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# Geographic smoothing of solar photovoltaic electric power production in the western USA

Kelly Klima<sup>1</sup>, Jay Apt<sup>2,3,a)</sup>, Mahesh Bandi<sup>4</sup>, Paul Happy<sup>5</sup>, Clyde Loutan<sup>6</sup> *and* Russell Young<sup>5</sup>

<sup>1</sup> RAND Corporation, Santa Monica, CA 90401

<sup>2</sup> Department of Engineering and Public Policy, Carnegie Mellon University, Pittsburgh, PA 15213, USA<sup>3</sup> Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213, USA

<sup>4</sup> Okinawa Institute of Science and Technology Graduate University, Okinawa, Japan 904-0495

<sup>5</sup> SoCore Energy, Chicago, IL 60654

<sup>6</sup> California Independent Systems Operator (CAISO), Folsom, CA 95630

<sup>a)</sup> **Author to whom correspondence should be addressed:** [apt@cmu.edu](mailto:apt@cmu.edu)

## Abstract

We examined the geographic smoothing of solar photovoltaic generation from 15 utility-scale plants in California, Nevada, and Arizona and from 19 commercial building installations in California. This is the first comparison of geographic smoothing from utility-scale and building-mounted PV, and the first examination of solar PV smoothing in this region. Our research questions were: (1) how does geographic smoothing of commercial building rooftop PV compare to that for utility scale PV?, (2) is the geographic smoothing found for utility-scale plants the same for the western US as in India? and (3) how does the geographic smoothing for PV compare to that of wind? By examining the power output of these generators in the frequency domain, we quantified the smoothing obtained by combining the output of geographically separated plants. We found that utility-scale and commercial rooftop plants exhibited similar geographic smoothing, with 10 combined plants reducing the amplitude of fluctuations at 1 hour to 18-28% of those seen for a single plant. We find that combining a few PV sites together reduces fluctuations, but that the point of quickly diminishing returns is reached after  $\sim 5$  sites, and that for all locations and plant sizes considered, PV does not exhibit as much geographic smoothing as is seen for combining wind plants. We present preliminary theoretical arguments for why geographic smoothing of PV plants is less effective than for wind plants. The slope of the high-frequency part of the PV power spectrum can at best be geographically smoothed (steepen) to an asymptotic spectrum of  $f^{-2}$ . This limit for PV has considerably less smoothing than for wind's geographic smoothing, shown theoretically and from observed data to be  $f^{-2.33}$ .

## I. Introduction

Solar photovoltaic (PV) generation has rapidly increased over the years creating unique operating challenges as depicted by the California ISO's (CAISO) "Duck Curve." By the end of 2016, the CAISO Balancing Authority had over 10,000 MW of utility installed solar and almost 6,000 MW of behind-the-meter rooftop Solar PV. Solar production provided 8.1% of California's electricity generation in 2016<sup>1</sup> and is showing rapid market share growth elsewhere in the world. More PV capacity was installed in the USA in 2016 than any other generation technology. Solar power production has significant short-term variability caused by clouds, diurnal intermittency, and seasonal variability. It has long been recognized that a portion of the short-term variability in solar PV plant generation (and in wind generation) might be mitigated by summing the outputs from geographically dispersed plants, or "geographic smoothing". Such reduction of PV's fluctuations may reduce the need for compensation by other generators, storage, and demand response.<sup>2</sup> Both theorists and practitioners have been interested in the effects of this variability and intermittency on power systems.<sup>3,4,5,6,7,8</sup> The authors examined the smoothing from both utility-scale and commercial rooftop PV installations in the western USA, and compare them against results from earlier research in the Indian state of Gujarat and to wind's smoothing.

Several researchers have used observed PV data to examine geographic smoothing of the output of solar PV plants output. Observations of output data from plants is preferred to work using measured or simulated solar irradiance, in part because real solar plants have power electronics that respond to changes differently than do irradiance monitors. For example, the inverters have a maximum power output that limits responses to cloud focusing; they also have finite response rates. Wiemken et al.<sup>9</sup> examined the standard deviation and generation duration curves of the average daily power generation from 100 rooftop solar PV systems in Germany using 5 minute time resolution data. Murata, Yamaguchi, and Otani<sup>10</sup> used a metric of the largest PV output fluctuation in a given time interval to examine geographic smoothing of PV generators in Japan, and found modest decreases in their output fluctuation coefficient as up to 20 small (0.1-5.6 kW) generators were aggregated. Mills and coauthors<sup>11</sup> used the ramp rate of utility-scale plants in Arizona, finding a reduction in the 1-min and 10-min extreme ramp rates when geographically separated plants are aggregated. Marcos et al.<sup>12</sup> used data from 7 PV plants with a total installed capacity of 20 MW and separated by 6-360 km to examine the reduction in the largest observed power fluctuation by adding plants together at discrete periods of 1, 4, 20, 60 and 600 seconds by adding plants together. They found that adding together the first few plants together quickly reduced fluctuations, but that adding additional plants showed limited value. Lave, Stein and Ellis<sup>13</sup> examined the sum of residential rooftop PV in Ota City, Japan (2.1 MW total) and a 19 MW plant in Colorado. The plants were similar in geographic extent (the utility-scale plant covered roughly twice the area as the rooftop installations). Using the maximum output change in 1 second over a single day at each site, they found that (1) the ramp rates were reduced by roughly an order of magnitude for the entire plant as compared to those at a single point, and (2) the relative variability decayed exponentially as additional houses, or (in the case of the utility-scale plant) inverters were added. They found that the smoothing decreased quickly as the first few houses or inverters are added, then decreased much more slowly as additional units were added.

