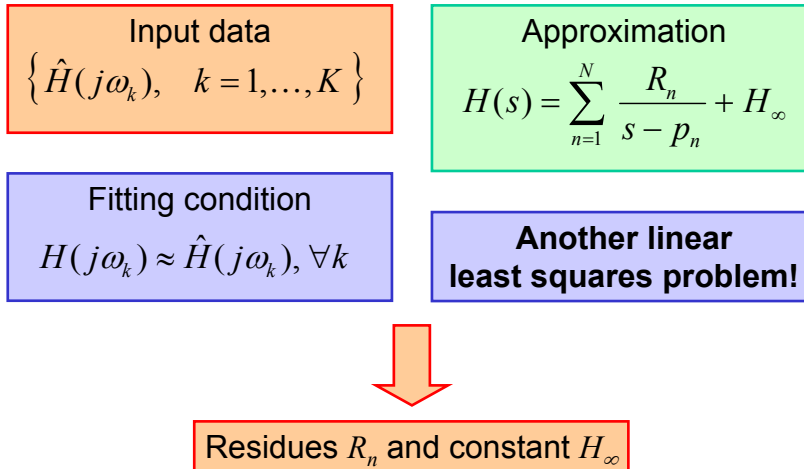
$$\mathbf{c} = (c_1 \quad c_2 \quad \dots \quad c_N)^T$$

