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We propose an implementation scheme for holonomic, i.e., geometrical, quantum information processing based on semiconductor nanostructures. Our quantum hardware consists of coupled semiconductor macroatoms addressed/controlled by ultrafast multicolor laser-pulse sequences. More specifically, logical qubits are encoded in excitonic states with different spin polarizations and manipulated by adiabatic time control of the laser amplitudes. The two-qubit gate is realized in a geometric fashion by exploiting dipole-dipole coupling between excitons in neighboring quantum dots.

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In the past few years, the promise to outperform classical protocols for information manipulation has attracted a huge interest in quantum information processing (QIP).[1] Unfortunately, processors working according to the rules of quantum mechanics are, even in principle, extremely delicate objects: On one hand, the unavoidable coupling with uncontrollable degrees of freedom (the environment) spoils the unitary nature of the dynamical evolution, i.e., decoherence. On the other hand, extreme capabilities in quantum-state control are required; indeed, typically even very small manipulation imperfections will eventually drive the processing system into a ''wrong'' output state. It is therefore clear that any general strategy that appears to be able to cope with this sort of inherent fragility of QIP is worthwhile of serious consideration.

So far, quantum error-correction,[2] error-avoiding,[3] and error-suppression techniques,[4,5] have been developed at the theoretical level. They are mainly devoted to stabilize quantum information against computational errors induced by coupling with the environment, and are based on either the idea of hiding information to the detrimental effects of noise or to dynamically get rid of the noise itself. All of these strategies require extra physical resources in terms of either qubits or additional manipulations.

A further, conceptually fascinating, strategy for the stabilization of quantum information is provided by the topological approach.[6,7] In such QIP schemes, gate operations depend just on topological—i.e., global—features of the control process, and are therefore largely insensitive to local inaccuracies and fluctuations. This approach can be regarded as a sort of ''digitalization'' of a continuous dynamical system and it allows in principle a very appealing liberty in the control process to be implemented.

As a matter of fact, such topological schemes are so far quite abstract: information has to be encoded in highly nonlocal quantum states of many-body systems interacting in an exotic fashion. A significant intermediate step in this direction is given by the so-called ''holonomic'' quantum computation (HQC).[8,9] In this framework quantum information is encoded in an n-fold degenerate eigenspace of a family of quantum Hamiltonians depending on dynamically controllable parameters λ. Quantum gates are enacted by driving λ's along suitable loops γ within the manifold. The nontrivial dependence of Hamiltonian eigenvectors on λ results in nontrivial transformations of the initially prepared state. Such transformations—known as *holonomies*—generalize into the non-Abelian case, the celebrated Berry's phase.[10] When the loops undergo in an adiabatic way, holonomies can be explicitly computed in terms of the Wilczek-Zee gauge connection,[11] and conditions for achieving universality are simply stated.[8]

As for the topological schemes, the built-in fault-tolerant features of the holonomic approach are related to the fact that *the holonomies depend on some global geometrical feature, e.g., area, of the γ, and not on the way the loops are actually realized*.

Quantum gates based on (Abelian) Berry phases have been experimentally realized using nuclear-magnetic-resonance schemes,[12] and recently proposed for mesoscopic Josephson junctions[13] and anyonic excitations in Bose-Einstein condensates.[14] Nonadiabatic realizations of Berry's phase logic gates have been studied as well.[15,16] More recently, schemes for the experimental implementation of non-Abelian holonomic gates have been proposed for atomic physics,[17] ion traps,[18] Josephson junctions,[19] Bose-Einstein condensates,[20] and neutral atoms in cavity.[21]

We propose the implementation scheme for the realization of a universal set[22] of non-Abelian holonomic quantum gates in semiconductor nanostructures.[23] As we shall see, in the proposed strategy a central role is played by the holonomic structure introduced in Refs. 17 and 18, as well as by the exciton-exciton interaction mechanism exploited in the all-optical semiconductor-based QIP scheme proposed in Ref. 24. The proposed quantum hardware is given by an array of semiconductor quantum dots (QD's),[25] often referred to as macroatoms; our computational degrees of freedom are interband optical excitations, also called excitonic transitions. Indeed, an exciton is a Coulomb-correlated electron-hole pair produced by promoting an electron from the valence band with total angular momentum $J_{tot}=3/2$ to the conduc-

tion band with $J_{tot}=1/2$. For a GaAs-based quantum dot structure, the confining potential along the growth (z) direction breaks the symmetry and lifts the degeneracy in the valence band;[23,25] the states ($|J_{tot},J_z\rangle$) of the quadruplet $J_{tot}=3/2$ are then energetically separated into $J_z=\pm 3/2$—heavy holes (HH)—and $J_z=\pm 1/2$—light holes (LH).