Methods that used the step change over a particular interval or the standard deviation of output fluctuations in a period of time gave information on fluctuations at selected time scales. However, there is a method that allows the determination of how much smoothing is obtained at any desired time scale. The power output of a PV plant contains fluctuations at many frequencies. Curtright and Apt<sup>14</sup> obtained 10 seconds time resolution data over two years from a 4.6 MW utility-scale array in Arizona, and used the Fourier Transform to decompose the time domain data into the frequency domain, and found that the power spectrum shows a falloff toward higher frequencies that goes roughly as frequency  $f^{-1.3}$  (in addition to the expected peak at a frequency corresponding to 24 hours and its harmonics).

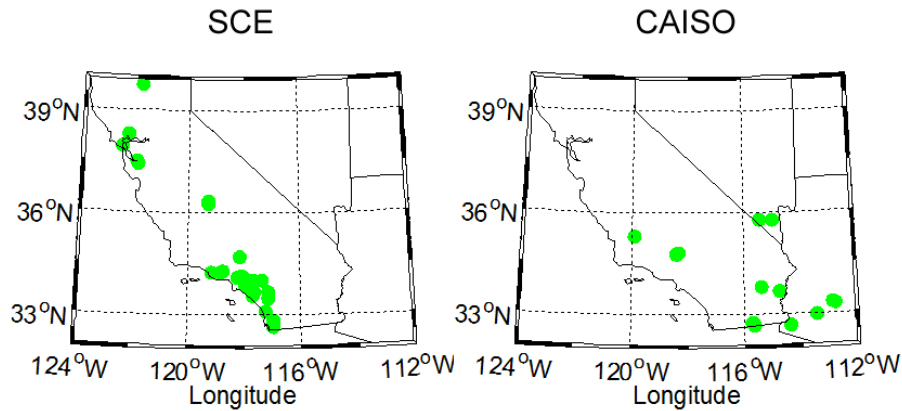
Klima and Apt<sup>15</sup> used information from the frequency domain to examine geographic smoothing of 20 utility-scale (3-221 MW) plants in the Indian state of Gujarat at 1-2 minute time resolution. They found that “Interconnecting approximately 20 plants yields a 25%-45% reduction in variability depending on [the] frequency examined.” This technique allowed the decrease in variability by adding plants together to be examined at any desired time scale. Similar to wind<sup>16</sup>, there is more smoothing at time scales of 10 minutes than at time scales of 6 hours. Also similar to wind, there is considerably more amplitude in the variations at long time scales. In agreement with Marcos et al. and with Lave, Stein and Ellis found quickly diminishing returns as more plants were added. However, the Gujarat data showed quite modest geographic smoothing for these plants even over 400 km. At time scales of one hour, for example, the Gujarat data showed that only half the amplitude of the variability at this time scale was eliminated; for wind plants with a similar geographic spread 95% of the variability was eliminated.

In this paper, the authors used both utility-scale and rooftop PV data from locations in the US states of California, Nevada and Arizona to study smoothing by using the frequency domain method. As far as we are aware, this is the first examination of the geographic smoothing of PV in the important PV region of the western USA. We believe it is also the first comparison of geographic smoothing from utility-scale and building-mounted PV.

Our research questions were: (1) how does geographic smoothing of commercial building rooftop PV compare to that for utility scale PV?; and (2) is the geographic smoothing found for utility-scale plants the same for the western US as in India? The analysis showed that geographic separation provides a similar degree of smoothing for rooftop PV and for larger PV plants. In both cases, solar PV benefits from considerably less geographic smoothing than wind. We found that geographic smoothing observed for PV in the western US is more substantial than that seen in Gujarat.