EMC  
GROUP

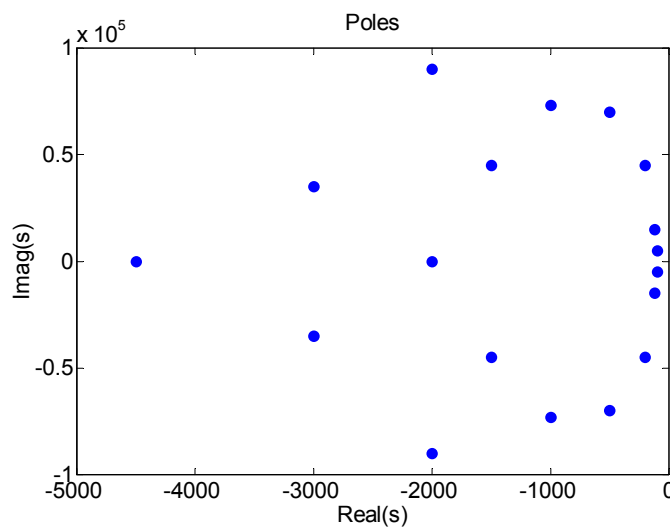




## Vector Fitting: residues



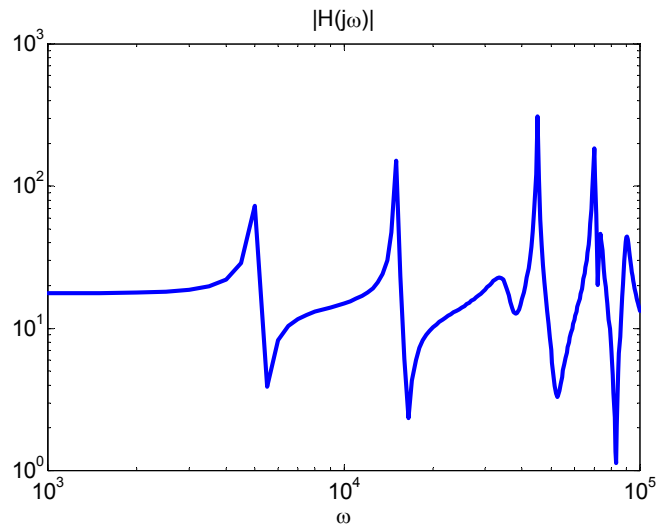
## Example 1: 18 random poles/residues



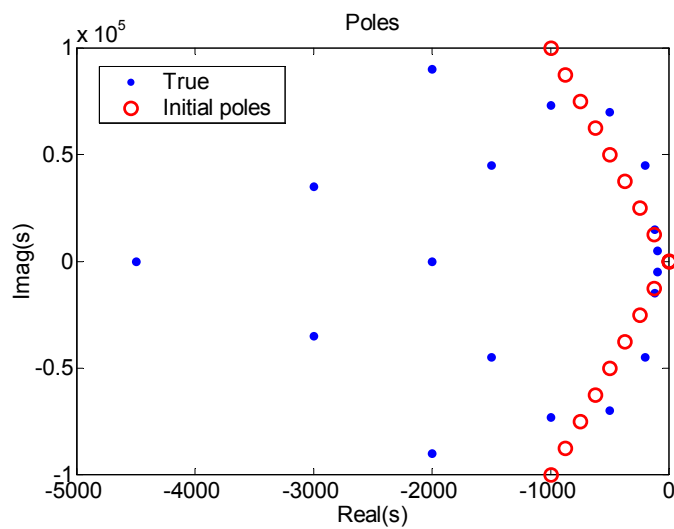




## Example 1

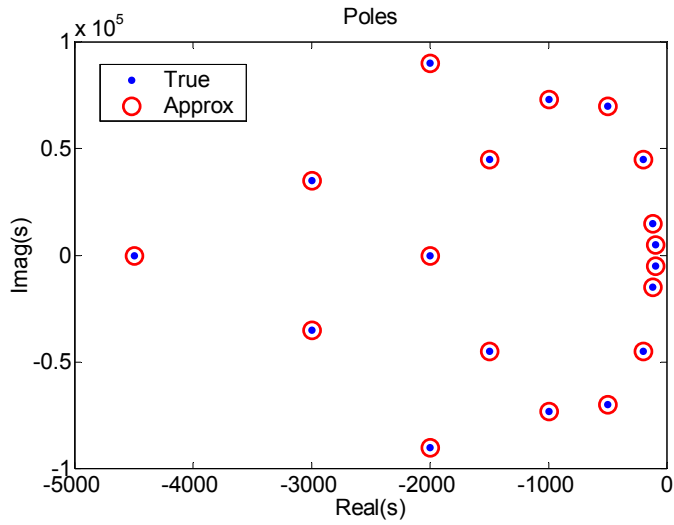


## Example 1

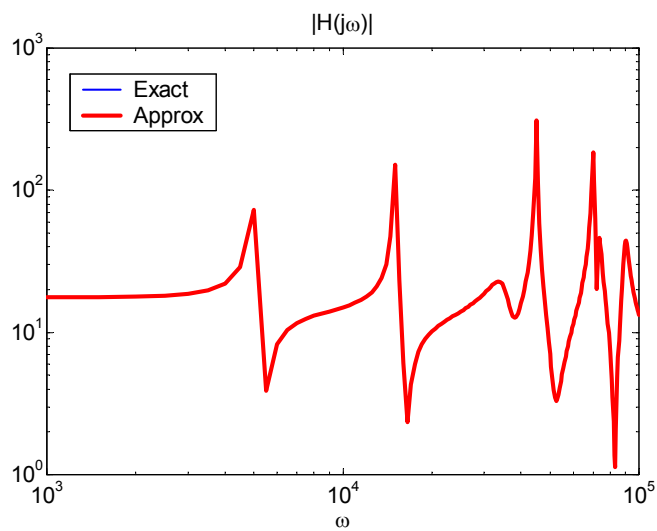




## Example 1

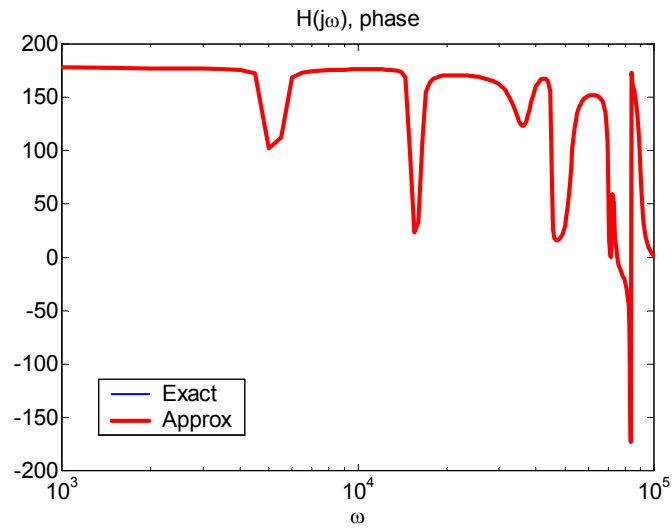


## Example 1





## Example 1



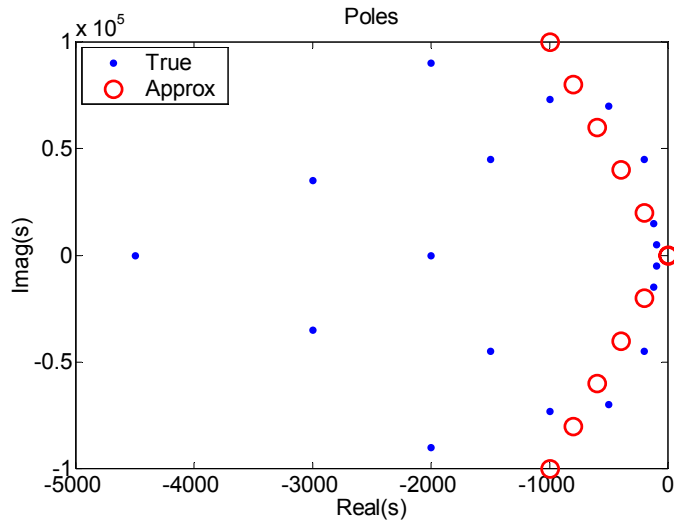
## Example 2

**Same 18-pole rational function**

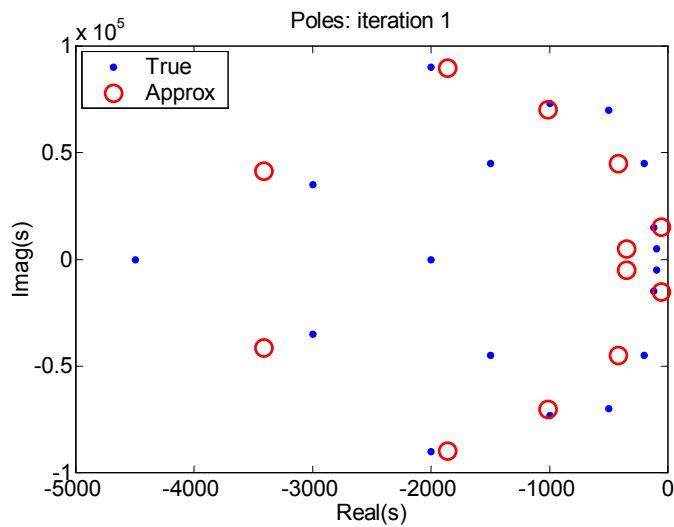
**Reduced-order fitting (12th order)**



## Example 2

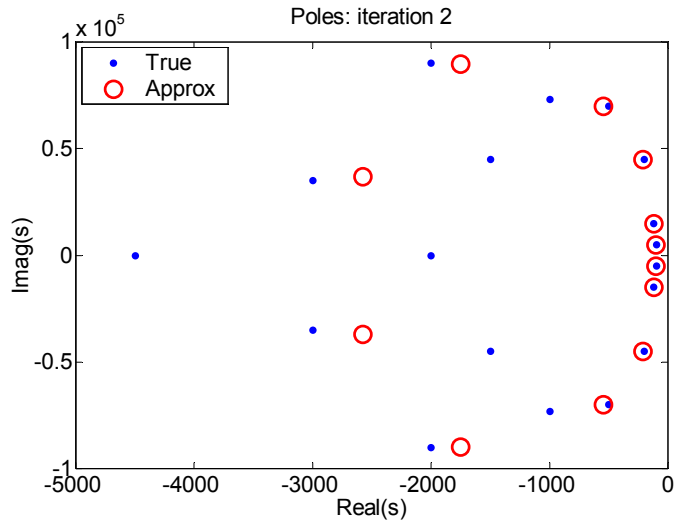


## Example 2

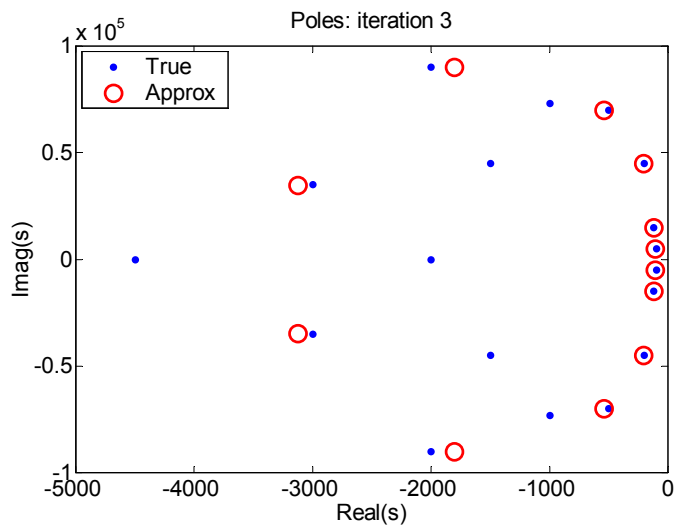




## Example 2

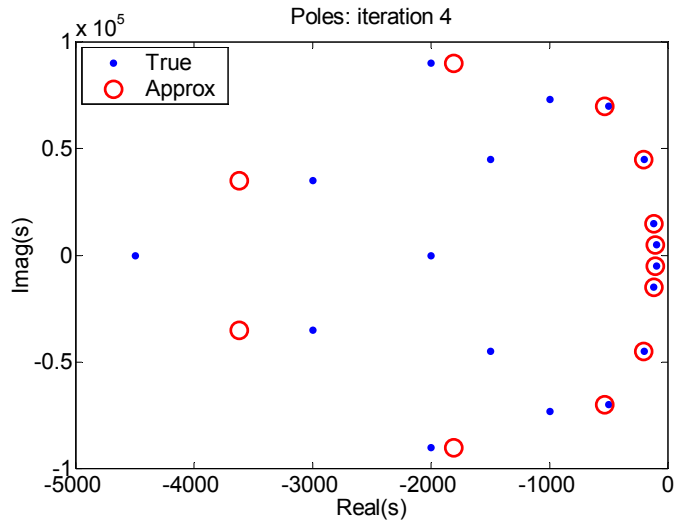


## Example 2

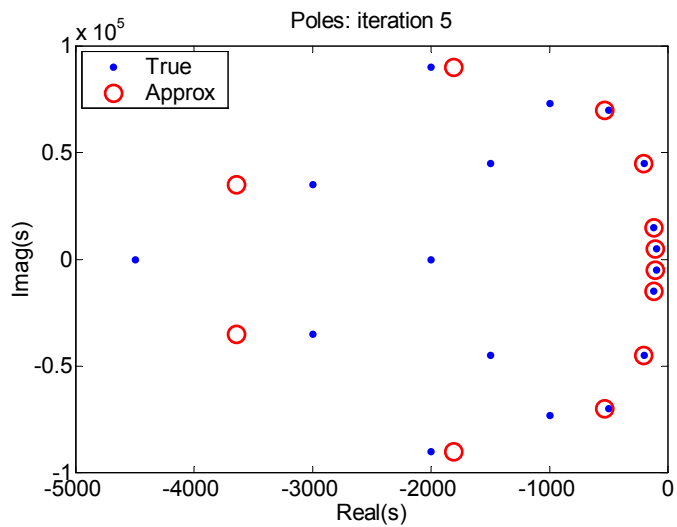




## Example 2

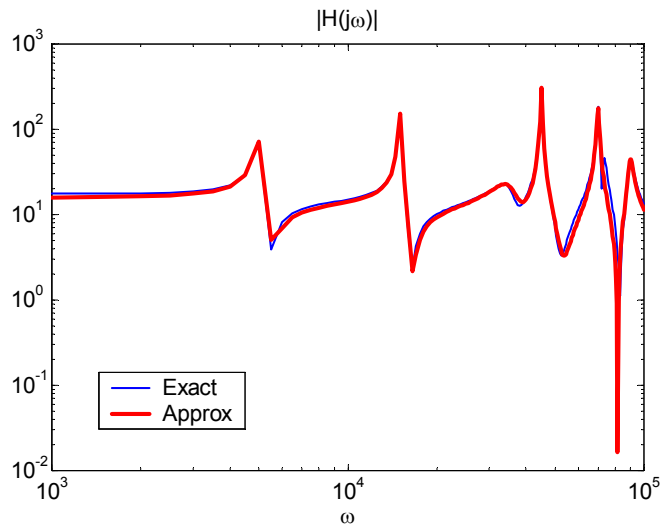


## Example 2





## Example 2



## Example 3

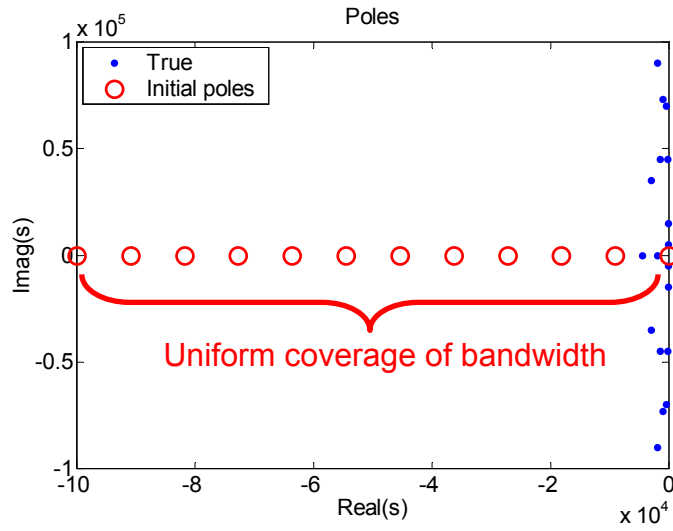
**Same 18-pole rational function**

**Reduced-order fitting (12th order)**

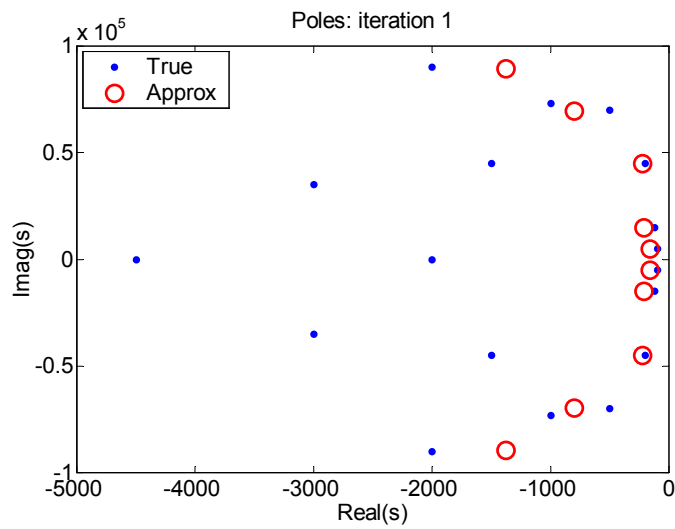
**Different starting poles (real poles)**



