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Author	Sudarshan Ananth, Stefano Kovacs, Yuki Sato, Hidehiko Shimada
journal or publication title	Journal of High Energy Physics
volume	2018
number	135
year	2018-07-20
Publisher	Springer Berlin Heidelberg
Rights	(C) 2018 The Author(s).
Author's flag	publisher
URL	http://id.nii.ac.jp/1394/00000771/

doi: info:doi/10.1007/JHEP07(2018)135

Published for SISSA by Springer

Received: June 5, 2018
Accepted: July 4, 2018
Published: July 20, 2018

Towards a tensionless string field theory for the $\mathcal{N} = (2,0)$ CFT in $d = 6$

Sudarshan Ananth,[a] Stefano Kovacs,[b] Yuki Sato[c,d] and Hidehiko Shimada[e]

[a] *Indian Institute of Science Education and Research, Dr. Homi Bhabha road, Pune 411008, India*

[b] *Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland*

[c] *Department of Physics, Nagoya University, Chikusaku, Nagoya 464-8602, Japan*

[d] *Department of Physics, Faculty of Science, Chulalongkorn University, Thanon Phayathai, Pathumwan, Bangkok 10330, Thailand*

[e] *Mathematical and Theoretical Physics Unit, Okinawa Institute of Science and Technology, 1919-1 Tancha, Onna-son, Okinawa 904-0495 Japan*

E-mail: ananth@iiserpune.ac.in, skovacs@stp.dias.ie, ysato.phys@gmail.com, shimada.hidehiko@gmail.com

Abstract: We describe progress in using the field theory of tensionless strings to arrive at a Lagrangian for the six-dimensional $\mathcal{N} = (2,0)$ conformal theory. We construct the free part of the theory and propose an ansatz for the cubic vertex in light-cone superspace. By requiring closure of the $(2,0)$ supersymmetry algebra, we fix the cubic vertex up to two parameters.

Keywords: Conformal Field Theory, Field Theories in Higher Dimensions, M-Theory, String Field Theory

ArXiv ePrint: 1805.10297

https://doi.org/10.1007/JHEP07(2018)135

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Contents

1 Introduction

The possible existence of a superconformal field theory with $(2, 0)$ supersymmetry in six dimensions was first pointed out in [1]. A string theory origin for such a conformal field theory (CFT) was proposed in [2] and the theory was then identified as a candidate for the description of the low-energy dynamics of M5-branes, important but elusive degrees of freedom (DOF) in M-theory [3]. In recent years, the theory has also played a crucial role in various developments in mathematical physics, with particular attention being devoted to the classification of BPS observables and the study of their properties both in six dimensions and, upon compactification, in lower dimensions.[1]

The $\mathcal{N} = (2, 0)$ theory is also interesting from the point of view of the theory space of quantum field theory. This space is governed by the renormalisation group flow [5] in which fixed points, i.e. conformal field theories [6], are an essential feature. It is known that six dimensions is the highest dimension of spacetime that permits a theory with superconformal symmetries [1]. The very existence of a six-dimensional CFT is surprising because power-counting makes it difficult to write down interacting theories (except for a scalar ϕ^3 coupling, which does not satisfy the requirement of positive definiteness of the energy) involving a dimensionless constant in dimensions higher than four.

Despite the importance of the $\mathcal{N} = (2, 0)$ theory and the attention it has attracted in recent years, there is no consensus on whether it should admit a Lagrangian formulation. Various obstructions exist to the realisation of superconformal symmetry in a conventional six-dimensional local field theory. Several Lagrangian constructions have been proposed, including the matrix model approach involving a low-energy limit [7, 8], the dimensional deconstruction approach [9], and the decompactification limit of $d = 5$ maximally supersymmetric Yang-Mills theory [10, 11]. For other proposals, see [12–17] and references therein. Another interesting approach is based on the idea of the conformal bootstrap [18], which does not rely on the existence of a Lagrangian.

Although the use of the bootstrap method may render a Lagrangian description unnecessary, having an explicit Lagrangian formulation is desirable for a better understanding of the fundamental DOF of the $(2, 0)$ theory. Such a description would also clarify the relationship of the $(2, 0)$ CFT in $d = 6$ to lower dimensional maximally supersymmetric theories and in particular the $\mathcal{N} = 4$ super Yang-Mills (SYM) theory in four dimensions. Moreover, although the $(2, 0)$ CFT is inherently non-perturbative, as implied by its M-theory origin, a Lagrangian description should make it possible to construct reliable weak-coupling approximation schemes valid in special sectors and/or for special observables, such as near-BPS quantities. These ideas were exploited in [19, 20] in the case of the ABJM theory [21] — the maximally supersymmetric CFT in three dimensions, associated with coincident M2-branes — which is also intrinsically strongly coupled. In [19], using the AdS/CFT correspondence, a perturbative analysis of the spectrum in a special sector of the ABJM theory was successfully compared to the dual AdS description provided by the pp-wave matrix model [22].

In this paper we propose developing a Lagrangian for the $\mathcal{N} = (2, 0)$ theory in six dimensions, using String Field Theory (SFT) in light-cone gauge. The use of light-cone gauge

[1] A review by G. Moore including a detailed list of references can be found in ref. [4].

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is key to our approach since it allows us in principle to determine the interacting theory by a fairly straightforward — albeit technically involved — closure of the supersymmetry algebra [23, 24].

It has been argued that the six-dimensional $(2,0)$ theory contains tensionless string DOF. In particular, in the M-theory construction in which the $(2,0)$ theory describes the low-energy dynamics of a collection of M5-branes, the strings arise from M2-branes stretched between M5-branes. When the M5-branes are coincident the M2-branes reduce to closed strings in the world-volume of the M5-branes. Such strings are tensionless as their tension is proportional to the (constant) M2-brane tension times the separation between the M5-branes. While of course this construction does not imply that the fundamental DOF in the effective theory describing the world-volume dynamics of coincident M5-branes should be tensionless strings, it is certainly natural to consider such a possibility.

In the case of the four-dimensional $\mathcal{N}=4$ SYM theory, open strings ending on N coincident D3-branes give rise to matrix-valued point-like DOF. Similarly, when considering a stack of N coincident M5-branes, there are $N\times N$ configurations of M2-branes ending on the M5-branes, with each cylindrical M2-brane degenerating to a closed string constrained to the six-dimensional world-volume of the M5-branes. Therefore we obtain a six-dimensional matrix-valued closed string theory, that we will formulate using the language of string field theory in light-cone gauge.

The approach that we propose in this paper is to construct directly a theory of tensionless strings in six dimensions, using the light-cone string field theory formalism, rather than to take the tensionless limit in a theory with tension. The main reason leading us to this choice is that the zero tension limit of an ordinary tensile string theory is problematic and not well understood.[2] This is analogous to the case of general quantum field theories, in which taking a zero mass limit often requires careful analysis. The most appropriate procedure to study such a limit would involve computing physical observables and then taking the limit on these. However, the conventional first quantised formulation of string theory, in our present understanding, only allows one to compute S-matrix elements, whereas the good observables in a conformal field theory such as the one we are trying to construct are expected to be local correlation functions. Since local correlators in tensile string theory are not understood and, further, S-matrix elements in the tensionless limit can be singular and at least not straightforward to define, we propose to construct the $(2,0)$ CFT directly as a tensionless string theory in six dimensions rather than trying to define it as the tensionless limit of some string theory with tension.

The fact that the tensile strings and the $(2,0)$ CFT should have fundamentally different natural observables also supports our choice to use a second-quantised, string field theory,

[2]The zero tension limit of ordinary tensile string theory has been studied by many authors in connection with higher spin gauge theories. For an overview and references see [25]. The tensionless limit of bosonic covariant SFT [26] was studied in [27], where the possibility of formulating the $(2,0)$ CFT as the zero tension limit of SFT was also mentioned. Early work on tensionless strings includes [28–39]. Some discussions on the tensionless limit can be found in [40] and references therein.

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formulation.[3] This formalism should prove better suited to the study of the observables of a CFT. Further support for such an approach follows from the analogy with the case of point particles. The world-line (first quantised) formalism is not straightforward for the study of massless particles, which instead are simple to describe in the field theory (second quantised) language.

Our approach may be compared to the standard treatment of Yang-Mills theory. As is well known, it is easier to work with massless Yang-Mills theory directly, rather than thinking of it as a limit of a theory of massive interacting vector particles, the essential reason being the gauge symmetry of the theory in the massless case. One of course also uses the second-quantised field theory formalism, rather than a first-quantised formulation, for Yang-Mills theory.

A particular virtue of our approach regards the dimension of the coupling constant. In traditional field theory, the dimension of the coupling constant depends on the dimension of spacetime. This renders the program of writing down an interacting $d = 6$ Lagrangian, in particular with the correct supersymmetry, very difficult. In contrast, the physical dimensions of the coupling constant do not depend on the spacetime dimension in SFT and therefore, in principle, no obstruction arises from power counting arguments. We elaborate on this point in section 5.1.

Another promising feature in our proposal is related to dimensional reduction. The six-dimensional (2,0) theory is expected to reduce to the $\mathcal{N} = 4$ SYM theory in four dimensions when compactified on a torus. The coupling constant of the reduced theory, g_{YM}, is given by the formula $\frac{1}{g_{YM}^2} \sim \frac{R_1}{R_2}$, where R_1 and R_2 are the two compactification radii. Although the dependence on R_1, in this formula, can be easily understood in terms of a standard Kaluza-Klein reduction, the dependence on R_2 is much harder to understand in the context of an ordinary local field theory. Using (tensionless) string DOF, on the other hand, means that wrapped strings play a role in the reduction, thus introducing a distinction between the two compactification radii. This may lead to a mechanism for generating the required dependence on R_2 in the formula for the four-dimensional coupling constant.

The choice of light-cone gauge allows one to focus exclusively on the physical DOF and in this gauge symmetry constraints can be more directly implemented, so that one can restrict or even determine the theory purely from symmetry considerations. This idea of determining the interacting Hamiltonian by requiring the closure of the symmetry algebra has proven extremely fruitful in the past [41–44]. In particular, the entire $\mathcal{N} = 4$ SYM theory — for which the light-cone superspace formulation was first obtained in [23, 45] — can be derived from closure of the superconformal algebra [46]. The action describing light-cone superstring field theory in ten dimensions has also been derived to cubic order in this way in [47–50] and the full Lorentz symmetry of the theory up to cubic order was verified in [51]. We also recall that light-cone gauge bosonic string field theory was developed in [52–58] and a detailed study of the Lorentz invariance of the theory was presented in [53, 59–64].

[3]The distinction between the first quantised and the second quantised formulations is important at the interacting level. For the free part, the two descriptions are directly related to each other, in particular in the light-cone gauge.

Our aim is to construct an interacting theory of tensionless strings having the right amount of supersymmetry and a dimensionless coupling constant (which is a necessary condition for the scale invariance of the model) in six space-time dimensions.[4] In this paper, we present the construction of the quadratic and cubic parts of the SFT action. We formulate an ansatz, which we justify by using (part of) the restrictions imposed by the closure of the supersymmetry algebra. The cubic vertices that we obtain still contain two arbitrary parameters. Our construction is based upon the light-cone superspace formulation of the free particle with $(2,0)$ supersymmetry in six dimensions [65, 66].

Our approach combines features of both the light-cone formulation of $\mathcal{N}=4$ SYM and the supersymmetric closed SFT. It is similar to the former since our aim is to formulate a theory with tensionless (massless), matrix-valued DOF and sixteen supercharges, while it resembles the latter because we are trying to construct a theory of closed strings.

This paper is organised as follows. In section 2, we review the relevant symmetries of the theory and explain our notation, with further details in appendices A and B. In section 3, we introduce the string field, and give the free part, i.e. the part which is quadratic in the string fields, of the symmetry charges. In section 4, we explain the notation necessary for describing the cubic interaction part, and introduce the two essential ingredients, the overlap and the insertion. Section 5 presents the ansatz for the cubic vertices, and shows that the ansatz is consistent with the supersymmetry algebra. A discussion of power counting is also presented. In section 6 we conclude with a discussion. Details involved in some of the definitions and computations are deferred to several appendices.

2 Symmetries and notation

The theory we are interested in exhibits $\mathcal{N}=(2,0)$ super-Poincaré symmetry and its superconformal extension. The associated R-symmetry is USp(4) [1, 67, 68].

We choose the metric with signature $(-,+,\ldots,+)$ and introduce the light-cone coordinates

$$x^{+}=\frac{1}{\sqrt{2}}\left(x^{0}+x^{5}\right), \qquad x^{-}=\frac{1}{\sqrt{2}}\left(x^{0}-x^{5}\right). \tag{2.1}$$

We denote the four transverse directions by x^{α}, $\alpha=1,2,3,4$. x^{+} plays the role of time implying that $-P_{+}=P^{-}$ is the light-cone Hamiltonian. As is often done, we work on a surface defined by $x^{+}=0$.

An SO(4) subgroup of the original SO(1,5) Lorentz symmetry, acting on the transverse directions x^{α}, remains manifest. We introduce capital indices, $A,B,\ldots=1,2,3,4$, for the R-symmetry and lower case undotted and dotted indices, $a,b,\ldots$, $\dot{a},\dot{b},\ldots=1,2$, to represent the SO(4)=SU(2)$\times$SU(2) spinor indices.

The generators of the super-Poincaré algebra split into two varieties. The kinematical generators

$$P^{+}, Q_{KaA}, P_{\alpha}, M^{\alpha\beta}, M^{+\alpha}, M^{+-}, \tag{2.2}$$

[4] We expect that, as in the case of $\mathcal{N}=4$ SYM in four-dimensions, the classical scale invariance is not broken by quantum effects.

which do not pick up corrections in the interacting theory, and the dynamical generators

$$P^-, Q_{D\dot{a}A}, M^{-\alpha}, \tag{2.3}$$

which do. When there is a possible ambiguity, such as in the case of the supercharges, we use subscripts, K and D, to differentiate between kinematical and dynamical generators. Dynamical generators transform fields non-linearly, while kinematical generators act linearly on the fields. In this light-cone formalism, the super-Poincaré algebra imposes strong constraints on the theory, including on the Hamiltonian, P^-. These symmetry algebra constraints are what we will use to determine the interacting Hamiltonian. The entire super-Poincaré symmetry algebra is presented in appendix B.

We will not consider the closure of the full superconformal algebra and will instead focus on just the super-Poincaré part of the algebra. We believe that this part of the superalgebra, together with the requirement of a dimensionless coupling constant, is sufficient to determine the ansatz. It would also be interesting to examine the entire superconformal symmetry, as was done previously for $\mathcal{N} = 4$ SYM [46].

3 The free theory

Our study of the free theory begins with the superfield functional

$$\phi^I_{P^+}[x^\alpha(\sigma), \theta^{aA}(\sigma)]\,. \tag{3.1}$$

We do not write the dependence on the time coordinate x^+ explicitly. The string field depends on the total momentum P^+ and not on the momentum density $p^+(\sigma)$, because the choice of the light-cone gauge condition implies that $p^+(\sigma)$ does not depend on σ [69]. The fermionic coordinates θ^{aA} carry both R-symmetry and SO(4) spinor indices. As explained in the introduction, we expect to have $N \times N$ matrix-valued string fields when we have N M5-branes. We use indices $I, J, \ldots$ to label these matrix DOF. We will later fix a Lie algebra and assume $I, J, \ldots$ to be Lie algebra indices running from 1 to the dimension of the Lie algebra. The σ coordinate takes values in an interval of length $[\sigma]$. We choose

$$-[\sigma]/2 < \sigma < [\sigma]/2\,. \tag{3.2}$$

The length $[\sigma]$ is taken to be proportional to P^+ and the coefficient of proportionality is denoted by p^+, i.e.

$$\frac{P^+}{[\sigma]} = p^+\,. \tag{3.3}$$

p^+ is a conventional constant and it is a c-number (it commutes with everything). The fermionic coordinates θ^{1A} and θ^{2A} are related by complex conjugation,

$$\overline{\theta^{aA}} = B^{\bar{a}}{}_b B^{\bar{A}}{}_B \theta^{bB}, \tag{3.4}$$

where $B^{\bar{a}}{}_b$ is proportional to the ϵ-tensor. For our definition of tensor structures such as the B's associated with the light-cone little group SO(4) and the R-symmetry group USp(4), see appendix A. We will refer to θ^{1A} as θ and θ^{2A} as $\bar{\theta}$ below when appropriate.

