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Modeling with Advanced Engineering Applications: The Overview of CI MATH 4990, the Propagation of Heat along the Human Arm in Electric Arc Phenomenon

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ABSTRACT. This creative inquiry seeks to educate students of all STEM majors in the development of mathematical models of physical phenomenon using a multi-disciplinary approach to examining the physical phenomenon behind the propagation of heat through the human arm by means of the electric arc phenomenon. With an end goal of effectively modeling this phenomenon, the creative inquiry involves students of all majors while increasing research, presentation, and mathematical skills in areas directly related to their course of study.

1. Introduction

The course CI MATH 4990 had the main goal of guiding its students, all members of STEM majors, in the direction of a better understanding of mathematical models and how to apply these to physical phenomenon as well as throughout their respective majors. Our course deals with the problem of heat propagation along the human arm involving the electric arc phenomenon. This problem, in and of itself, is quite profound. There are many factors which must be considered in the approach of this problem. So, since the propagation of heat along the human arm in electric arc phenomenon is so complex, we divided this issue into its basic components and measured out multi-disciplinary groups of students to get a better understanding of the fundamentals behind the entirety.

First, we dealt with the bounds and conditions of our human arm, and for the sake of our problem, our students treated this arm as a viscoelastic medium much like an insulated cylinder which is open on both ends. Second, we considered the electric arc itself, which is a decaying and increasingly powerful electric phenomenon. Third, we took note of what a viscoelastic material is, how it behaves, and under our conditions, how it would be expected to act. Fourth, we considered general heat propagation along various materials so as to build up the understanding and a foundation to move forward from. Finally, we delved into mechanical vibrations and how they affect the system that we are considering, the human arm.

2. Methods

Our course, CI MATH 4990, split the problem of propagation of heat along the human arm under the electric arc phenomenon into multiple, related problems so that we might approach the whole of the topic from various directions and with a more rounded perspective. So, we divided the project into the following groups: heat propagation, viscoelasticity, electric arc, and mechanical

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vibrations. All of these groups followed a similar format: they all proceeded to research and understand their topics so as to bring this cultivated knowledge before the class and to present it to us all in an organized and intelligible manner. All of these presentations were followed by learning activities and these were meant to test the class so as to assure a solidity of gained knowledge.

The students within these respective groups all used their own time, outside of class, and special skills derived of their various disciplines in order to obtain information on their topics of research. Once the members became familiar and versed with their topics of discussion, they came in and presented their topic, in a PowerPoint format, to the class as a whole. After a group presented, every member in the class would take time to ask questions of the group so as to better understand the topics themselves. Likewise, the group in question was tasked with preparing a list of questions to give to the class so as to test them for their attentiveness and understanding of the new material.

Each presentation period was made to last roughly fifteen minutes with an equal amount of time after to test the students on what they had learned during. The time after each presentation was divided thusly, first was a five minute session dedicated to a verbal test and second was a ten minute test utilizing a written format. The verbal tests were monitored by the supervisors of the course and, similarly, the written tests were graded for accuracy by the supervisors and the presenting group. This post-testing was meant to reinforce the material covered by each group.

3. Presentation

The heat propagation group covered topics such as: types of heat transfer, heat model and equation, heat gradient, and a brief talk on Fourier's Law of Heat Transfer. Types of heat transfer can be split into conductive, radiation, and convection. Each of these heat transfer types allow for heat to be transferred in a different manner- conductive through contact of two objects, radiation via electromagnetic radiation, and convection through interactions between an object and its environment. The model used by this group proposed treating our environment like an insulated rod with open ends, such that the ends allow for propagation of heat flow, the equation for this model and its heat gradient is given by a one-dimensional heat equation $u_t = ku_{xx}$ Further, the heat propagation group went on to detail non-homogenous partial differential equations. This group used its multidisciplinary skills to look at the important aspects of their topic and bring them together for the class. We were presented with methods of solving partial differential equations, an integral aspect in dealing with all of the issues within the overall problem of the propagation of heat along the human arm.

Next, the viscoelastic group presented and introduced their topic by basic definitions and then examples for a more in-depth understanding. The viscoelastic group first defined what a viscoelastic material is by the components of its name: "A material that exhibits resistivity to shear stress and material memory after stress reduction." The group went on to detail types of viscoelasticity and how they relate to our modeling course, they focused on models as closely related to the human arm as possible. The response of viscoelastic materials to stress and strain showed marked resemblance to step responses seen in circuits- this was hinted at by the group as they mentioned that viscoelastic materials can be modeled as electrical systems. This group detailed the Maxwell model, Kelvin-Voight model, and a Standard Linear Solid model to give different understandings of the factors involved in a viscoelastic material- the kind of material that we are treating as being the medium of our heat transfer.Lastly, the viscoelastic group finished their presentation by showing graphs of stress and strain over time.

Another group that brought a unique set of information to our course was the electric arc group. This group dealt with the characteristic of electric arcs through materials and their general behavior. Firstly, beginning with a definition, this group introduced electrical arcs as currents which pass between two mediums by ionizing the space in-between these two mediums and exhibiting a particularly interesting quality of negativity resistivity. Negative resistivity means that these arcs pass more and more currents as time increases and in so doing increase the heat of the medium being arced to. Usually, these arcs break down on their own due to either 1) heat building up and breaking down the system or 2) too much current and not enough of the source to supply it. These arcs are integral to heat transfer in that they respond to the temperature of the system and increase or decrease in intensity based on the system temperature.