A properly tailored laser excitation may promote electrons from the valence to the conduction band in an energy-selective fashion.[26] For the HH, the only allowed transitions are $|\frac{3}{2},\frac{3}{2}\rangle\rightarrow|\frac{1}{2},\frac{1}{2}\rangle$,$|\frac{3}{2},-\frac{3}{2}\rangle\rightarrow|\frac{1}{2},-\frac{1}{2}\rangle$. Here, the first transition is produced by light with left-circular polarization (usually referred to as σ^-), while the second transition is produced by light with right-circular polarization (σ^+). In contrast, due to the different structure of their wave functions for the LH, we have more allowed transitions.[23] As for the HH, we have $|\frac{3}{2},\frac{1}{2}\rangle\rightarrow|\frac{1}{2},-\frac{1}{2}\rangle$,$|\frac{3}{2},-\frac{1}{2}\rangle\rightarrow|\frac{1}{2},\frac{1}{2}\rangle$.[27] These transitions may be induced by light propagating along the z direction with circular (left or right) polarization. Moreover, for light propagating along the x-y plane with polarization along $z(\sigma^0)$ the following transitions are also allowed (and experimentally observed:[28] $|\frac{3}{2},\frac{1}{2}\rangle\rightarrow|\frac{1}{2},\frac{1}{2}\rangle$,$|\frac{3}{2},-\frac{1}{2}\rangle\rightarrow|\frac{1}{2},-\frac{1}{2}\rangle$. As a result, we see that by exciting LH electrons with three different kinds of light —left- and right-circular polarization as well as linear polarization along z—we can induce three different transitions with the same energy: $|G\rangle\mapsto|E^\alpha\rangle$, ($\alpha=\pm,0$), where $|G\rangle$ denotes the ground state of the semiconductor crystal. The allowed optical transitions as well as the corresponding energy-level structure for HH and LH are schematically depicted in Fig. 1(a). For the case of a laser excitation, resonant with the three degenerate LH transitions, the corresponding light-matter interaction Hamiltonian is of the form

$$H_{int}=-\hbar\sum_{\mu=0,\pm}(\Omega_{\mu,LH}|E^\mu\rangle\langle G|+\text{H.c.}). \tag{1}$$

This Hamiltonian has the same structure as the one for trapped-ion internal levels analyzed in Ref. 18. Indeed, for each value of the Rabi couplings Ω's, it admits a couple of *dark* states, i.e., two states $|D_\alpha(\Omega)\rangle$ ($\alpha=0,1$) corresponding to a zero eigenvalue. These dark states, in a distinguished point in the Ω space will encode our qubit. The quantum manipulations will be realized by the holonomies $P\exp\oint_\gamma A$ associated to the Wilczeck $u(2)$-valued connection A defined by $(A_\mu)_{\alpha\beta}=\langle D_\alpha|\partial/\partial\Omega^\mu|D_\beta\rangle$ ($\alpha,\beta=0,1$; $\mu=0,\pm$). Our computational basis is given by $|1\rangle:=|E^+\rangle$ and $|0\rangle:=|E^-\rangle$. The state $|E^0\rangle$ will play the role of an *ancilla*, used, as an auxiliary resource.

