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Estimating Prevalence of Sexual Abuse by an Acquaintance with an Optional Unrelated Question RRT Model

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ABSTRACT. The main focus of this paper is on validating an Optional Unrelated Question RRT Model using real survey data. The model to be used in this study is designed to simultaneously estimate the mean of a sensitive variable and the sensitivity level of the underlying sensitive question without using the traditional split sample approach. The data were collected via a survey conducted by the authors on a sample of undergraduate female students in the age group of 17-21 years at a college of University of Delhi, India, in January 2015. The binary research question of interest was "Have you ever been a victim of sexual abuse by friend or family member?" and the quantitative research question was "How many days in a typical month do you watch pornographic clips/videos/movies on movie channels, WhatsApp, YouTube, Internet etc.?"

1. Introduction

Sexual abuse is forcing of undesired sexual behavior by one person on another person. Although sexual abuse is perpetrated more on young children and pre-teens, older kids or adolescents also, particularly girls are often victims of this menace. The perpetrators of sexual abuse are typically viewed as strangers who are psychologically disturbed, pathological men (Donat and D'Emilio, 1992). However, these acts are often committed by parents, neighbors, relatives, extended family members and friends (Kacker and Mohsin, 2007). The main causes of sexual abuse include socio-economic factors, anger, power, sexual pleasure and psychopathic tendencies. In recent years, there has been an increasing attention to sexual coercive experiences of young people in developing countries, including India. In the Indian context, sexual abuse and violence is primarily observed in young girls (Malhotra, 2010), though data on its prevalence is limited. In India, 4.5 percent of girls aged between 15-19 years have been subjected to sexual abuse (UNICEF, 2015). However, the studies conducted in urban India have shown that 42 percent of adolescent girls reported of being touched against their will, and most commonly reported perpetrators were neighbors (Jaya and Hindin, 2007).

A related problem is that of watching pornographic material. The children being quite vulnerable get easily provoked by watching pornographic clips/movies and would be instigated to have a firsthand experience. In the seventies, it was claimed that "pornography is the theory and assault is the practice" but consequent studies could not establish this 'beyond reasonable doubt' (Bhattacharya, 2013). A public interest litigation (PIL) was filed by an advocate Kamlesh Vaswani in the Supreme Court of India calling for the criminalization of the viewing of porn stating that porn

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is a "moral cancer" that corrupts the society, and ready availability of online porn is a leading cause of rape cases (Bhattacharya, 2013). In a Delhi rape case of a five year old girl, the police gave a statement that the rapist arrested for sexual assault had porn clips on his mobile phone and before the assault he allegedly watched the clips (Bhattacharya, 2013). Sexual abuse often does not result in lasting physical injuries or produce clear observable evidence; however it can be associated with various psychological and behavioral problems into adulthood (Canter and Paige, 2004). Since the sexual abuse questions are sensitive to many students, they might tend to evade such kind of discussions, or provide inaccurate responses.

Randomized Response Technique models (RRT models), introduced by Warner (1965), are commonly used in surveys involving sensitive questions. It is a research method which allows respondents to respond to questions on sensitive issues truthfully while maintaining confidentiality. Greenberg et al. (1969) modified the Warner's model by introducing the unrelated question technique. Instead of requesting the respondent to reply affirmatively or negatively to the sensitive research question, the alternatives were stated differently. Here, the respondents face a randomization device in which the sensitive research question occurs with known probability p and an unrelated question which has no possible embarrassment occurs with probability $(1-p)$. However, a question may be sensitive to one respondent but not sensitive to another. Taking this into account, Gupta et al. (2002) introduced optionality in the randomized response models. Respondents for whom the research question is not personally sensitive are instructed to directly answer the research question. Rest of the respondents provide their responses using the chosen RRT model.

In an optional model of this type, there are two parameters of interest- the sensitivity level of the question (proportion of respondents in the population who consider the question sensitive) and prevalence of the sensitive characteristic in the population. Estimating these two parameters simultaneously often requires a split-sample approach, and hence a much larger total sample size. Sihm et al. introduced a modified optional RRT model that eliminates the need for a split sample approach. In their approach, the sensitivity level is estimated first by using the usual Greenberg et al. (1969) model and then the prevalence of the sensitive characteristic is estimated by using the Gupta et al. (2013) optional RRT model. Thus both parameters are estimated sequentially using the same sample.