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3.1 Chiral derivatives, supersymmetry and level-matching

There are two different formulations of supersymmetric theories in terms of light-cone superfields. In one approach, one uses superfields which depend only on θ (or $\bar{\theta}$). For $\mathcal{N}=4$ SYM in four dimensions, this approach was introduced in [23]. The formulation of spacetime supersymmetric SFT by Green, Schwarz and Brink [47–50] also belongs to this class of models. In the other approach, one uses superfields depending on both θ and $\bar{\theta}$, and certain chirality constraints are imposed, as was done for $\mathcal{N}=4$ SYM in [45]. While the former choice has the advantage of being direct, in the latter, formulae for the charges and the power-counting procedure [70] are more transparent because fermionic coordinates enter in supercovariant combinations.

We adopt the latter approach. Our superfields depend on both θ and $\bar{\theta}$, i.e. θ^{1A} and θ^{2A}. We impose the fundamental chirality constraint on our superfield for each value of σ,

$$d_{1A}(\sigma)\phi = 0\,, \tag{3.5}$$

where the chiral derivative is defined by

$$d_{aA}(\sigma) = \frac{\delta}{\delta\theta^{aA}(\sigma)} + \frac{p^+}{\sqrt{2}}\theta^{bB}(\sigma)\epsilon_{ba}C_{BA}\,. \tag{3.6}$$

C_{BA} is defined in appendix A.

One can solve the constraint (3.5),

$$\phi_{P^+}(x^\alpha,\theta,\bar{\theta}) = e^{\frac{1}{\sqrt{2}}p^+\int\theta^A\bar{\theta}_A d\sigma}\,\Psi_{P^+}(x^\alpha,\bar{\theta})\,. \tag{3.7}$$

Here Ψ is an arbitrary superfield depending only on $\bar{\theta}$, which can be identified with the superfield in an approach analogous to [23, 47–50].

The superstring field is a natural extension of the superfield for a superparticle in six-dimensional spacetime constructed in [65, 66]. If one focusses on the dependence of the string field on the zero-mode part of $x(\sigma)$ and $\theta(\sigma)$, one obtains the superfield for the superparticle (for each value of the index I). The superfield corresponds to the tensor multiplet [67] of $(2,0)$ supersymmetry, and gives the light-cone superfield corresponding to the $\mathcal{N}=4$ SYM theory in four-dimensions [45] upon dimensional reduction. This gives additional support to our idea that the superstring field is a natural choice for the construction of the $(2,0)$ theory.[5] In particular, it incorporates the self-duality property of the theory, because the tensor multiplet includes a two-form gauge field with self-dual field strength. Although our formulation is based on closed string DOF, it is nevertheless non-gravitational since the tensor multiplet does not contain any field of spin 2.

We introduce the local supersymmetry generator

$$q_{aA}(\sigma) = \frac{\delta}{\delta\theta^{aA}(\sigma)} - \frac{p^+}{\sqrt{2}}\theta^{bB}(\sigma)\epsilon_{ba}C_{BA}\,, \tag{3.8}$$

[5]In the degenerate case of a single M5-brane [71], the $(2,0)$ CFT is conventionally believed to be a free theory of fields belonging to the tensor multiplet. Putting $N=1$ in our case also leads to a free theory with very many light degrees of freedom including the tensor multiplet associated with the zero mode. There is no immediate contradiction here since, being free, these fields are completely decoupled.

which satisfies the following anti-commutation relations

$$[q_{aA}(\sigma), q_{bB}(\sigma')] = -\sqrt{2}p^+\epsilon_{ab}C_{AB}\delta(\sigma-\sigma')\,, \tag{3.9}$$

$$[q_{aA}(\sigma), d_{bB}(\sigma')] = 0\,, \tag{3.10}$$

$$[d_{aA}(\sigma), d_{bB}(\sigma')] = \sqrt{2}p^+\epsilon_{ab}C_{AB}\delta(\sigma-\sigma')\,. \tag{3.11}$$

Here and in the rest of the paper we use square brackets to denote both commutators and anti-commutators, depending on the Grassmann parity of the operators involved. We also define

$$p_\alpha(\sigma) = -i\frac{\delta}{\delta x^\alpha(\sigma)}\,. \tag{3.12}$$

A level matching condition should be imposed on the string fields, which ensures that the state be invariant under shifts of σ. The condition is related to the requirement of global existence of x^-,

$$\int \frac{\partial x^-}{\partial\sigma}d\sigma = 0\,, \tag{3.13}$$

where the bosonic contribution to $\partial_\sigma x^-$ is [69]

$$\frac{\partial x^-}{\partial\sigma} = \frac{1}{p^+}p_\alpha\frac{\partial x^\alpha}{\partial\sigma}\,. \tag{3.14}$$

When fermionic DOF are incorporated, the level matching condition becomes

$$\left(\int\left(p_\alpha\frac{\partial x^\alpha}{\partial\sigma} - i\frac{\partial\theta^{aA}}{\partial\sigma}(\sigma)\frac{\delta}{\delta\theta^{aA}(\sigma)}\right)d\sigma\right)\phi = 0 \tag{3.15}$$

and we have

$$\frac{\partial x^-}{\partial\sigma} = \frac{1}{p^+}\left(p_\alpha\frac{\partial x^\alpha}{\partial\sigma} - i\frac{\partial\theta^{aA}}{\partial\sigma}(\sigma)\frac{\delta}{\delta\theta^{aA}(\sigma)}\right)\,, \tag{3.16}$$

which defines $x^-(\sigma)$ up to the zero-mode part

$$X^- = \frac{1}{[\sigma]}\int x^-(\sigma)d\sigma\,. \tag{3.17}$$

3.2 Generators

We are now in a position to write down the "free" part of the various generators in our algebra. To simplify our presentation, we will use the language of the first quantised theory: we present the various charges as operators acting on the string fields. The charges in the second quantisation formulation can be written down basically by sandwiching the first quantised charge between $\bar{\phi}$ and ϕ in the usual way.

We begin by noting that the fist-quantised Hamiltonian for the tensionless string in the light-cone gauge is simply

$$P^- = \int \frac{1}{2p^+}\left(p^\alpha(\sigma)\right)^2 d\sigma\,, \tag{3.18}$$

and does not contain the usual $(\partial_\sigma x^\alpha)^2$ term which is proportional to the square of the tension [69]. This formula is unchanged even if one includes fermionic DOF. Equation (3.18) shows that, while an ordinary tensile string can be understood as a collection of harmonic oscillators, a tensionless string is a collection of free particles. Each part of the string moves independently and all terms involving ∂_σ vanish, except for the important level matching conditions (3.15) and the associated formula for the x^- coordinate (3.16). This makes the construction of the generators (except for $M^{-\alpha}$) quite easy; we can start from the superparticle case [65, 66] and we can then simply add the σ-dependence. These properties may be considered as a direct realisation of the idea of string bits [72, 73].

For the supersymmetry charges we have

$$Q_{KaA} = \int q_{aA}(\sigma) d\sigma \,, \tag{3.19}$$

$$Q_{D\dot{a}A} = \int \frac{1}{\sqrt{2}} q_{bA}(\sigma) \frac{1}{p^+} \epsilon^{bc} p^\alpha(\sigma) \sigma^\alpha{}_{c\dot{a}} d\sigma \,. \tag{3.20}$$

Other Poincaré generators include

$$M^{+\alpha} = \int -x^\alpha(\sigma) p^+ d\sigma = -X^\alpha P^+ \,, \tag{3.21}$$

$$M^{\alpha\beta} = \int \left[x^\alpha(\sigma) p^\beta(\sigma) - x^\beta(\sigma) p^\alpha(\sigma) - i\frac{\sqrt{2}}{8}\frac{1}{p^+} \sigma^{\alpha\beta a}{}_c \epsilon^{cb} C^{-1AB} q_{aA}(\sigma) q_{bB}(\sigma) \right] d\sigma \,, \tag{3.22}$$

and

$$M^{+-} = -\frac{1}{2}\left(X^- P^+ + P^+ X^-\right) - \int \frac{i}{2} \theta^{aA}(\sigma) \frac{\delta}{\delta\theta^{aA}(\sigma)} d\sigma \,. \tag{3.23}$$

All three Lorentz generators in (3.21)–(3.23) are kinematical. The only dynamical Lorentz generator is

$$\begin{aligned} M^{-\alpha} = \int \Bigg[& x^-(\sigma) p^\alpha(\sigma) - \frac{1}{2}\left(x^\alpha(\sigma) p^-(\sigma) + p^-(\sigma) x^\alpha(\sigma)\right) + \frac{i}{2}\theta^{aA}(\sigma) \frac{\delta}{\delta\theta^{aA}(\sigma)} \frac{p^\alpha(\sigma)}{p^+} \\ & + \frac{\sqrt{2}}{8} i \frac{p^\gamma(\sigma)}{(p^+)^2} q_{aA}(\sigma) q_{bB}(\sigma) \sigma^{\alpha\gamma ab} C^{-1AB} \Bigg] d\sigma \,. \end{aligned} \tag{3.24}$$

The algebra satisfied by these generators is presented in appendix B. We have explicitly verified the commutators without taking care of ordering issues in the definition of products of operators, i.e. only at the level of the Poisson brackets. Useful formulae and an outline of the computation of the commutator $[M^{-\alpha}, M^{-\beta}]$ are presented in appendix C.

The action of the charges on the superfield does not spoil the chirality constraint (3.5) because the charges are written in terms of q's which anti-commute with chiral derivatives, $[q(\sigma), d(\sigma')] = 0$. For M^{+-} and $M^{-\alpha}$, which contain θ and $\frac{\delta}{\delta\theta}$ explicitly, the consistency with the chirality constraint needs to be checked. Using arguments similar to those in appendix C, one can show

$$[M^{+-}, d_{aA}(\sigma)] = \frac{i}{2} d_{aA}(\sigma) - i\partial_\sigma(\sigma d_{aA}(\sigma)) \,, \tag{3.25}$$

$$[M^{-\alpha}, d_{aA}(\sigma)] = -\frac{i}{2}\frac{p^\alpha(\sigma)}{p^+} d_{aA}(\sigma) + i\partial_\sigma \left(\left(\int_{-[\sigma]/2}^{\sigma} p^\alpha(\sigma')\, d\sigma' - \frac{P^\alpha}{2} \right) d_{aA}(\sigma) \right) , \tag{3.26}$$

as a consequence of the fact that d_{aA} transforms as a density. This yields

$$[M^{+-}, d_{aA}(\sigma)]\phi = 0\,, \quad [M^{-\alpha}, d_{aA}(\sigma)]\phi = 0\,, \tag{3.27}$$

which assures the consistency of the action of the generators with the chirality constraint.

4 The interacting theory: overlap and insertions

We now wish to introduce interactions in this formalism with the focus being on cubic interactions. We label the three strings using indices $r, s = 1, 2, 3$. String 3 is chosen to be the long one with strings 1 and 2 connecting to it or string 3 splitting into 1 and 2. The range of $\sigma_1, \sigma_2, \sigma_3$ is denoted by $[\sigma_1], [\sigma_2], [\sigma_3]$ respectively. We require that

$$[\sigma_1] + [\sigma_2] = [\sigma_3]\,, \tag{4.1}$$

which also follows from the fact that $[\sigma]$ is proportional to the conserved momentum P^+, so that (4.1) is equivalent to

$$P_1^+ + P_2^+ = P_3^+\,. \tag{4.2}$$

It is convenient to introduce σ which takes value in the whole interval $I = I_3$. The whole interval I consists of two "intervals" I_1 and I_2 respectively for strings 1 and 2. We use the following scheme

$$I = I_3 = [-[\sigma_3]/2, [\sigma_3]/2]\,, \tag{4.3}$$

$$I_1 = [-[\sigma_1]/2, [\sigma_1]/2]\,, \tag{4.4}$$

$$I_2 = [[\sigma_1]/2, [\sigma_3]/2] + [-[\sigma_3]/2, -[\sigma_1]/2]\,. \tag{4.5}$$

Each σ_r takes values within $[-[\sigma_r]/2, [\sigma_r]/2]$ for $r = 1, 2, 3$. σ and σ_r $(r = 1, 2, 3)$ are related by

$$\sigma_3 = \sigma\,, \tag{4.6}$$

$$\sigma_1 = \sigma \text{ for } \sigma \in I_1\,, \tag{4.7}$$

$$\sigma_2 = \sigma - [\sigma_3]/2 \text{ or } \sigma_2 = \sigma + [\sigma_3]/2 \text{ for } \sigma \in I_2\,. \tag{4.8}$$

Following the work on superstring theory in the spacetime supersymmetric formalism [47–50], we introduce the two building blocks used to construct the cubic interactions: the overlap and the insertions. The overlap is a delta functional connecting the third string to the first and second strings. Local insertions of operators at the interaction point are also necessary. These same ingredients (the overlap and the insertions) can be defined in the tensionless case as well.

As usual, it is easier to work with discretely labelled variables by introducing mode expansions. We introduce the Fourier components of $x_r(\sigma_r)$ by

$$x_r(\sigma_r) = \sum_n x^{rn} e^{in\frac{2\pi}{[\sigma_r]}\sigma_r}\,. \tag{4.9}$$

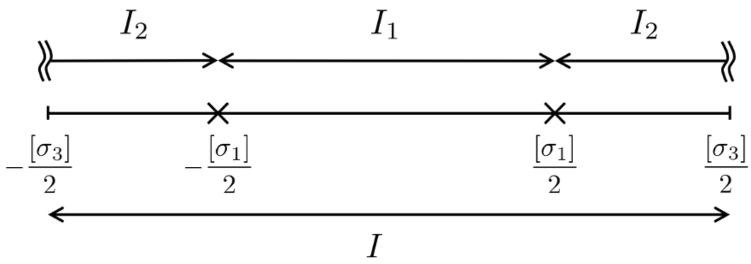

Figure 1. The σ-coordinates of closed strings $1, 2$ and 3 are defined on intervals I_1, I_2 and $I = I_3$. The crosses indicate the interaction point.

The canonical conjugate of x^n, p_n, is

$$p_{rn} = \int p_r(\sigma_r) e^{in\frac{2\pi}{[\sigma_r]}\sigma_r} d\sigma_r \tag{4.10}$$

and p_{r0} is the total transverse momentum P_r (we omit α indices). The Fourier modes for $r = 1, 2$ and for $r = 3$ respectively define two sets of basis vectors. We define a matrix A relating the basis associated with the third string to that associated with the first and second strings by

$$x^{rn} = A^{rn}{}_{3m} x^{3m} \quad (r = 1, 2)\,. \tag{4.11}$$

We have

$$A^{rn}{}_{3m} = \frac{1}{[\sigma_r]} \int_{\sigma \in I_r} e^{-i\frac{2\pi}{[\sigma_r]} n\sigma_r} e^{+i\frac{2\pi}{[\sigma_3]} m\sigma_3} d\sigma\,. \tag{4.12}$$

The overlap for the bosonic DOF is expressed as

$$V_B = \prod_{r=1,2} \prod_n \delta(x^{rn} - A^{rn}{}_{3m} x^{3m})\,. \tag{4.13}$$

For the fermionic component, we use

$$V_F = \prod_{r=1,2} \prod_{a=1,2} \prod_n \delta(\theta^{ran} - A^{rn}{}_{3m} \theta^{3am})\,. \tag{4.14}$$

Our philosophy in this paper is very similar, in spirit, to that followed in [46]. In order to build a consistent interacting theory, we start with an ansatz for the dynamical supersymmetry generators. We allow the entire symmetry algebra to constrain our ansatz and finally use the fact that the Hamiltonian for the interacting theory can be written as the "square" of the dynamical supercharge.