To achieve single-qubit universality is sufficient to enact a couple of noncommuting single-qubit gates U_1 and U_2.[22] Following Ref. 18, for the first gate we choose $\Omega_-=0$, $\Omega_+=-\Omega\sin(\theta/2)e^{i\varphi}$, and $\Omega_0=\Omega\cos(\theta/2)$. The dark states are given by $|E^-\rangle$ and $|\psi\rangle=\cos(\theta/2)|E^+\rangle+\sin(\theta/2)\,e^{i\varphi}|E^0\rangle$. By evaluating the connection associated to this two-dimensional degenerate eigenspace, it is not difficult to see that the unitary transformation $U_1=e^{i\phi_1|E^+\rangle\langle E^+|}$ ($\phi_1=\frac{1}{2}\oint\sin\theta\,d\theta d\psi$) can be realized as holonomy. For the second gate, we choose $\Omega_-=\Omega\sin\theta\cos\varphi$, $\Omega_+=\Omega\sin\theta\sin\varphi$, and $\Omega_0=\Omega\cos\theta$. The dark states are now given by $|\psi_1\rangle=\cos\theta\cos\varphi|E^-\rangle+\cos\theta\sin\varphi|E^+\rangle-\sin\theta|E^0\rangle$ and $|\psi_2\rangle=\cos\varphi|E^+\rangle-\sin\varphi|E^-\rangle$. In this case, the unitary transformation $U_2=e^{i\phi_2\sigma_y}$, where $\phi_2=\oint\sin\theta d\theta d\psi$ can be implemented.

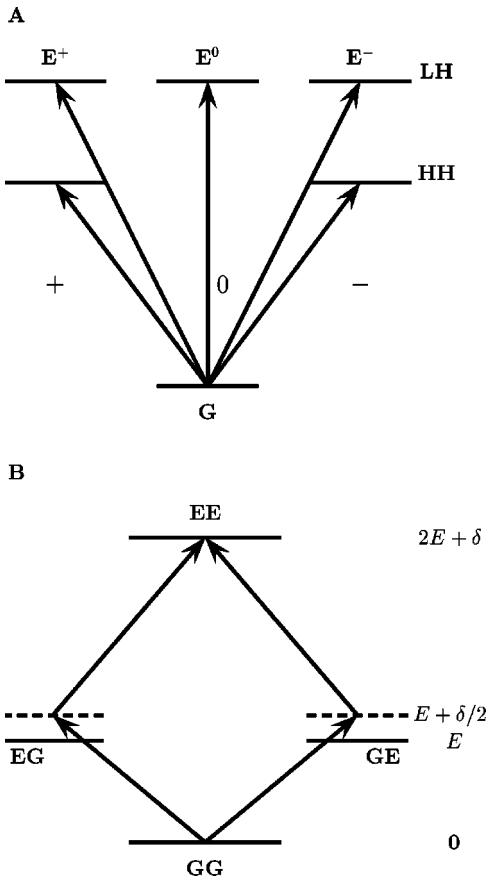

FIG. 1. Schematic diagram of the energy-level structure of LH and HH valence-band states (a) and of a typical two-photon process (b) in GaAs-based semiconductor macroatoms.

For the implementation of the two-qubit gate, we resort to the exciton-exciton dipole coupling in semiconductor macromolecules proposed in Ref. 24. Indeed, if we have two Coulomb-coupled quantum dots, the presence of an exciton in one of them (e.g., in dot b) produces a shift in the energy level of the other one (e.g., dot a) from E to $E+\delta$; the total energy in the process is $2E+\delta$. Let us consider the two dots in the ground state $|GG\rangle$; if we shine them with light resonant with $E+\delta/2$, we should be able to produce two excitons $|EE\rangle$. This is a second-order—two-photon—process, i.e., it involves a virtual transition to the intermediate states $|EG\rangle$ and $|GE\rangle$ [see Fig. 1(b)]. Due to energy conservation this is the only possible transition (the first-order—or single-photon—absorption is at energy E). Using different polarizations ($\sigma_+,\sigma_-,\sigma_0$), all the degenerate second-order transitions $|GG\rangle\rightarrow|E^\alpha E^\beta\rangle$, ($\alpha,\beta=0,+,-$) can be excited.