The mechanical vibrations group covered the topics of stress and strain, hysteresis effect, heat explosion, modeling heat transfer using Fourier's law, modeling heat transfer using Maxwell-Catteneo equation, and self-heating. Stress and strain, ties in with the previous group which covered viscoelasticity, and was further defined in this group's presentation. Stress and strain is defined as a relationship between an applied force to a material and its resulting deformation. Hysteresis effects were defined as a relation between the loading and the unloading of a true viscoelastic material. Further, hysteresis effects can be seen as a loop where the area between the curves of the stress and strain of the material in question is the energy lost in the deformation of the material. Heat explosions, detailed by this group, connected with self-heating and showed that a material can exhibit catastrophic thermal failure under high loading or cyclic loading conditions. Heat explosions, though capable of being exhibited, normally require considerable energy and in our environment are much more unlikely, as they would have to be forced, even though we are dealing with an insulated system. This group also covered how to use Fourier and Maxwell-Catteneo equations to model heat transfer under different conditions. Further, we concluded that Maxwell-Catteneo equations are a much more realistic approach to modeling heat transfer and heat flow within a viscoelastic material.

4. Post-Presentation Testing

The testing that was conducted after each group presentation was two part: first was the verbal portion and second was the written portion. The verbal portion was a simple question and answer session; in this part each student was not tested individually but rather the class was tested as a whole on their memory of the presentation. On the written portion, however, we were able to discern which students were paying close attention and who were taking the time to understand the presentation as this test asked simple and basic questions related to the individual slides that were brought before the class. These post-presentation tests were dubbed "learning activities" for this course, CI MATH 4990.

The tests were graded on a pass-fail basis where the student in question either had a grasp on the subject that was presented to them or they were completely oblivious. We did not grade as a means to harm the students but rather we were testing to solidify that which was presented to them. Incentives were offered to the class for performing well on these post-presentation tests. One incentive was a good grade for adequate performance. Another large bonus for outstanding, class-wide marks was an end of the semester dinner. The members of the class performed well enough to obtain the primary and secondary bonuses.

5. What was Gained

The use of learning activities reinforced all introduced topics and made for lessening the loss of information between presenter and attendee for each class session and each new subject. Likewise, these tests forced our students to know each and every other group's information as well as their own. Many members of the course commented as to how having to know about another method of modeling heat transfer deepened their own ability to model by their respective means. So, these learning activities show great promise and we plan to implement them in all further levels of this course. Also, these learning activities will be more regulated in further levels of the course and require for an increased grasp on each subject.

Through their work with gaining knowledge on and learning how to model these various aspects of heat transfer, the members of CI MATH 4990 obtained valuable analytical tools and experience with modeling realistic conditions. Many members within these groups noted that they gained valuable knowledge that can be used in their respective majors, ranging from pure math to industrial engineering and all the way to mechanical engineering and electrical engineering. One member, through their efforts with modeling heat transfer using Fourier has gained skills with maple in solving partial differential equations, an invaluable ability for later work. Also, this same member has stated that after taking this course that they have a brighter outlook and wish to pursue graduate school as they have a new found interest in research. A mechanical engineering major had mentioned that this work with heat transfer has given them confidence and readiness for a later course that they will have to take dealing with this exact topic. An electrical engineering major also said how they felt that they had a more peculiar outlook on materials after undergoing this course and that they wish to pursue more abstract views on electrical systems thanks to insights provided from this course.

All of our groups gained a considerable amount of knowledge in their respective assigned topics whilst also gaining the appropriate information from other groups and their focuses in order to be capable of connecting the dots, per say, and became more capable of attacking the problem of propagation of heat along the human arm by means of the electric arc phenomenon. All groups were highly competitive and attempted to outmatch one and other, so each week the presentation quality and depth increased considerably. Likewise, having multidisciplinary groups working on these various topics allowed for them to view their subject from different angles as was a notable characteristic of the presentations for all groups. There would always be a sharing of the presentation as certain members would be more knowledgeable on a specific subtopic as it is within their background. The method that was used to introduce each topic within our problem, the presentation by group, was an overall success. Similarly, CI MATH 4990 was a success as all members of our class now have the capacity to discuss their topics at much higher levels than they could at the beginning.

6. Future Work

Our future plans are to engage these same students in a much more rigorous study of our problem of the propagation of heat along the human arm by means of the electric arc phenomenon and to further delve into the mathematical models related to our problem. Also, we plan to tie in the analytical methods taught to each student, from their respective majors, into this course so as to bring a more meaningful connection with the material of CI MATH 4990. Our goals are to expand our modeling tools and to explore new methods of modeling. Likewise, we plan to delve

further into computer simulations and modeling using MATLAB and Maple. One of our groups has already gained a considerable amount of invaluable skills resulting from their use of Maple in order to solve partial differential equations. So, it is our hope that in the future we might bring these same skills to the rest of the class and prepare an even stronger group of students which will be capable of modeling more intense problems and situations. We propose to bring these skills to our class both in terms of our problem and their respective majors- as it is our goal to deepen their skills in mathematical modeling.

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