In general, the delta function (the overlap) is not sufficient to construct the dynamical charges in light-cone gauge field theory and one has to "insert" operators such as derivatives in x and their fermionic counterparts acting on the overlap part. This is the case both for $\mathcal{N} = 4$ SYM in four dimensions [23, 45] and for superstring field theory [47–50]. In string theory it is not possible to insert the operators at an arbitrary point in σ. The insertion should only act at the interaction point.

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The insertion operator we choose is represented by the functions $w_r(\sigma)$ ($r = 1, 2$), which have delta function like singularities at the interaction point,

$$w_1(\sigma_1) = \delta\left(\sigma_1 - \frac{[\sigma_1]}{2}\right) = \delta\left(\sigma_1 + \frac{[\sigma_1]}{2}\right), \tag{4.15}$$

$$w_2(\sigma_2) = -\delta\left(\sigma_2 - \frac{[\sigma_2]}{2}\right) = -\delta\left(\sigma_2 + \frac{[\sigma_2]}{2}\right), \tag{4.16}$$

where we assume that the delta functions satisfy appropriate periodicity conditions. In the mode number representation, we have

$$w^{1n} = \frac{1}{[\sigma_1]}(-1)^n, \tag{4.17}$$

$$w^{2n} = \frac{1}{[\sigma_2]}(-1)^{n+1}. \tag{4.18}$$

The rationale for this choice is described in appendix D.1.

Now that we have an overlap and an insertion, we are in a position to write down an ansatz for the dynamical supersymmetry generator, describing a cubic interaction between the tensionless string fields. This is the focus of the next section.

5 Ansatz for cubic interaction terms

In general dynamical charges have an expansion, which in the case of Q_D, for instance, takes the form

$$Q_D^{(0)} + Q_D^{(1)} + Q_D^{(2)} + \cdots. \tag{5.1}$$

Here $Q_D^{(0)}$ is the free part, quadratic in the string fields, $Q_D^{(1)}$ is the cubic interaction part, containing three string fields, and so forth. The form of the ansatz is chosen so as to satisfy the super-Poincaré algebra (listed in appendix B) order by order in terms of the number of fields involved. The cubic part of a dynamical charge consists of two terms respectively involving $\overline{\phi\phi}\phi$ and $\overline{\phi}\phi\phi$. Since one of them can be easily recovered from the other by the hermiticity conditions presented in appendix B, we will hereafter only write the $\overline{\phi}\phi\phi$ part. Our ansatz for $Q_D^{(1)}$ is

$$\begin{aligned} Q_{D\dot{a}A}^{(1)} = f^I{}_{JK} \int & \overline{\phi_{P_3^+}{}_I}[x_3, \theta_3] \\ & \times \left((p_\alpha \cdot w)(\sigma^{\alpha b}{}_{\dot{a}} d_{bA} \cdot w)(p^+)^{\lambda_0}(P_1^+)^{\lambda_1}(P_2^+)^{\lambda_2}(P_3^+)^{\lambda_3}\delta(P_1^+ + P_2^+ - P_3^+)V\right) \\ & \times \phi_{P_1^+}{}^J[x_1, \theta_1]\phi_{P_2^+}{}^K[x_2, \theta_2] \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r. \end{aligned} \tag{5.2}$$

Here we assume that $f^I{}_{JK}$ are the structure constants of a Lie algebra. For the case of N M5-branes in flat spacetime, $f^I{}_{JK}$ should correspond to $\mathrm{U}(N)$. $\lambda_0, \cdots, \lambda_3$ are parameters to be determined. Below we will partially fix them by requiring invariance under rescaling of the σ coordinate and using power counting arguments. In (5.2) $V = V_B V_F$ and

$$p_\alpha \cdot w = \int p_\alpha(\sigma) w(\sigma) d\sigma = p_{\alpha rn} w^{rn}\,, \tag{5.3}$$

$$d_{bA} \cdot w = \int d_{bA}(\sigma) w(\sigma) \mathrm{d}\sigma = d_{bArn} w^{rn}\,. \tag{5.4}$$

The form of the ansatz is fixed basically by requiring that it has the correct index structure.

If one exchanges $r = 1$ and $r = 2$ and the dummy indices J, K in the above formula, the result will have λ_1 and λ_2 exchanged. Furthermore one has a factor of -1 from each w (compare (4.15)–(4.18)) and a factor of -1 from f. This implies that we must avoid choosing $\lambda_1 = \lambda_2$ to have a non-vanishing ansatz. The ansatz for $P^{-(1)}$ will be determined below from the supersymmetry algebra.

5.1 Power counting in SFT

We briefly discuss the power counting analysis of the cubic vertex. The first step is to notice that the appearance of θ and $\bar{\theta}$ is accompanied by factors of p^+, so that the integral measure for the fermionic coordinates is dimensionless.[6] The fermionic coordinates only contribute to the physical dimensions through q's and d's and we will omit the dependence on the θ coordinates of the string field in this subsection.

The dimension of the string fields turns out to be infinite. We thus introduce a regularisation where we discretise the σ variables by M string bits

$$\phi_{P^+}(x_1^\alpha, \cdots, x_M^\alpha)\,. \tag{5.5}$$

The dimension of the string field can be determined by noting that it can be considered as the wave function of the first-quantised string theory.[7] Thus the normalisation factor

$$\int |\phi_{P^+}(x_1^\alpha, \cdots, x_M^\alpha)|^2 dP^+ d^4x_1 \cdots d^4x_M\,, \tag{5.6}$$

should be dimensionless implying that the string field ϕ has dimension

$$[\phi] = \frac{1}{2} \times (4M - 1)\,, \tag{5.7}$$

which depends on the number of bits.

In the bit representation, the overlap delta functional V is

$$V = \prod_{n=1}^{M_1} \delta\left(x_{3n} - x_{1n}\right) \prod_{n'=1}^{M_2} \delta\left(x_{3(M_1+n')} - x_{2n'}\right)\,. \tag{5.8}$$

[6] This is because of the anti-commutation relations in superstring theory in the light-cone gauge, $[\theta(\sigma), \bar{\theta}(\sigma')] \sim \frac{1}{p^+}\delta(\sigma - \sigma')$.

[7] We note that in general one can redefine the string field by multiplying it by factors of P^+. We do not introduce such a redefinition. This choice is related to shifts of the operators M^{+-} and $M^{-\alpha}$ in (3.23) and (3.24) and it is reflected in the hermitian ordering between X^- and P^+ and p^- and x^α respectively.

The schematic form (omitting factors irrelevant to the power counting) of the supercharge $Q_D^{(1)}$, after carrying out the $\mathcal{D}x_1\mathcal{D}x_2$ integrals using the delta functions, is

$$Q_D^{(1)} \sim \int \prod_{n=1}^{M_3} d^4x_{3n} dP_1^+ dP_2^+ dP_3^+ \overline{\phi_3}\phi_1\phi_2 \\ \times p\cdot wq\cdot w\delta(P_1^+ + P_2^+ - P_3^+) \left(p^+\right)^{\lambda_0} \left(P_1^+\right)^{\lambda_1} \left(P_2^+\right)^{\lambda_2} \left(P_3^+\right)^{\lambda_3} . \tag{5.9}$$

We note that we are not introducing any dimensionful coupling constant here; this is a requirement we impose on the SFT in order to construct a scale invariant theory.

Requiring that both sides of (5.9) have dimension $\frac{1}{2}$, we find

$$\lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 = -\frac{3}{2} . \tag{5.10}$$

An essential feature in the power-counting analysis of SFT presented above is that the M-dependent term in the dimension arising from the string fields

$$[\phi_1] + [\phi_2] + [\phi_3] = \frac{1}{2}\times(4M_1 - 1) + \frac{1}{2}\times(4M_2 - 1) + \frac{1}{2}\times(4M_3 - 1) , \tag{5.11}$$

is exactly cancelled by the dimension of the measure

$$\left[\prod_{n=1}^{M_3} d^4x_{3n}\right] = -4M_3 , \tag{5.12}$$

because of the conservation of the number of bits

$$M_1 + M_2 = M_3 , \tag{5.13}$$

for the cubic vertex.

We observe that this cancellation implies that the dimensional analysis is independent of the number of transverse directions, as can be seen from (5.11) and (5.12). This is in sharp contrast with the dimensional counting in traditional field theories. The power counting in SFT is favourable compared to that in usual QFT in this sense.

In the SFT case under consideration the free part of the action contains a term which schematically can be written as

$$\int \overline{\phi_{P^+}}(x_1,\ldots,x_M) \sum_{n=1}^{M} \left(\frac{\partial}{\partial x_n^\alpha}\right)^2 \phi_{P^+}(x_1,\ldots,x_M) dP^+ d^4x_1 \cdots d^4x_M . \tag{5.14}$$

Comparing this formula in the case $M = M_3$ to the cubic vertex (5.9) we see that the terms quadratic and cubic in the fields essentially have the same structure; the difference only lies in how we group the string bits into different string fields. This is the origin of why the power counting analysis does not depend on the spacetime dimension. This in turn reflects the basic feature of string theory that locally string interaction and string propagation cannot be distinguished.

This result may have been expected as it is well known that the coupling constant in string theory is dimensionless irrespective of the spacetime dimension. The property of

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possessing a dimensionless coupling constant potentially makes tensionless string theory a natural framework for constructing theories with conformal symmetry.

The parameter λ_0 is fixed considering a rescaling of the σ coordinate under which $[\sigma]$ becomes $\alpha[\sigma]$. Under this transformation p_α turns into p_α/α, i.e. it transforms as a density. p^+, $d_{bA}(\sigma)$, and $w(\sigma)$ are also densities. Taking into account the two σ integrals involved in the definition of $p \cdot w$ and $q \cdot w$, we see that

$$\lambda_0 = -2\,. \tag{5.15}$$

Combining this with (5.10), we have

$$\lambda_1 + \lambda_2 + \lambda_3 = \frac{1}{2}\,. \tag{5.16}$$

5.2 Computation of commutators

We explicitly work out the commutators to show that the ansatz is consistent with the superalgebra.

An issue in the computation is the potential singularity which can occur because of the multiplication of operators at the same point in σ-space. To perform the computations in a well defined manner we use a regularisation scheme, analogous to that introduced in [53], in which operators are smeared. For most of the commutators a computation done using smeared operators, in the limit $\epsilon \to 0$ (where ϵ is the length scale associated with smearing), produces a result which is identical to that of a formal computation without regularisation. For the computation of the commutator $[Q_D, P^-]$, however, smearing makes a difference. Also, it is necessary to evaluate the result of the computation, which includes delta functionals, by means of test functionals. In this section, we avoid the explicit introduction of smearing. Details regarding smearing and test functionals are discussed in appendix E.

We begin with the commutation relation,

$$[Q_{KaA}, Q_{Db B}] = (\sigma^\alpha)_{a\dot{b}} C_{AB} P_\alpha\,. \tag{5.17}$$

When expanded, this implies

$$\left[Q_K^{(0)}, Q_D^{(1)}\right] = 0\,, \tag{5.18}$$

since the kinematical generators Q_K and P have no non-linear parts.

To compute the commutator $\left[Q_K^{(0)}, Q_D^{(1)}\right]$, we note that in general, the commutator between a symmetry generator $\mathcal{O}$ and the string field (at the linearised level) is given by

$$\left[\mathcal{O}^{(0)}, \phi_{P^+}\right] = -\mathcal{O} \cdot \phi_{P^+}\,. \tag{5.19}$$

Here $\mathcal{O}^{(0)}$ appearing on the l.h.s. denotes the linear part (quadratic in terms of the fields) of the charge in the second-quantised formulation. On the r.h.s. $\mathcal{O}\cdot$ denotes how these operators act on the field (as a ket-vector) from the left in the first-quantisation formulation. The commutator between the charges and $\bar{\phi}$ can be computed by taking the complex conjugate of (5.19). Apart from the case of a few exceptional operators,[8] one can show that

$$\left[\mathcal{O}^{(0)}, \overline{\phi_{P^+}}\right] = \overline{\phi_{P^+}} \cdot \mathcal{O} \,, \tag{5.20}$$

where $\cdot\mathcal{O}$ denotes the action of the operator from the right on the complex conjugate of the field (as a bra-vector). For instance, in the present case, we have

$$\left[Q^{(0)}_{KaA}, \phi_{P^+}\right] = -Q_{KaA} \cdot \phi_{P^+} = -\int q_{aA}(\sigma) d\sigma \; \phi_{P^+} \,, \tag{5.21}$$

$$\left[Q^{(0)}_{KaA}, \overline{\phi_{P^+}}\right] = \overline{\phi_{P^+}} \cdot Q_{KaA} = -\int d_{aA}(\sigma) d\sigma \; \overline{\phi_{P^+}} \,. \tag{5.22}$$

Since the operator $Q^{(0)}_K$ acts on the string fields,

$$\begin{aligned}\left[Q^{(0)}_{KaA}, Q^{(1)}_{D\dot{b}B}\right] &= f^I{}_{JK} \int \left(\overline{\phi_{P_3^+}}_I \cdot Q_{KaA}\right) (\cdots V) \, \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r \\ &+ f^I{}_{JK} \int \overline{\phi_{P_3^+}}_I \, (\cdots V) \, Q_{KaA} \cdot \left(\phi_{P_1^+}{}^J \phi_{P_2^+}{}^K\right) \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r \,, \end{aligned} \tag{5.23}$$

where

$$(\cdots V) = (p_\alpha \cdot w)(\sigma^{\alpha b}{}_{\dot{a}} d_{bA} \cdot w)(p^+)^{\lambda_0} (P_1^+)^{\lambda_1} (P_2^+)^{\lambda_2} (P_3^+)^{\lambda_3} \delta(P_1^+ + P_2^+ - P_3^+) V \,. \tag{5.24}$$

Using the associativity property we rewrite (5.23) as

$$f^I{}_{JK} \int \overline{\phi_{P_3^+}}_I \left(Q^3_{KaA} \cdot (\cdots V) + (\cdots V) \cdot (Q^1_{KaA} + Q^2_{KaA})\right) \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r \,, \tag{5.25}$$

where Q^r_K with $r = 1, 2, 3$ denotes the operator Q_K acting on the r-th string field. Moving $Q^{1,2}_K$ to the left of $(\cdots V)$, (5.25) becomes

$$\begin{aligned} & f^I{}_{JK} \int \overline{\phi_{P_3^+}}_I \left(\int_{I_3} q_{aA}(\sigma) d\sigma + \int_{I_1} d_{aA}(\sigma) d\sigma + \int_{I_2} d_{aA}(\sigma) d\sigma\right) (\cdots V) \; \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \\ & \quad \times \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r \,. \end{aligned} \tag{5.26}$$

From this we can show

$$\begin{aligned} & \left(\int_{I_3} q_{aA}(\sigma) d\sigma + \int_{I_1} d_{aA}(\sigma) d\sigma + \int_{I_2} d_{aA}(\sigma) d\sigma\right) (\cdots V) \\ & = \sum_{r=1,2} \int_{I_r} \left[d_{aA}(\sigma), (\cdots)\right] d\sigma V - (\cdots) \left(\int_{I_3} q_{aA}(\sigma) d\sigma + \int_{I_1} d_{aA}(\sigma) d\sigma + \int_{I_2} d_{aA}(\sigma) d\sigma\right) V \\ & = 0 \,. \end{aligned} \tag{5.27}$$

[8] Exceptional ones are M^{+-} and $M^{-\alpha}$. For these, the ordering of θ and $\frac{\delta}{\delta\theta}$ has to be worked out carefully.

