September 1, 2015

Matching Public Support for Private Product-Innovating R&D: A Theoretical Analysis

by

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Abstract

This paper develops a theoretical model of product innovation where R&D effort by a monopolist firm is endogenous and its outcome uncertain. The government attempts to aid such efforts with a matching grant. We consider different scenarios depending on whether two parties act simultaneously, act sequentially, or take part in a dynamic cooperative game with a trigger strategy. We also consider cases (i) when the products are exported, (ii) when the firm lobbies for R&D subsidy, and (iii) when the firm is foreign owned. We characterize situations when government intervention increases the chances of product innovation and when it does not.

JEL Classification: C71, C72, F10, O31
Key words: product innovation, R&D, matching subsidy.

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1 Introduction

Research and Development (R&D) is important not only to firms, but also to the society at large. However, there are different types of R&D investments. Firms can engage in product R&D, which attempts to develop new goods, and in process R&D, which attempts cost-reducing innovations in manufacturing process. Product R&D deserves particular attention because it makes up the majority of total R&D investment and it is responsible for many breakthroughs that spur economic growth (Gilbert, 2006). According to the National Science Foundation, in 1981 about 75% of all industry R&D was product R&D (NSF 2004).

It is believed that a firm's level of R&D investment often differs from that of social optimum, and therefore many nations have attempted various policies intended to stimulate private R&D investment. Cohen and Noll (1995) and Stiglitz and Wallsten (1999) provide survey of the United States government programs of technology partnerships, and theories supporting them. These partnerships include public funding of industry R&D projects and private research ventures, as well as, collaborations between industry and government scientists. The idea is to generate research that yields commercial products and innovations. Others examine industry specific programs of R&D.[1] Economic justification for these programs is straightforward: although some of the projects could benefit society they are not being developed due to the low profitability from the firm's point of view. By helping the firm, a government could make these projects privately profitable.

The purpose of this paper is to answer the following research questions regarding product R&D. First is basic question. Should a government help domestic firms to develop product? As we shall show in some circumstances it is best for the government not to get involved. Can government support encourage firms to undertake projects that it would not attempt otherwise? We find this to be the case in some other circumstances. Does repeated interactions between the firm and the government help innovation which would not be attempted otherwise? Once again we find that the answer can be yes.

One of the ways that government could help and at the same time encourage firms to innovate is by issuing a matching grant, where private investments are proportionally matched by public contributions. There are many state organizations that provide help to product innovating firms in this manner. For example, Florida High Tech Corridor Council (FHTCC), and Massachusetts Life Sciences Center (MLSC). Since these organizations were established in the late 1990 as a part of President Clinton's economic campaign in 1993. These two organizations have participated in thousands of research project and cooperated with hundreds of companies in a variety of industries ranging from Aerospace to renewable energy in attempts to develop new products. Just two of them have contributed close to a hundred million dollars that have been matched by a couple of hundred millions of corporate dollars toward product innovation. Overall, their economic impact is estimated to exceed a few billion dollars. The primary focus of these organizations is to foster cooperation between national universities and their industry partners. Every year, many companies use these matching grants to leverage their R&D budgets with academic partnership to develop new

[1] Cohen and Noll (1995b) study cooperative agreements. Irwin and Klenow (1996) study the effect of Sematech. Yager and Schmidt (1997) study the Institutions of Advanced Technology Program. Wallsten (2000) takes a closer look at Small Business Innovation Research Program.

commercial goods. These organizations match private investments at different levels, ranging from one public to three private dollars to one-to-one basis. Although these matching grant programs have become popular, the theoretical literature on this topic is limited.

The broader literature on product innovation considers a number of issues such the role of buyers' information on quality (Chiang and Masson, 1988), the role of the nature of competition (Motta, 1993; De Bondt and Vendekerckhove, 2010; Schmutzler, 2010), and uncertainty about product quality (Bagwell and Staiger, 1989). There is also a large empirical and theoretical literature on the effect of competition within an industry on the product innovation by firms (see, Gilbert, 2006b; Spiegel and Tooks, 2008; Askenazy et al., 2008; Castellacci, 2009; Chen and Schwartz, 2010; Aghion et al., 1998, 2002, 2005; Belleflamme and Vergari, 2011; Karaman and Lahiri, 2014; Ahmed and Lahiri, 2014). Some like Aghion et al. consider a patient-race model while other like Grossman and Helpman (1991) use a quality-ladder model. In contrast, we consider the product-innovating R&D with uncertain outcome *a la* Grossman and Helpman (1991) by a monopolist and focus of the role of the government under different assumptions on the latter's objective function.