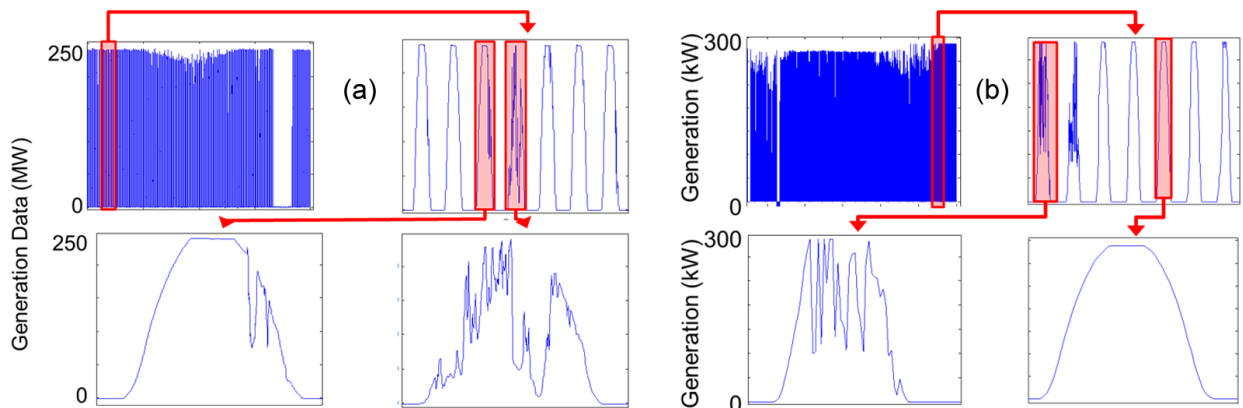
## II. Data

For the US utility-scale PV plants, we used proprietary data provided by the California Independent Systems Operator (CAISO) for one year at 1-minute resolution for 15 plants in California, Nevada, and Arizona. Plant sizes were 125-315 MW. Southern California Edison (SCE) and SoCore Energy, LLC provided one year of proprietary 15-minute resolution data for 19 rooftop PV arrays of 80-520 kW placed on commercial buildings such as drugstores. Plant locations for the CAISO and SCE data are shown in Figure 1.



**Figure 1.** Locations of solar PV systems. The green dots represent sites with data that were used. The CAISO data covered  $\sim 300 \times 800$  km, and the Southern California Edison (SCE) data  $\sim 175 \times 900$  km. The CAISO data are utility-scale PV arrays, while the SCE data are rooftop arrays on commercial buildings.

Sample generation characteristics for the CAISO data and SCE rooftop data are shown in Figure 2. Many of the SCE commercial rooftop inverters were sized so that the output reached the rated capacity on sunny days, and thus has a characteristic flat profile near local noon on clear days during the spring, summer, and fall months (or a “saturated profile”). Comparing geographic smoothing between the CAISO and SCE data only for January 2016 avoided any distortions that may arise from the saturated output.



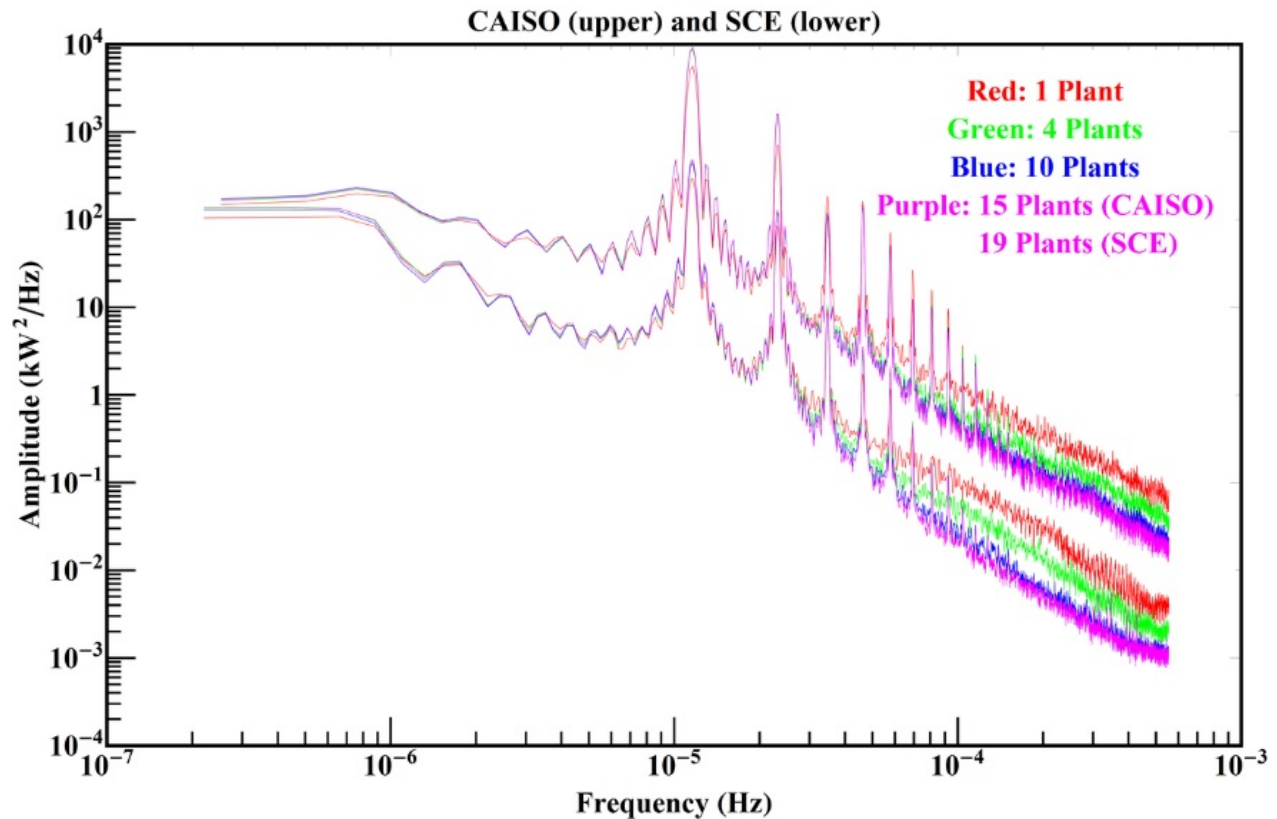
**Figure 2.** Generation data for one representative CAISO large plant (a) and for an SCE commercial building rooftop plant (b) for one year beginning July 1, 2015, one week, a partly cloudy day, and a clear day. The power electronics for both are sized so that the maximum output saturates on sunny summer days.