For the second term in the second line of (5.27), we used

$$\left(\theta^{ra}(\sigma)-\theta^{3a}(\sigma)\right)V_F=0\,,\quad \left(\frac{\delta}{\delta\theta^{ra}(\sigma)}+\frac{\delta}{\delta\theta^{3a}(\sigma)}\right)V_F=0\,, \tag{5.28}$$

where $\sigma\in I_r$ with $r=1,2$, and, for the first term,

$$\int_{I_1} w(\sigma)d\sigma+\int_{I_2} w(\sigma)d\sigma=0\,. \tag{5.29}$$

This important property of w is also used for the commutators $[M^{+\alpha},Q_D]$ and $[M^{+\alpha},P_D^-]$, which can be verified using similar manipulations.

The commutator $[Q_D,M^{\alpha\beta}]$ can also be verified directly. This is expected since (3.22) has the correct index structure ensuring the correct transformation of $Q_D^{(1)}$ under the SO(4) little group.

The commutation relation

$$[Q_{D\dot{a}A},Q_{D\dot{b}B}]=\sqrt{2}\epsilon_{\dot{a}\dot{b}}C_{AB}P^-\,, \tag{5.30}$$

yields

$$\left[Q^{(0)}_{D\dot{a}A},Q^{(1)}_{D\dot{b}B}\right]+\left[Q^{(1)}_{D\dot{a}A},Q^{(0)}_{D\dot{b}B}\right]=\sqrt{2}\epsilon_{\dot{a}\dot{b}}C_{AB}P^{-(1)}\,. \tag{5.31}$$

Evaluating the l.h.s. , one obtains

$$\begin{aligned}P^{-(1)}=2\sqrt{2}\,f^I{}_{JK}\int\overline{\phi_{P_3^+}}_I\Big((p_\alpha\cdot w)^2(p^+)^{\lambda_0}(P_1^+)^{\lambda_1}(P_2^+)^{\lambda_2}(P_3^+)^{\lambda_3}\delta(P_1^++P_2^+-P_3^+)V\Big)\\ \times\,\phi_{P_1^+}{}^J\phi_{P_2^+}{}^K\prod_{r=1}^3 dP_r^+\mathcal{D}\theta_r\mathcal{D}x_r\,,\end{aligned} \tag{5.32}$$

where we used

$$\left(p_r^\alpha(\sigma)+p_3^\alpha(\sigma)\right)V_B=0\,, \tag{5.33}$$

for $\sigma\in I_r$ with $r=1,2$.

Commutators involving M^{+-} can also be verified and lead to the same condition (5.16) obtained from the power counting analysis. We note that taking the commutator of the boost generator M^{+-} with another operator essentially amounts to counting the number of P^+'s contained in the operator. One also has to take into account the "intrinsic weight", $-\frac{1}{2}$, of the string field under boosts which can be read off from (3.23).

The commutator $[Q_D,P^-]=0$ requires a careful analysis using smearing and test functionals, because p^2 terms in $P^{-(0)}$ acting on the overlap part, combined with $p{\cdot}w$ in $Q^{(1)}$ may result in unwanted non-zero contributions. An outline of this calculation is presented in appendix E.4. The result justifies our choice of the insertion w explained in appendix D.1.

The commutators involving the Lorentz generator $M^{-\alpha}$ are more subtle and we have not completed their analysis. We expect that the computation of the commutator between $M^{-\alpha}$ and P^- will fix the λ parameters, since the analogous parameters of the tensile superstring field theory were fixed in this way in [51].

6 Conclusions and discussion

In this paper we have used light-cone string field theory to formulate an interacting theory of tensionless strings in six dimensions, with the purpose of obtaining a Lagrangian description of the $(2,0)$ superconformal field theory. Our proposal is motivated by the M-theory picture in which the $(2,0)$ CFT arises from the low-energy dynamics of coincident M5-branes. In this M-theory construction, M2-branes stretched between coincident M5-branes yield degrees of freedom consisting of (matrix valued) tensionless closed strings confined to the world-volume of the M5-branes. We have argued that string field theory, in its light-cone form, is the most suitable language to study these interacting tensionless strings.

The most appealing feature of a formulation of the $(2,0)$ CFT as a tensionless string field theory is the fact that it may allow us to avoid the obstacles, associated with power counting arguments, which impede the construction of local renormalisable interacting QFT's in dimension larger than four. The use of stringy degrees of freedom has also interesting implications in connection with the relation between the $(2,0)$ CFT in $d=6$ and the four-dimensional $\mathcal{N}=4$ SYM theory. The latter is obtained upon dimensional reduction on a torus and we have suggested that wrapped string configurations may play a central role in the emergence of the four-dimensional Yang-Mills coupling constant.

In this paper we introduced our formalism and we presented the free part of the SFT action, together with an ansatz for the cubic interaction part. These are only the first steps towards obtaining a viable formulation of the six-dimensional $(2,0)$ CFT. There remain multiple issues to be clarified, both of a technical nature — in the construction of the tensionless SFT — and of a more conceptual nature — in relation to its interpretation as a description of the $(2,0)$ CFT.

In order to complete the construction of the interacting SFT to cubic order, it is important to finish the analysis of the ansatz for the $M^{-\alpha}$ Lorentz generators and their commutators with the other charges. We expect that this should allow us to completely fix our ansatz, determining the λ parameters. By a more comprehensive study of the constraints imposed by the full superconformal algebra, one can presumably deduce the full anti-symmetry and the Jacobi identity for the parameters ${f^I}_{JK}$, thus characterising them as structure constants of a Lie algebra, as was done for $\mathcal{N}=4$ SYM in [46].

Our study of the free part of the superalgebra has only been carried out at the level of the Poisson brackets, without taking care of ordering issues in the definition of operator products. It is clearly desirable to repeat these calculations at the quantum level. For this purpose it may be necessary to make a more systematic use of smearing and test functionals, following the approach discussed in appendix E.

The most important issues that remain to be addressed are, however, more conceptual and concern the interpretation of our six-dimensional tensionless SFT as describing the dynamics of the $(2,0)$ CFT. The fundamental physical properties of a CFT formulated in this manner need to be investigated. As a theory of tensionless strings our model contains a very large number of light degrees of freedom, whose properties and behaviour need to be understood. The most crucial aspects to focus on are the identification of the correct observables in the theory and how to describe them in the SFT language. Clarifying these aspects is essential in order to understand the very nature of the resulting CFT.

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On general grounds, one expects the proper observables to be correlation functions of local operators organised in superconformal multiplets. Within the formulation proposed in this paper such local operators should be built from the string field. It is possible that there be a vast redundancy in our formulation, so that, in spite of the seemingly very large number of degrees of freedom contained in the string field, the set of physical observables built from them is similar to those found in more familiar conformal theories in lower dimensions. It is also possible, however, that the construction that we presented give rise to a much broader set of observables compared to more conventional CFT's and that the system described by our tensionless string field is fundamentally different from the known examples of conformal theories.

There are several ways to gain insights into the properties of observables in the theory we constructed. It can be very useful to consider special sectors in which one has independent means of guessing the structure of the relevant observables. Particularly interesting in this respect are large R-charge states in M-theory in $\mathrm{AdS}_7 \times S^4$. According to the AdS/CFT correspondence, the $(2,0)$ CFT has a dual description in terms of M-theory in $\mathrm{AdS}_7 \times S^4$, which possesses a large R-charge sector analogous to that considered in [19], described by the BMN matrix model. The spectrum of the BMN matrix model includes states associated with near-BPS fluctuations of spherical membranes. Then the AdS/CFT duality implies that there exist a large R-charge sector in the six-dimensional $(2,0)$ CFT containing operators corresponding to fluctuations of spherical membranes. Recalling the properties of the analogous sector in the duality between type IIB string theory in $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ SYM, we can speculate about the characteristics of a set of large R-charge degrees of freedom in the $(2,0)$ CFT. In the $\mathrm{AdS}_5/\mathrm{CFT}_4$ case one considers so-called BMN operators [22], which are constructed as traces of products of a large number of the matrix-valued elementary fields of the $\mathcal{N} = 4$ SYM theory. The position in the sequence of fields inside such traces can be understood as being associated with the σ coordinate in the dual string. In the case of the $(2,0)$ theory the states with large R-charge we are interested in are membrane fluctuations and thus one has two σ coordinates to identify in the relevant CFT operators. Since the $(2,0)$ theory contains tensionless string degrees of freedom, it is natural to build the analog of the BMN operators as traces of products of matrix fields defined on a loop space, which is the configuration space of tensionless strings. In this way one may introduce two σ-coordinates: one associated with a given "point" in the loop space, the other labelling the order of the matrix fields in the product. Our string field precisely provides a matrix valued field on a loop space. Thus the consideration of a BMN-like sector suggests that operators written as traces of products of string fields may be a natural choice of observables in the $(2,0)$ CFT. Although in general it is not straightforward to define a theory built on a loop space, SFT provides a rather successful example of such a theory. This is actually one of the motivations that led us to study the SFT approach proposed in this paper.

When considering the problem of identifying the observables of the $(2,0)$ theory, it is clearly important to take into account as much as possible the constraints from symmetry arguments and consistency requirements. The bootstrap program [18] is a way of systematically implement these constraints to obtain, in particular, bounds on the spectrum. Provided that the observables in our formulation are not of a qualitatively different nature from those of more standard CFT's, any constraints established using the bootstrap method should be satisfied in our case as well.

Further guidance in characterising the observables in our formulation of the $(2,0)$ CFT can be provided by the study of the compactification of the theory to lower dimensions, in particular to $\mathcal{N}=4$ SYM in $d=4$. Understanding how to derive the $\mathcal{N}=4$ SYM theory in this way is in its own right an important issue, that is essential to address in order to establish the validity of our formulation. Wrapped string configurations are expected to give rise to the SYM degrees of freedom in $d=4$. However, the fate of unwrapped strings upon compactification remains to be clarified. Unless there is a mechanism for the decoupling of these configurations, it would appear that our formulation of the $(2,0)$ CFT may give rise to tensionless strings in four dimensions. There is also a related issue associated with the presence of an infinite number of flat directions (one for each mode of the tensionless string) in the action, which may produce severe IR divergences. There seem to be two possible scenarios in connection to the compactification of our tensionless SFT to $d=4$ — either there is a mechanism explaining the decoupling of the extra light degrees of freedom or there exists a new description of $\mathcal{N}=4$ SYM in four dimensions containing tensionless strings. It would be interesting to study the possible connections of such a formulation to the loop equation [74–76], i.e. the Schwinger-Dyson equation for Wilson loop operators, in $\mathcal{N}=4$ SYM. Because of scale invariance, the string arising from the Wilson loop may be expected to be tensionless. For our purpose it is natural to consider the loop equation defined in light-cone superspace [45]. Various types of loop equations for $\mathcal{N}=4$ SYM, mainly in the context of the AdS/CFT correspondence, were considered in [77–83].

Another important issue to understand is whether or not a critical dimension exists for tensionless strings. The analysis of the critical dimension is expected to be different compared to the case of ordinary tensile strings.[9] This is because the nature of the UV divergences in σ-space and the normal ordering, which underlie the calculation of the critical dimension, are different in the tensionless case. Moreover, in the case we are interested in the coupling constant should be of order 1 and thus the free and the interaction parts may mix when discussing possible anomalies in the Lorentz symmetry.

The possible mixing between contributions of different orders has another important implication. It may allow us to determine the magnitude of the coupling constant by requiring the cancellation of the quantum anomaly in the symmetry algebra. In the case of the bosonic open-closed light-cone gauge string field theory, it is known that the Lorentz anomaly of the string field theory (not that of the first quantised theory) determines the relationship between the various coupling constants in the theory [62–64]. The situation in the case of the $(2,0)$ CFT may be analogous to that of the Chern-Simons theory, in which the coupling constant is constrained to be an integer by the requirement that the path integral be uniquely defined. Another way to fix the coupling constant is to work out the reduction discussed above to four-dimensional $\mathcal{N}=4$ SYM.

[9] The Lorentz anomaly in the first-quantised formulation of light-cone gauge tensile string theory in six dimensions was computed in [84]. In [27, 29] it was argued that there is no critical dimension for tensionless bosonic string theory, i.e. the theory is consistent for any number of spacetime dimensions.

Our formulation of the (2,0) theory as a tensionless string theory for the low-energy dynamics of M5-branes is analogous to the description of the low-energy dynamics of parallel D-branes in terms of SYM theories. In view of this, we expect to have the analog of the well-known realisation of the Higgs mechanism in a system of D-branes. Each matrix element of our matrix-valued string field contains the tensor multiplet arising from the zero mode part of $x(\sigma)$ and $\theta(\sigma)$. The 5 scalar fields in the tensor multiplet describe transverse fluctuations of the M5-branes and a vacuum expectation value for the scalars in the i-th diagonal element in the matrix-valued string field corresponds to the position of the i-th M5-brane. It is interesting to study the theory around configurations in which these scalar fields have non-zero vacuum expectation values. The theory should then describe the low energy limit of parallel, but non-coincident, M5-branes. There are two scales involved in this construction, the M2-brane tension (or equivalently the 11-dimensional Planck length) and the separation between the M5-branes (or equivalently the scalar vacuum expectation value). The tension of the strings arising from M2-branes stretched between M5-branes is the product of the membrane tension and the distance between the M5-branes. One should consider the low energy limit by simultaneously sending to zero the separation between any two M5-branes, in such a way as to keep the tension of the strings finite when measured in terms of the relevant energy scale. Equivalently, one sends the eleven-dimensional Planck energy to infinity, while tuning the distances between M5-branes, so that the string tension remains finite. Let us consider, for definiteness, the case in which N M5-branes are divided into two groups of N_1 and N_2 coincident branes, with $N = N_1 + N_2$. The configuration is then represented by a block diagonal matrix. In the original $N \times N$ matrix one can identify $N_1 \times N_1$ and $N_2 \times N_2$ diagonal blocks and two off-diagonal blocks of size $N_1 \times N_2$ and $N_2 \times N_1$ respectively. The scaling limit should decouple both the bulk gravity dynamics and the degrees of freedom associated with fluctuations of the M2-branes in the directions transverse to the M5-branes. In this limit the DOF contained in the block diagonal elements should be tensionless strings and those contained in the block off-diagonal elements should be tensile strings with a tension proportional to the vacuum expectation value (or equivalently the distance between the two sets of M5-branes). This coupled system of tensionless and tensile strings should arise by expanding our SFT around the configuration with non-zero vacuum expectation values. In this situation the cubic and higher order vertices in the Hamiltonian give rise to additional contributions to the part quadratic in the string fields. Checking that these quadratic terms produce the correct free Hamiltonian for the block off-diagonal tensile strings provides a non-trivial test of the form of the interaction vertices.

One may also study M5-branes in a spacetime with a compactified transverse direction, that can be realised considering an infinite number of copies of M5-branes, in a way analogous to the description of D-branes in a compactified spacetime by SYM [85]. In this way one may obtain a SFT formulation of the theory describing the decoupling limit of NS5-branes, i.e. the little string theory with (2,0) supersymmetry [86]. For a review of little string theory, see [87]. The SFT description would contain tensionless strings as well as an infinite variety of tensile strings with tensions proportional to an integer multiple of the compactification radius.

In this paper we constructed the cubic vertex for a tensionless string field theory in six dimensions. It is important to study the possible higher order terms in the Hamiltonian.