We provide a theoretical framework that can answer our research questions, and analyze optimal choice of matching subsidy, as well as, optimal level of private investment. We consider different scenarios depending on whether the private firm and the government act simultaneously, act sequentially, or take part in a dynamic cooperative game with a trigger strategy. We also consider cases (i) when the products are exported, (ii) when the firm lobbies for R&D subsidy, and (iii) when the firm is foreign owned. We compare results across the scenarios. In some of the cases, we find that whereas the sequential and the simultaneous moves results in the same outcome. We also find that although the dynamic setting does not lead to an increase in total R&D expenditure, it can lead to positive efforts on R&D, while in the absence of repeated interactions no R&D investment would take place. Finally, we find that government support does not always fully crowd out private investments; sometime it can in fact increase private investment.

The layout of the paper is as follows. In section 2 we first of all set up and analyze the benchmark model where the firms decides on the the level of R&D investment, and the government decides on the matching grant, simultaneous. This model is then extended in section 2.1 to allow the government to pre-commit on the matching subsidy before the firm decides on R&D investment. In section 3 we consider a repeated game, and in section 4 some concluding remarks are made.

2 Private R&D Innovation with matching support

We consider an economy with two goods: one good that already exist and the other one that is demanded but has not been developed yet. The goods are imperfect substitutes. Demand and price for good i are represented by D_i and P_i respectively. The utility function of the representative consumer is:

$$U = \alpha_1 D_1 + \alpha_2 D_2 - \frac{\beta(D_1^2 + D_2^2) + 2\gamma D_1 D_2}{2} + y, \tag{1}$$

where the degree of product differentiation is captured by γ which satisfies $\beta > \gamma > 0$, and y is the consumption of the numeraire good.

From (1) we can derive the following inverse demand functions:

$$p_1 = \alpha_1 - \beta D_1 - \gamma D_2 \quad \text{and} \quad p_2 = \alpha_2 - \beta D_2 - \gamma D_1, \tag{2}$$

where α_i captures the maximum amount that the consumer is willing to pay for good i.

The producer side of the economy is represented by one monopolistic firm. The firm has technology to produce good 1, and invests in R&D to develop technology to produce good 2. The R&D investment level, e. This private R&D investment and a matching share s from the government determines the probability, $q(e(1+s))$, of it being successful in developing a new good. We consider the following functional form of the probability function:

$$q(e(1+s)) = \frac{e(1+s)}{1+e(1+s)}. \tag{3}$$

Expected profit is given by:

$$E[\pi] = q(e(1+s))\pi^s + (1 - q(e(1+s)))\,\pi^n \tag{4}$$

where π^s and π^n are profits when R&D is successful and when it is not successful respectively.

Once the outcome of R&D becomes known the firm makes output decisions and profit is realized. If R&D achieves its objective, the firm chooses how much of good 1 and how much of good 2 to produce. However, if R&D fails, then the firm chooses quantity of good 1 whose production technology has been available prior to R&D. All R&D activities are done by an independent lab, and to start with we assume that the the R&D investment decision by the firms and the government's decision on the matching share s are done simultaneously. That is, the timing of the game is described as follows.

Stage 1:

- The government chooses matching share
- The firm chooses level of private investment
- R&D is done by an independent lab

Stage 2:

- The firm chooses output

We use backward induction to solve the game. In the last stage of the game, the success of R&D, as well as expected profit and expected social welfare, are realized.

In the last stage there are two possible outcomes:

$$\pi^s = (p_{1s} - c_1)x_{1s} + (p_{2s} - c_2)x_{2s} - e, \quad \pi^n = (p_{1n} - c_1)x_{1n} - e, \tag{5}$$

where p_{ij} is price and x_{ij} is quantity of good i when the outcome of the R&D j with $j = s$ (success) or $j = n$ (failure), c_i is the constant unit cost of producing good i, and e is cost of the R&D

investment. Since R&D is done by an independent laboratory, profit under each state of the world is not directly affected by the matching share that government contributes.

