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Training, Memory and Universal Scaling in Amorphous Frictional Granular Matter

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Abstract –We report a joint experimental and theoretical investigation of cyclic training of amorphous frictional granular assemblies, with special attention to memory formation and retention. Measures of dissipation and compactification are introduced, culminating with a proposed scaling law for the reducing dissipation and increasing memory. This scaling law is expected to be universal, and insensitive to the details of the elastic and frictional interactions between the granules.

"Memory" in materials physics is usually associated with the existence of macroscopic hysteretic responses [1]. Two distinct states, separated by a potential barrier larger than the thermal energy scale, can be used as a memory encoding mechanism; magnetic hysteresis being the most famous example from physics and the basis for all magnetic information storage media [2]. Memory formation and retention *via* hysteresis is a non-equilibrium process because the system requires external forcing to get across the energy barrier from one state to the other. Consequently, the notion of "universality" which provides fundamental understanding of equilibrium transitions is very difficult to find in non-equilibrium hysteretic processes.

Friction-induced hysteresis as well as memory in amorphous granular media is well known in the context of rock geophysics [3] and engineering [4], but the mechanism of memory formation, its training, and eventual retention are not yet well understood. Furthermore, the strong protocol dependence and the granular pack's preparation history immediately dash any hope of observing universal behavior in these systems. Here we focus on memory that is induced by training a frictional granular matter by cyclic loading and unloading [5–11]. In each such cycle dissipation leads to hysteresis, but with repeated cycles the dissipation diminishes until the system retains memory of an asymptotic loaded state that is not forgotten even under complete unloading. We report and explain a universal power law associated with the reduced dissipation and increase in memory which is expected to hold irrespective of the details of the microscopic interactions.

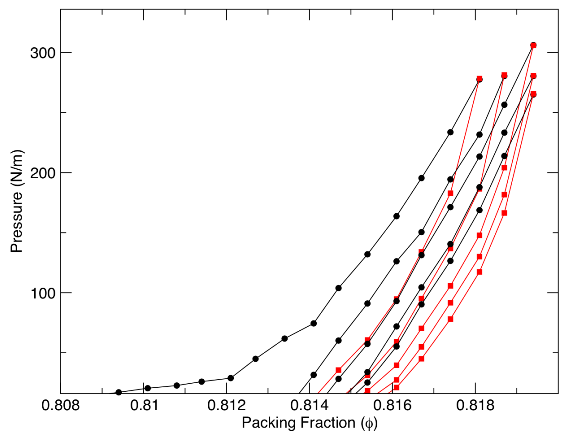

Fig. 1: Representative coarse-grained ($\Delta\Phi = 5\times10^{-3}$) hysteresis loops obtained experimentally upon uniaxial compression and decompression of an amorphous configuration of frictional disks as seen in Fig. 2. The pressure P is measured in N/m, and Φ is dimensionless. Compression legs are in black and decompression in red.

The phenomenon under study is best introduced by the experimental plots of pressure vs. packing fraction obtained by compressing and decompressing uniaxially an array of frictional disks [9], cf. Fig. 1. The experimental set up is detailed in the SI. A typical example of the experimental cell is shown in the upper panel of Fig. 2. Two opposing boundaries separated by chamber length L were movable while the other two (transverse) boundaries were held fixed. The two opposing movable boundaries provided uni-axial pack compression, through which the packing fraction Φ was controlled. Accordingly, we define the packing fraction Φ as the ratio of total area occupied by the disks to the chamber area bounded within the four boundaries, two of which are movable. Fig. 1 displays a typical series of consecutive loops of compression - decompression loops; the coarse-grained data was collected in quasi-static steps $\Delta\Phi = 5\times10^{-3}$ for representational purposes. The qualitative experimental observation is that the area in consecutive hysteresis loops diminishes monotonically while the packing fraction is increasing with every loop. This indicates that the system is compacted further with every loop and this process is accompanied by a reduction in the dissipation. The experimental results left however two open questions: (i) whether asymptotically the dissipation vanished, such that every compression became purely elastic and the decompression to zero pressure left the system with perfect memory of the stressed configuration; and (ii) whether there is anything universal in the way that the areas of the loops approaches its asymptote, be them finite or zero. To answer these question we performed numerical simulations that lead to the conclusion that (i) asymptotically the hysteresis loops are still dissipative due to frictional losses, but the structural rearrangements disappear and the neighbor list becomes invariant; and (ii) that the area A_n under the nth hysteresis loop (which is a direct measure of the dissipation) decays as a power law to an asymptotic value according to

$$A_n = A_\infty + Bn^{-\theta}\ , \quad \theta \approx 1. \qquad (1)$$

Here A_∞ represents the dissipation due to frictional slips that exist even in the asymptotic loop, and it depends on the material properties. The second term in Eq. (1) is due to the successive compactification of the sample, and the constant B is also expected to depend on material properties. The form of this law however is universal, expected to hold independently of the details of the microscopic interaction between the granules.