In the case of tensile bosonic string field theory in light-cone gauge, it is known that cubic and quartic vertices are sufficient to reproduce the correct S-matrix [54, 55]. For the light-cone superstring field theory constructed in [47–50] the necessity of quartic couplings was discussed in [88–91], but there seems to be no definitive answer to the question of whether higher order vertices are present in the theory. It is still premature to draw any conclusions about the structure of higher-order terms in our model, although the similarity and close relationship to $\mathcal{N} = 4$ SYM may suggest that the action should stop at quartic order.

The SFT description we proposed in this paper applies to a special sector of M-theory, i.e. the low energy fluctuations of coincident M5-branes. The tensionless string DOF we studied arise from membranes stretched between coincident M5-branes. The matrix model of M-theory [92, 93], which is a good candidate for the formulation of the full M-theory, can be considered as the matrix-regularised version of membrane theory [92, 94, 95]. Within this framework it is possible that our SFT construction may eventually be superseded by a description in terms of regularised DOF.

Although additional work is required to establish whether our tensionless string field theory approach will lead to a valid formulation of the six-dimensional $(2, 0)$ CFT, we believe that the ideas presented in this paper deserve to be further studied. If successful, this proposal would extend the realms of both string theory and QFT. We hope that our work provides the first steps and the necessary tools to pursue this line of investigation.

Acknowledgments

We are grateful to Pierre Ramond for sharing unpublished results with us. We thank Lars Brink and Pierre Ramond for discussions. Part of this work was done when HS was at the Okayama Institute for Quantum Physics and at KEK. HS is grateful to his colleagues there in particular Fumihiko Sugino, Satoshi Iso and Shotaro Shiba for many useful discussions, comments and encouraging remarks. We would also like to thank Koji Hashimoto, Nobuyuki Ishibashi, Hiroshi Isono, Hikaru Kawai, Yoichi Kazama, Seok Kim, Ryuichiro Kitano, Shota Komatsu, Hiroshi Kunitomo, Tsunehide Kuroki, Shun'ya Mizoguchi, Norisuke Sakai, Tadakatsu Sakai, Ashoke Sen, Shigeki Sugimoto, Stefan Theisen, Seiji Terashima, Satoshi Yamaguchi and Tamiaki Yoneya for discussions, encouraging remarks and useful comments. The work of YS was supported by Building of Consortia for the Development of Human Resources in Science and Technology and by CUniverse research promotion project by Chulalongkorn University (grant reference CUAASC). The work of HS was supported by JSPS KAKENHI Grant Numbers JP16H06490. The work of SA is partially supported by a DST-SERB grant (EMR/2014/000687).

A Tensors with R-symmetry and spinor indices

Six-dimensional $\mathcal{N} = (2, 0)$ supersymmetry is described, for example, in [67, 68].

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A.1 R-symmetry USp(4)

For the R-symmetry USp(4) tensors we use an anti-symmetric and non-degenerate 4×4 matrix C,

$$C_{AB} = -C_{BA}\,. \tag{A.1}$$

It is related to the B matrix used in the complex conjugation by

$$C = B^T A\,, \tag{A.2}$$

i.e.

$$C_{AB} = B^{\bar{C}}{}_A A_{\bar{C}B}\,, \tag{A.3}$$

where one can choose a representation in which A equals the Kronecker delta. The B matrix satisfies

$$B^* B = -1\,, \tag{A.4}$$

i.e.

$$B^{\bar{A}}{}_B \overline{B^{\bar{B}}{}_C} = -\delta^{\bar{A}}{}_{\bar{C}}\,. \tag{A.5}$$

A.2 Light-cone little group SO(4)

We define SU(2) anti-symmetric ϵ tensors with the convention

$$\epsilon^{12} = 1\,, \tag{A.6}$$

$$\epsilon_{12} = 1\,. \tag{A.7}$$

We introduce the σ-matrices

$$(\sigma^\alpha)^{\dot{a}b} = -(\sigma^\alpha)^{b\dot{a}}\,, \quad (\sigma^\alpha)_{\dot{a}b} = -(\sigma^\alpha)_{b\dot{a}}\,, \tag{A.8}$$

related to each other by

$$\sigma^{\alpha a\dot{b}} = +\epsilon^{ac}\epsilon^{\dot{b}\dot{d}}\sigma^\alpha_{c\dot{d}}\,, \tag{A.9}$$

$$\sigma^\alpha_{c\dot{d}} = \sigma^{\alpha a\dot{b}}\epsilon_{ac}\epsilon_{\dot{b}\dot{d}}\,. \tag{A.10}$$

They satisfy the algebra

$$\sigma^{\alpha a\dot{c}}\sigma^\beta{}_{\dot{c}b} + \sigma^{\beta a\dot{c}}\sigma^\alpha{}_{\dot{c}b} = \delta^a{}_b\,, \tag{A.11}$$

$$\sigma^{\alpha\dot{a}c}\sigma^\beta{}_{c\dot{b}} + \sigma^{\beta\dot{a}c}\sigma^\alpha{}_{c\dot{b}} = \delta^{\dot{a}}{}_{\dot{b}}\,. \tag{A.12}$$

An explicit representation is

$$\sigma^{\alpha a\dot{b}} = (-\sigma^1, \sigma^2, -\sigma^3, i1)\,, \tag{A.13}$$

$$\sigma^{\alpha\dot{a}b} = (\sigma^1, \sigma^2, \sigma^3, -i1)\,. \tag{A.14}$$

We define

$$\sigma^{\alpha\beta a}{}_b = \frac{1}{2}\left(\sigma^{\alpha a\dot{c}}\sigma^\beta{}_{\dot{c}b} - \sigma^{\beta a\dot{c}}\sigma^\alpha{}_{\dot{c}b}\right)\,, \tag{A.15}$$

$$\sigma^{\alpha\beta\dot{a}}{}_{\dot{b}} = \frac{1}{2}\left(\sigma^{\alpha\dot{a}c}\sigma^\beta{}_{c\dot{b}} - \sigma^{\beta\dot{a}c}\sigma^\alpha{}_{c\dot{b}}\right) \tag{A.16}$$

and

$$\sigma^{\alpha\beta ab} = \sigma^{\alpha\beta a}{}_{c}\epsilon^{cb}\,, \tag{A.17}$$

which satisfy

$$\sigma^{\alpha\beta\, cd} = \sigma^{\alpha\beta\, dc}\,. \tag{A.18}$$

We introduce 2×2 matrices $B^{\bar{\dot{a}}}{}_{\dot{c}}$, $B^{\bar{a}}{}_{c}$, whose components are equal to those of $-i\sigma^2$. We have

$$B^{-1\,c}{}_{\bar{a}}\overline{\sigma^{\alpha a\dot{b}}}B^{-1\,\dot{d}}{}_{\bar{\dot{b}}} = +\sigma^{\alpha c\dot{d}}\,, \tag{A.19}$$

$$B^{\bar{\dot{a}}}{}_{\dot{c}}\overline{\sigma^{\alpha}{}_{\dot{a}b}}B^{\bar{b}}{}_{d} = +\sigma^{\alpha}{}_{\dot{c}d}\,, \tag{A.20}$$

$$B^{-1\,c}{}_{\bar{a}}\overline{\sigma^{\alpha\beta\,a}{}_{b}}B^{\bar{b}}{}_{d} = +\sigma^{\alpha\beta\,c}{}_{d}\,, \tag{A.21}$$

$$\overline{\epsilon^{cd}}B^{-1\,e}{}_{\bar{d}} = +B^{\bar{c}}{}_{d}\epsilon^{de}\,, \tag{A.22}$$

$$B^{-1\,d}{}_{\bar{b}}B^{-1\,c}{}_{\bar{a}}\overline{\sigma^{\alpha\beta\,ab}} = \sigma^{\alpha\beta\,cd}\,. \tag{A.23}$$

B Superalgebra

$$[(Q_K)_{aA}, (Q_K)_{bB}] = -\sqrt{2}\epsilon_{ab}C_{AB}P^{+}\,, \tag{B.1}$$

$$[(Q_K)_{aA}, (Q_D)_{\dot{b}B}] = (\sigma^{\alpha})_{a\dot{b}}C_{AB}P_{\alpha}\,, \tag{B.2}$$

$$[(Q_D)_{\dot{a}A}, (Q_K)_{bB}] = -(\sigma^{\alpha})_{b\dot{a}}C_{AB}P_{\alpha}\,, \tag{B.3}$$

$$[(Q_D)_{\dot{a}A}, (Q_D)_{\dot{b}B}] = \sqrt{2}\epsilon_{\dot{a}\dot{b}}C_{AB}P^{-}\,, \tag{B.4}$$

$$[M^{+\alpha}, (Q_D)_{\dot{a}A}] = -\frac{i}{\sqrt{2}}(Q_K)_{bA}\epsilon^{bc}(\sigma^{\alpha})_{c\dot{a}}\,, \tag{B.5}$$

$$[M^{-\alpha}, (Q_K)_{aA}] = \frac{i}{\sqrt{2}}(Q_D)_{\dot{b}A}\epsilon^{\dot{b}\dot{c}}(\sigma^{\alpha})_{\dot{c}a}\,, \tag{B.6}$$

$$[M^{\alpha\beta}, (Q_K)_{aA}] = -\frac{i}{2}(Q_K)_{bA}(\sigma^{\alpha\beta})^{b}{}_{a}\,, \tag{B.7}$$

$$[M^{\alpha\beta}, (Q_D)_{\dot{a}A}] = -\frac{i}{2}(Q_D)_{\dot{b}A}(\sigma^{\alpha\beta})^{\dot{b}}{}_{\dot{a}}\,, \tag{B.8}$$

$$[M^{+-}, (Q_K)_{aA}] = \frac{i}{2}(Q_K)_{aA}\,, \tag{B.9}$$

$$[M^{+-}, (Q_D)_{\dot{a}A}] = -\frac{i}{2}(Q_D)_{\dot{a}A}\,, \tag{B.10}$$

$$[M^{+-}, M^{+\alpha}] = iM^{+\alpha}\,, \tag{B.11}$$

$$[M^{+-}, M^{-\alpha}] = -iM^{-\alpha}\,, \tag{B.12}$$

$$[M^{+\alpha}, M^{-\beta}] = -iM^{\alpha\beta} + i\delta^{\alpha\beta}M^{+-}\,, \tag{B.13}$$

$$[M^{\alpha\beta}, M^{\pm\gamma}] = i(\eta^{\alpha\gamma}M^{\pm\beta} - \eta^{\beta\gamma}M^{\pm\alpha})\,, \tag{B.14}$$

$$[M^{\alpha\beta}, M^{\gamma\delta}] = i(\eta^{\beta\gamma}M^{\delta\alpha} - \eta^{\alpha\gamma}M^{\delta\beta} - \eta^{\beta\delta}M^{\gamma\alpha} + \eta^{\alpha\delta}M^{\gamma\beta})\,, \tag{B.15}$$

$$[M^{+-}, P^{+}] = iP^{+}\,, \tag{B.16}$$

$$[M^{+-}, P^{-}] = -iP^{-}\,, \tag{B.17}$$

$$[M^{+\alpha}, P^{-}] = -iP^{\alpha}\,, \tag{B.18}$$

$$[M^{+\alpha}, P^{\beta}] = -iP^{+}\delta^{\alpha\beta}\,, \tag{B.19}$$

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$$[M^{-\alpha}, P^{+}] = -iP^{\alpha}\,, \tag{B.20}$$
$$[M^{-\alpha}, P^{\beta}] = -iP^{-}\delta^{\alpha\beta}\,, \tag{B.21}$$
$$[M^{\alpha\beta}, P^{\gamma}] = i(P^{\beta}\delta^{\gamma\alpha} - P^{\alpha}\delta^{\gamma\beta})\,. \tag{B.22}$$

All other commutators not listed here vanish.

Our convention is that all bosonic charges M's and P's are hermitian, while Q_K and Q_D satisfy the hermiticity conditions

$$\overline{Q_{KaA}} = -Q_{KbB}\; B^{-1b}{}_{\bar{a}} B^{-1B}{}_{\bar{A}}\,, \tag{B.23}$$
$$\overline{Q_{D\dot{a}A}} = Q_{D\dot{b}B}\; B^{-1\dot{b}}{}_{\bar{\dot{a}}} B^{-1B}{}_{\bar{A}}\,. \tag{B.24}$$

C Computation of $[M^{-\alpha}, M^{-\beta}]$

We verified explicitly the commutators between the charges for the free part of the theory presented in section 3. We work at the level of Poisson brackets, i.e. we ignore ordering issues in the definition of products of operators.

In this appendix we show how to compute the commutators of the free part of the symmetry charges focussing on the most involved commutator

$$[M^{-\alpha}, M^{-\beta}] = 0\,, \tag{C.1}$$

as an example.

For the free part, we can work solely in the first quantised language,

$$M^{-\alpha} = \int_0^{[\sigma]} \Bigg(x^{-}(\sigma)p^{\alpha}(\sigma) - x^{\alpha}(\sigma)p^{-}(\sigma) + \frac{i}{2}\theta^{aA}(\sigma)\frac{\delta}{\delta\theta^{aA}}(\sigma)\frac{p^{\alpha}(\sigma)}{p^{+}} + \frac{\sqrt{2}}{8}i\frac{p^{\gamma}(\sigma)}{(p^{+})^2}q_{aA}(\sigma)q_{bB}(\sigma)\sigma^{\alpha\gamma ab}C^{-1AB}\Bigg)d\sigma\,. \tag{C.2}$$

For simplicity we choose the range of σ to be $[0,[\sigma]]$; the computation goes through also in the convention used in the main text.

The essential simplification which occurs for the tensionless string theory is that a good part of the computation is completely parallel to the computation for the superparticle case. This is because each charge presented in section 3 is an integral of the charge density which does not involve σ-derivatives. Dropping the σ dependence from the charge density, we get the charge for the superparticle case. Thus for example $M^{-\alpha}$ for the superparticle is

$$M^{-\alpha} = x^{-}p^{\alpha} - x^{\alpha}p^{-} + \frac{i}{2}\theta^{aA}\frac{\partial}{\partial\theta^{aA}}\frac{p^{\alpha}}{p^{+}} + \frac{\sqrt{2}}{8}i\frac{p^{\gamma}}{(p^{+})^2}q_{aA}q_{bB}\sigma^{\alpha\gamma ab}C^{-1AB}\,. \tag{C.3}$$

The definition of q is the same as (3.8) except that there is no σ-dependence for the superparticle case. By a slight abuse of notation, we use for the variables characterising the superparticle, $x^+, p^-, x^\alpha, p_\alpha, \theta^{aA}$, the same symbols used in the string case. The commutation relations between these variables are

$$[x^+, p^-] = -i\,, \tag{C.4}$$
$$[x^\alpha, p^\beta] = i\delta^{\alpha\beta}\,, \tag{C.5}$$
$$\left[\frac{\partial}{\partial\theta^{aA}}, \theta^{bB}\right] = \delta^a{}_b\delta^A{}_B\,. \tag{C.6}$$

Comparing these to the commutation relations in the tensionless superstring theory

$$[X^+, P^-] = -i\,, \tag{C.7}$$
$$[x^\alpha(\sigma), p^\beta(\sigma')] = i\delta^{\alpha\beta}\delta(\sigma-\sigma')\,, \tag{C.8}$$
$$\left[\frac{\delta}{\delta\theta^{aA}}(\sigma), \theta^{bB}(\sigma')\right] = \delta^a{}_b\delta^A{}_B\delta(\sigma-\sigma')\,, \tag{C.9}$$

we see that if $x^-(\sigma)$ is not involved, the computation of commutators for the tensionless string case is completely parallel to the superparticle case; the commutators between the charge densities of the tensionless string are given simply by the commutators between the charges of the particle multiplied by $\delta(\sigma-\sigma')$.

The only charge[10] which contains $x^-(\sigma)$ is $M^{-\alpha}$. Hence one needs to perform additional computations to verify the commutation relations involving this generator. In section C.1 we present the computation of the commutator $[M^{-\alpha}, M^{-\beta}]$ in the superparticle case and in C.2 we explain the modifications necessary to deal with the tensionless superstring case.