This paper presents a field test of the Sihm et al. model covering both the binary and the quantitative response situations. The field work validation of RRT and Optional RRT models was successfully done before by Ostapczuk et al. (2009) and by Gill et al. (2013). However our approach in this study does not rely on the usual split sample approach.

Using the Sihm et al. model, a field survey was conducted at a college of University of Delhi, India, in which research questions were "Have you ever been a victim of sexual abuse by friend or family member?" and "How many days in a typical month do you watch pornographic clips/videos/ movies on movie channels, WhatsApp, YouTube, Internet etc.?" The estimates of population mean and prevalence obtained by this method are compared to results obtained by using the face-to-face interview method and the confidential survey method using independent samples from the same population. Estimates of sensitivity level are only obtained by optional RRT methods since sensitivity is not an issue for non-optional models. Our expectation is that the estimates obtained by the Sihm et al. optional RRT method should fall between those obtained by the confidential survey method and the face-to-face survey method since there is complete loss of privacy in the face-to-face survey and no loss of privacy in confidential surveys. This would indicate that the optional RRT models help recover some of the lost truth resulting from the face-to-face survey condition.

2. Optional Unrelated Question RRT Models - Sihm et al.

Gupta et al. (2013) proposed a generalization of the original unrelated question models (Greenberg et al., 1969, 1971) by giving respondents the option of responding to the sensitive question directly if they feel that the question is not too sensitive to be answered directly, otherwise they can give a scrambled response using Greenberg et al. (1969) model for binary response and using Greenberg et al. (1971) model for the quantitative response.

Sihm et al. proposed a modified version of the Gupta et al. (2013) model where the sensitivity level of the question and prevalence of the sensitive characteristic in the population are estimated simultaneously, as described earlier.

2.1. Binary Response Situation

Each respondent answers two questions. Question 1 asks if the respondent considers the research question too sensitive for a face-to-face interview. Question 2 asks the real research question (for example if the respondent has been victim of a sexual abuse by a friend or family member). We will use the following notation:
Let

- π_a be the known prevalence of an unrelated question when estimating the sensitivity level of the underlying research question using Question 1,
- π_b be the known prevalence of an unrelated question when estimating the prevalence of the underlying binary research question using Question 2,
- π be the unknown proportion of population that belongs to the sensitive group,
- p_a be the known probability of the respondent selecting the question about sensitivity in Question 1,
- p_b be the known probability of the respondent selecting the sensitive question in Question 2,
- w be the sensitivity level of the survey question in the population,
- P_{y1} be the probability of "yes" response from a respondent to Question 1 in a simple random sample with replacement of size n,
- P_{y2} be the probability of "yes" response from a respondent to Question 2 in the same simple random sample with replacement of size n.

We have

$$P_{y1} = p_a w + (1 - p_a)\pi_a. \tag{2.1}$$

Solving for w, we get

$$w = \frac{P_{y1} - (1 - p_a)\pi_a}{p_a}. \tag{2.2}$$

Thus, the estimator of w is given by,

$$\widehat{w} = \frac{\widehat{P}_{y1} - (1 - p_a)\pi_a}{p_a}, \tag{2.3}$$

where $\widehat{P}_{y1}$ is the sample proportion of 'yes' responses to Question 1.
It is clear that $\widehat{w}$ is an unbiased estimator of w with

$$Var(\widehat{w}) = \frac{P_{y1}(1 - P_{y1})}{n{p_a}^2}. \tag{2.4}$$

Similarly,

$$P_{y2} = (1-w)\pi + w\{\pi p_b + (1-p_b)\pi_b\}. \tag{2.5}$$

Equation (2.5) can be rearranged as,

$$P_{y2} - \pi = w(1-p_b)(\pi_b - \pi). \tag{2.6}$$

Solving for π, we get

$$\pi = \frac{P_{y2} - (1-p_b)w\pi_b}{1-(1-p_b)w}. \tag{2.7}$$

Thus we have the estimator of π given by,

$$\widehat{\pi} = \frac{\widehat{P}_{y2} - (1-p_b)\widehat{w}\pi_b}{1-(1-p_b)\widehat{w}}, \tag{2.8}$$

where $\widehat{w}$ is obtained from equation (2.3) and $\widehat{P}_{y2}$ is the sample proportion of 'yes' responses to Question 2.
After applying first order Taylor's expansion to equation (2.8), we have