The details of the numerical set up are provided in the SI. An example of an initial configuration is shown in the lower panel of Fig. 2. We assign Hertzian normal force $F^{(n)}_{ij}$ and a Mindlin tangential force $F^{(t)}_{ij}$ [12] to each binary contact ij. The tangential force is always limited by the Coulomb law

$$F^{(t)}_{ij} \le \mu F^{(n)}_{ij}\ . \qquad (2)$$

In uniaxial straining the pressure is increased by pushing two opposite walls of the system towards each other. In each cycle we first reach a chosen maximal pressure by quasi-static steps. After each compression step, the system is allowed to relax to reach a new mechanical equilibrium in which the global stress tensor is measured by averaging the dyadic products between all the binary contact forces and the vectors connecting the centers of mass in a given volume. The trace of this stress tensor is the new pressure P. After a full compression leg, a cycle is completed by decompressing back to zero pressure, where the next compression cycle begins. The packing fraction Φ is monitored throughout this process. Each such cycle traces a hysteresis loop in the $P-\Phi$ plane, see Fig. 3 as an example. The area within each hysteresis loop is a measure of the dissipation, which in general stems from two sources. One is plastic events in which the neighbor lists change in an irreversible fashion, and the other is due to frictional losses when the frictional tangential force exceeds the allowed Coulomb limit. The training of the system is exemplified by the fact that the dissipation as measured by the area A_n of the nth cycle reduces with n and reaches an asymptotic value when $n\to\infty$. Measuring the area in the nth loop we find that it follows a power law decay in the form of Eq. (1) The data supporting this power law are exhibited in Fig. 4. From this data we can conclude that $\theta\approx1$ and that the scaling law appears universal with respect to changes in the value of μ. A further evidence of universality is obtained by changing the size distribution of disks, choosing a multi-dispersed system with radii ratios 1, 1.1, 1.2 and 1.4. Identical power laws were found. To understand the scaling law we need to

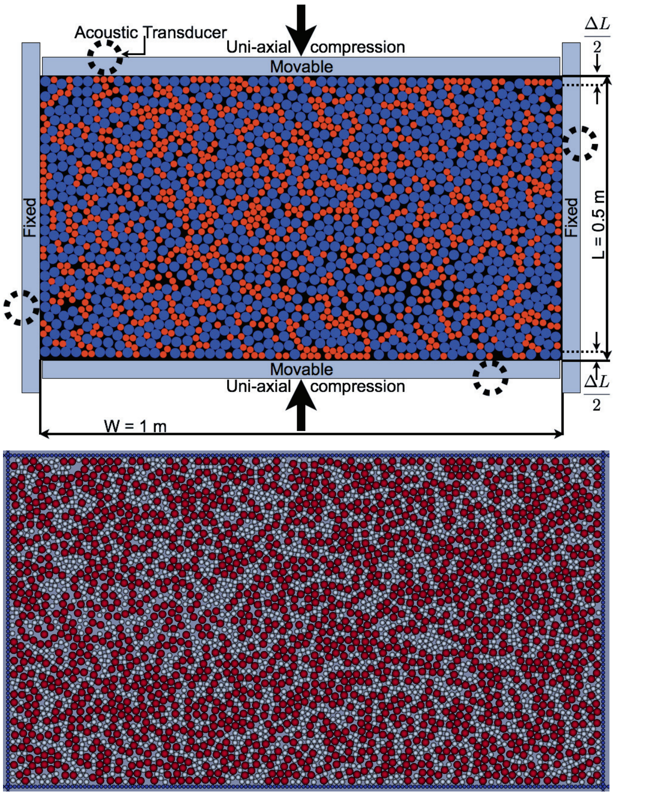

Fig. 2: Upper panel: An example of a typical initial configuration in the experiment. Lower panel: An example of a typical initial configuration in the numerical simulation.