### III. Methods

As described fully in Refs. 15 and 16, the generation data in the frequency domain can be used to determine the geographic smoothing at any desired frequency. To briefly summarize the

method described earlier, we used Fourier Transform to decompose and examine the generation data in the frequency domain, where the power spectral density (PSD) at a particular frequency indicated the relative amount of variability at the corresponding time interval. To handle the observed uneven time steps in the data, we used the Lomb periodogram<sup>17</sup> as coded in Press et al.<sup>18</sup>. Although we find no difference between the Lomb periodogram procedure and a standard Fourier Transform procedure, we also used the Lomb periodogram for all data sets to ensure our procedure was similar between datasets,

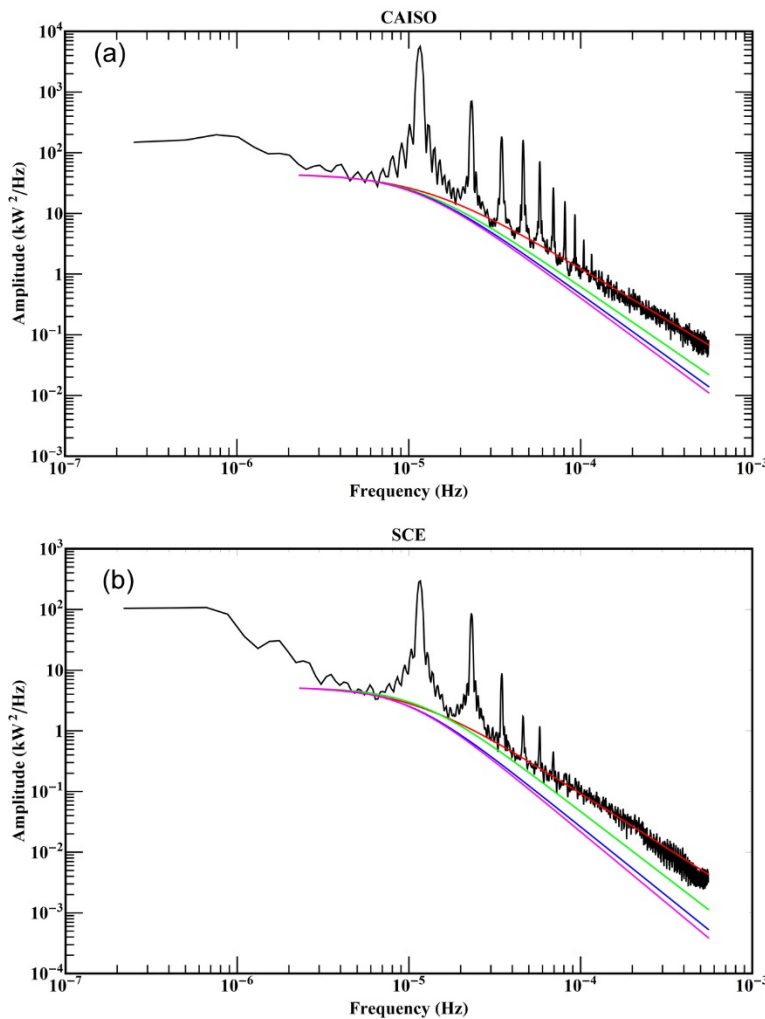
As for the India utility-scale PV data<sup>15</sup>, we conducted analysis to understand the potential for smoothing variability in plants. First, we created groups of all possible combinations of plants (regardless of size or geographic distance). For each group, we summed the generation of the plants in the time domain. Then, we calculated the PSD for each group of plants. Figure 3 shows representative PSDs for CAISO and SCE, and allows the reader to visually compare the slopes. The slopes for the CAISO dataset ranged from  $f^{-1.59}$  to  $f^{-1.64}$ , and the SCE dataset are from  $f^{-1.78}$  to  $f^{-1.91}$ .



**Figure 3.** Power spectral densities for a representative plant in CAISO and SCE (red), the sum of 4 plants (green), 10 plants (blue), and all plants (purple). The largest peak is at the frequency corresponding to 24 hours ( $1.2 \times 10^{-5}$  Hz), the second-largest to 12 hours, and the remaining peaks are higher harmonics. The SCE data are snapshots taken at 15-minute intervals, so the Nyquist frequency corresponds to 30 minutes. The PSDs for the 1-minute CAISO data have been truncated at the same frequency. 32-segment averaging has been used.

We investigated how interconnecting combinations of plants in each data set can potentially provide smoothing (Figure 4). The procedure (as in Ref. 15 and 16) was to compare the line of best fit for the two PSDs at particular frequencies by taking the ratio of the single plant value to that of the interconnected plants value. If no smoothing occurs when solar plants are interconnected, the result should be close to one at all frequencies. If there is a reduction in variability then there will be frequencies for which the fraction is less than one. This procedure allowed the determination of the fraction of the amplitude fluctuations that are smoothed by adding a given number of plants at any given frequency (for example, the frequency corresponding to 6 hours or to 1 hour, Figure 5).

As for utility-scale PV generation examined in earlier research (Ref. 14 and 15), the spectra were relatively flat at frequencies lower than  $\sim 10^{-5}$  Hz, and fell off at higher frequencies.