$$\widehat{\pi} = \widehat{\pi}(P_{y2,w}) + \left.\frac{\partial\widehat{\pi}(\widehat{P}_{y2},\widehat{w})}{\partial\widehat{P}_{y2}}\right|_{(P_{y2},w)}.(\widehat{P}_{y2} - P_{y2}) + \left.\frac{\partial\widehat{\pi}(\widehat{P}_{y2},\widehat{w})}{\partial\widehat{w}}\right|_{(P_{y2},w)}.(\widehat{w} - w), \tag{2.9}$$

which gives

$$\widehat{\pi} = \frac{P_{y2} - (1-p_b)w\pi_b}{1-(1-p_b)w} + \frac{\widehat{P}_{y2} - P_{y2}}{1-(1-p_b)w} + \frac{(1-p_b)(P_{y2}-\pi_b)(\widehat{w}-w)}{\left\{1-(1-p_b)w\right\}^2}. \tag{2.10}$$

The approximate mean of $\widehat{\pi}$ is then given by,

$$E(\widehat{\pi}) = \frac{P_{y2} - (1-p_b)w\pi_b}{1-(1-p_b)w} = \pi. \tag{2.11}$$

The variance of $\widehat{\pi}$ (using 2.4) is given by,

$$Var(\widehat{\pi}) = \frac{1}{\left\{1-(1-p_b)w\right\}^2}\left\{\frac{P_{y2}(1-P_{y2})}{n}\right\} + \frac{(1-p_b)^2(P_{y2}-\pi_b)^2}{\left\{1-(1-p_b)w\right\}^4}\left\{\frac{P_{y1}(1-P_{y1})}{np_a^2}\right\}. \tag{2.12}$$

2.2. Quantitative Response Situation

Just as in the binary case, each respondent answers two questions using two different randomization devices. Using first randomization device the respondent answers the sensitive question "Is the research question too sensitive to be answered directly in face-to-face surveys?" or an innocuous question. Using second randomization device the respondent answers the sensitive question (for example, "How many days in a typical month do you watch pornographic clips/videos/movies on movie channels, WhatsApp, YouTube, Internet etc.?" or an innocuous question. Question1 estimates the underlying sensitivity level from the sample whereas Question 2 estimates the mean of the sensitive question.

As established in the binary case, the estimate of sensitivity level w is given by,

$$\widehat{w} = \frac{\widehat{P}_{y1} - (1-p_a)\pi_a}{p_a}, \tag{2.13}$$

with its variance as,

$$Var(\widehat{w}) = \frac{P_{y1}(1-P_{y1})}{np_a{}^2}. \tag{2.14}$$

We introduce the following additional notations.
Let

- $\mu_Y \; and \; \sigma_Y^2$ be the known mean and variance of an innocuous question,
- $\mu_X \; and \; \sigma_X^2$ be the unknown mean and variance of the sensitive question of the population,
- Z be the reported quantitative response to Question 2 from a respondent.
  Then,

$$Z = \begin{cases} X \; with \; probability \; (1-w)+wp_b \quad (Sensitive \; question), \\ \\ Y \; with \; probability \; w(1-p_b) \quad (Innocuous \; question). \end{cases}$$

Thus the mean and variance of Z are given by,

$$E(Z) \; = \mu_Z \; = \; \big\{(1-w)+wp_b\big\}\mu_X + w(1-p_b)\mu_Y, \tag{2.15}$$

and

$$Var(Z) = \big\{(1-w)+wp_b\big\}E(X^2) + w(1-p_b)E(Y^2) - \mu_Z^2, \tag{2.16}$$

$$= \big\{(1-w)+wp_b\big\}(\mu_X^2+\sigma_X^2) + w(1-p_b)(\mu_Y^2+\sigma_Y^2) - \mu_Z^2. \tag{2.17}$$

From equation (2.15),

$$\mu_X \; = \; \frac{\mu_Z - (1-p_b)\mu_Y w}{1-(1-p_b)w}. \tag{2.18}$$

Thus μ_X can be estimated by,

$$\widehat{\mu}_X \; = \; \frac{\widehat{\mu}_Z - (1-p_b)\mu_Y \widehat{w}}{1-(1-p_b)\widehat{w}}, \tag{2.19}$$

where $\widehat{w}$ is the unbiased estimator for w obtained from equation (2.13).
After applying first order Taylor's expansion to equation (2.19), we have