In each of the two cases firm chooses how much of each available good to produce by maximizing profit. In the case of R&D achieving its objectives, the firm chooses amount of x_{1s} and x_{2s} by setting $\partial E[\pi]/\partial x_{1s} = 0$ and $\partial E[\pi]/\partial x_{2s} = 0$. When R&D fails, the firm chooses amount of x_{1n} by setting $\partial E[\pi]/\partial x_{1n} = 0$. The first order conditions for each state of the world are

$$\begin{aligned} \partial E[\pi]/\partial x_{1s} &= \alpha_1 - \beta x_{1s} - \gamma x_{2s} - c_1 - \beta x_{1s} - \gamma x_{2s} = 0, \\ \partial E[\pi]/\partial x_{2s} &= \alpha_2 - \beta x_{2s} - \gamma x_{1s} - c_1 - \beta x_{2s} - \gamma x_{1s} = 0, \\ \partial E[\pi]/\partial x_{1n} &= \alpha_1 - \beta x_{1n} - c_1 - \beta x_{1n} = 0. \end{aligned} \tag{6}$$

The optimal profits under different outcomes of R&D and the expected profit are given by

$$\begin{aligned} \pi^s &= \beta x_{1s}^2 + 2\gamma x_{1s} x_{2s} + \beta x_{2s}^2 - e, \quad \pi^n = \beta x_{1n}^2 - e, \\ E[\pi] &= \frac{e(1+s)}{e(1+s)+1}(\beta x_{1s}^2 + 2\gamma x_{1s} x_{2s} + \beta x_{2s}^2) + \frac{1}{e(1+s)+1}(\beta x_{1n}^2) - e. \end{aligned} \tag{7}$$

Similarly, consumers' surplus (CS) under different R&D outcomes and the expected consumers' surplus are

$$\begin{aligned} \mathrm{CS}^s &= \frac{(\alpha_1 - p_{1s})x_{1s}}{2} + \frac{(\alpha_2 - p_{2s})x_{2s}}{2}, \quad \mathrm{CS}^n = \frac{(\alpha_1 - p_{1n})x_{1n}}{2}, \\ E[\mathrm{CS}] &= \frac{e(1+s)}{e(1+s)+1}\mathrm{CS}^s + \frac{1}{e(1+s)+1}\mathrm{CS}^n. \end{aligned} \tag{8}$$

We now turn to the specification of the government's objective function. The government maximizes a linear combination of expected profits, expected consumers' surplus and subsidy payments which is paid for by lump-sum taxation of the representative consumer

$$E[W] = \theta_1 E[\pi] + \theta_2 E[\mathrm{CS}] - s * e, \tag{9}$$

where θ_1 and θ_2 are two parameters. We shall consider a number of special cases depending on specific values of these two parameters, and these are:

Case 1: The benchmark case: $\theta_1 = \theta_2 = 1$. Here the government's objective function coincides with social welfare.

Case 2: The case of export oriented production: $\theta_1 = 1$ and $\theta_2 = 0$.

Case 3: The case of lobbying by the producer: $\theta_1 > 1$ and $\theta_2 = 1$. In this case (9) is best described as a political support function (see Long and Vousden (1991)). It can also be seen as a reduced form of campaign contribution model *a la* Grossman and Helpman (1994).

Case 4: The case of foreign direct investment: $\theta_1 = 0$ and $\theta_2 = 1$.

Substituting the solutions of the output levels and prices from (2) and (6) into (7) and (8) and then those in turn into (9), we get

$$E[\pi] = \frac{e(1+s)}{e(1+s)+1}(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2) + \frac{1}{e(1+s)+1}(\beta x_{1n}^2) - e \tag{10}$$

$$E[W] = \frac{e(1+s)}{e(1+s)+1}(\theta_1 + 0.5\theta_2)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2) + \frac{1}{e(1+s)+1}(\theta_1 + 0.5\theta_2)(\beta x_{1n}^2) - e(\theta_1 + s). \tag{11}$$