C.1 Superparticle case

We write the generator as

$$M^{-\alpha} = X^\alpha + Y^\alpha\,, \tag{C.10}$$

where

$$X^\alpha = x^-p^\alpha - x^\alpha p^- + \frac{i}{2}\theta^{aA}\frac{\partial}{\partial\theta^{aA}}\frac{p^\alpha}{p^+}\,,\qquad Y^\alpha = \frac{\sqrt{2}}{8}i\frac{p^\gamma}{(p^+)^2}q_{aA}q_{bB}\sigma^{\alpha\gamma ab}C^{-1AB}\,. \tag{C.11}$$

It is easy to show

$$[X^\alpha, X^\beta] = 0\,, \tag{C.12}$$
$$[X^\alpha, Y^\beta] = -\frac{\sqrt{2}}{8}\frac{p^\gamma p^\alpha}{(p^+)^3}q_{aA}q_{bB}\sigma^{\beta\gamma ab}C^{-1AB} - \frac{\sqrt{2}}{8}\frac{p^-}{(p^+)^2}q_{aA}q_{bB}\sigma^{\alpha\beta ab}C^{-1AB}\,. \tag{C.13}$$

We also get

$$\begin{aligned}[Y^\alpha, Y^\beta] &= \left(\frac{\sqrt{2}}{8}i\right)^2 \times 2\times\frac{p^\gamma}{(p^+)^2}\frac{p^\delta}{(p^+)^2}\times\sigma^{\underline{\alpha}\gamma ab}C^{-1LM}\sigma^{\underline{\beta}\delta cd}C^{-1NP}\times q_{aL}[q_{bM}, q_{cN}]q_{dP}\\ &= \frac{1}{16}\frac{p^\gamma p^\delta}{(p^+)^4}\times\sigma^{\underline{\alpha}\gamma ab}C^{-1LM}\sigma^{\underline{\beta}\delta cd}C^{-1NP}\times q_{aL}q_{dP}\times\sqrt{2}\epsilon_{bc}C_{MN}p^+\\ &= -\frac{\sqrt{2}}{8}\frac{1}{(p^+)^3}\times q_{aL}q_{dP}\times C^{-1LP}\times\left(\sigma^{\underline{\alpha}\gamma ad}p_\gamma p^{\underline{\beta}} - p_\gamma p^\gamma\sigma^{\alpha\beta ad}\right)\,,\end{aligned} \tag{C.14}$$

[10] M^{+-} depends only on the zero-mode X^-.

where the underlined indices are understood to be anti-symmetrised with no multiplicative coefficient. Adding up these contributions, we obtain $[M^{-\alpha}, M^{-\beta}] = 0$ for the superparticle case.

In the computation we use the following formulae and the general formulae listed in appendix A

$$\sigma^{\underline{\alpha}\gamma ab}\epsilon_{bc}\sigma^{\underline{\beta}\delta cd} = -\sigma^{\underline{\alpha}\gamma a}{}_{b}\sigma^{\underline{\beta}\delta b}{}_{c}\epsilon^{cd}\,, \tag{C.15}$$

$$\sigma^{\alpha\gamma}\sigma^{\beta\delta} = \sigma^{\alpha\gamma\beta\delta} + \sigma^{\alpha\delta}\delta^{\gamma\beta} - \sigma^{\gamma\delta}\delta^{\alpha\beta} - \sigma^{\alpha\beta}\delta^{\gamma\delta} + \sigma^{\gamma\beta}\delta^{\alpha\delta} + \delta^{\alpha\delta}\delta^{\gamma\beta} - \delta^{\alpha\beta}\delta^{\gamma\delta}\,, \tag{C.16}$$

$$\begin{aligned}\sigma^{\underline{\alpha}\gamma}\sigma^{\underline{\beta}\delta} &= 2\sigma^{\alpha\gamma\beta\delta} + \sigma^{\alpha\delta}\delta^{\gamma\beta} - 2\sigma^{\alpha\beta}\delta^{\gamma\delta} + \sigma^{\gamma\beta}\delta^{\alpha\delta} - \sigma^{\beta\delta}\delta^{\gamma\alpha} - \sigma^{\gamma\alpha}\delta^{\beta\delta} + \delta^{\alpha\delta}\delta^{\gamma\beta} \\ &\quad - \delta^{\beta\delta}\delta^{\gamma\alpha}\,,\end{aligned} \tag{C.17}$$

$$\sigma^{\underline{\alpha}\gamma}\sigma^{\underline{\beta}\delta}p_\gamma p_\delta = 2\sigma^{\alpha\gamma}p_\gamma p^\beta - 2\sigma^{\alpha\beta}p_\gamma p^\gamma - 2\sigma^{\beta\gamma}p_\gamma p^\alpha\,. \tag{C.18}$$

In the last three equations the spinor indices are suppressed.

C.2 Contribution involving $x^-(\sigma)$ in $[M^{-\alpha}, M^{-\beta}]$

As already explained, most of the terms appearing in the computation of $[M^{-\alpha}, M^{-\beta}]$ for the tensionless superstring case can be simply obtained from the corresponding terms in the computation for the superparticle.

The exceptions are the terms involving x^-, since x^- is defined non-locally in terms of other dynamical variables (3.16). More practically, the calculations in the string and in the particle cases differ because p^+ is a c-number in the string case and we do not have the analogue of the commutator

$$[x^-(\sigma), p^+(\sigma')] = -i\delta(\sigma - \sigma')\,. \tag{C.19}$$

The term involving x^- in the $[M^{-\alpha}, M^{-\beta}]$ commutator is

$$[A^{-\alpha}, M^{-\beta}]\,, \quad \text{with} \quad A^{-\alpha} = \int_0^{[\sigma]} x^-(\sigma')p^\alpha(\sigma')\;d\sigma'\,. \tag{C.20}$$

This commutator can be computed by rewriting the generator $A^{-\alpha}$ following Mandelstam [53],

$$\begin{aligned}A^{-\alpha} &= X^-P^\alpha + \int_0^{[\sigma]} x^-(\sigma')\left(p^\alpha(\sigma') - \frac{P^\alpha}{[\sigma]}\right)d\sigma' \\ &= X^-P^\alpha + \int_0^{[\sigma]} x^-(\sigma')\;\partial_{\sigma'}\int_0^{\sigma'}\left(p^\alpha(\sigma'') - \frac{P^\alpha}{[\sigma]}\right)d\sigma'' d\sigma' \\ &= X^-P^\alpha - \int_0^{[\sigma]} \partial_{\sigma'}x^-(\sigma')\left(\int_0^{\sigma'} p^\alpha(\sigma'')\;d\sigma'' - \frac{P^\alpha}{[\sigma]}\sigma'\right)d\sigma'\,,\end{aligned} \tag{C.21}$$

where $\partial_\sigma x^-$ is given by (3.16).

The computation of $[A^{-\alpha}, M^{-\beta}]$ can be done systematically by noting the following observation about the commutator $[A^{-\alpha}, f]$ for a generic dynamical variable f. We denote by $[A^{-\alpha}, f]_{\rm cov}$ the commutator based on the covariant commutation relation, i.e. the commutation relations (C.19), (C.8) and (C.9). The computation of $[A^{-\alpha}, f]_{\rm cov}$ can be done in

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a way which is completely parallel to the superparticle case. Thus the difference between $[A^{-\alpha}, f]$ and $[A^{-\alpha}, f]_{\text{cov}}$ is of interest.

It is known that this difference can be understood as the effect of the compensating gauge transformation on f [69]. The generator $A^{-\alpha}$ (using the covariant commutation relation) transforms p^+. This breaks the light-cone gauge condition and one needs a compensating gauge transformation to go back to the light-cone gauge slice. The variation of p^+ computed using the covariant commutation relation is proportional to

$$[A^{-\alpha}, p^+]_{\text{cov}} = -ip^\alpha(\sigma)\,. \tag{C.22}$$

Since p^+ transforms as a density under σ-reparametrisations, we have

$$\delta p^+(\sigma) = -\partial_\sigma(p^+\delta\sigma(\sigma)) = -p^+\partial_\sigma\delta\sigma(\sigma)\,. \tag{C.23}$$

Comparing (C.22) with (C.23) we find that $\delta\sigma$ associated with the compensating gauge transformation is proportional to

$$u^\alpha(\sigma) = \frac{1}{p^+}\int_0^\sigma p^\alpha(\sigma')\, d\sigma'\,, \tag{C.24}$$

where the integration constant is fixed by $\delta\sigma(0) = 0$. We have

$$[A^{-\alpha}, f(\sigma)] = [A^{-\alpha}, f(\sigma)]_{\text{cov}} + i\partial_\sigma f(\sigma)u^\alpha(\sigma) \qquad \text{if } f \text{ is a scalar}\,, \tag{C.25}$$

$$[A^{-\alpha}, f(\sigma)] = [A^{-\alpha}, f(\sigma)]_{\text{cov}} + i\partial_\sigma(f(\sigma)u^\alpha(\sigma)) \qquad \text{if } f \text{ is a density}\,. \tag{C.26}$$

The second terms on the r.h.s. correspond to the compensating gauge transformations. Indeed, for $f = p^+$ the r.h.s. of (C.26) vanishes.[11]

Later, we will need to evaluate $\left[A^{-\alpha}, \int_0^{[\sigma]} f(\sigma)\, d\sigma\right]$. We have

$$\left[A^{-\alpha}, \int_0^{[\sigma]} f(\sigma)\, d\sigma\right] = -i\frac{P^\alpha}{p^+}f([\sigma]) + \int_0^{[\sigma]} [A^{-\alpha}, f(\sigma)]\, d\sigma\,. \tag{C.27}$$

To obtain the first term, we regularise the integral in terms of a Riemann sum,

$$\int_0^{[\sigma]} f(\sigma)\, d\sigma \cong \sum_{m=1}^{M} f\left(\frac{[\sigma]}{M}m\right)\frac{[\sigma]}{M}\,, \tag{C.28}$$

and use

$$[X^-, [\sigma]] = \left[X^-, \frac{P^+}{p^+}\right] = -\frac{i}{p^+}\,. \tag{C.29}$$

In particular, if f is a density, we obtain

$$\left[A^{-\alpha}, \int_0^{[\sigma]} f(\sigma)\, d\sigma\right] = \int_0^{[\sigma]} [A^{-\alpha}, f(\sigma)]_{\text{cov}}\, d\sigma\,. \tag{C.30}$$

[11] There are two conventions for the light-cone gauge in string theory. The convention we are using in which p^+ is a constant is suitable when discussing interactions of strings [52]. There is another convention, used in [69], in which $[\sigma]$ is a constant (such as 2π). The form of the compensating gauge transformation depends on this convention. In the convention of [69], we need another contribution to the r.h.s. of (C.24) which is linear in σ.

using (C.26).

We compute $[A^{-\alpha}, M^{-\beta}]$ by successively verifying the generic formulae (C.25) and (C.26) for various building blocks of $M^{-\beta}$. For instance, we verify (C.25) for $f = x^-$, and (C.26) for $f = p^\beta$, and then (C.26) for $f = x^- p^\beta$. Finally by using (C.30) we obtain

$$[A^{-\alpha}, M^{-\beta}] = [A^{-\alpha}, M^{-\beta}]_{\text{cov}} \,. \tag{C.31}$$

This, combined with the computation for the superparticle in appendix C.1, implies $[M^{-\alpha}, M^{-\beta}] = 0$.

The following formulae are used in the computation. $x^-(\sigma)$ can be written as [69]

$$x^-(\sigma) = X^- + \int_0^{[\sigma]} \left(\frac{\sigma'}{[\sigma]} - \theta(\sigma' - \sigma)\right) \frac{1}{p^+} \left(p^\beta(\sigma')\partial_\sigma x^\beta(\sigma') - i\partial_\sigma \theta^{aA}(\sigma') \frac{\delta}{\delta\theta^{aA}}(\sigma')\right) d\sigma' \,, \tag{C.32}$$

which can be confirmed by differentiating with respect to σ and using (3.16). When computing $[A^{-\alpha}, x^-(\sigma)]$ the integral over σ in (C.32) should be dealt with in a manner similar to the manipulations used above for the computation of (C.27). Another important formula is

$$[X^-, x^\beta(\sigma)] = \left[X^-, \sum_n x^{\beta n} e^{in\frac{2\pi}{[\sigma]}\sigma}\right] = i\partial_\sigma x^\beta(\sigma)\frac{\sigma}{P^+} \,. \tag{C.33}$$

We also use

$$[X^-, p^\beta(\sigma)] = i\partial_\sigma \left(p^\beta(\sigma)\frac{\sigma}{P^+}\right) \,, \tag{C.34}$$

$$[X^-, \theta^{aA}(\sigma)] = i\partial_\sigma \theta^{aA}(\sigma)\frac{\sigma}{P^+} \,, \tag{C.35}$$

$$\left[X^-, \frac{\delta}{\delta\theta^{aA}}(\sigma)\right] = i\partial_\sigma \left(\frac{\delta}{\delta\theta^{aA}}(\sigma)\frac{\sigma}{P^+}\right) \,. \tag{C.36}$$

D Overlap and insertion

D.1 Insertion operator

In this appendix we motivate the use of $w(\sigma)$ defined in the main text (4.15)–(4.18) as the insertion and we discuss an alternative possibility.

One should insert operators at the interaction point, since there is no other special point on the string world-sheet. It is necessary here to distinguish the immediate left/right of the interaction point, since the very concept of interaction point may be considered as defined by the change of left/right from the point of view of the $r = 1, 2$ strings and the $r = 3$ string.

One could in general consider any linear combination

$$a_1 \boldsymbol{e}_1 + a_2 \boldsymbol{e}_2 + a_3 \boldsymbol{e}_3 + a_4 \boldsymbol{e}_4 \,, \tag{D.1}$$

of the four delta function approximations $\boldsymbol{e}_1, \ldots, \boldsymbol{e}_4$ depicted schematically in figure 2.

As explained in section 5.2 below (5.29) it is desirable to have

$$a_1 + a_2 + a_3 + a_4 = 0 \,, \tag{D.2}$$

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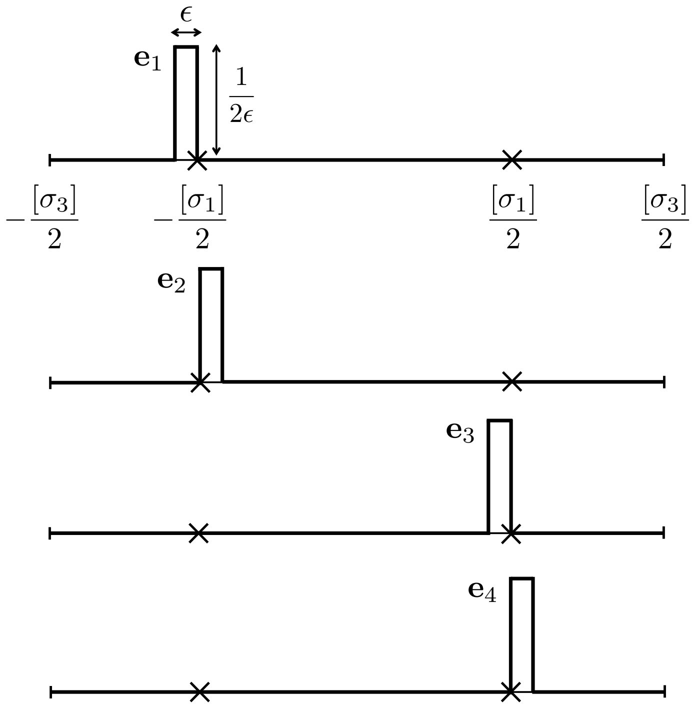

Figure 2. The smeared delta functions localised near the interaction point (indicated by the crosses) $\boldsymbol{e}_i$ with $i = 1, \cdots, 4$.

in order to eliminate some unwanted contributions in the computation of commutators.