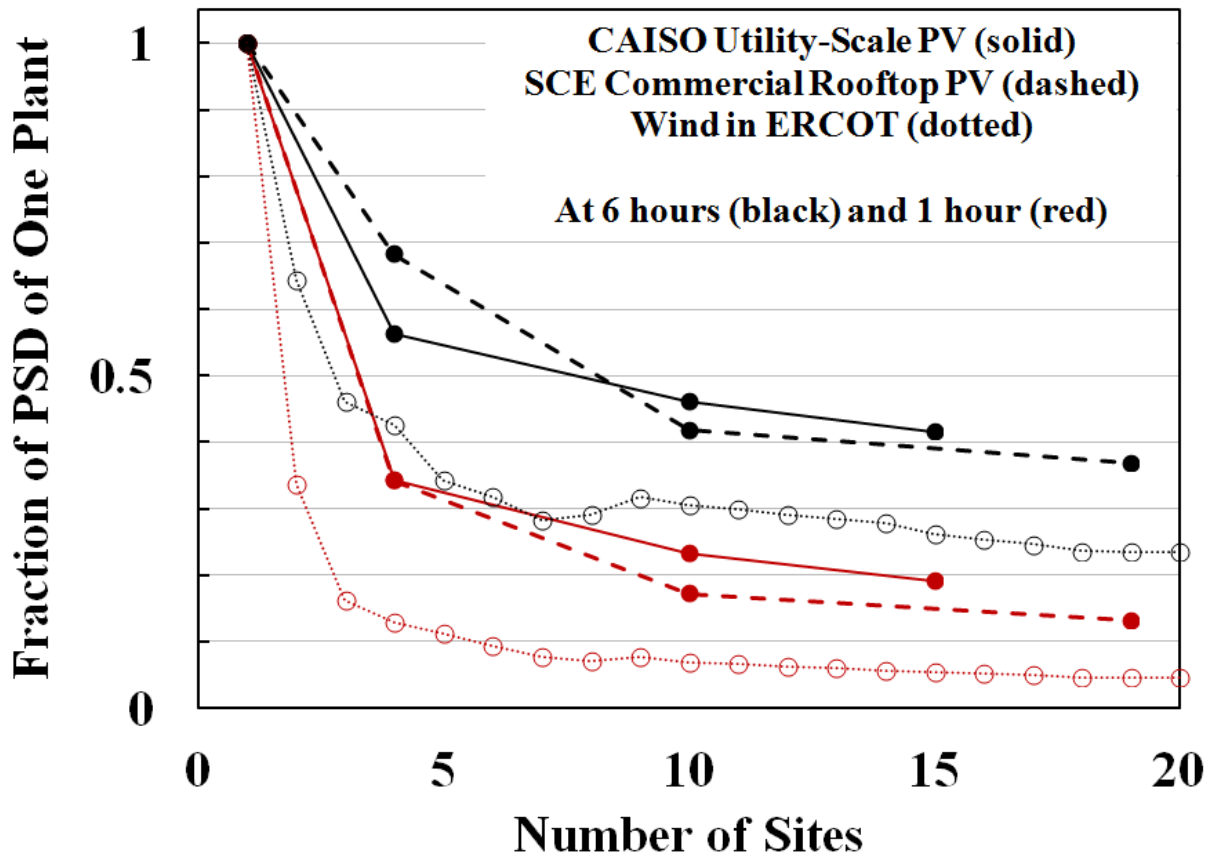


**Figure 4.** Power spectral density (PSD) plots for one CAISO utility-scale PV plant (a) and one SCE commercial rooftop plant (b). As for Figure 3, the largest peak is at the frequency corresponding to 24 hours ( $1.2 \times 10^{-5}$  Hz), the second-largest to 12 hours, and the remaining peaks are higher harmonics. Colored lines are the fits of the form  $A/(1+Bf^\alpha)$  to the PSDs for one plant (red), 4 plants (green), 10 plants (blue), and 15 plants for CAISO or 19 plants for SCE (purple). PSDs for the combination of plants are not shown for clarity.



#### IV. Results

We compared the smoothing observed for the utility-scale CAISO plants in California, Nevada, and Arizona (~300 x 800 km) to that for SCE rooftop PV in California (~175 x 900 km) and to the observed smoothing for wind plants over a similar geographic extent in Texas. Utility-scale and commercial rooftop PV plants exhibited similar geographic smoothing, but it was considerably less than what was observed for wind.<sup>16</sup>



**Figure 5.** Comparison of smoothing at 6 hours in black and 1 hour in red among the 120-315 MW utility-scale plants in CAISO (solid lines) with the 5-400 kW SCE commercial rooftop plants in California (dashed lines) and with the smoothing found by adding geographically separated ERCOT wind plants over 500 km<sup>16</sup> (dotted lines with open circles). The CAISO data covered ~300 x 800 km, and the Southern California Edison (SCE) data ~175 x 900 km. The ERCOT wind data covered ~200 x 500 km. A full year of data is used for all PV plants shown here, beginning July 1, 2015; the wind data of Ref. 16 shown here was for the full year 2008.

When geographically separated PV plants are aggregated together, the slope of the summed power spectrum steepens, meaning that the summed power has less variability at shorter time scales. Nearly identical steepening were observed when rooftop plants are summed the same as utility-scale plants (Figure 4).