In stage 1 of the game, the firm maximizes (10) with respect to e and the government maximizes (11) with respect to s, simultaneously, giving the following first-order conditions:

$$\frac{\partial E[\pi]}{\partial e} = \frac{(1+s)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2 - \beta x_{1n}^2)}{(e(1+s)+1)^2} - 1 = 0, \tag{12}$$

$$\frac{\partial E[W]}{\partial s} = \frac{e(\theta_1 + 0.5\theta_2)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2 - \beta x_{1n}^2)}{(e(1+s)+1)^2} - e = 0. \tag{13}$$

Solving (12) and (13) simultaneously the Nash optimal private investment and optimal matching share are obtained as

$$e_{sim} = \frac{\sqrt{(\theta_1 + 0.5\theta_2)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2 - \beta x_{1n}^2)} - 1}{\theta_1 + 0.5\theta_2}, \tag{14}$$

$$s_{sim} = \theta_1 + 0.5\theta_2 - 1. \tag{15}$$

From (15) we see that under case 1 ($\theta_1 = \theta_2 = 1$), $s_{sim} = 0.5$. That is, the public to private input ratio in the R&D is 1:2. From (14) it also follows that the firm would invest in product innovation, i.e., $e_{sim} > 0$, if and only if the following is true

$$\pi^s - \pi^n > \frac{1}{\theta_1 + 0.5\theta_2} = \Omega \text{ (say)}. \tag{16}$$

That is, for the firm to have enough incentive to invest the difference in profits between success and failure has to be sufficiently high. $\pi^s - \pi^n > 0$ is not enough because of the inherent risk involved.

When the government does not support R&D, by setting $s = 0$ in (12) we find that the firm will invest in R&D if and only if $\pi^s - \pi^n > 1$. In the case of export-oriented production (case 2), $\Omega = 1$, the government support makes no difference to the private incentive to invest. In the case of foreign direct investment (case 4), $\Omega > 1$ and the level of optimal matching subsidy is negative. In this case the firm is more likely to invest in R&D without any government intervention. In the other two cases (case 1 and case 3), $\Omega < 1$,and the firm would invest in product innovation for a wider range of parameter in the presence of government support compared to the range when the government does not intervene.

Finally, it can be verified that the total amount of R&D expenditure $e(1+s)$ is larger with government support than without it if and only if $\theta_1 + 0.5\theta_2 > 1$. This condition is satisfied in cases 1 and 2, it is not satisfied in case 4, and in case 2 government subsidy is zero and therefore it is irrelevant.

These results are summarized in the following proposition.

Proposition 1 *Suppose that the firm decides on the R&D investment and government decides on the level of matching subsidy simultaneously. Then we have the following.*

1. *When the government maximizes social welfare (case 1) and when the firm lobbies the government for support (case 3), optimal matching subsidy is positive, and government support increases the range of parameter values over which the the firm will invest in R&D. Under case 1, the optimal matching share is 50%,*

2. *In the case of export-oriented production (case 2), the government does not support R&D ($s_{sim} = 0$),*

3. *In the case of foreign direct investment (case 4), the government taxes R&D, and the firm is more likely to invest without government intervention, and*

4. *Total R&D expenditure is higher with government support under cases 1 and 3.*

Given the risk of not succeeding in R&D efforts, a firm would not invest in R&D if potential success does not bring sufficiently large benefit. Clearly, if the government supports such private efforts with matching subsidy, the cost of downside risk is mitigated somewhat and the firm would invest in R&D with support when it would not have without the support. However, subsidy is financed by a tax on the representative consumer and has a welfare cost. When the products are only for exports, the marginal benefit of a subsidy to the producer cancels with the cost to the taxpayers, and no intervention is optimal. When the firm is foreign, the benefit to the firm is not a part of the welfare, and benefit to the consumers' is outweighed by the cost to the taxpayers and the optimal policy is to tax R&D efforts.