Furthermore it can be shown, using the method of the test functional discussed in appendix E, that

$$\boldsymbol{e}_1 - \boldsymbol{e}_2 + \boldsymbol{e}_3 - \boldsymbol{e}_4 \,, \tag{D.3}$$

gives vanishing contribution as an insertion operator. Intuitively, this combination vanishes, because it vanishes from the perspective of both the $r = 1, 2$ strings and the $r = 3$ string. In other words, in the limit $\epsilon \to 0$, the above vanishes as a distribution both acting on well-behaved periodic functions defined on I and also on I_1 and I_2.

Hence we are left with a two-dimensional vector space which is spanned by w (4.15)–(4.16) used in the main text

$$w = -\boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3 - \boldsymbol{e}_4 \,, \tag{D.4}$$

and v defined by

$$v = \boldsymbol{e}_1 + \boldsymbol{e}_2 - \boldsymbol{e}_3 - \boldsymbol{e}_4 \,, \tag{D.5}$$

or equivalently

$$v(\sigma) = \delta\left(\sigma + \frac{[\sigma_1]}{2}\right) - \delta\left(\sigma - \frac{[\sigma_1]}{2}\right) \,, \tag{D.6}$$

i.e.

$$v^{3m} = \frac{2i}{[\sigma_3]} \sin\left(m\pi \frac{[\sigma_1]}{[\sigma_3]}\right) . \tag{D.7}$$

As explained in detail in appendix E, w must be used instead of v, since this choice assures the vanishing of the commutator $[Q_D, P^-]$ to cubic order.

D.2 Some mathematical properties of the overlap and the insertions

In this section we compile mathematical properties of v and w with the overlap, which is associated with subtleties related to the interaction point. The formulae in this section are not used in the main text. We nonetheless present them, since they may play a role in case the need arise to improve the ansatze presented in the main text. The formulae also somewhat clarify the relation of the insertion we used for tensionless strings and the insertion used in [47–50] for tensile superstring field theories.

We will focus on the bosonic sector and denote the overlap by V omitting the subscript B. Analogous properties hold for the fermionic sector as well.

We first introduce another basis-changing matrix (in the opposite direction compared to (4.11)) defined by

$$x^{3n} = \left(A^{-1}\right)^{3m}{}_{rn} x^{rn} , \tag{D.8}$$

where we hereafter use the convention in which the repeated index r is summed over $1, 2$. We will see below that the notation A^{-1} is somewhat inaccurate.

In [47–50], the form of the bosonic insertion Z is fixed by the requirement that it satisfy

$$[Z, x(\sigma_3) - x(\sigma_r)] = 0 , \tag{D.9}$$
$$[Z, p(\sigma_3) + p(\sigma_r)] = 0 , \tag{D.10}$$

for $\sigma \in I_r$ $(r = 1, 2)$ in our notation. Let us consider a Z which is a linear combination of x^{rm} $(r = 1, 2, 3)$,[12]

$$Z = \sum_{r=1,2,3} \sum_m z_{rm} x^{rm} . \tag{D.11}$$

We need only consider (D.10) which can be re-expressed as

$$[Z, p_{rn} + A^{-1\,3m}{}_{rn} p_{3m}] = 0 , \tag{D.12}$$

or

$$[Z, p_{3m} + A^{rn}{}_{3m} p_{rn}] = 0 , \tag{D.13}$$

depending on the basis we use.

If we employ, say, the latter condition, this implies

$$z_{3m} = -A^{rn}{}_{3m} z_{rn} . \tag{D.14}$$

Hence for any given z_{rn} $(r = 1, 2)$ we have an insertion

$$Z = z_{rn} \left(A^{rn}{}_{3m} - x^{rn}\right) , \tag{D.15}$$

satisfying the condition (D.13).

However, if the Z obtained above acts on the overlap operator V, we have

$$z_{rn} \left(A^{rn}{}_{3m} - x^{rn}\right) \prod \delta\left(x^{rn} - A^{rn}{}_{3m} x^{3m}\right) = 0 . \tag{D.16}$$

[12]The arguments below go through with little modification even if we consider a general linear combination of both x's and p's.

Thus all solutions of (D.13) seem to give a vanishing result when acting on V and thus cannot be employed as the insertion.

This seemingly paradoxical result could actually have been anticipated. The conditions, (D.9) and (D.10), mean that the $r = 1, 2$ strings and the $r = 3$ string are stitched together. This is the same condition which defines V. Thus it is natural that the objects satisfying (D.9) and (D.10) annihilate V. The stitching conditions, however, could fail at the interaction point, where we expect them to become ill-defined. Thus any object which does not annihilate V and satisfies (D.9) and (D.10) is necessarily associated with the interaction point.

This ill-defined nature at the interaction point is reflected in the fact that the infinite-dimensional matrix ${A^{rn}}_{3m}$ has an eigenvector with zero eigenvalue,

$$ {A^{rn}}_{3m} v^{3m} = 0 \,, \tag{D.17} $$

where v is defined in (D.5). This can be verified directly using the formula

$$ {A^{1m_1}}_{3m_3} = (-1)^{m_1} \frac{1}{\pi[\sigma_1]\left(\frac{m_3}{[\sigma_3]} - \frac{m_1}{[\sigma_1]}\right)} \sin\left(\pi \frac{[\sigma_1]}{[\sigma_3]} m_3\right) \,, \tag{D.18} $$

$$ {A^{2m_2}}_{3m_3} = (-1)^{m_2+1} \frac{1}{\pi[\sigma_2]\left(\frac{m_3}{[\sigma_3]} - \frac{m_2}{[\sigma_2]}\right)} \sin\left(\pi \frac{[\sigma_1]}{[\sigma_3]} m_3\right) \,. \tag{D.19} $$

A geometrical understanding of this condition is as follows. v is a well defined delta function (as a distribution) in the space of well-behaved (i.e. periodic with no gap) functions on the interval I associated with the third string. However it gives vanishing contribution when acting on well-behaved functions defined on I_1, I_2 corresponding to the first and the second strings.

Similarly we have

$$ {A^{-1\,3m}}_{rn} w^{rn} = 0 \,, \tag{D.20} $$

which again can be verified directly and has a similar geometrical interpretation.

The existence of v, w means that the following expression

$$ V' = \prod_m \delta(x^{3m} - {(A^{-1})^{3m}}_{rn} x^{rn}) \,, \tag{D.21} $$

which formally is equivalent to V (up to an overall factor), is actually subtly different from V.

Indeed, it can be shown that whereas

$$ (x\cdot v)V \,, \quad (p\cdot v)V \,, \quad (x\cdot w)V' \,, \quad (p\cdot w)V' \,, \tag{D.22} $$

are non-zero, the other combinations are equal to zero

$$ (x\cdot w)V = 0 \,, \quad (p\cdot w)V = 0 \,, \quad (x\cdot v)V' = 0 \,, \quad (p\cdot v)V' = 0 \,. \tag{D.23} $$

To understand this, it is instructive to consider the following integral

$$ X = \int f(\boldsymbol{x})\delta(\boldsymbol{x} - A\boldsymbol{x}')g(\boldsymbol{x}')d^3\boldsymbol{x}d^3\boldsymbol{x}' \,, \tag{D.24} $$

where the matrix A is defined by

$$A = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} \tag{D.25}$$

and the "wave functions" $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ decay sufficiently fast for $|\boldsymbol{x}| \to \infty$. Carrying out the x', y' integral in the usual manner, we obtain

$$\begin{aligned} X &= \int f(x,y,z)\delta(z-0)g(x,y,z')dxdydzdz' \\ &= \int f(x,y,0)\left(\int g(x,y,z')dz'\right)dxdy\,. \end{aligned} \tag{D.26}$$

We see that in the last expression the integral over z' is performed first and acts only on g. Thus the wave function g in the z'-direction is averaged over. Hence whereas inserting $\partial_{z'}$ acting on g into (D.24) gives 0, the insertion of z' gives, in general, a non-vanishing contribution. On the other hand, the z-variable of f is bound firmly to 0. Hence in (D.24) the insertion of ∂_z acting on f is non-vanishing, while inserting z gives a vanishing result.

It is interesting to note that when one performs a Fourier transformation and uses the p-representation instead of x-representation, the role of (V, V'), (A, A^{-1}), (v, w) is respectively exchanged in (D.22) and (D.23). In particular, the momentum representations of V, V' are

$$V = \prod_m \delta\left(p_{3m} - A^{rn}{}_{3m} p_{rn}\right)\,, \tag{D.27}$$

$$V' = \prod_{r=1,2}\prod_n \delta\left(p_{rn} - A^{-1\;3m}{}_{rn} p_{3m}\right)\,, \tag{D.28}$$

up to an overall constant.

The list of non-zero insertions (D.22) shows that one can choose insertions which satisfy relations such as (D.13) but are non-vanishing when acting on the overlap. These relations may be useful to construct an ansatz of the cubic vertices satisfying the superalgebra. However, there is a caveat associated with the smearing procedure explained in appendix E.

As discussed in appendix E, it seems that we need to introduce a smearing of the insertions, say, $\tilde{p}\cdot w = p\cdot\tilde{w}$. It turns out that the identities (D.17), (D.20), and hence (D.23), become invalid for any finite smearing. For example,

$$\lim_{\epsilon\to 0} A\tilde{v} \neq 0\,, \tag{D.29}$$

while it is true that

$$Av = 0\,, \tag{D.30}$$

and

$$\lim_{\epsilon\to 0} \tilde{v} = v\,. \tag{D.31}$$

Thus the limit involved in the infinite sum over the mode numbers in the computation of Av does not commute with the limit $\epsilon \to 0$. This is because there is a number of order $\sim \frac{1}{\epsilon}$ of terms contributing to the sum Av, each of which behaves as ϵ.

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Thus, although (D.23) seems to prohibit the use of some insertions (since they vanish), the introduction of the smearing makes it possible to use them. Also, when smearing is introduced, one can ignore the subtle difference between V and V'. This is natural since the difference is associated with the singularity strictly at the interaction point.

E Smearing and test functionals

E.1 Computation of commutators with smearing

In the computations of the commutators described in section 5.2, we encounter the multiplication of operators defined at the same point in σ-space. In order to perform the computation in a well defined manner we introduce a regularisation of the operators by using a smearing procedure.

Here we will define the smearing procedure and compute, as an example, the commutator

$$\left[Q^{(0)}_{D\dot{a}A}, Q^{(1)}_{D\dot{b}B}\right] + \left[Q^{(1)}_{D\dot{a}A}, Q^{(0)}_{D\dot{b}B}\right] = \sqrt{2}\epsilon_{\dot{a}\dot{b}} C_{AB} P^{-(1)} \tag{E.1}$$

using the smeared operators.

We define a smeared version of the momentum density $p(\sigma)$ by

$$\tilde{p}(\sigma) = \int f(\sigma, \sigma') p(\sigma') d\sigma' \,. \tag{E.2}$$

One can choose, as the kernel function $f(\sigma, \sigma')$, any regularisation of the Dirac delta function. For definiteness, we choose

$$f(\sigma, \sigma') = \begin{cases} \frac{1}{2\epsilon} & \text{for } \ \sigma - \epsilon \le \sigma' \le \sigma + \epsilon \\ 0 & \text{otherwise} \end{cases} \,, \tag{E.3}$$

where $\epsilon \ll 1$ is the parameter of the smearing. If σ is close to the interaction point and/or the boundary of the interval on which σ is defined, the above formula should be modified appropriately so that the correct periodicity is maintained.

To regularise the terms in the supercharge that are quadratic and cubic in the string field one replaces $p(\sigma)$ in (3.20) and (5.2) by its smeared version $\tilde{p}(\sigma)$,

$$Q^{(0)}_{D\dot{a}A} = \int \frac{1}{\sqrt{2}} \tilde{q}_{bA}(\sigma) \frac{1}{p^+} \epsilon^{bc} \tilde{p}^{\alpha}(\sigma) \sigma^{\alpha}{}_{c\dot{a}} d\sigma \,, \tag{E.4}$$

$$\begin{aligned} Q^{(1)}_{D\dot{a}A} = f^I{}_{JK} \int & \overline{\phi_{P_3^+}}_I \\ & \times \Big((\tilde{p}_\alpha \cdot w)(\sigma^{\alpha b}{}_{\dot{a}} \tilde{d}_{bA} \cdot w)(p^+)^{\lambda_0} (P_1^+)^{\lambda_1} (P_2^+)^{\lambda_2} (P_3^+)^{\lambda_3} \delta(P_1^+ + P_2^+ - P_3^+) V \Big) \\ & \times \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r \,. \end{aligned} \tag{E.5}$$

The computation of $[Q^{(0)}_D, Q^{(1)}_D]$ involves

$$[\tilde{d}_{aA}(\sigma), \tilde{d}_{bB}(\sigma')] = 2p^+ \epsilon_{ab} C_{AB} f'(\sigma - \sigma') \,, \tag{E.6}$$

where f' is given by the convolution integral,

$$f'(\sigma,\sigma') = \int f(\sigma,\sigma'')f(\sigma',\sigma'')d\sigma'' = \begin{cases} -\frac{|\sigma-\sigma'|}{(2\epsilon)^2} + \frac{1}{2\epsilon} & \text{for } \sigma - 2\epsilon \le \sigma' \le \sigma + 2\epsilon \\ 0 & \text{otherwise} \end{cases}, \tag{E.7}$$

satisfying

$$\int f'(\sigma,\sigma')d\sigma' = 1\,. \tag{E.8}$$

Using (E.7) as well as (E.3), the resulting commutator can be written as

$$\begin{aligned} \left[Q^{(0)}_{D\dot{a}A}, Q^{(1)}_{D\dot{b}B}\right] = 2C_{AB}\epsilon_{\dot{a}\dot{b}} f^I{}_{JK} \int \overline{\phi_{P_3^+}}_I &\times \left((\tilde{p}_\alpha \cdot w)(\tilde{p}'_\alpha \cdot w)(p^+)^{\lambda_0}(P_1^+)^{\lambda_1}(P_2^+)^{\lambda_2}(P_3^+)^{\lambda_3}\delta(P_1^+ + P_2^+ - P_3^+)V \right) \\ &\times \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r\,, \end{aligned} \tag{E.9}$$

where

$$\tilde{p}'_\alpha(\sigma) = \int f'(\sigma,\sigma')p_\alpha(\sigma')d\sigma'\,, \tag{E.10}$$

is a smeared version of $p_\alpha(\sigma)$.

From (E.9), one obtains

$$\begin{aligned} &P^{-(1)} \\ &= 2\sqrt{2} f^I{}_{JK} \int \overline{\phi_{P_3^+}}_I \left((\tilde{p}_\alpha \cdot w)(\tilde{p}'_\alpha \cdot w)(p^+)^{\lambda_0}(P_1^+)^{\lambda_1}(P_2^+)^{\lambda_2}(P_3^+)^{\lambda_3}\delta(P_1^+ + P_2^+ - P_3^+)V \right) \\ &\quad \times \phi_{P_1^+}{}^J \phi_{P_2^+}{}^K \prod_{r=1}^{3} dP_r^+ \mathcal{D}\theta_r \mathcal{D}x_r\,. \end{aligned} \tag{E.11}$$

E.2 Test functionals

We also occasionally have to deal with complicated expressions involving delta functions at the interaction point and delta functionals connecting the first and second strings to the third string. In order to deal with these expressions, it is often useful to introduce a set of test functionals and see how these expressions act on those test functionals.