This process may be described by the following (effective) two-photon Hamiltonian

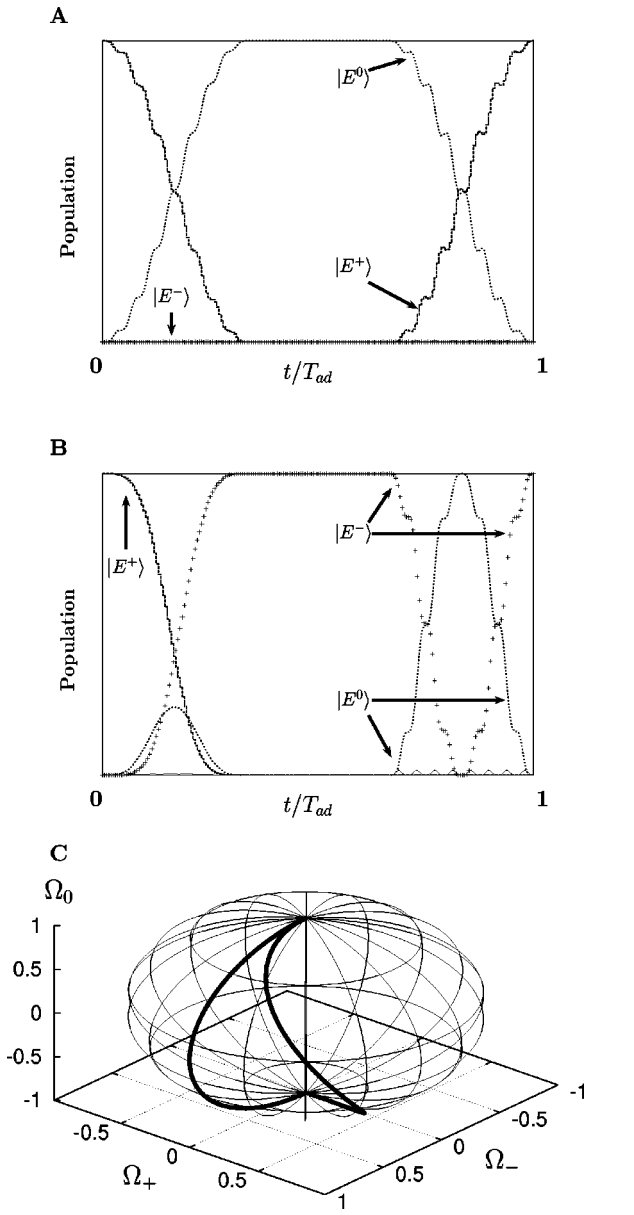

FIG. 2. (a) Simulated time evolution of the HQC gate 1 with $\phi_1=\pi/4$ and initial state $|E^+\rangle$. (b) Simulated time evolution of the HQC gate 2 with $\phi_2=\pi/2$ and initial state $|E^+\rangle$. (c) Simulated quantum evolution of gate 2 in the control parameter manifold $(\Omega^-,\Omega^+,\Omega^0)$. In these simulated experiments we have chosen $\Omega^{-1}=50$ fs and $T_{ad}=7.5$ ps (see text).

$$H_{int}=-\frac{2\hbar^2}{\delta}\sum_{\alpha,\beta=0,+}(\Omega_\alpha\Omega_\beta|E^\alpha,E^\beta\rangle\langle G,G|+\text{H.c.}), \tag{2}$$

where $\Omega_{+,0}$ is the Rabi frequency for the single-photon process within second-order perturbation theory. Here we have a three-dimensional dark-state manifold; by evaluating the associated $u(3)$-valued connection form one can check in a straightforward way that universal control is this dark space can be achieved in a fully holonomic fashion.[28] An explicit result will be shown later on.

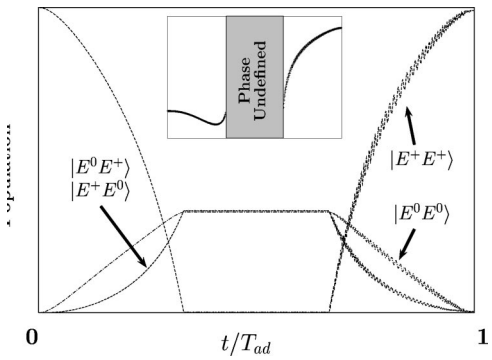

FIG. 3. Simulated control shift over the state $|E^+\rangle^{\otimes 2}$. The inset shows (where it is defined) the quantity φ^+ where $\varphi^+ := \text{Arg}\langle\Psi(t)|E^+E^+\rangle/|\langle\Psi(t)|E^+E^+\rangle|$ The values of the parameters are $\delta=5$ meV, $|\Omega_{0,+}|=\delta/5$, $T_{ad}=0.8$ ns. The gate fidelity $F=|\langle E^+E^+|\Psi(T_{ad})\rangle|^2=0.9899$.