2.1 Sequential Game

Hitherto we assumed that in stage 1 of the game the firm decides on the R&D investment level and the government decides on the matching subsidy level simultaneously. In this subsection, we shall assume that the government pre-commits on the matching subsidy before the firm decides on the investment level. To be more specific, we assume that the government moves before the firm. In particular, the timing of the game is given by:

Stage 1:

- The government chooses s that maximize expected social welfare

Stage 2:

- The firm chooses e to maximize expected profit
- An independent lab conducts $e(1 + s)$ amount of R&D activity

Stage 3:

- The firm makes output decisions

We once again use backward induction to solve the problem. In the third stage of the game expected profit as well as expected social welfare are determined. Since stage 3 is as before, from the previous section we know that expected profit is

$$E[\pi] = \frac{e(1+s)}{e(1+s)+1}(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2) + \frac{1}{e(1+s)+1}(\beta x_{1n}^2) - e. \tag{17}$$

In the second stage, the firm chooses the amount of private R&D investment, e, that maximizes expected profit given in (17). The first-order condition is

$$\frac{\partial E[\pi]}{\partial e} = \frac{(1+s)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2 - \beta x_{1n}^2)}{(e(1+s)+1)^2} - 1 = 0, \tag{18}$$

From the above first-order conditions we can derive the firm's reaction function as:

$$e = \frac{\sqrt{(1+s)(\beta x_{1s}^2 + 2\gamma x_{1s}x_{2s} + \beta x_{2s}^2 - \beta x_{1n}^2)} - 1}{1+s}, \tag{19}$$

with

$$\frac{de}{ds} = \frac{(1+s)^2}{1 - 0.5\sqrt{(\pi^s - \pi^n)(1+s)}}. \tag{20}$$

In the first stage of the game, the government chooses s by maximizing the objective function given by (11), taking into account the firm's reaction function given above. That is

$$\frac{dE[W]}{ds} = \frac{\partial E[W]}{\partial s} + \frac{\partial E[W]}{\partial e}\frac{de}{ds} = 0, \tag{21}$$

where

$$\frac{\partial E[W]}{\partial e} = \frac{(1+s)(\theta_1 + 0.5\theta_2)(\pi^s - \pi^n)}{(e(1+s)+1)^2} - (\theta_1 + s), \tag{22}$$

and de/ds is given by (20).

We shall now examine how the value of optimal s in this case (s_{seq}) compares with the case where the firm and the government move simultaneously. We know that

$$s_{seq} \gtreqless s_{sim} \quad \Longleftrightarrow \quad \left.\frac{\partial E[W]}{\partial s}\right|_{(e,s)=(e_{sim},s_{sim})} \gtreqless 0.$$

Substituting (14) and (15) in (20) and (22) and then from (21) we get

$$\begin{aligned} \left.\frac{dE[W]}{ds}\right|_{(e,s)=(e_{sim},s_{sim})} &= \left.\left(\frac{\partial E[W]}{\partial e}\frac{de}{ds}\right)\right|_{(e,s)=(e_{sim},s_{sim})} \\ &= \frac{1-\theta_1}{(\theta_1 + 0.5\theta_2)^2}\left(1 - 0.5\sqrt{(\pi^s - \pi^n)(\theta_1 + 0.5\theta_2)}\right). \end{aligned} \tag{23}$$

We shall now examine the sign of the right-hand side of (23) under the four cases considered in the preceding section. In case 1 ($\theta_1 = \theta_2 = 1$) and in case 2 ($\theta_1 = 1,\ \theta_2 = 0$), $dE[W]/ds = 0$ and therefore we have $s_{seq} = s_{sim}$. In case 3 ($\theta_1 > 1,\ \theta_2 = 1$) we have

$$\left.\frac{dE[W]}{ds}\right|_{(e,s)=(e_{sim},s_{sim})} = \frac{1-\theta_1}{(\theta_1 + 0.5\theta_2)^2}\left(1 - 0.5\sqrt{(\pi^s - \pi^n)(\theta_1 + 0.5)}\right),$$

and thus

$$s_{seq} \gtreqless s_{sim} \qquad \Longleftrightarrow \qquad \pi^s - \pi^n \gtreqless \frac{8}{2\theta_1 + 1}.$$

Similarly, in case 4 ($\theta_1 = 0,\ \theta_2 = 1$), it can be show that

$$s_{seq} \gtreqless s_{sim} \qquad \Longleftrightarrow \qquad \pi^s - \pi^n \lesseqgtr 8.$$

The above results are summarized in the following proposition.