$$\widehat{\mu}_X = \widehat{\mu}_X(\mu_Z, w) + \frac{\partial \widehat{\mu}_X(\widehat{\mu}_Z, \widehat{w})}{\partial \widehat{\mu}_Z}\bigg|_{(\mu_Z,w)}.(\widehat{\mu}_Z - \mu_z) + \frac{\partial \widehat{\mu}_X(\widehat{\mu}_Z, \widehat{w})}{\partial \widehat{w}}\bigg|_{(\mu_Z,w)}.(\widehat{w}-w), \tag{2.20}$$

$$= \frac{\mu_Z - (1-p_b)\mu_Y w}{1-(1-p_b)w} + \frac{\widehat{\mu}_Z - \mu_Z}{1-(1-p_b)w} + \frac{(1-p_b)(\mu_Z-\mu_Y)(\widehat{w}-w)}{\big\{1-(1-p_b)w\big\}^2}. \tag{2.21}$$

The mean of $\widehat{\mu}_X$ is given by,

$$E(\widehat{\mu}_X) \; = \frac{\mu_Z - \mu_Y(1-p_b)w}{1-(1-p_b)w} \; = \widehat{\mu}_X. \tag{2.22}$$

The variance of $\widehat{\mu}_X$ is given by,

$$Var(\widehat{\mu}_X) = \frac{1}{\big\{1-(1-p_b)w\big\}^2}\left\{\frac{Var(Z)}{n}\right\} + \frac{(1-p_b)^2(\mu_Z-\mu_Y)^2}{\big\{1-(1-p_b)w\big\}^4}\left\{\frac{P_{y1}(1-P_{y1})}{np_a^2}\right\}. \tag{2.23}$$

3. Field Work Validation

Sihm et al. have presented extensive simulation results to show that their proposed model works well. The main objective of this study is to offer a field work validation of this model.

TABLE 3.1. Estimates of Sexual Abuse Prevalence.

Method	$\widehat{\pi}$	$\widehat{Var}(\widehat{\pi})$
Confidential Method	0.21	0.00102
Face to Face Survey	0.15	0.00078
Optional RRT Model	0.39	0.002738

3.1. Initial Survey Attempt

In September 2014, an initial field survey of 492 students was conducted by the authors at a college of University of Delhi, India. The respondents were briefed about the general RRT methodology and optional unrelated question RRT models. Respondents were given a hand out about the survey one-week prior to the actual implementation. Respondents were told that participation was voluntary and no incentives were provided. The entire sample was divided equally in three randomly selected groups. The binary sensitive question used in the study was "Have you ever been a victim of sexual abuse by friend or family member?" and the corresponding innocuous question was "Were you born between January 1 and March 31?" The randomization device chosen was a deck of cards in which there were 20 cards of which 12 cards were "numbered cards" (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10) and 8 cards were "face cards" (J, Q, K). If the respondent faced a "numbered card" (which would happen with probability, p_a = p_b =$\frac{12}{20}$), then she has to answer the sensitive question in both cases.

If the respondent answers that the main research question is not sensitive in first randomization device then she has to ignore the card drawn in second randomization device and answer the main research question directly, otherwise she answers the question as per the randomization device.

3.1.1. Survey Results

In the confidential survey, 35 of the 164 subjects replied with a "yes" answer, whereas in face-to-face survey, 25 of the 164 subjects replied affirmatively. In the optional unrelated question RRT method, 79 out of 164 subjects responded positively for Question1 while 59 out of 164 subjects responded positively for Question 2. Using the above mentioned recorded responses, the prevalence of sensitive character and its variance are estimated in Table 3.1. For the optional model, we get $\widehat{w} = 0.60$ and $\widehat{Var}(\widehat{w}) = 0.00422$.

It is significant to observe that the survey results are counter intuitive. Ideally, since the confidential survey method and optional unrelated question RRT method maintain the privacy of the respondents, the numerical estimates of prevalence of sexual abuse obtained in both these survey techniques should be close. But the survey condition where an optional unrelated question RRT model is used produced a very high estimate of the prevalence of sexual abuse. We suspect it is due to the poorly prepared randomization device used here. This inadequacy might have occurred because of insufficient randomization of the deck which may be due to improper shuffling or because of the small size of the deck (20 cards only). Since the results obtained for binary model were not very promising, we did not attempt validation of the quantitative model.

TABLE 3.2. Estimates of Sexual Abuse Prevalence.