No significant difference in geographic smoothing were observed between physically large utility-scale PV arrays and small commercial rooftop arrays. This suggests, for at least the two sets of capacity ranges examined, that geographic smoothing is more likely to be a function of the number of plants as opposed to the geographic size of the arrays. Unfortunately the geographic areas of the arrays were not available for this study, so we were unable to test hypotheses on smoothing as a function of geographic size of the plant (e.g., Marcos et al., 2011<sup>19</sup>). If a future study were able to obtain these data, it could examine policy implications for capacity or geographic array area ranges over which power could be smoothed.

Using identical techniques to examine the geographic smoothing of utility-scale PV plants in the Indian state of Gujarat<sup>16</sup>, the geographic smoothing observed for solar PV in both CAISO and SCE is more substantial than that that seen in Gujarat. One hypothesis to explain the difference is that if the weather in the Gujarat desert region is consistently clear that there may be fewer fluctuations to smooth than in the southwest USA. We explored this hypothesis by noting that the 10-year average monthly cloud fraction in Gujarat is indeed considerably lower than for the regions in the USA we examined, but only in eight months of the year. In June and September, the two areas have similar cloud fractions, and in July and August Gujarat is cloudier. When we performed our analysis by dividing the data into one set comprised of June through September and another with the remaining months, we saw no significant difference in the geographic smoothing. Thus we are forced to reject this hypothesis and simply note that there is considerably less geographic smoothing for utility-scale plants in Gujarat than for the plants in the USA.

## V. Discussion: Towards a Theory of PV Geographic Smoothing

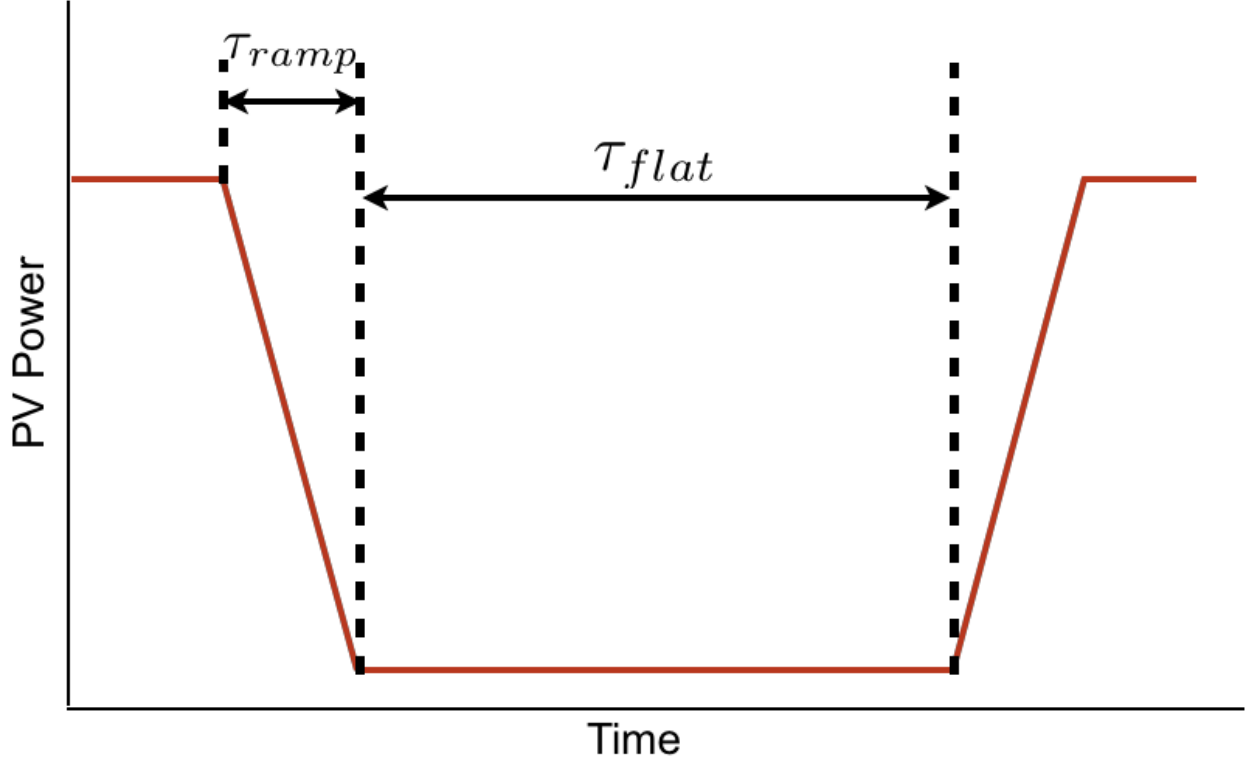
Solar PV plants experience less geographic smoothing than do wind plants (Figure 5). Wind and solar photovoltaic power fluctuations arise from quite different physical considerations. Wind speed varies around a usually non-zero mean value, which in turn causes wind power to fluctuate through the dimensional relation  $P \leq \left(\frac{16}{27}\right)\left(\frac{1}{2}\right)\rho Av^3$ , where  $\left(\frac{16}{27}\right)$  is the theoretical maximum energy fraction extractable from wind,  $\rho$  is the air density,  $A$  is the turbine rotor swept area, and  $v$  is the wind speed. Whereas the wind power fluctuations are amplified due to the cubic dependence on wind speed, they still fluctuate about a mean (or steady) power. The largest length scales of atmospheric flows spanning hundreds of kilometers and representing the lowest frequencies (inverse daylight time  $\sim 1.2 \times 10^{-5}$  Hz) influence all smaller length scales, hence higher wind speed fluctuation frequencies.<sup>20</sup> Consequently, the largest atmospheric flow scales of hundreds of kilometers represent the correlation length for wind power fluctuations, and therefore control the extent to which the geographic smoothing occurs in combined aggregate power of wind plants situated large distances from each other.