Proposition 2 *Suppose that the firm decides on the R&D investment after government decides on the level of matching subsidy. Then we have the following.*

1. *When the government maximizes social welfare (case 1: $\theta_1 = \theta_2 = 1$) or when the product is produced solely for exports (case 2: $\theta_1 = 1,\ \theta_2 = 0$), we have $s_{seq} = s_{sim}$.*
2. *In the case of lobbying by the producer (case 3: $\theta_1 > 1,\ \theta_2 = 1$), we have*

$$s_{seq} \gtreqless s_{sim} \qquad \Longleftrightarrow \qquad \pi^s - \pi^n \gtreqless \frac{8}{2\theta_1 + 1}.$$

3. *In the case of foreign direct investment (case 4: $\theta_1 = 0,\ \theta_2 = 1$), we have*

$$s_{seq} \gtreqless s_{sim} \qquad \Longleftrightarrow \qquad \pi^s - \pi^n \lesseqgtr 8.$$

When the matching subsidy is at the Nash optimal level, i.e., $s = s_{sim}$, and $\theta_1 = 1$, equations (10) and (11) are the same but for a multiplicative term. Thus, the investment level e that maximizes private profits also maximizes social welfare, i..e., $(\partial E[W]/\partial e)|_{s=s_{sim}} = 0$. This happens for a number of reasons. First, output levels under different states of the world do not depend on the level of investment because of the time structure of the game. Second, e and $1+s$ always appear together in a multiplicative form $e(1+s)$. Finally, when $\theta_1 = 1$, the private cost of investment to the firm e and the taxpayer's cost of subsidizing R&D have the same weight in the welfare function. In this case, by pre-committing itself on matching subsidy, the government cannot alter the equilibrium. When $\theta_1 > 1$, $(\partial E[W]/\partial e)|_{s=s_{sim}} < 0$, and thus $s_{seq} > s_{sim}$ if the firm's reaction function is downward sloping, which puts a lower bound on $\pi^s - \pi^n$. Similarly, the rest of the results can be explained.

3 Cooperative Dynamic Game

In the previous section we have seen that by committing itself to a matching share before the firm decides on the R&D investment, a government cannot increase the range of parameters over which R&D investments would take place when $\theta_1 = 1$, i.e., when the the government maximizes social welfare or the firm does not lobby (proposition 2). In this section we explore the possibility of the firm and government achieving greater efficiency by choosing values of private R&D investment and matching share through cooperation, which are different from the Nash equilibrium and that are self-enforcing when $\theta_1 = 1$. For this, we consider a stationary dynamic matching R&D game, a similar to one used by Bagwell and Staiger(1997) in a different context, which is defined by the infinite repetition of the static matching game described above. Typically, R&D partnerships do not end on one project, and the matching game repeats over and over again.

In each period, the firm and the government observe all previous selections of private investments and matching shares and then simultaneously choose the level of private investment and the matching share respectively. The game is stationary in the sense that none of the parameters of the model change over time. We focus on a particular class of subgame perfect equilibria for the stationary dynamic matching game. Specifically, we consider equilibria in which (i) firm and government select feasible and stationary values of private investment and matching share respectively along their equilibrium paths, meaning that in equilibrium each party chooses the same value in each period, and (ii) if a deviation from these equilibrium values occur, then in the next period and forever thereafter players revert to the Nash equilibrium values of private investment and matching share of the static matching game. The latter is the punishment or trigger strategy employed by each. We shall refer to the subgame perfect equilibrium which yields the highest possible values of the expected profit and expected social welfare while satisfying restrictions (i) and (ii) as the most-cooperative equilibrium of the stationary dynamic matching game. The corresponding private investment and matching share are then termed the most-cooperative values for the stationary dynamic matching game.

It is possible to support a cooperative solution that is mutually beneficial, since any attempt to deviate from the current period values of private investment or matching share will be greeted with retaliatory (Nash) values of private investment and matching share respectively in future periods. Therefore, a cooperative private investment and a cooperative matching share can be supported in equilibrium for the stationary dynamic matching game if one time incentive to cheat is sufficiently small relative to the discounted future value of maintaining a cooperative relationship between firm and government.