Method	$\widehat{\pi}$	$\widehat{Var}(\widehat{\pi})$
Confidential Method	0.138461	0.00061174
Face to Face Survey	0.082051	0.00038600
Optional RRT Model	0.119021	0.00067577

3.1.2. Modified Survey Attempt

The above study was again repeated at the same college of University of Delhi, India for both the binary and the quantitative responses during January 2015 with a modified randomization device. The modified randomization device for the binary model was a deck of 100 cards in which the sensitive question "Is the research question too sensitive to be answered directly in face-to-face survey?" is asked with 60% probability and the innocuous question "Were you born between January 1 and March 31?" is asked with 40% probability. Recall that we had naively used a deck of 20 cards only in the first attempt. The second randomization device was also a deck of 100 cards where the sensitive question "Have you ever been a victim of sexual abuse by friend or family member?" is asked with 60% probability and the innocuous question "Were you born between January 1 and March 31?" is asked with 40% probability. The larger deck was expected to create greater degree of randomness as it could lead to proper shuffling. It would also increase trust of respondents in the randomization device.

Prior to participation, all students were given a short lecture about the general RRT methodology, questions to be asked in the study, and were assured of total confidentiality of their response. There were a total of 585 volunteers for the survey who were divided equally and randomly in three groups. Respondents were surveyed by one of the three methods: the optional unrelated question RRT method described above, direct face-to-face interview and the confidential survey method.

3.1.3. Survey Results

In the confidential survey, 27 out of 195 subjects gave a "yes" response whereas in the face-to-face survey, 16 out of 195 subjects replied affirmatively. With the optional unrelated question RRT method, 40 out of 195 subjects responded positively for Question1 while 25 out of 195 subjects responded positively for Question 2. Using the above mentioned recorded responses, the estimated prevalence of sensitive characteristic and its estimated variance are in Table 3.2. For the optional model, we get $\widehat{w} = 0.1752$ and $\widehat{Var}(\widehat{w}) = 0.002322$.

For binary method, the estimate of π is highest when obtained by the confidential survey method ($\widehat{\pi} = 0.138461$) and lowest in the face-to-face survey ($\widehat{\pi} = 0.082051$) with optional unrelated question RRT method estimate being in the middle, as expected ($\widehat{\pi} = 0.119021$).

3.1.4. Quantitative Model Using Modified Scrambling Device

As the results obtained for the binary model seemed to be promising, an attempt was made for the validation of quantitative model also using the same randomization device. The sensitive question used in the quantitative response model was "How many days in a typical month do you watch pornographic clips/videos/movies on movie channels, WhatsApp, YouTube, Internet etc?" and the corresponding innocuous question was "What is the number listed on your card?" The numbers

TABLE 3.3. Estimates of mean number of days the Respondent Watch Porn Clips.

Method	$\widehat{\mu_X}$	$\widehat{Var}(\widehat{\mu_X})$
Confidential Method	3.216	0.825404
Face to Face Survey	1.560	1.026010
Optional RRT Model	2.765	0.011500

listed on the card ranged from 0 to 4 following a binomial distribution with $n = 4$ and $p = 0.2$. Using the responses recorded during the survey, estimated mean of the sensitive question and its estimated variance are in Table 3.3. With the optional model, we get $\widehat{w} = 0.26066$ and $\widehat{Var}(\widehat{w}) = 0.002716$.

The estimate of μ_X obtained by optional unrelated question RRT method ($\widehat{\mu_X} = 2.765$) is closer to the estimate given by the confidential survey ($\widehat{\mu_X} = 3.216$).The lowest point estimate of μ_X is obtained by the face-to-face survey ($\widehat{\mu_X} = 1.56$) which is expected since this method provides the least anonymity.

4. Discussions

The authors wanted to validate the findings of Sihm et al. optional unrelated question RRT model in real situations where the respondents have the tendency to evade sensitive questions. It was observed from this study that optional unrelated question RRT method helps recover some of the lost truth resulting from face-to-face survey condition. It is also interesting that these students found the question on sexual abuse less sensitive ($\widehat{w} = 0.1752$) as compared to the question on watching pornographic material ($\widehat{w} = 0.26066$). A possible explanation might be that sexual abuse is something forced on someone but watching porn is an individual choice. Since sensitivity level is estimated by optional unrelated RRT method only, there is no comparison of sensitivity levels for the three survey methods.

An important message from this study is that complex survey designs need to be implemented carefully. Our first attempt did not succeed due to inadequate randomization device used whereas the second attempt worked much better with a better randomization device.

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