The test functionals should be sufficiently general. The set of the test functionals we choose is, for a single string,

$$\phi_k[x] = e^{-\frac{\alpha}{4}p^+ \int x(\sigma)^2 d\sigma} \times e^{i\int k(\sigma)x(\sigma)d\sigma}\,, \tag{E.12}$$

where $k(\sigma)$, which is a smooth periodic function of σ, and α are the parameters of the test functional.

When dealing with string interactions, we use

$$\phi_r[x_r] = e^{-\frac{\alpha}{4}p^+ \int x_r(\sigma_r)^2 d\sigma_r} \times e^{i\int k_r(\sigma_r)x_r(\sigma_r)d\sigma_r}\,, \qquad r = 1,2,3\,, \tag{E.13}$$

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where $k_r(\sigma_r)$ are the parameters of the test functional. Each $k_r(\sigma_r)$ is a smooth periodic function defined on $\sigma_r \in [-[\sigma_r]/2, +[\sigma_r]/2]$.

These test functionals are generalised Gaussian wave packets. This is natural for a tensionless string, which is a collection of free particles associated with each value of σ. The probability distributions $|\phi_k|^2$ at each point in σ are uncorrelated. The distribution corresponds to Gaussian white noise (used for example in describing Brownian motion). The width of the Gaussian is proportional to α^{-1}. The factor of p^+ in the exponent makes it invariant under trivial rescalings of the σ coordinate. Also, one has the same Gaussian weight locally for all strings when an interaction is considered, since p^+ is common to all three strings due to momentum conservation implying $P_1^+ + P_2^+ = P_3^+$.

We evaluate the expressions by sandwiching them between test functionals. We first consider basic building blocks in such an analysis. By standard manipulations of Gaussian integrals (involving completing the square in the exponent and a shift of the integration contour in the complex plane), we obtain

$$\begin{aligned}
\int \overline{\phi'_k}\ p(\sigma)\ \phi_k \mathcal{D}x &= \int \left(\frac{i}{2}\alpha p^+ x(\sigma) + k(\sigma)\right) e^{-\frac{\alpha}{2}p^+\int x^2 d\sigma + i\int (k-k')x d\sigma}\mathcal{D}x \\
&= \int \left(\frac{i}{2}\alpha p^+ x + k\right)(\sigma) e^{-\frac{\alpha}{2}p^+\int\left(x - i\frac{k-k'}{\alpha p^+}\right)^2 d\sigma}\mathcal{D}x \times e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma} \\
&= \int \left(\frac{i}{2}\alpha p^+\left(x + i\frac{k-k'}{\alpha p^+}\right) + k\right)(\sigma) e^{-\frac{\alpha}{2}p^+\int x^2 d\sigma}\mathcal{D}x \times e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma} \\
&= \frac{k+k'}{2}(\sigma) \times \mathcal{N} e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma}\,.
\end{aligned} \tag{E.14}$$

Here $\mathcal{N}$ is an (infinite) normalisation constant, which may be absorbed into the definition of the test functionals $\phi(k)$.

We further have,

$$\begin{aligned}
&\int \overline{\phi'_k}\ p(\sigma)p(\sigma')\phi_k \mathcal{D}x \\
&\quad= \int \left(\left(\frac{i}{2}\alpha p^+ x + \frac{k+k'}{2}\right)(\sigma)\left(\frac{i}{2}\alpha p^+ x + \frac{k+k'}{2}\right)(\sigma') + \frac{\alpha}{2}p^+\delta(\sigma'-\sigma)\right) \\
&\quad\quad \times e^{-\frac{\alpha}{2}p^+\int x^2 d\sigma}\mathcal{D}x \times e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma} \\
&\quad= \left(\frac{\alpha}{4}p^+\delta(\sigma-\sigma') + \frac{k+k'}{2}(\sigma)\frac{k+k'}{2}(\sigma')\right) \times \mathcal{N} e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma}\,.
\end{aligned} \tag{E.15}$$

These results can be understood as following from Wick's theorem with non-zero one point functions. Namely, we can write

$$\langle p(\sigma)\rangle = \frac{k+k'}{2}(\sigma)\,, \tag{E.16}$$

$$\begin{aligned}
\langle p(\sigma)p(\sigma')\rangle &= \langle p(\sigma)\rangle\langle p(\sigma')\rangle + \overbracket{p(\sigma)p(\sigma')} \\
&= \frac{k+k'}{2}(\sigma)\frac{k+k'}{2}(\sigma') + \frac{\alpha}{4}p^+\delta(\sigma-\sigma')\,,
\end{aligned} \tag{E.17}$$

where we omit the common factor $\mathcal{N} e^{-\frac{(k-k')^2}{2\alpha p^+}d\sigma}$.

This pattern continues and we have, e.g.,

$$
\begin{aligned}
&\langle p(\sigma)p(\sigma')p(\sigma'')\rangle \\
&\quad = \langle p(\sigma)\rangle\langle p(\sigma')\rangle\langle p(\sigma'')\rangle + \langle p(\sigma)\rangle p(\sigma')p(\sigma'') + p(\sigma)\langle p(\sigma')\rangle p(\sigma'') + p(\sigma)p(\sigma')\langle p(\sigma'')\rangle \\
&\quad = \frac{k+k'}{2}(\sigma)\frac{k+k'}{2}(\sigma')\frac{k+k'}{2}(\sigma'') \qquad\qquad (\text{E.18}) \\
&\qquad + \frac{k+k'}{2}(\sigma)\frac{\alpha}{4}p^+\delta(\sigma'-\sigma'') + \frac{k+k'}{2}(\sigma')\frac{\alpha}{4}p^+\delta(\sigma-\sigma'') + \frac{k+k'}{2}(\sigma'')\frac{\alpha}{4}p^+\delta(\sigma-\sigma')\,.
\end{aligned}
$$

The use of Wick contractions here is reminiscent of that in the treatment of the Brownian motion. It may also play a similar role, for the tensionless string theory, to the simplifications via CFT techniques in ordinary string theory [96, 97].

If we consider a smeared version of $\int p^2 d\sigma$,

$$\int f(\sigma,\sigma')p(\sigma)p(\sigma')d\sigma d\sigma'\,, \tag{E.19}$$

for a generic kernel $f(\sigma,\sigma')$, we have,

$$\left\langle \int f(\sigma,\sigma')p(\sigma)p(\sigma')d\sigma d\sigma' \right\rangle = \int f(\sigma,\sigma')\frac{k+k'}{2}(\sigma)\frac{k+k'}{2}(\sigma')d\sigma + \int f(\sigma,\sigma)\frac{\alpha}{4}p^+ d\sigma\,. \tag{E.20}$$

The first term in this expression has a well defined limit when $\epsilon \to 0$. The second term, on the other hand, depends on the choice of the kernel function and generically is of order $\frac{1}{\epsilon}$. It is natural to drop the second term when evaluating these expressions. This is analogous to taking the normal order in tensile string theory. The steps used in defining a normal ordered form are: (i) regularisation of the product of operators, for instance by point-splitting, (ii) evaluation of matrix elements, (iii) subtraction of divergent terms. In our case the analog of step (i) is smearing, (ii) involves the sandwiching by test functionals and (iii) corresponds to discarding the second term in the above formula.

E.3 Sample computation using test functionals

In order to discuss $[Q_D, P^-]$, it is instructive first to consider the following expression

$$\int \bar{\phi}_3 \left(\int p_3(\sigma_3)^2 d\sigma_3 - \int p_1(\sigma_1)^2 d\sigma_1 - \int p_2(\sigma_2)^2 d\sigma_2 \right) V\phi_1\phi_2 \mathcal{D}x_1\mathcal{D}x_2\mathcal{D}x_3\,. \tag{E.21}$$

Formal application of (5.33) seems to imply that this expression vanishes. However, whether that is true has to be carefully examined because of the singularity associated with the multiplication of p's at the same point in the above formula.

We first introduce the smearing to the above,

$$\int \bar{\phi}_3 \left(\int \tilde{p}_3(\sigma_3)^2 d\sigma_3 - \int \tilde{p}_1(\sigma_1)^2 d\sigma_1 - \int \tilde{p}_2(\sigma_2)^2 d\sigma_2 \right) V\phi_1\phi_2 \mathcal{D}x_1\mathcal{D}x_2\mathcal{D}x_3\,, \tag{E.22}$$

where $\tilde{p}_r$ is the smeared momentum density defined for the r-th string.

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For brevity, we introduce $p_{12}(\sigma)$, defined on the whole interval I, which coincides with $p_r(\sigma_r)$ for $\sigma \in I_r$ $(r = 1, 2)$. Similarly, we also define $k_{12}(\sigma)$ out of $k_1(\sigma_1)$ and $k_2(\sigma_2)$. We have

$$\begin{aligned}
&\int \tilde{p}_3(\sigma_3)^2 d\sigma_3 - \int \tilde{p}_1(\sigma_1)^2 d\sigma_1 - \int \tilde{p}_2(\sigma_2)^2 d\sigma_2 \\
&\quad = \int \left(\tilde{p}_3(\sigma)^2 - \tilde{p}_{12}(\sigma)^2\right) d\sigma \\
&\quad = \int \left(p_3(\sigma)p_3(\sigma')f_3(\sigma,\sigma') - p_{12}(\sigma)p_{12}(\sigma')f_{12}(\sigma,\sigma')\right) d\sigma d\sigma' \,.
\end{aligned} \tag{E.23}$$

Here $f_3(\sigma,\sigma')$ and $f_{12}(\sigma,\sigma')$ are kernels for the smearing associated with the third string and the first-second strings. f_3 and f_{12} are different because they should obey different periodicity conditions. They are the same except when σ and σ' are sufficiently close (of the order of the length scale ϵ of smearing) to the interaction point.

Using (5.33) and eliminating the delta functional V, (E.22) becomes

$$\int \bar{\phi}_{k_3} \int p(\sigma)p(\sigma')\left(f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma')\right) d\sigma d\sigma' \phi_{k_{12}} \mathcal{D}x \,. \tag{E.24}$$

Using the short-hand notation introduced in the previous subsection, we have

$$\begin{aligned}
&\left\langle \int p(\sigma)p(\sigma')(f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma'))d\sigma d\sigma' \right\rangle \\
&\quad = \left\langle \int \left(\frac{\alpha}{4}p^+\delta(\sigma-\sigma') + k(\sigma)k(\sigma')\right)(f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma'))d\sigma d\sigma' \right\rangle \\
&\quad = \left\langle \int \left(k(\sigma)k(\sigma')\right)\left(f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma')\right) d\sigma d\sigma' \right\rangle \,,
\end{aligned} \tag{E.25}$$

where $k(\sigma) = \frac{k_{12}(\sigma)+k_3(\sigma)}{2}$. To obtain the last line we used $f_3(\sigma,\sigma) = f_{12}(\sigma,\sigma)$.

The expression $f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma')$ is non-zero only if σ is sufficiently near the interaction point. Examining the behaviour of this expression for each possible case of σ (the left/right of the first/second interaction points on I) and of σ', we find that, effectively,

$$f_3(\sigma,\sigma') - f_{12}(\sigma,\sigma') \sim \epsilon\tilde{v}(\sigma)\tilde{v}(\sigma') \,, \tag{E.26}$$

for $\epsilon \ll 1$, where $\tilde{v}(\sigma)$ is a smeared version of $v(\sigma)$ (a linear combination of delta functions having singularities at the vicinity of the interaction point) defined in (D.5). Here we omitted an unimportant numerical constant in the r.h.s.

Using this, (E.22) becomes finally

$$\epsilon v \cdot k v \cdot k \,, \tag{E.27}$$

and thus goes to zero when $\epsilon \to 0$. Thus, we have shown that, for the case of (E.21), formal manipulations using (5.33) are indeed justified by means of the smearing and the test functionals.

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E.4 $[Q_D, P^-]$ via smearing and test functionals

Now we consider $[P^-, Q_D] = 0$. There are two contributions in the cubic order, $[P^{-(0)}, Q_D^{(1)}]$ and $[P^{-(1)}, Q_D^{(0)}]$. The latter can be computed in the manner presented in section 5.2 and vanishes. For the former, one can perform a similar computation which yields an expression of the following form

$$\tilde{p} \cdot w \tilde{q} \cdot w \int \left(\tilde{p}_3(\sigma)^2 - \tilde{p}_{12}(\sigma)^2 \right) d\sigma V \,, \tag{E.28}$$

where we omit all unimportant factors. We have to verify that this expression vanishes which needs to be justified using smearing and the test functionals.

Firstly, we notice that the fermionic insertion $\tilde{q} \cdot w$ plays no important role. It will give a non-singular and non-zero contribution if we introduce appropriate fermionic contributions in the definition of the test functionals.

Thus we will focus on, by using test functionals,

$$\int \bar{\phi}_3 \tilde{p} \cdot w \int \left(\tilde{p}_3(\sigma)^2 - \tilde{p}_{12}(\sigma)^2 \right) d\sigma V \phi_1 \phi_2 \mathcal{D}x_1 \mathcal{D}x_2 \mathcal{D}x_3 \,. \tag{E.29}$$

We proceed in a manner similar to the previous subsection.Using (5.33) and eliminating V, (E.29) can be recast into

$$\int \overline{\phi_{k_3}} \tilde{p} \cdot w \int p(\sigma) p(\sigma') \left(f_3(\sigma, \sigma') - f_{12}(\sigma, \sigma') \right) d\sigma d\sigma' \phi_{k_{12}} \mathcal{D}x \,. \tag{E.30}$$

In the short-hand notation this becomes, using $\tilde{p} \cdot w = p \cdot \tilde{w}$,

$$\begin{aligned} &\int d\sigma d\sigma' d\sigma'' \tilde{w}(\sigma'') \left(f_3(\sigma, \sigma') - f_{12}(\sigma, \sigma') \right) \langle p(\sigma'') p(\sigma) p(\sigma') \rangle \\ &= \int d\sigma d\sigma' d\sigma'' \tilde{w}(\sigma'') \left(f_3(\sigma, \sigma') - f_{12}(\sigma, \sigma') \right) \\ &\quad \times \Big(\langle p(\sigma) \rangle \langle p(\sigma') \rangle \langle p(\sigma'') \rangle + \langle p(\sigma) \rangle \overbracket{p(\sigma') p(\sigma'')} + \overbracket{p(\sigma) \langle p(\sigma') \rangle p(\sigma'')} + \overbracket{p(\sigma) p(\sigma')} \langle p(\sigma'') \rangle \Big) \,. \end{aligned} \tag{E.31}$$

Using (E.18), and then $f_{12}(\sigma, \sigma) = f_3(\sigma, \sigma)$ and (E.26), this becomes, omitting an unimportant overall numerical factor,

$$\sim \epsilon (\tilde{k} \cdot v)^2 \tilde{k} \cdot w + \frac{\alpha}{2} p^+ \epsilon k \cdot \tilde{v} \tilde{v} \cdot \tilde{w} \,. \tag{E.32}$$

The first term vanishes in the limit $\epsilon \to 0$. This is also the case for the second term because $v \cdot w = 0$.

An important point here is that had we chosen to construct the ansatz in terms of v, the second term would have become $2\epsilon \tilde{k} \cdot v \tilde{v} \cdot \tilde{v}$. This gives a finite contribution, since $\tilde{v} \cdot \tilde{v} \sim \frac{1}{\epsilon}$. This would be inconsistent with the superalgebra. This justifies our use of w, rather than v, for insertions in our ansatz of the dynamical supercharge.

We also notice that formal application of (5.33) to (E.29) yields zero automatically irrespective of the choice of v or w in the insertion. The smearing and the test functional method we developed show that such formal application is not allowed due to the singularity associated with multiplication of $p(\sigma)$'s at the same point. A contribution to the commutator $[P^-, M^{-\alpha}]$ in light-cone gauge bosonic string theory arising by essentially the same mechanism is discussed in [53]. There the critical dimension $d = 26$ follows from requiring that the contribution vanishes.

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