Let us first examine the incentives for the firm to cheat or not. For a fixed cooperative matching share and level of private investment given the class of the subgame-perfect equilibria upon which we focus, if the firm is to deviate and choose a e which is different from the cooperative equilibrium value e_{co}, then it will deviate to its best-response simultaneous Nash level of private investment, e_{sim}. Thus the gain from cheating is given by

$$\Omega_f(e_{co}, s_{co}) \equiv E[\pi(e_{sim}, s_{co})] - E[\pi(e_{co}, s_{co})].$$

When the firm cheats, its future expected profit s, and the cost of cheating can be computed as follows. The following expression defines the one-period value of cooperation:

$$\omega_f(e_{co}, s_{co}) \equiv E[\pi(e_{co}, s_{co})] - E[\pi(e_{sim}, s_{sim})]$$

Then the cost of cheating to the firm is $\omega_f(e_{co}, s_{co})\delta/(1-\delta))$ since once the firm defects and chooses the level of private investment different to the cooperative value, cooperative matching share is thereafter replaced by the Nash matching share. Hence, cooperative condition, or know as "no-defect" condition is that the benefit of cheating has to be less than the discounted future value of cooperation, or:

$$\Omega_f(e_{co}, s_{co}) \leq \frac{\delta}{1-\delta}\omega_f(e_{co}, s_{co}),$$

where $\delta \in (0, 1)$ is the time discount factor, i.e., it is $1/(1 + r)$ where r is the discount rate.

We now turn to the incentives of the government to cheat not. For a fixed cooperative private investment, matching share, and the class of subgame-perfect equilibria upon which we focus, if government deviates and selects $s \neq s_{co}$, then it will deviate to its best-response Nash matching share, s_{sim}. Thus, the gain from cheating for the government is given by:

$$\Omega_g(e_{co}, s_{co}) \equiv E[W(e_{co}, s_{sim})] - E[W(e_{co}, s_{co})]$$

When government cheats, however, it also causes future welfare to drop, and one-period cost of cheating is equal to:

$$\omega_g(e_{co}, s_{co}) \equiv E[W(e_{co}, s_{co})] - E[W(e_{sim}, s_{sim})]$$

Similarly to the firm, government would choose to cooperate if one time incentive to cheat is less than the discounted value of the future cooperation. Government's cooperative or "no-defect" condition can be written as:

$$\Omega_g(e_{co}, s_{co}) \leq \frac{\delta}{1 - \delta} \omega_g(e_{co}, s_{co})$$

Any cooperative private investment and matching share that satisfy both firms and governments no-defect conditions can be supported in the subgame-perfect equilibrium of the stationary dynamic matching R&D game.

When $\theta_1 = 1$ (an assumption that we are making here), a special feature of the model is that in the simultaneous game the government achieves the level of the total R&D investment that maximizes expected social welfare even without cooperation. We can check this by maximizing expected social welfare with respect to total investment, by setting $\partial E[W]/\partial e(1 + s) = 0$. First order condition of this maximization problem is:

$$\frac{\partial E[W]}{\partial(e(1 + s))} = \frac{(1 + 0.5\theta)(\pi^s - \pi^n)}{(e(1 + s))^2} - 1 = 0.$$

Solving the first order condition for $e(1+s)$ reveals that optimal amount of R&D investment, from government's point of view, is equal to the Nash total investment, i.e.,

$$[e(1 + s)]_{soc} = \sqrt{(1 + 0.5\theta_2)(\pi^s - \pi^n)} - 1$$

Therefore, $E[W(e_{sim}, s_{sim})]$ is the maximum value of the social welfare.[2] It means that social welfare cannot be increased by cooperation. However, if the total amount of R&D investment in cooperative equilibrium is equal to that of the non-cooperative game then the government would

[2]From second order condition we know that Nash investment is a global maximum, because

$$\frac{\partial^2 E[W]}{\partial e(1 + s) \partial e(1 + s)} = -\frac{2(1 + 0.5\theta)(\pi^s - \pi^n)}{(e(1 + s))^3} < 0. \tag{24}$$

be indifferent whether to cooperate or not. Hence, the government cooperation condition for simultaneous cooperative game can be rewritten as:

$$e_{co}(s_{co}+1) = e_{sim}(s_{sim}+1).$$

The firm's gain from cheating and value of cooperation can be calculated as

$$\begin{aligned}\Omega_f(e_{co}, s_{co}) &= \frac{e_{sim}(e_{sim}(s_{sim}+1)+1)(s_{co}-s_N)}{(1+s_{sim})(e_{sim}(s_{co}+1)+1)} - (e_{sim}-e_{co}) \\ \omega_f(e_{co}, s_{co}) &= (e_{sim}-e_{co}).\end{aligned}$$