Solar PV plants on the other hand are globally correlated by their common energy source, the sun. The solar radiation amplitude exhibits continuous spatial (latitude dependent) variation as one moves from the equator towards the poles due to change in the angle at which solar radiation is incident upon the earth's surface. The temporal variation in this amplitude occurs over long, seasonal time scales due to the tilt in earth's axis as it revolves around the sun; these long time scales are of little interest to the energy community. However, at a chosen latitude, all PV plants situated along the circle of latitude – the abstract east-west circle connecting all

locations around the earth (ignoring elevation) at a given latitude – become correlated through time lags determined by the earth's diurnal rotation about its axis over the 24 hour time scale and the longitudinal distance between plants. PV plants located on the earth's surface facing the sun are positively correlated with concomitant time lags whereas plants on the night side exhibit negative correlation, which nonetheless is still a correlation in the power output. Circles of latitude are always parallel to each other unlike circles of longitude, which are always great circles with the center of the earth in the middle; circles of latitude therefore get smaller as the distance from equator increases. Therefore the time lags are controlled not only by longitudinal distance between PV plants, but also by the latitudinal location. Whereas the seasonal and diurnal variations in PV output occur over timescales too long relative to the electrical grid response timescale or the ISO's dynamic load balancing timescales, the important point to bear in mind here is the global correlation length spanning the circle of latitude, unlike wind power where it extends to  $\sim 100$ s of kilometers. As a result, the baseline solar PV spectrum is determined by the seasonal and diurnal low frequency behavior controlled by latitudinal and longitudinal positioning of a PV plant.

The disruption of this baseline spectrum at the high-frequency end due to solar occlusion by low altitude cloud passage is of particular interest to the solar PV community. Cloud passage disrupts the global correlation by causing a negative square wave dip in local PV power output. In sharp contrast with wind, solar PV fluctuations are strongly asymmetric about the (mean) clear sky index. Decreases in power output due to cloud passage contribute large magnitude fluctuations below the clear sky index (leading order effect), while cloud focusing contributes very small magnitude fluctuations above the clear sky index (higher order effect), in addition to background diffuse radiation which provides a rather constant DC offset (also higher order effect). Although, further theoretical work is required to fully understand the difference between PV and wind geographic smoothing, some quick estimates may be made.

Consider a PV plant of linear dimension  $l_{PV}$  over which a cloud passes at mean speed  $\bar{v}$  and casts a shadow of linear dimension  $l_C$ . Assuming the cloud completely occludes sunlight, i.e. a simple ON-OFF state, one observes a negative ramp in PV plant output as the cloud's shadow traverses to overlap the area occupied by PV panels and the PV output remains at a lower magnitude until the cloud shadow exits PV plant area when the power ramps up again (see fig. 6 for a schematic representation). We now define two length scales  $l_{ramp}$  and  $l_{flat}$ , where  $l_{ramp}$  represents the shortest of length scales between  $l_C$  and  $l_{PV}$ , and  $l_{flat}$  is the longest of length scales between  $l_C$  and  $l_{PV}$ . This provides two corresponding timescales  $\tau_{ramp} = l_{ramp}/\bar{v}$  and  $\tau_{flat} = (l_{flat} - 2l_{ramp})/\bar{v}$ .



**Figure 6.** Schematic representation of the effect of a cloud passing over a photovoltaic generator.

From the schematic in fig. 6, we see the primary contribution to PV spectrum from cloud passage at the high frequency end comes from  $\tau_{ramp}$ . In other words, all frequencies  $f \geq 1/\tau_{ramp}$  contribute to the high-frequency portion of the power spectrum. This tells us that there exists a critical frequency  $f_c \equiv 1/\tau_{ramp}$  at which the PV spectrum changes slope. All frequencies  $f < f_c$  exhibit a shallower slope determined by seasonal and diurnal oscillations. All frequencies  $f > f_c$  are determined by local cloud passage above the PV plant. Although the PV plant's linear dimension  $l_{PV}$  remains fixed, the size of the clouds can vary above and below  $l_{PV}$ , hence the critical frequency  $f_c$  is unfortunately not a constant. But for large utility scale PV plants at locations that experience small cloud sizes  $l_C < l_{PV}$ ,  $f_c$  can become a constant and scales as  $f_c = 1/\tau_{ramp} = \bar{v}/l_{PV}$ . Since the area of the PV plant is  $A_{PV} = l_{PV}^2$ , we see  $f_c \sim 1/\sqrt{A_{PV}}$  as reported for PV plants in Spain.<sup>21</sup> In the opposite limit of small-scale rooftop PV generators, we can be confident that  $l_{PV} < l_C$ , in which case  $f_c$  becomes a stochastic variable dependent on the size of the shadowing cloud.