### Example 3



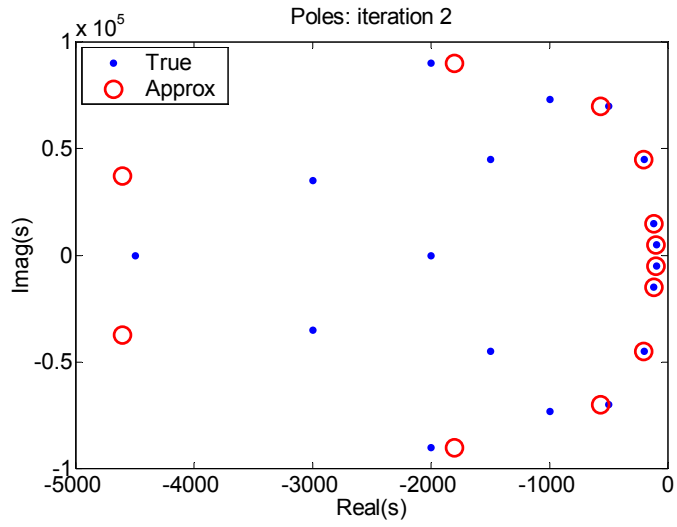
### Example 3



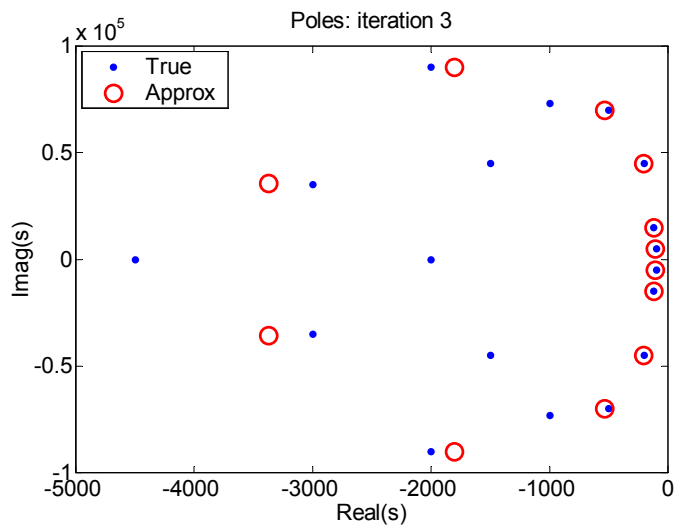




### Example 3

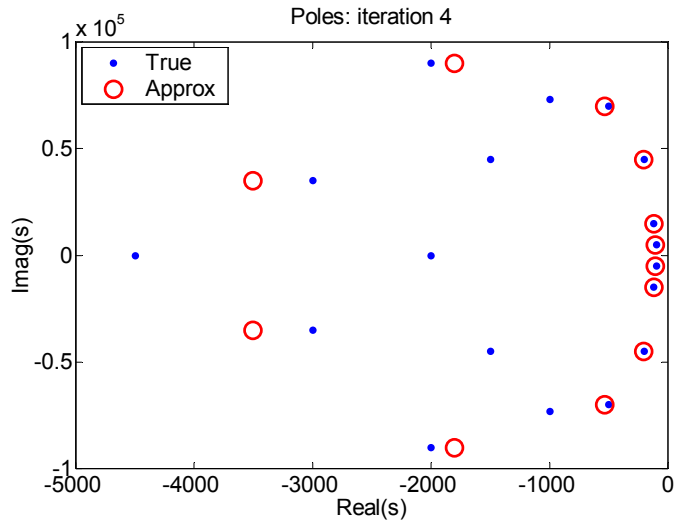


### Example 3

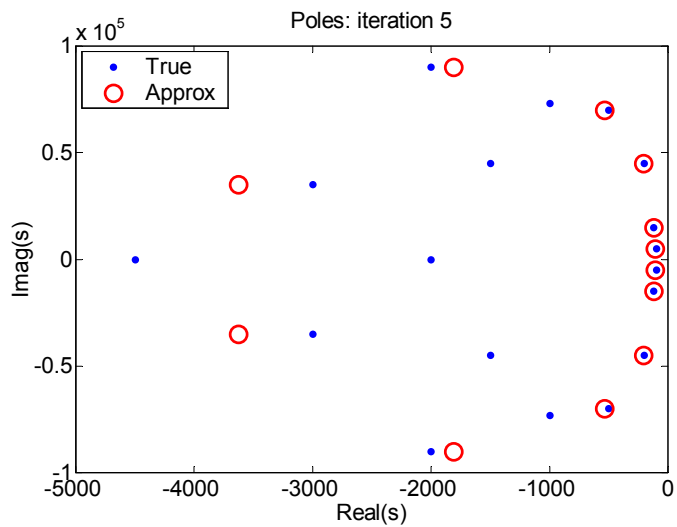




### Example 3

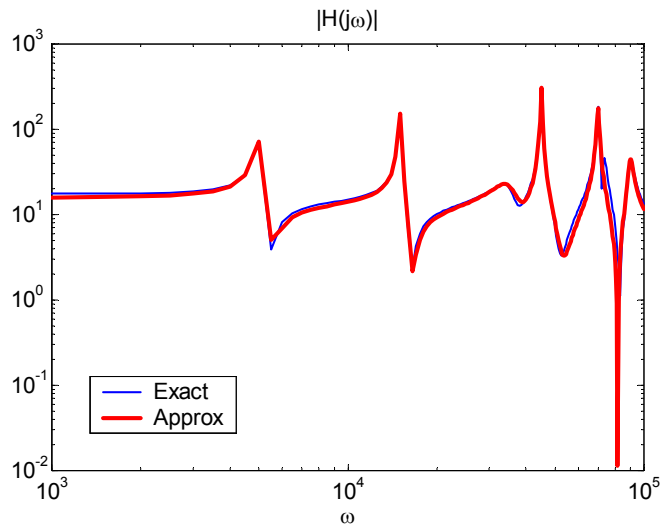


### Example 3

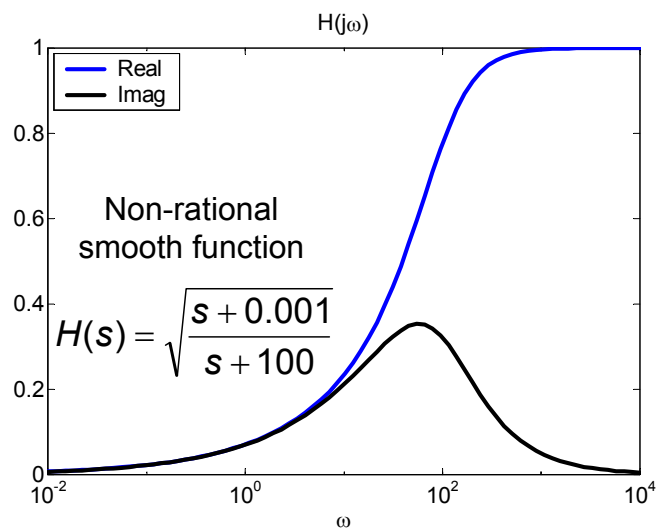


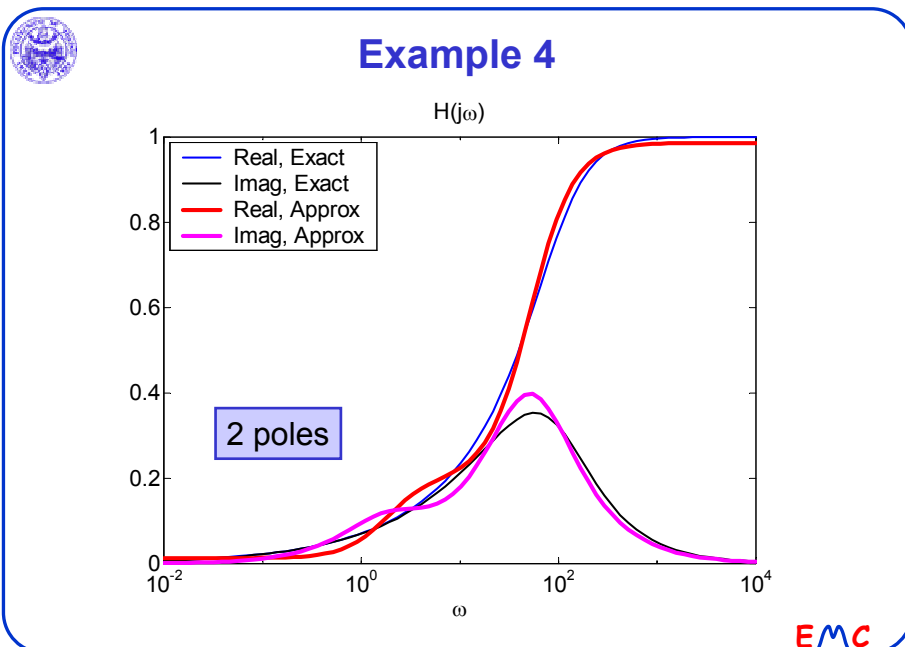
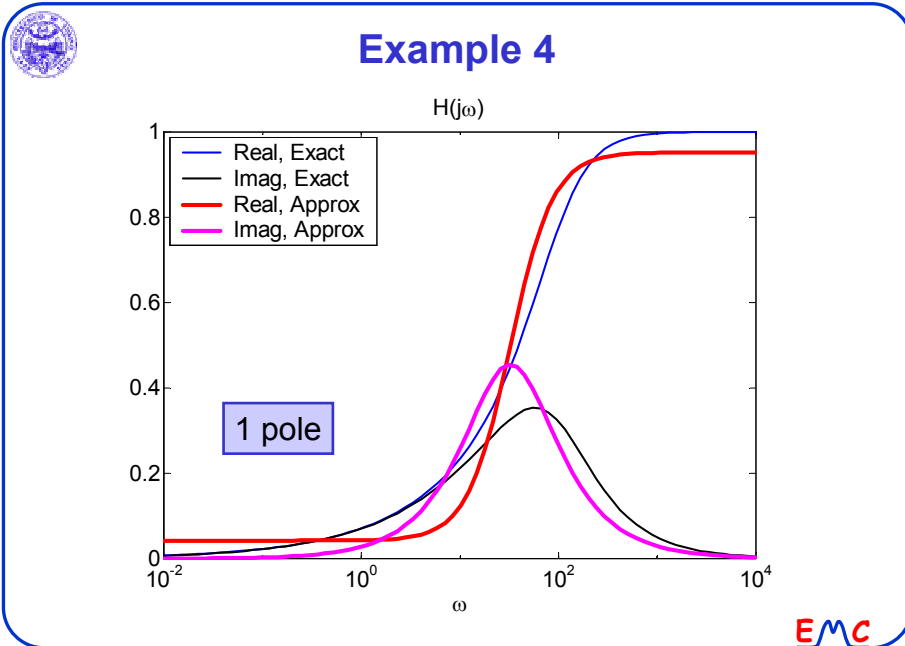


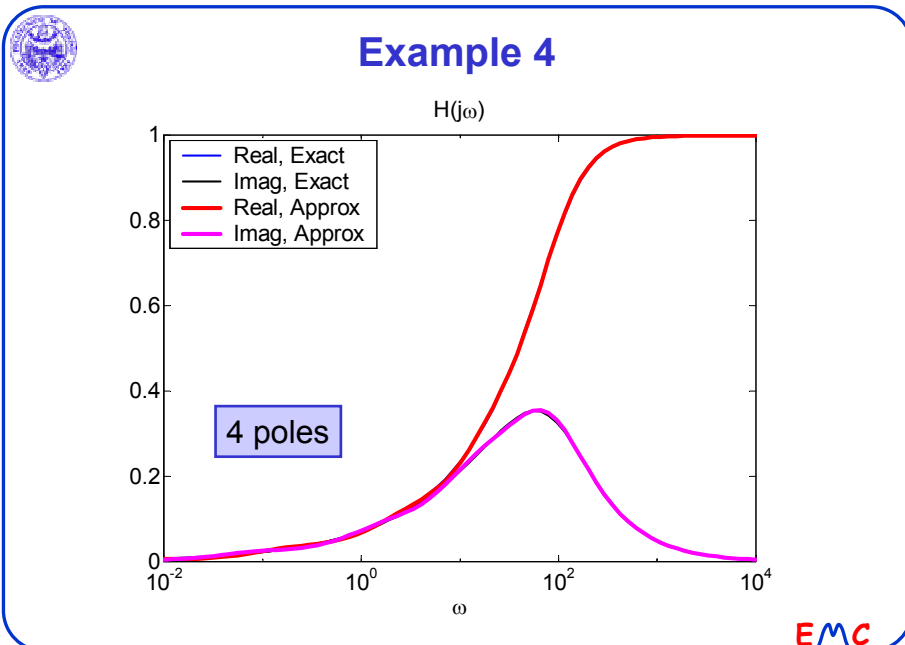
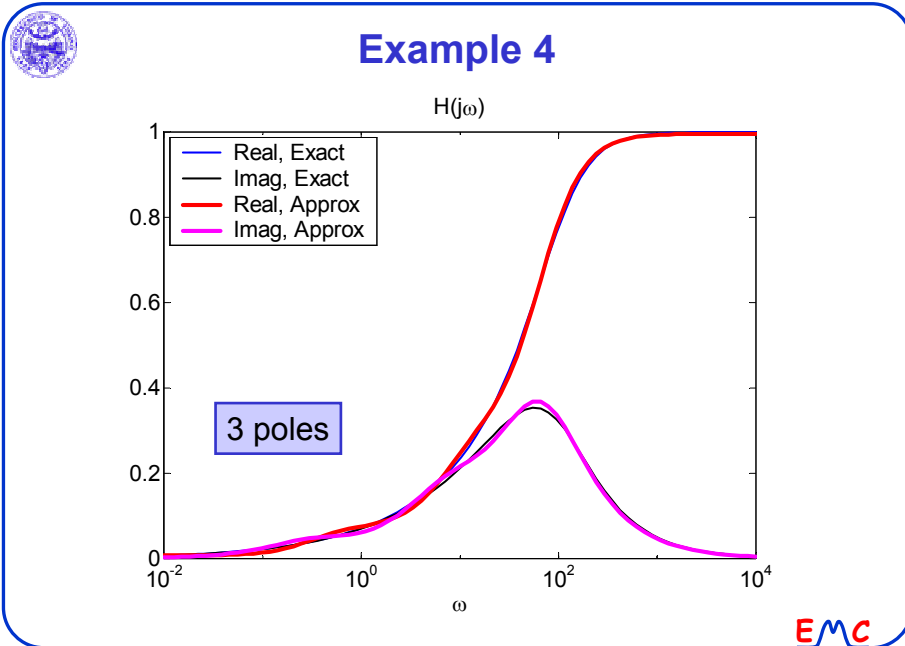
### Example 3