Thus, cooperative equilibrium exists when the following system holds:

$$\begin{aligned}e_{co}(s_{co}+1) &= e_{sim}(s_{sim}+1) \\ \frac{e_{sim}(e_{sim}(s_{sim}+1)+1)(s_{co}-s_N)}{(1+s_{sim})(e_{sim}(s_{co}+1)+1)} &\leq \frac{1}{1-\delta}(e_N - e_{co}).\end{aligned}$$

It is easy to check that the second of the above condition will be satisfied with equality. Solving the system yields the following solutions

$$e_{co} = e_{sim}(1-\delta(e_{sim}(s_{sim}+1)+1)), \quad s_{co} = \frac{s_{sim}+\delta(e_{sim}(s_{sim}+1)+1)}{1-\delta(e_{sim}(s_{sim}+1)+1)}.$$

Three points to note from the above solution. First, as long as $e_{sim} > 0$, both e_{co} and s_{co} are strictly positive if and only if the following restriction on the value of δ is satisfied:

$$\delta < \frac{1}{e_{sim}(s_{sim}+1)+1} = \frac{1}{\sqrt{(\pi^s-\pi^n)(1+0.5\theta_2)}},$$

Second, under the same restriction on δ, $e_{co} < e_{sim}$ and $s_{eco} > s_{sim}$. Finally, as we have seen before, $e_{sim} > 0$ if and only if

$$\pi^s - \pi^n > \frac{1}{1+0.5\theta_2}.$$

Therefore, R&D investment would take place under cooperative if $\pi^s - \pi^n$ fall in the range,

$$\frac{1}{1+0.5\theta_2} < \pi^s - \pi^n < \frac{1}{(1+0.5\theta_2)\delta^2}. \tag{25}$$

To summarize, if the parameters are in the range described in (25), the government will be indifferent between a cooperative and non-cooperative game, but the firm will prefer the co-operative game as it will spend less on R&D and the government will pay more subsidy. Total R&D expenditure will remain the same. Thus in this range the cooperative solution will prevail. If $1/(1+0.5\theta_2) > \pi^s - \pi^n$, there will be no R&D investment at all, and if $\pi^s - \pi^n > 1/((1+0.5\theta_2)\delta^2)$, there will be no cooperation and the non-cooperative equilibrium will prevail. Interestingly, if the goods are produced solely for exports, i.e., if $\theta_2 = 0$, the government would not provide any matching subsidy (see proposition 1(2)). Thus, the existence of cooperation simply enhances the possibility of public support for product R&D for export-oriented firms. Also, the less patient the firm is (small δ), the larger is the matching share that government chooses, and the more likely that a cooperative equilibrium would prevail. These results are stated in the following proposition.

Proposition 3 *Suppose that the firm is domestically owned and it does not lobby the government for subsidy ($\theta_1 = 1$). Then a cooperative equilibrium will prevail if the parameters are in the range described in (25). The less patient the firm is, the larger is the matching share that government chooses, and the more likely that a cooperative equilibrium would prevail.*

4 Conclusion

This paper is first to develop a theoretical model of product R&D by a monopolist and a matching grant by the government. The relationship between the two parties is modeled in a game theoretic framework and three frameworks are considered and compared. In the first, the firm decides on the R&D investment level and the government determines the matching R&D subsidy simultaneously. In the second framework, they move sequentially: the government moves earlier than the firm. Finally, the third framework considers a repeated game between the two parties. We characterize the equilibrium in each of the three frameworks.

We consider four scenarios regarding the government's objective function: (i) it is the social welfare function, (ii) no consumers' surplus because the goods are exported, (iii) it is a political support function where the monopolist lobby the government for subsidy, and (iv) the monopolist firm is foreign owned. Some of our key findings are as follows. We find that under (i) and (ii) the simultaneous game and the sequential game leads to identical outcome. Moreover, under (ii) the government does not provide any matching subsidy. This non-participation by the government will not hold when a cooperative repeated game prevails over a non-cooperative one. We derive restrictions on the parameter space for a cooperative equilibrium to prevail.

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