So far, we have considered the effect of a single cloud on the PV power spectrum. Extending the simple ideas above to a train of clouds moving past a PV plant is not straightforward for two primary reasons. First, clouds come in a range of linear dimensions  $l$  that follow a power-law distribution of the form  $\Pi(l) \sim l^{-\alpha}$  where Wood et al.<sup>22</sup> reported the exponent  $\alpha = 1.66 \pm 0.04$  from satellite, aircraft, and modeling data over a range of  $l = 0.1 - 1500$  km. (The cloud shadow length  $l_C$  and cloud length  $l$  are related via the simple relation  $l_C = \gamma l$ , where  $\gamma$  is a constant that depends on the altitude of the cloud.) Through the simple linear transformation  $\tau_C = l_C/\bar{v}$ , we see that the distribution of cloud traversal time past a fixed

spatial location would also follow a power-law distribution  $\Pi(\tau_C) \sim (\bar{v} \tau_C)^{-\alpha}$ . Here  $\bar{v}$  may well be taken to be a constant from a long-time average, but becomes location dependent due to local geophysical flow conditions. A point to be borne in mind here is that although Woods et al. took all clouds into account in determining the cloud size distribution, for solar PV we are interested in only low altitude clouds that are capable of casting a shadow on the PV array. This fact may well change the slope of the distribution from  $\alpha = 1.66 \pm 0.04$  and needs to be determined in future work. This information would be readily available from the negative square wave dips measured for a long time series of power output from a PV plant, since the plant would experience dips from shadows cast by only low altitude clouds.

The second reason that complicates extension of the ideas developed for single cloud to a cloud train is the fact that clouds do not arrive at random intervals, but in clusters. Indeed, even the cloud arrival time statistics, commonly known as the waiting time distribution (to denote the waiting time between two successive cloud arrivals) is also known to be power-law distributed as measured directly from sensor and PV plant time series data in Hawaii and Germany by Tabar et al.<sup>23</sup> Power-law distributions are notorious for their non-analyticity which precludes one from developing a predictive model for solar PV fluctuation statistics and hence their spectra, except for rare special cases. The difficulty is readily seen in a simple manner. For a given quantity  $x$  with a probability density function  $\Pi(x)$ , its  $n^{\text{th}}$  moment is defined as  $\int_0^\infty x^n \Pi(x) dx$ . In order for this integral to converge and the  $n^{\text{th}}$  moment to be well defined, one requires that  $\Pi(x)$  should decay faster than  $x^n$ , else the integral will diverge. For power-law distributions, this requirement is tantamount to the statement that if  $\Pi(x) \sim x^{-(n+1)}$ , then only the first  $n$  moments of the distribution can be well defined. If  $\alpha = 1.66 \pm 0.04$  for  $\Pi(\tau_C)$ , all the moments of the distribution are not well defined. In that case, it is meaningless to measure even the mean (first moment) duration of cloud traversal past a PV plant, its variance (second moment) and so on.

Although the situation portrayed above may seem hopeless at first sight, one can still deduce a limiting case. Consider a number of utility scale PV plant outputs feeding the electrical grid. Suppose that whereas each individual PV plant may experience power-law distributed cloud traversal times of power-law distributed cloud sizes, when their outputs are combined in the aggregate, these correlated negative square waves look like a random train of negative square waves. In such a limit, we know the Fourier transform of a square wave is  $\frac{\sin(f_t)}{f_t}$ . For a random square wave train, the numerator averages to one because all phases of the sine waves are scrambled. Then the power spectral density is the absolute square of the Fourier transform which yields  $S(f) = 1/f^2$ . Ergo, we see that the slope of the high-frequency part of the PV power spectrum that is influenced by clouds can at best be smoothed (steepen) due to geographic smoothing to an asymptotic limiting spectrum of  $f^{-2}$ , which is steeper than the behavior of any single array. This asymptotic limit for PV is considerably shallower (less smoothing) than the asymptote for wind's geographic smoothing shown theoretically and from observed data to be<sup>20</sup>  $f^{-7/3}$  or  $f^{-2.33}$ .

## VI. Conclusion

We examined the geographic smoothing of solar photovoltaic generation from 15 utility-scale plants in California, Nevada, and Arizona and from 19 commercial building installations in California. We found that utility-scale and commercial rooftop plants exhibited similar

geographic smoothing, with 10 combined plants reducing the amplitude of fluctuations at 1 hour to 18-28% of those seen for a single plant. We find that combining a few PV sites together reduces fluctuations, but that the point of quickly diminishing returns is reached after  $\sim 5$  sites, and that for all locations and plant sizes considered, PV does not exhibit as much geographic smoothing as is seen for combining wind plants. We present preliminary theoretical arguments for why geographic smoothing of PV plants is less effective than for wind plants. Specifically, we see that the slope of the high-frequency part of the PV power spectrum that is influenced by clouds can at best be smoothed (steepen) due to geographic smoothing to an asymptotic limiting spectrum of  $f^{-2}$ . This asymptotic limit for PV is considerably shallower (less smoothing) than the asymptote for wind's geographic smoothing shown theoretically and from observed data to be  $f^{-7/3}$  or  $f^{-2.33}$ .

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