### Example 4

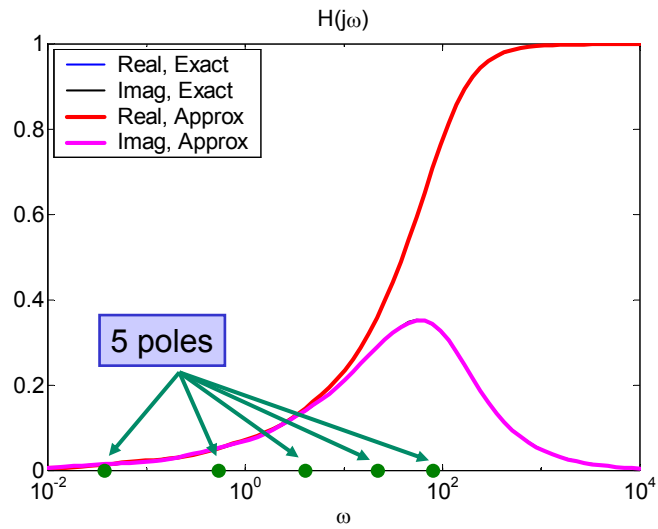




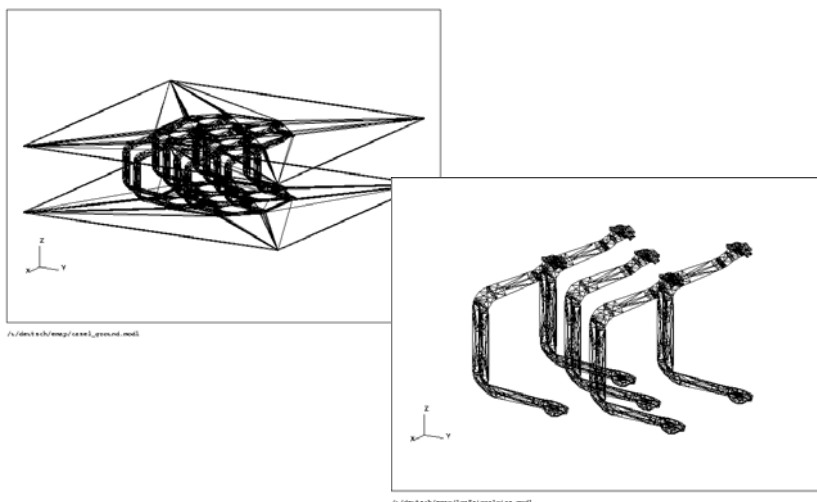




## Example 4



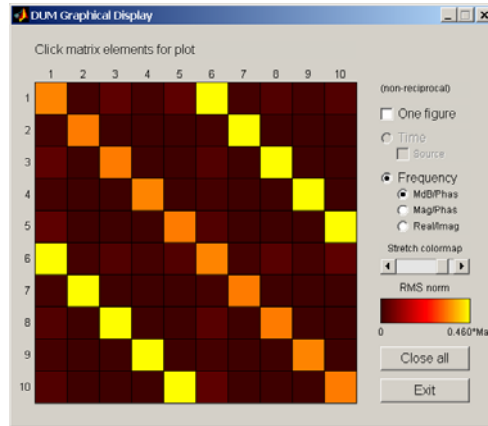
## Example 5: MCM-board connector



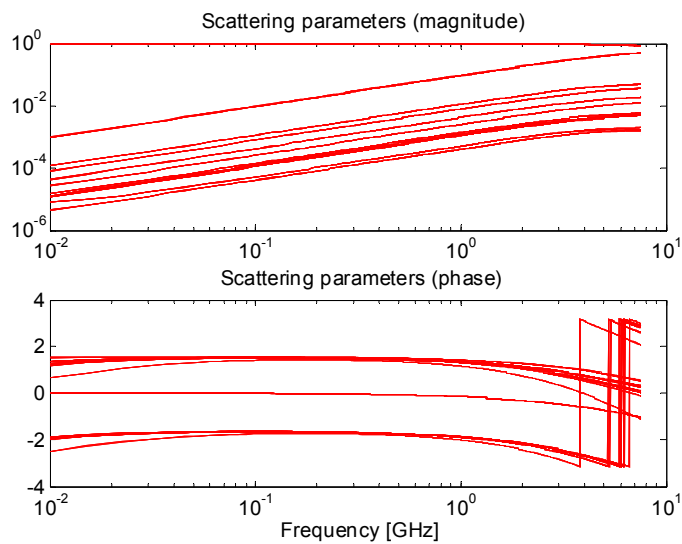


## Example 5: MCM-board connector

Data: 10-port structure, frequency-domain S-matrix



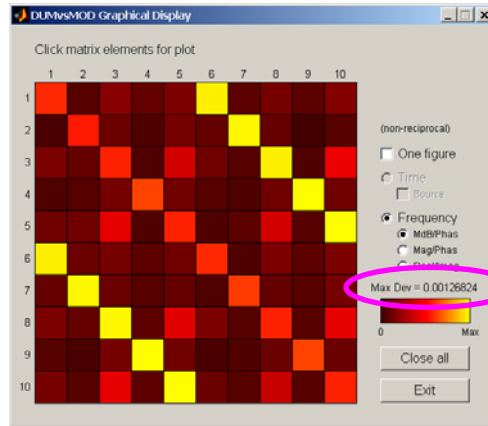
## Example 5: MCM-board connector





## Example 5: MCM-board connector

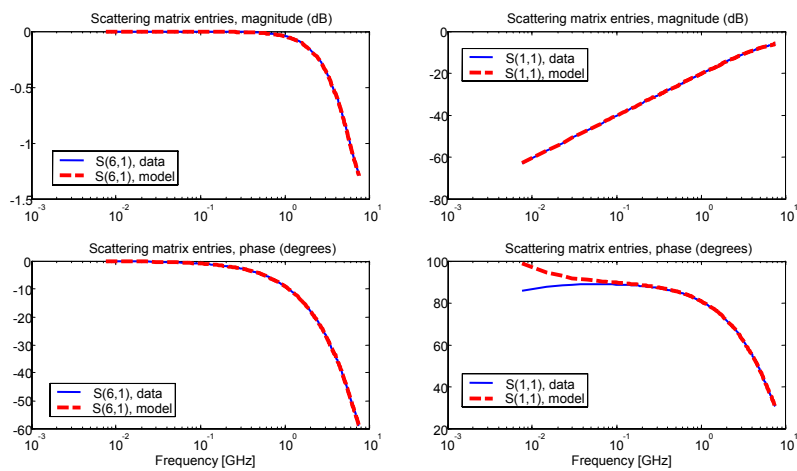
Macromodel: 4-poles



**Error:  
0.1%**



## Example 5: MCM-board connector

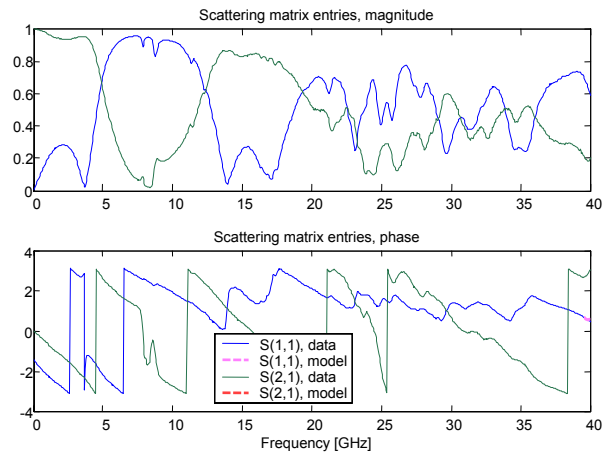






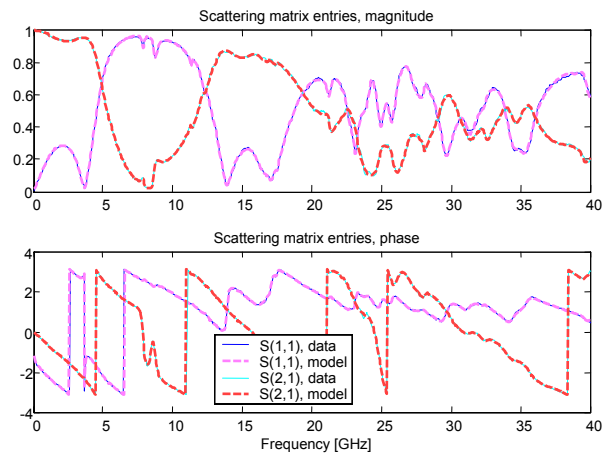
## Example 6: stripline+lauches

Data: measured S-parameters



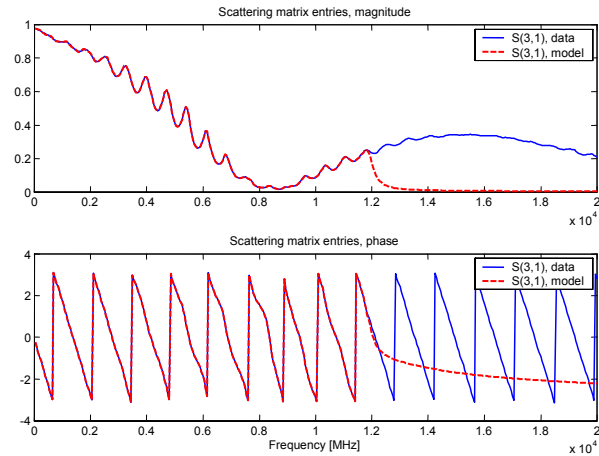
## Example 6: stripline+lauches

Macromodel: 60 poles

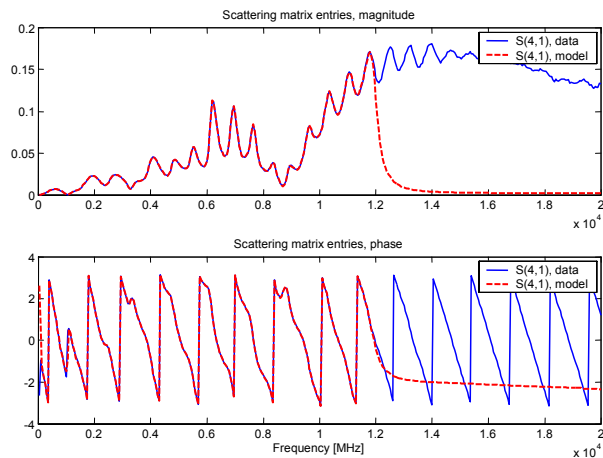




## Example 7: PCB path, measured VF with frequency-selective weighting

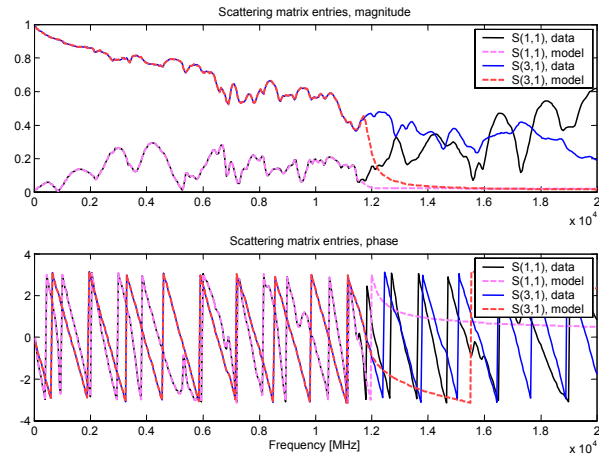


## Example 7: PCB path, measured VF with frequency-selective weighting





## Example 8: Connector, measured VF with frequency-selective weighting



## Vector Fitting: summary

- Tool for frequency-domain rational approximation
  - rational transfer functions (system identification)
  - rational transfer functions (reduced-order modeling)
  - non-rational transfer functions
- Data from full-wave simulations
- Direct frequency-domain measurements



## Vector Fitting: summary

[www.energy.sintef.no/produkt/VECTFIT/home.asp](http://www.energy.sintef.no/produkt/VECTFIT/home.asp)

- Very accurate and robust
- Only linear least squares + eigenvalues required
- Stability is not guaranteed
  - can be fixed by flipping real part during relocation
- Passivity is not guaranteed
  - can be fixed a posteriori (see later)



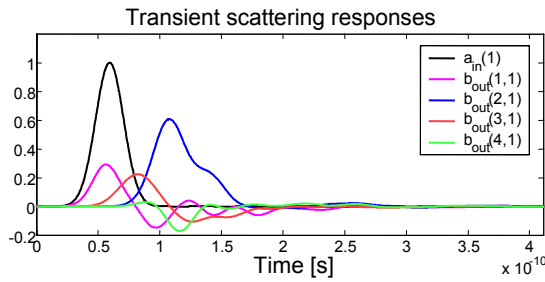
## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Time-domain macromodeling

Model identification from time-domain responses



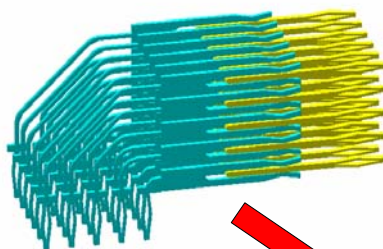
**IDENTIFICATION**



**EMC**  
GROUP

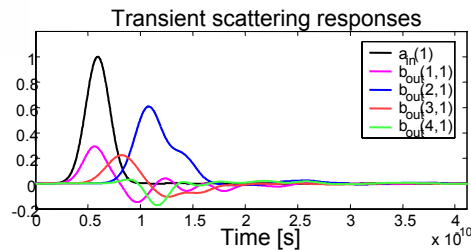


## Possible scenarios



Time-Domain full-wave  
simulation (FIT, FDTD)

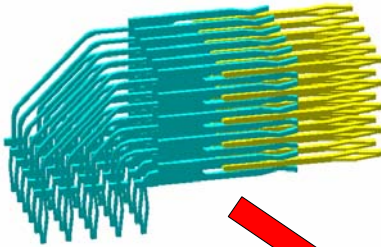
Port responses to  
transient excitations  
(usually gaussian)



**EMC**  
GROUP



## Possible scenarios

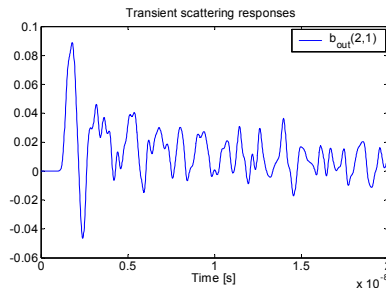


Port responses to transient excitations (usually gaussian)

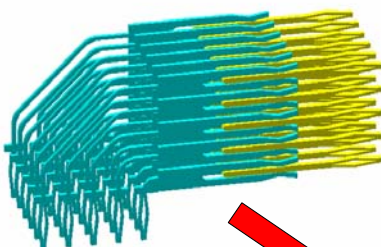


Time-Domain full-wave simulation (FIT, FDTD)

Truncated waveforms from short FDTD runs



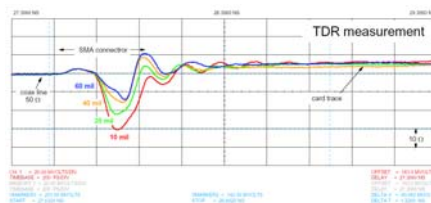
## Possible scenarios



Port responses to transient excitations (usually gaussian)



Time-domain measurements (work in progress)





## Time-Domain Macromodeling

Input pulse

$x(t)$  t – domain

Output responses

$y(t)$  t – domain

Transfer function

$$Y(s) = H(s)X(s)$$

Rational approximation

$$H(s) \approx H_\infty + \sum_n \frac{R_n}{s - p_n}$$

Unknowns:

- Poles  $p_n$
- Residues  $R_n$
- Constant  $H_\infty$



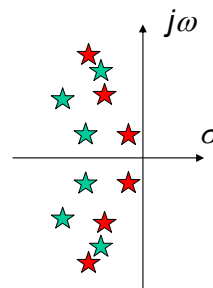
## Time-Domain Vector Fitting

Step 1. Find the dominant poles via “relocation”

Guess poles  
 $\{q_n\}$



New poles  
 $\{p_n\}$



How to do it using time-domain data?

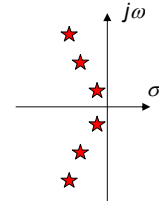
How to insure convergence to the right poles?



## Time-Domain Vector Fitting

1a. Start with initial poles:  $\{q_n\}$

1b. Define weight function: unknown  $\{k_n\}$



$$w(s) = 1 + \sum_n \frac{k_n}{s - q_n}$$

Starting poles

1c. Assume the following condition

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Poles of  $\mathbf{H}(s)$  = Zeros of  $w(s)$



## Time-Domain Vector Fitting

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n} \quad \text{Apply the input pulse } \mathbf{X}(s)$$

$$w(s)\mathbf{Y}(s) = \left( a + \sum_n \frac{b_n}{s - q_n} \right) \mathbf{X}(s) \quad \text{Compute inverse Laplace transform}$$

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$

$$\mathbf{x}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{x}(\tau) d\tau$$

$$\mathbf{y}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{y}(\tau) d\tau$$

Low-pass filtered input and output signals





## Time-Domain Vector Fitting

1d. Solve a linear least squares system for  $k_n$ ,  $a$ ,  $b_n$

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$

1e. Compute the zeros  $\{p_n\}$  of the weight function

$$w(s) = 1 + \sum_n \frac{k_n}{s - q_n} = \frac{\prod_n (s - p_n)}{\prod_n (s - q_n)}$$

These are the dominant poles!