To test the viability of the proposed HQC implementation scheme in state-of-the-art semiconductor nanostructures, we have performed a direct time-dependent simulation of gate 1 as well as gate 2. To this end, we have chosen $\Omega^{-1}=50$ fs and as evolution time $T_{ad}=7.5$ ps to satisfy adiabaticity. Moreover, we have chosen as initial state $|\psi(0)\rangle=|E^+\rangle$, and the loop such as to have $2\phi_1=\phi_2=\pi/2$. The computational states at the end of our adiabatic loop are $U_1|E^+\rangle=\exp[i(\pi/4)|E^+\rangle\langle E^+|]|E^+\rangle=(1+i)/\sqrt{2}|E^+\rangle$ for gate 1 and $U_2|E^+\rangle=\exp[i(\pi/2)\sigma_y]|E^+\rangle=|E^-\rangle$ for gate 2. Figure 2 shows the state populations during the quantum-mechanical evolution; as we can see, the state $|G\rangle$ is never populated (as expected in the adiabatic limit). For the case of gate 1 [see Fig. 2(a)], the $|E^-\rangle$ state is decoupled in the evolution, while the state $|E^+\rangle$ evolves to the ancilla state ($|E^0\rangle$) to eventually end in $|E^+\rangle$ (as we expect for the dark state). For the case of gate 2 [Fig. 2(b)], the initial state $|E^+\rangle$ evolves in $|E^-\rangle$, then in $|E^0\rangle$, and to end in $|E^-\rangle$; so we apply a NOT gate. Figure 2(c) shows the loop in the control parameters manifold $(\Omega^-,\Omega^+,\Omega^0)$ for gate 2.

We also performed a time-dependent simulation of a two-qubit gate, the effective Hamiltonian (2) has been used. Figure 3 shows how a controlled-phase shift over the state $|E^+\rangle^{\otimes 2}$ can be realized. It is important to notice here that the adiabaticity requirement along with the condition necessary for the validity of a second-order perturbative approximation implies that $T_{ad}\gg\delta/|\Omega_{0,+}|^2\gg 1/|\Omega_{0,+}|$. This means that the operation time for the two-qubit gates are necessarily longer than the ones for the single qubit. In view of the fast dephasing times in excitonic system, this latter fact would result in a lack of operation fidelity; this drawback has to be mitigated by a careful parameter optimization.

The simulated experiments in Fig. 2 clearly show that the proposed HQC implementation scheme is fully compatible with realistic parameters of state-of-the-art semiconductor nanostructures[29] as well as with current ultrafast laser technology,[26] prerequisite for its concrete realization. Indeed, our simulation shows that (i) one is able to work in the adiabatic limit, and (ii) our all-optical scheme allows for picosecond gating times; the ''ultralong'' exciton dephasing (on the nanosecond time scale) recently measured in state-

of-the-art QD structures[30] indicate that within the proposed HQC implementation scheme one should be able to perform a few operations within the dephasing time. In this respect, let us stress that our aim here is not to achieve the error rate threshold for massive fault-tolerant QIP, rather to demonstrate how highly nontrivial non-Abelian quantum phases can be used to realize elementary quantum-state manipulations in a semiconductor-based nanostructure.

In summary, we have proposed the implementation scheme for the realization of non-Abelian geometrical gates in semiconductor nanostructures. Our quantum hardware consists of state-of-the-art Coulomb-coupled semiconductor macroatoms; quantum bits are encoded in the dark states of polarization-selective excitonic transitions, driven by ultrafast laser pulses; the key ingredient for the implementation of the proposed two-qubit gate is dipole-dipole coupling between excitons in neighboring quantum dots. The proposed scheme combines the benefits of geometrical QIP with the distinguished characteristics of all-optical implementations in nanostructured semiconductors.

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