S. Grivet-Talocia, "Package Macromodeling via Time-Domain Vector Fitting", *IEEE Microwave Wireless Comp. Lett.*, Nov. 2003



## Time-Domain Vector Fitting

Step 2. Compute the residues

2a. Low-pass filter input signals with new poles

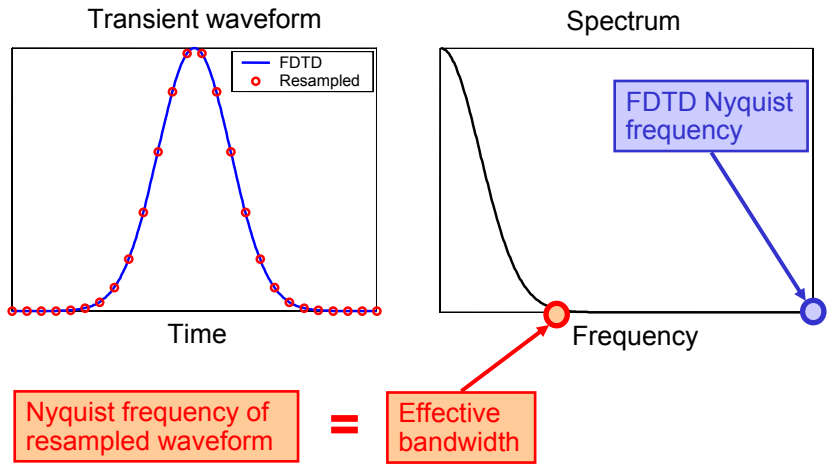
$$\tilde{\mathbf{x}}_n(t) = \int_0^t e^{p_n(t-\tau)} \mathbf{x}(\tau) d\tau$$

2b. Solve a linear least squares system for  $\mathbf{R}_n$  and  $\mathbf{H}_\infty$

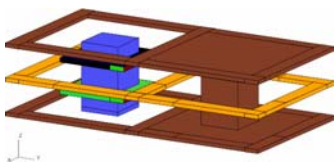
$$\mathbf{y}(t) = \mathbf{H}_\infty \mathbf{x}(t) + \sum_n \mathbf{R}_n \tilde{\mathbf{x}}_n(t)$$



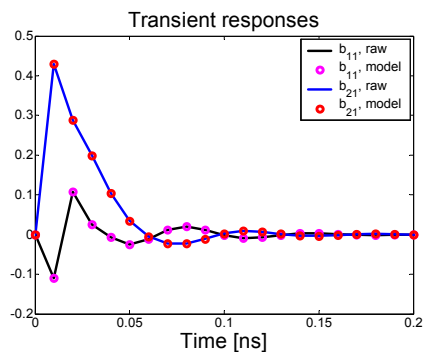
## Subsampling



## Example 1: single via

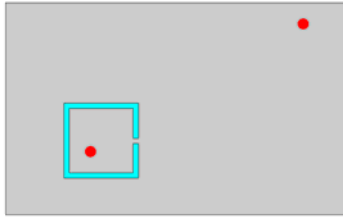


Raw data:  
Triangle Impulse Responses  
obtained by a transient PEEC  
solver (by Dr. Ruehli, IBM)

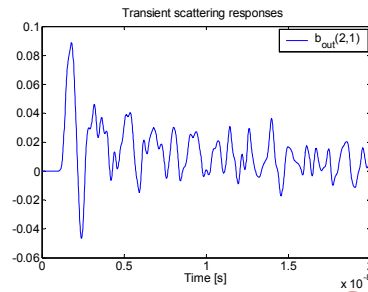
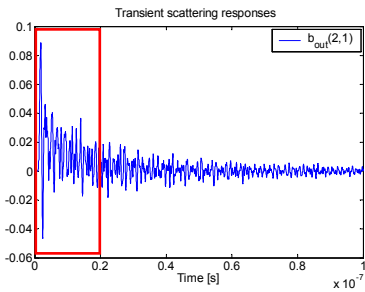




## Example 2: segmented power bus

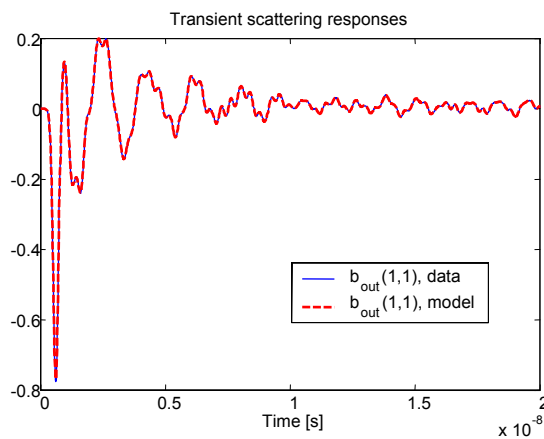


- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- $50 \Omega$  port terminations



## Example 2: segmented power bus

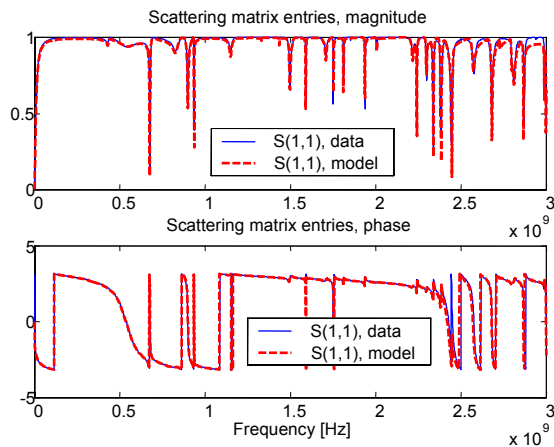
80-poles model (Time-Domain Vector Fitting)





## Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



## Example 2: segmented power bus

**Full-wave simulation time** (CST) to compute...

... **frequency scattering data**: **60 hours**

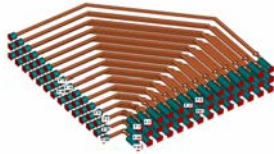
(wait until transients are finished for reliable FFT)

... **macromodel**: **6 hours**

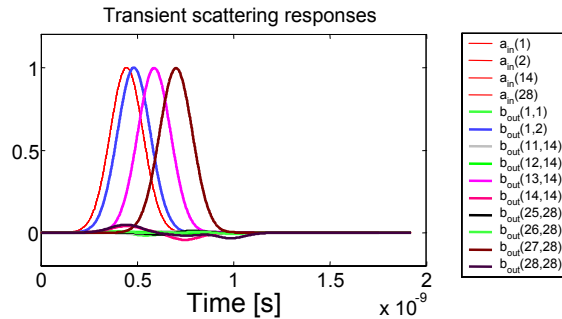
(can use truncated waveforms for TD-VF)



### Example 3: 42-pin connector



3x14 pins, 84 ports  
Characterized via FIT  
(CST Microwave Studio 4)  
(Courtesy: Erni - AdMOS)



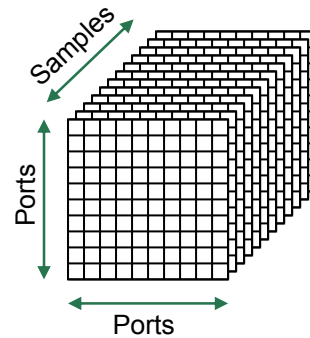
### Handling many ports

Frequency-Domain Vector Fitting

$$\left(1 + \sum_n \frac{k_n}{s - q_n}\right) \mathbf{H}(s) = \mathbf{a} + \sum_n \frac{b_n}{s - q_n}$$

Time-Domain Vector Fitting

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = \mathbf{a} \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$



Processing **all** responses may lead to a **large** system!

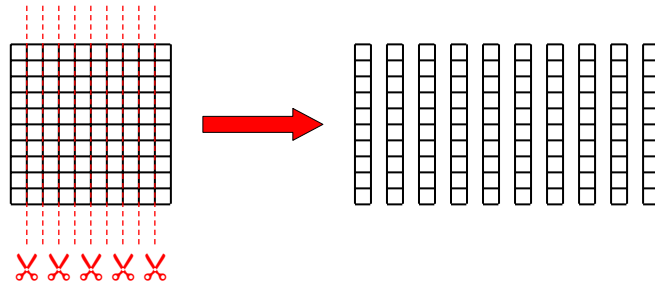


## Handling many ports

### 1. Split port responses into subsets

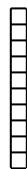
Transfer matrix  $\mathbf{H}(s)$

Subsets  $\{\mathbf{h}_k(s)\}$



## Handling many ports

### 2. Macromodel each subset via FD-VF or TD-VF



$$\mathbf{h}_k(s) \approx \mathbf{h}_{k,\infty} + \sum_n \frac{\mathbf{r}_{k,n}}{s - p_{k,n}}$$



Partial state-space representation

$$\begin{cases} \dot{\mathbf{w}}_k = \mathbf{A}_k \mathbf{w}_k + \mathbf{B}_k \mathbf{x}_k \\ \mathbf{y}_k = \mathbf{C}_k \mathbf{w}_k + \mathbf{D}_k \mathbf{x}_k \end{cases}$$

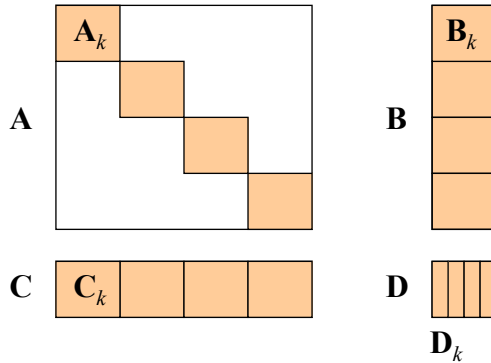


## Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$

All matrices  
can be  
constructed  
as sparse!

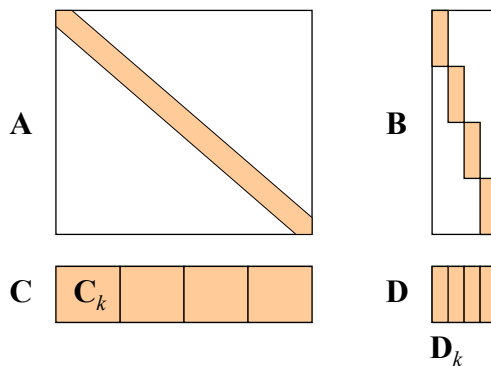


## Handling many ports

4. Assemble all partial models into a global model

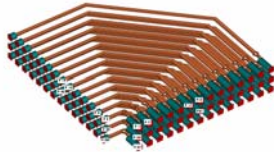
$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$

All matrices  
can be  
constructed  
as sparse!

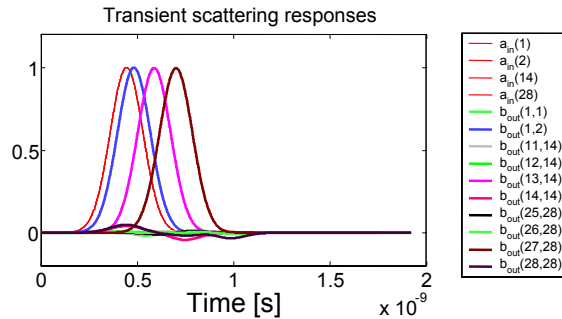




### Example 3: 42-pin connector

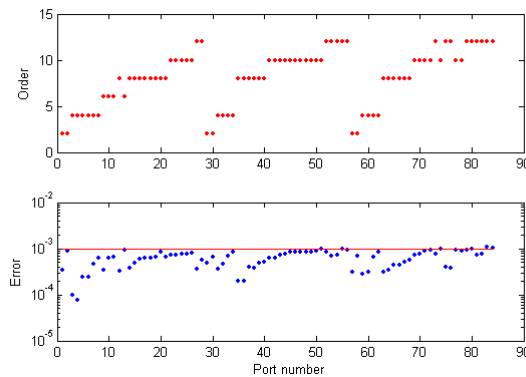


3x14 pins, 84 ports  
Characterized via FIT  
(CST Microwave Studio 4)  
(Courtesy: Erni - AdMOS)



### Example 3: model order selection

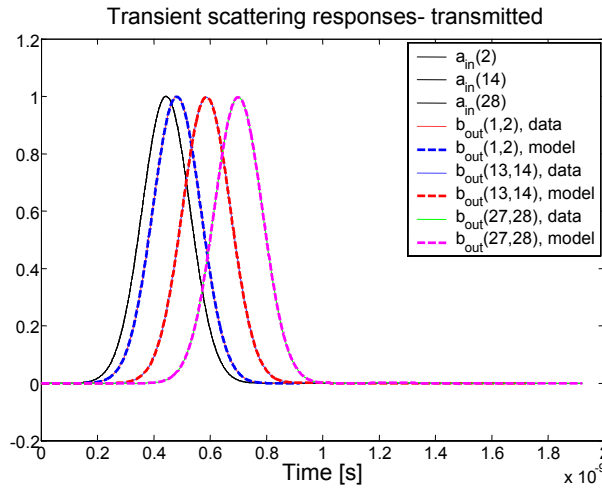
Automatic (iterative) order selection on each of the 84 subsets of port responses (reduced model complexity)



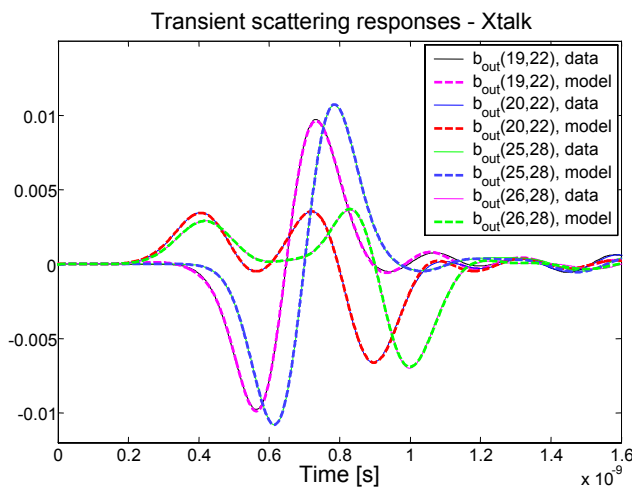




### Example 3: macromodel accuracy

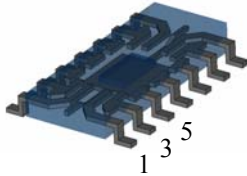


### Example 3: macromodel accuracy

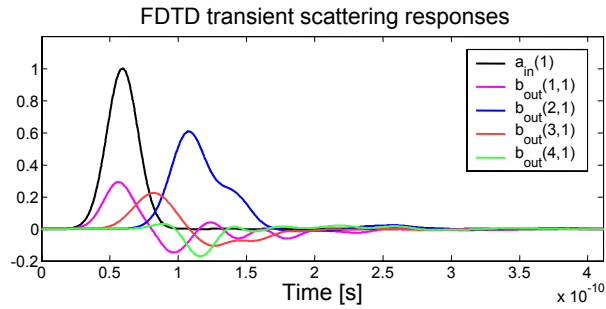




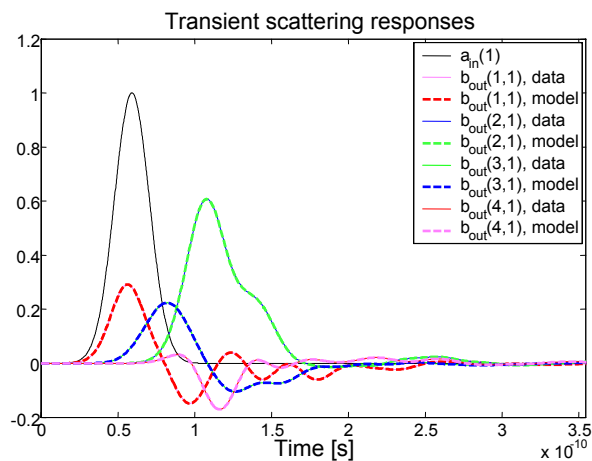
## Example 4: 14-pin package



14-pin SOIC package  
Simplified CAD for FDTD  
Bandwidth: 40 GHz  
50  $\Omega$  port terminations



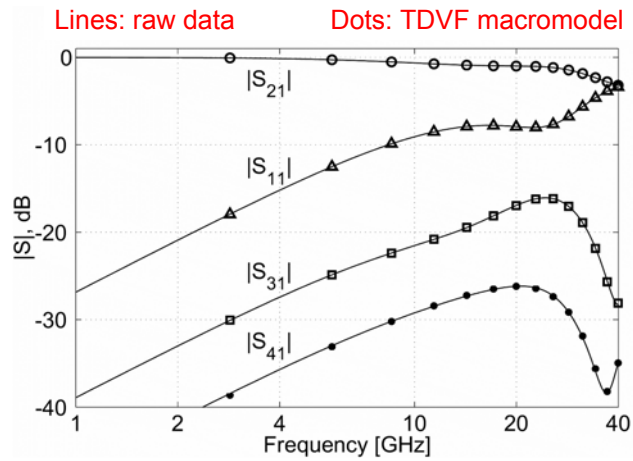
## Example 4: macromodel responses



No visible difference between data and model



## Example 4: macromodel responses



No visible difference between data and model



## Example 4: macromodel accuracy

Maximum deviation between model and data for all 28x28 responses



Largest:  
0.00074

TD-VF produces highly accurate macromodels



## Macromodel properties

### 😊 Accuracy

Good initial data  $\Rightarrow$  small approximation errors

### 😊 Stability

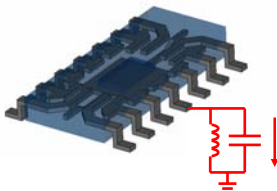
All poles with negative real part

### ☹️ Passivity

The macromodel may not be passive



## Example 4: change terminations

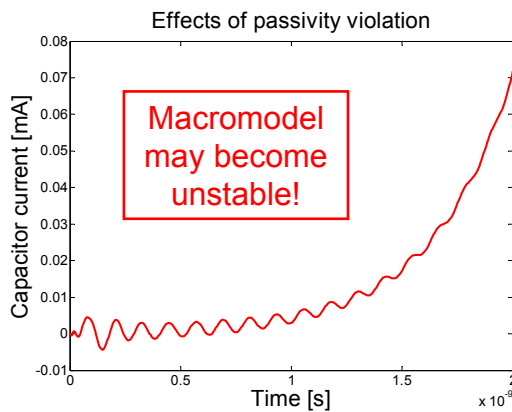


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \text{ }\Omega$$

$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$



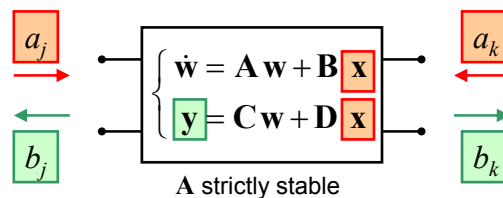


## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Passivity conditions Scattering representation



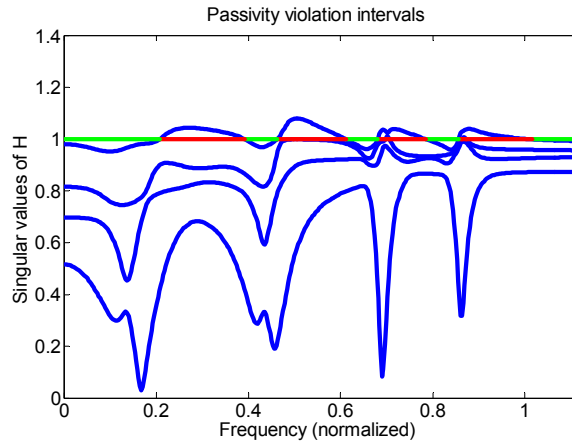
Scattering matrix: must be bounded real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

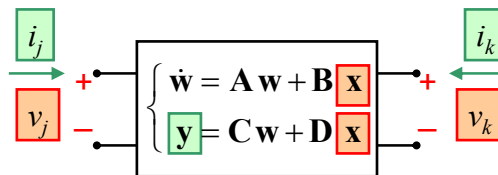
$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$



## Passivity conditions Scattering representation



## Passivity conditions Admittance representation



A strictly stable

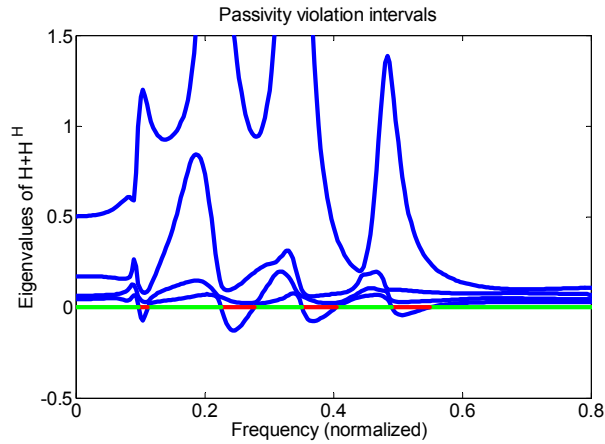
Admittance matrix: must be positive real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\left\{ \text{eigenvalues of } \left( \mathbf{H}(j\omega) + \mathbf{H}^H(j\omega) \right) \right\} \geq 0, \quad \forall \omega$$



## Passivity conditions Admittance representation



## Checking passivity Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

Several techniques can be used

**Frequency sweep test:** most straightforward

- Choose a set of frequency samples
- Compute  $\mathbf{H}$  and its singular values, and check
- **Time-consuming** for large models
- **May give wrong answers** due to poor sampling



## Checking passivity Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

Equivalent purely algebraic conditions:

- Linear Matrix Inequalities (**LMI**)
- Algebraic Riccati Equations (**ARE**)
- Eigenvalues of **Hamiltonian matrices**

S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, "Linear Matrix Inequalities in System and Control Theory, SIAM, Philadelphia, 1994



## Checking passivity Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

Linear Matrix Inequality (LMI)

$$\begin{pmatrix} \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{C}^T \mathbf{C} & \mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D} \\ \mathbf{B}^T \mathbf{P} + \mathbf{D}^T \mathbf{C} & \mathbf{D}^T \mathbf{D} - \mathbf{I} \end{pmatrix} \leq \mathbf{0} \quad \mathbf{P} = \mathbf{P}^T, \mathbf{P} > \mathbf{0}$$

Real matrix  $\mathbf{P}$  is the variable





## Checking passivity Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

### Algebraic Riccati Equation (ARE)

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{C}^T \mathbf{C} + (\mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D})(\mathbf{I} - \mathbf{D}^T \mathbf{D})^{-1} (\mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D})^T = \mathbf{0}$$

$$\mathbf{P} = \mathbf{P}^T$$

Real matrix  $\mathbf{P}$  is the variable



## Checking passivity Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

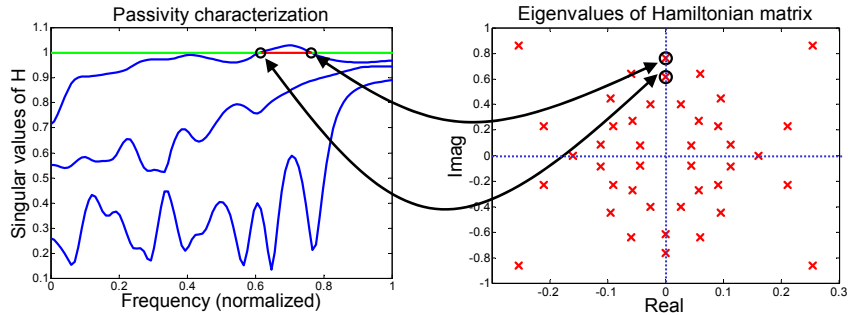
### Eigenvalues of Hamiltonian matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} - \mathbf{B}(\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{D}^T \mathbf{C} & -\mathbf{B}(\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{B}^T \\ \mathbf{C}^T (\mathbf{D} \mathbf{D}^T - \mathbf{I})^{-1} \mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T \mathbf{D} (\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{B}^T \end{pmatrix}$$

Real matrix  $\mathbf{M}$  must have no imaginary eigenvalues



## Checking passivity Scattering representation

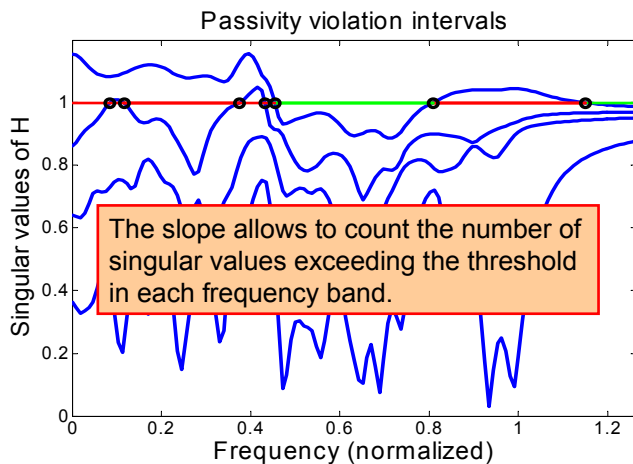


**Theorem** [Boyd, Balakrishnan, Kabamba, 1989]

$j\omega_0$  is an eigenvalue of  $\mathbf{M} \Leftrightarrow \sigma = 1$  is a singular value of  $\mathbf{H}(j\omega_0)$



## Checking passivity Scattering representation

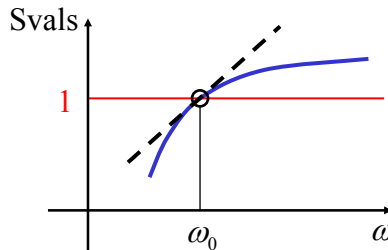




## Checking passivity Scattering representation

First-order perturbation of Hamiltonian eigenvalues

$$\text{Slope} = \text{Im} \left\{ \frac{\mathbf{w}^T \mathbf{v}}{\mathbf{w}^T \mathbf{M}' \mathbf{v}} \right\}$$

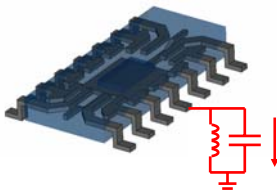


$\mathbf{w}, \mathbf{v}$ : Left and right eigenvectors of  $\mathbf{M}$  associated to  $\omega_0$

$\mathbf{M}'$ : Another Hamiltonian matrix (computed via  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ )



## Example 4: change terminations

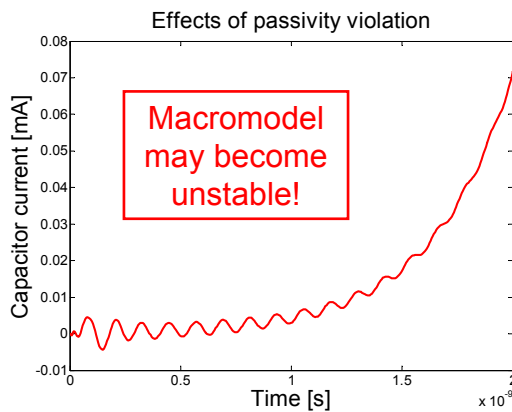


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \Omega$$

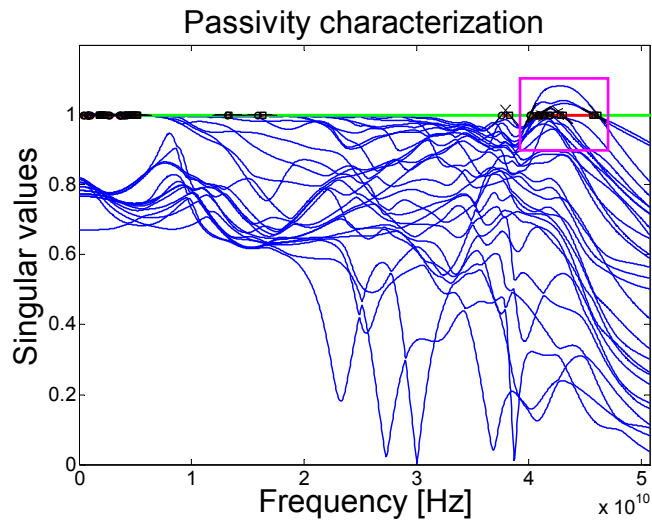
$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$

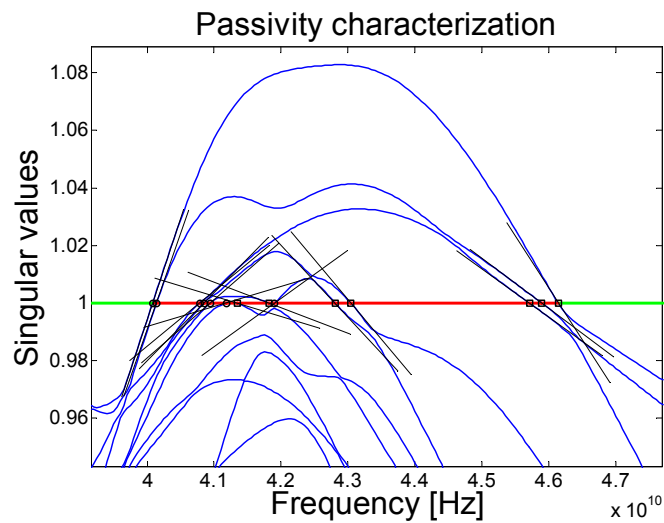




## Example 4: passivity characterization

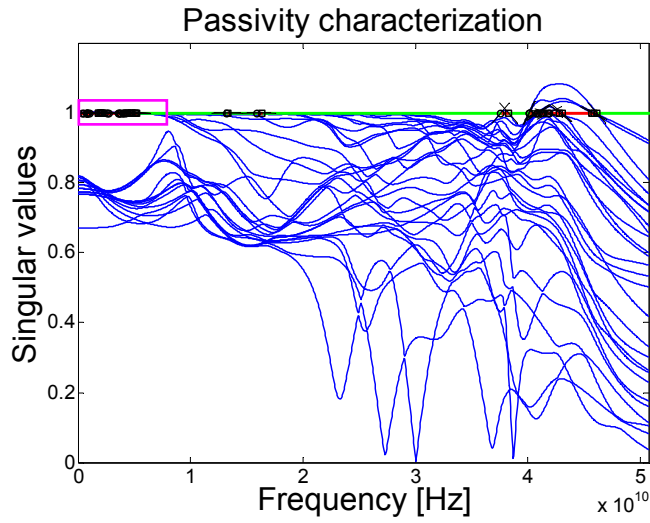


## Example 4: passivity characterization

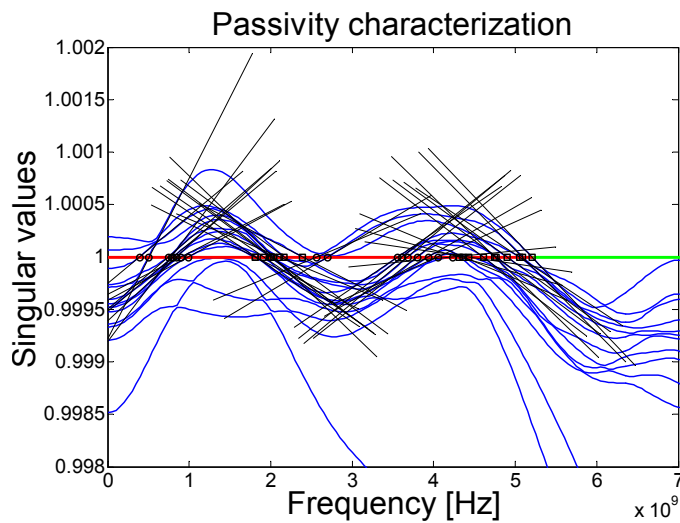




## Example 4: passivity characterization



## Example 4: passivity characterization





## Passivity enforcement

- Generate a **new passive macromodel**
- Apply **small correction** to **preserve accuracy**
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases} \longrightarrow \begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = (\mathbf{C} + \mathbf{dC}) \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$



## Passivity enforcement

Several different approaches are possible. Examples are

### Quadratic/convex optimization

[B.Gustavsen, A.Semlyen: IEEE Trans. Power Systems, vol.16, 2001]

[C.P.Coelho, J.Phillips, L.M.Silveira, IEEE Trans. CADICAS, vol.23, 2004]

### Trace parameterization/Semi-Definite Programming

[H.Chen, J.Fang: Proc. EPEP, 2003]

### Perturbation of residues

[D.Saraswat, R.Achar, M.Nakhla: Proc. EPEP, 2003]

### Perturbation of Hamiltonian eigenvalues

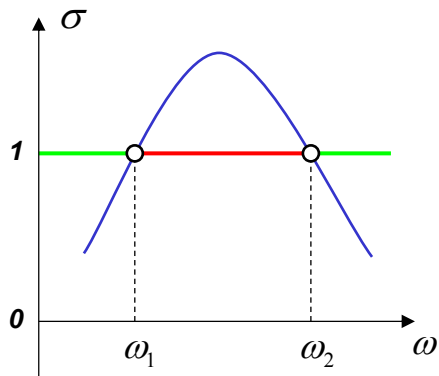
[S.Grivet-Talocia: Proc. SPI, 2003 and IEEE Trans. CAS (in press)]

**Many others... Hot research topic!**

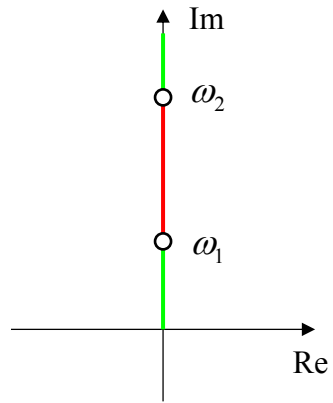


## Perturbation of Hamiltonian Eigs

Singular values of H

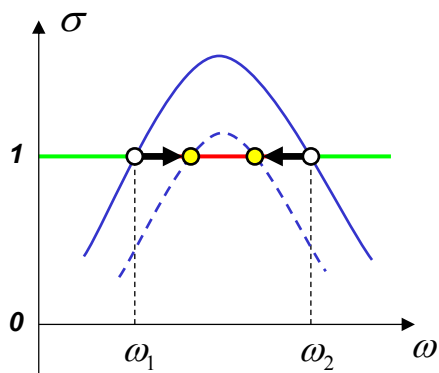


Eigenvalues of M

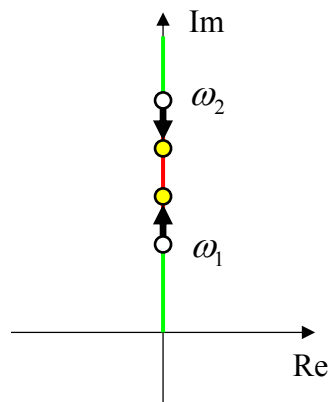


## Perturbation of Hamiltonian Eigs

Singular values of H

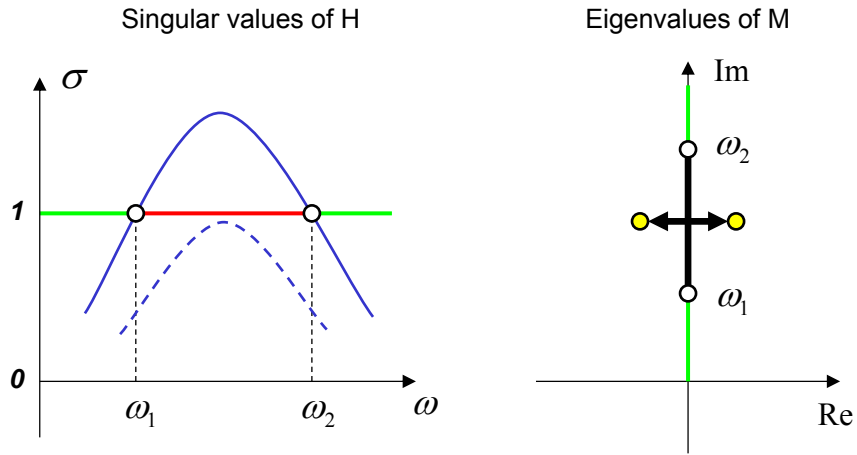


Eigenvalues of M





## Perturbation of Hamiltonian Eigs



## Perturbation of Hamiltonian Eigs

First-order perturbation of eigenvalues (again)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases}$$

Perturb state matrix  $\mathbf{C}$

$$\tilde{\mathbf{C}} = \mathbf{C} + \mathbf{dC}$$

$$\tilde{\mathbf{M}} \approx \mathbf{M} + \mathbf{dM} \quad (\text{first-order: } \mathbf{dM} \text{ is linear in } \mathbf{dC})$$

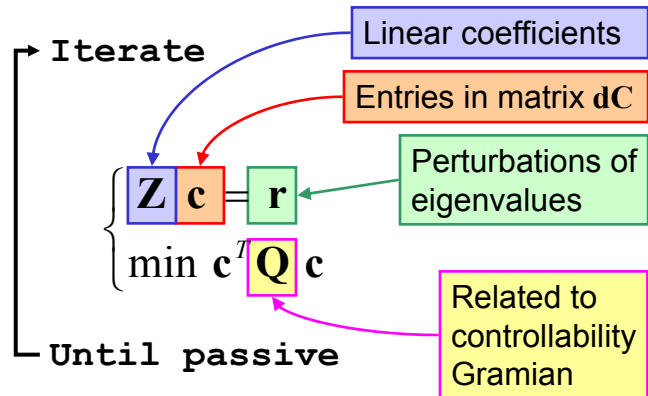
$$\mathbf{w}_m^T \mathbf{dM} \mathbf{v}_m \approx j(\tilde{\omega}_m - \omega_m) \mathbf{w}_m^T \mathbf{v}_m$$

Linear constraint on the correction matrix  $\mathbf{dC}$





## Perturbation of Hamiltonian Eigs



Minimizes the perturbation on the original responses



## Preserve accuracy of macromodel

Minimize this norm !

$$\sum_{i,j} \int_0^{\infty} (\tilde{h}_{i,j}(t) - h_{i,j}(t))^2 dt = \|\mathbf{dC} \mathbf{W} \mathbf{dC}^T\|_F^2$$

Induced perturbation in the impulse responses

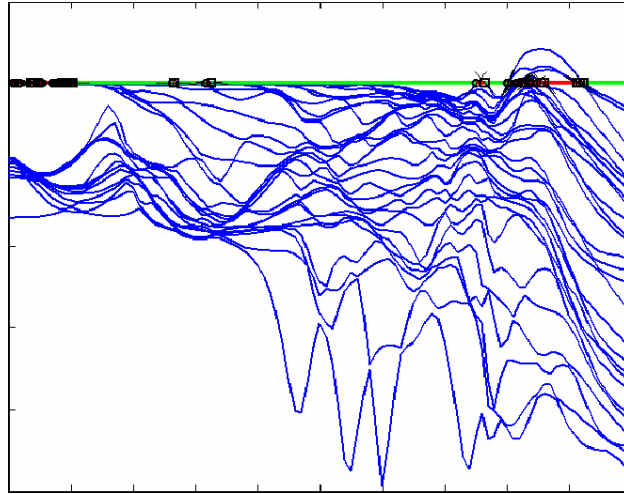
Weighted norm of state matrix perturbation

$\mathbf{W}$ : controllability Gramian

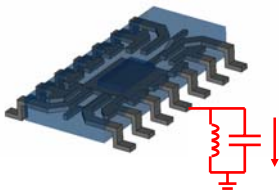
$$\mathbf{A}\mathbf{W} + \mathbf{W}\mathbf{A}^T = -\mathbf{B}\mathbf{B}^T$$



## Example 4: passivity compensation



## Example 4 : passivity compensation

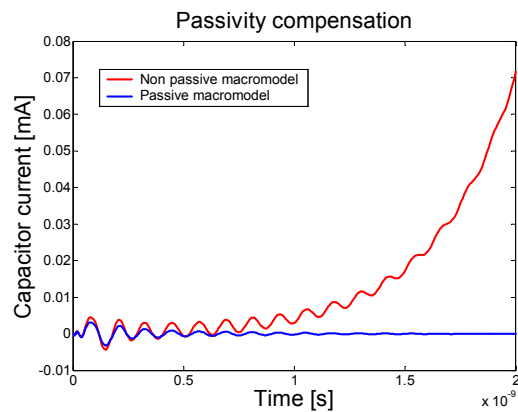


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \text{ }\Omega$$

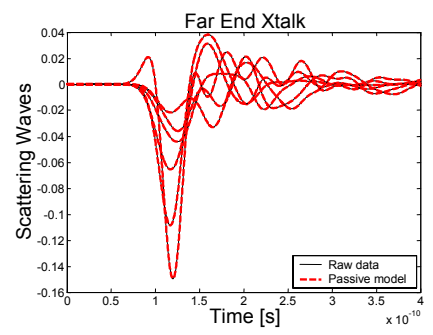
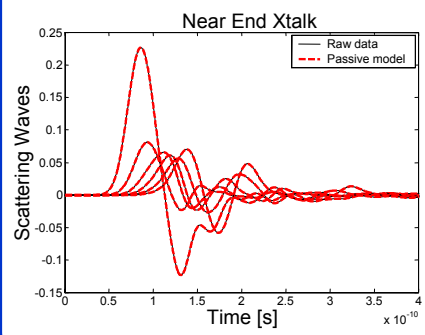
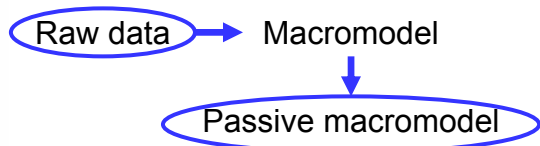
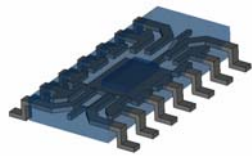
$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$

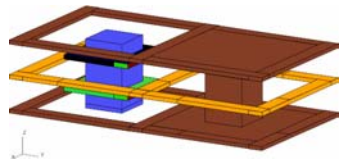




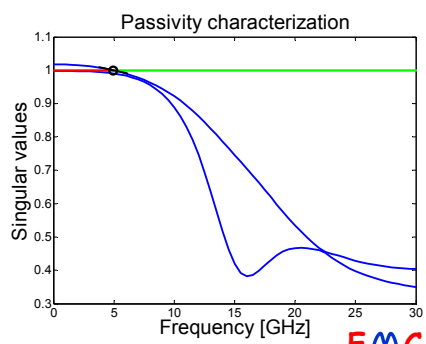
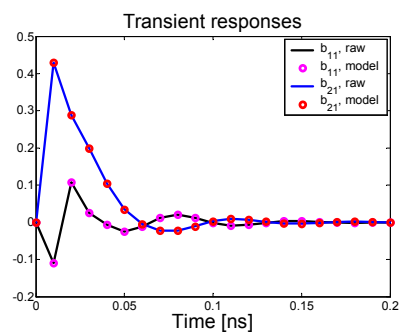
### Example 4: passive macromodel



### Example 1: single via, nonpassive

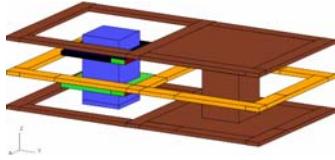


Raw data:  
Triangle Impulse Responses  
obtained by a transient PEEC  
solver (by Dr. Ruehli, IBM)

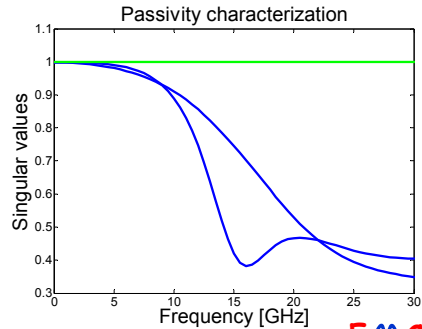
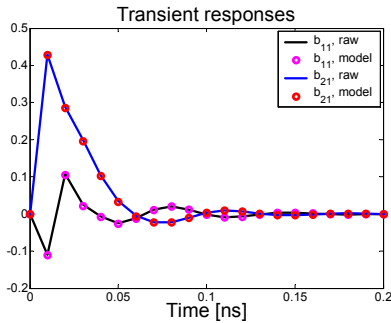




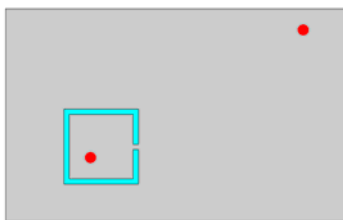
## Example 1: single via, passive



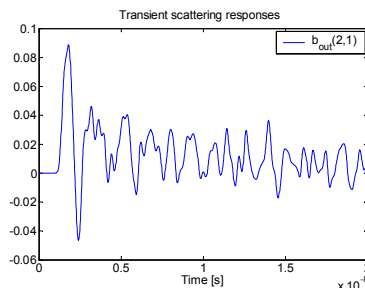
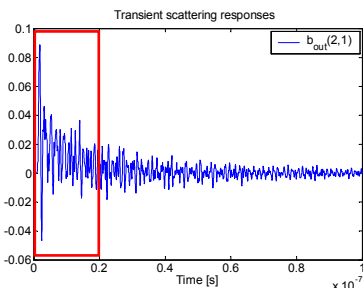
Raw data:  
Triangle Impulse Responses  
obtained by a transient PEEC  
solver (by Dr. Ruehli, IBM)



## Example 2: segmented power bus



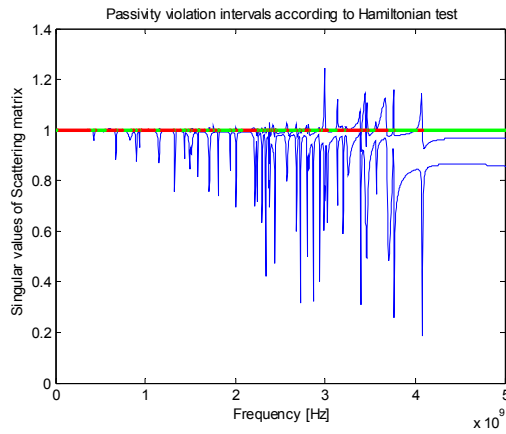
- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- 50  $\Omega$  port terminations





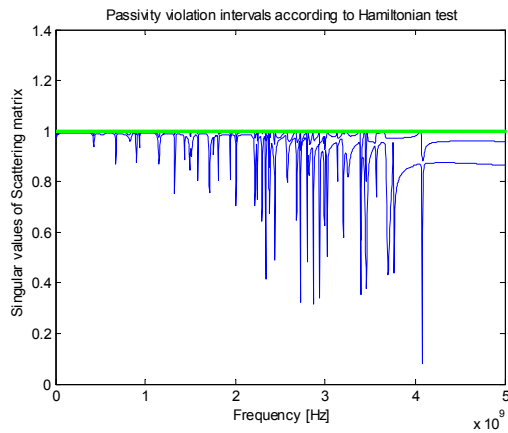
## Example 2: segmented power bus

### Passivity characterization



## Example 2: segmented power bus

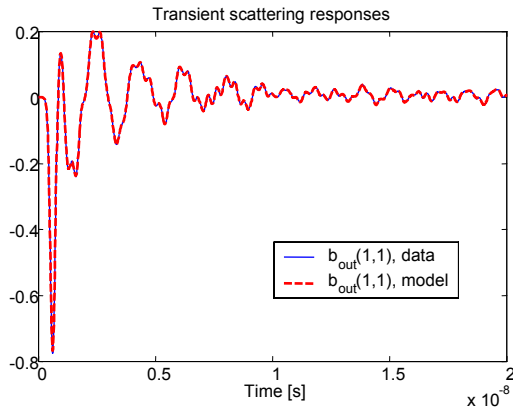
### Passivity compensation





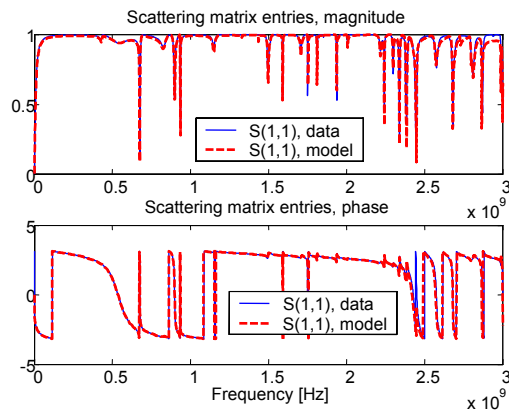
## Example 2: segmented power bus

80-poles **passive** model



## Example 2: segmented power bus

Comparison vs. frequency-domain scattering data





## More examples...

41 poles, 2 ports

Compensation

Accuracy

110 poles, 5 ports

Compensation

Accuracy

308 poles, 11 ports

Compensation

Accuracy



## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Macromodel implementation

### Main approaches

1. Synthesize an **equivalent circuit** in SPICE format  
No access to SPICE kernel  
Must use **standard circuit elements**
2. Direct SPICE implementation via **recursive convolution**  
**Laplace element**, most efficient
3. Other languages for **mixed-signal** analyses  
Verilog-AMS, VHDL-AMS, ...  
**Equation-based**

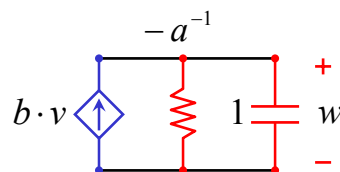
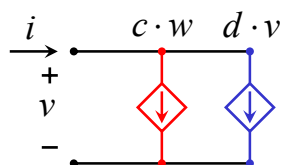


## SPICE synthesis

Admittance representation

One-port, one-pole

$$\begin{cases} \dot{w} = a w + b v \\ i = c w + d v \end{cases}$$







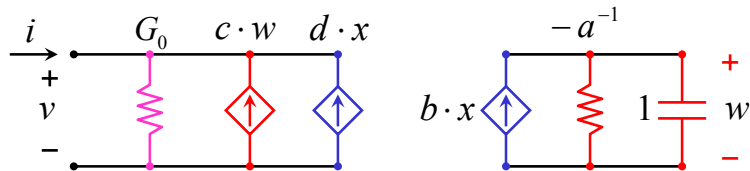
## SPICE synthesis

Scattering representation

One-port, one-pole

$$x = G_0 v + i, \quad y = G_0 v - i$$

$$\begin{cases} \dot{w} = a w + b x \\ y = c w + d x \end{cases}$$



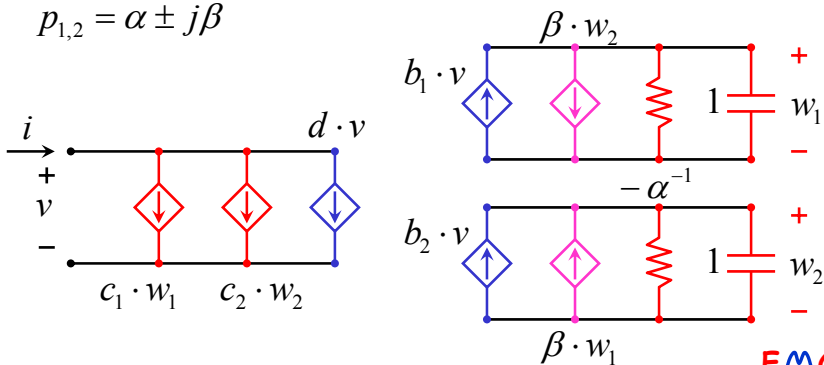
## SPICE synthesis

Admittance representation

One-port, two-poles (complex)

$$p_{1,2} = \alpha \pm j\beta$$

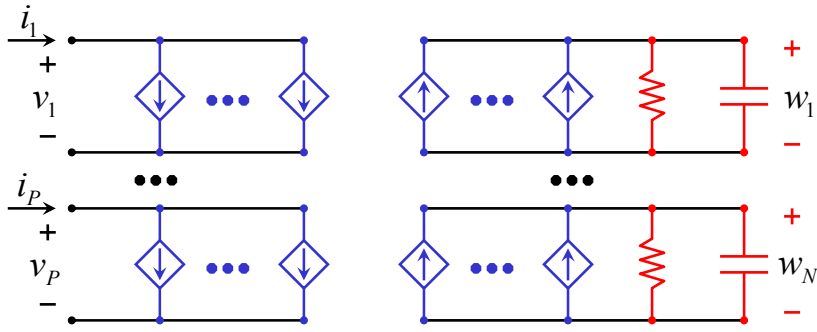
$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{b} v \\ i = \mathbf{c} \mathbf{w} + d v \end{cases}$$





## SPICE synthesis

$$\begin{array}{l} \text{Admittance representation} \\ \text{General state-space synthesis} \end{array} \left\{ \begin{array}{l} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{v} \\ \mathbf{i} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{v} \end{array} \right.$$



## Recursive convolutions

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \mathbf{D} + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

$$\mathbf{h}(t) = \mathbf{D}\delta(t) + \sum_n \mathbf{R}_n e^{p_n t} u(t)$$

$$\mathbf{y}(t) = \mathbf{D}\mathbf{x}(t) + \sum_n \mathbf{R}_n \int_0^t e^{p_n(t-\tau)} \mathbf{x}(\tau) d\tau$$



## Recursive convolutions

$$\tilde{\mathbf{y}}(t_k) = \int_0^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau$$

Discrete time

$$t_k = t_{k-1} + \Delta t_k$$

$$= \int_0^{t_{k-1}} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau$$

$$= e^{p\Delta t_k} \int_0^{t_{k-1}} e^{p(t_{k-1}-\tau)} \mathbf{x}(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau$$

$$\approx e^{p\Delta t_k} \tilde{\mathbf{y}}(t_{k-1}) + \frac{1 - e^{p\Delta t_k}}{p} \mathbf{x}(t_k)$$

Approximation!



## The macromodeling dream...

### Arbitrary characterization of the structure

- Equation-based or **Black-Box**
- Time or **frequency**, **simulation** or **measurement**

### Generation of a broadband macromodel

- Any order, **any number of ports**
- Any prescribed accuracy
- **Stable** and **passive** by construction
- **Efficient** (reduced-order and low-complexity)
- Fully automatic