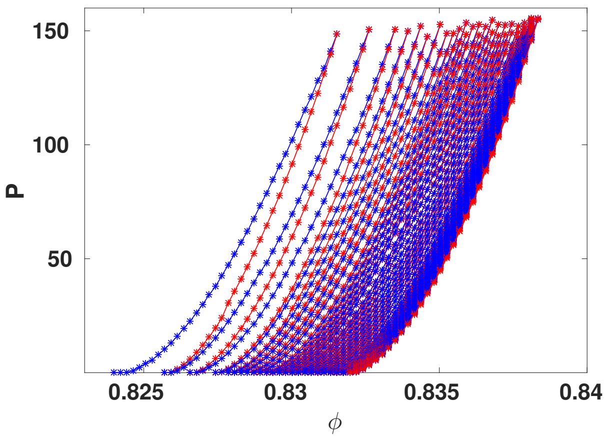

Fig. 3: A succession of hysteresis loops as measured in the numerical simulation. Blue symbols are compression legs and red symbols decompression legs. Here $\mu = 0.1$.

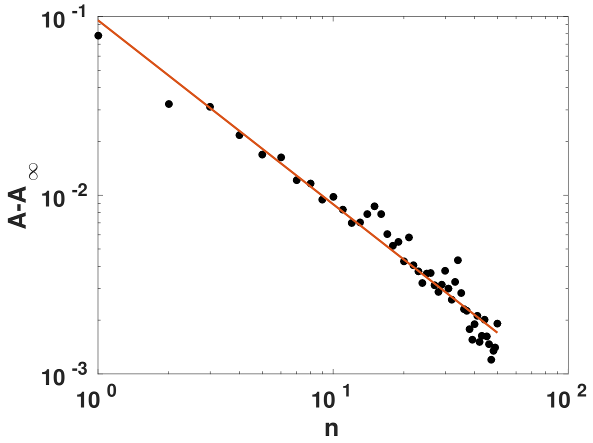

Fig. 4: The power law for the decaying areas under the hysteresis loops as measured in the numerical simulation. Here $\mu = 0.1$, Black dots are data and the red line is the best fitting power law $y = 0.095X^{-1.03}$. The observed lot is independent of μ, cf. the SI.

identify the two important processes that take place during the cyclic training. One is compactification. In every compression leg of the cycle the system compactifies, until a limiting Φ value is reached for the chosen maximal pressure. To quantify this process we can measure the volume fraction $\Phi_n(P_{\max})$ at the highest value of the pressure in the nth cycle. Define then a new variable

$$X_n \equiv \Phi_{n+1}(P_{\max}) - \Phi_n(P_{\max}) \ . \tag{3}$$

This new variable is history dependent in the sense that $X_{n+1} = g(X_n)$ where the function $g(x)$ is unknown at this point. This function must have a fixed point $g(x = 0) = 0$ since the series $\sum_n X_n$ must converge; for any given chosen maximal pressure there is a limiting volume fraction that cannot be exceeded. Near the fixed point, assuming analyticity, we must have the form

$$X_{n+1} = g(X_n) = X_n - CX_n^2 + \cdots \ . \tag{4}$$

The solution of this equation for n large is

$$X_n = \frac{C^{-1}}{n} \ . \tag{5}$$

This is the source of the second term in Eq. (1), which stems from the compactification and reduces the amount of dissipation due to irreversible plastic rearrangements. A direct measurement of X_n as a function of n is shown in the log-log plot presented in Fig. 5, supporting the generality of this power law. Without any reason for non-analyticity in the function $g(X_n)$ this conclusion is firm. It should be noted at this point that the scaling laws Eqs. (1) and (5) must contain some logarithmic corrections, since the harmonic series does not converge, but the series $\sum_n X_n$ must converge to get an asymptotic value of $\Phi_{\max}$. Indeed, the scaling laws measured above consistently show exponents slightly smaller than -1, and therefore the series of Φ_n converges. It is very likely that this small difference is due to logarithmic corrections to the scaling law which cannot be computed from the simple theory presented here.

The first term in Eq. (1), on the other hand, is due to the frictional dissipation. In uniaxial compression there is a shear component, and the shear stress loads the tangential contacts. Whenever the tangential force exceeds the Coulomb limit Eq. (2), the system dissipates some energy to a frictional slip. Even if the neighbor list becomes invariant at large values of n, the loading of the system is always accompanied by frictional slips. In our simulations we can measure the energy dissipated by friction slips, denoted as $\Delta E_n^{(f)}$ in the nth hysteresis loop. To have a non-dimensional measure we normalized this quantity by $\Delta E_1^{(f)}$, denoting the result as S_n. The dependence of this normalized dissipated energy on n is presented in Fig. 6. It is clear that the normalized dissipated energy due to frictional slips reduces rapidly to a stable value; this is the first term in Eq. (1).

Encouraged by this theory we returned to the experimental data to measure both X_n and A_n. To obtain the logarithmic n dependence of the area we needed to know the value of A_∞. A direct measurement of this area is unfeasible. But one recognizes that the total dissipation under the asymptotic loop should be the same with or without memory. Accordingly, we applied an acoustic perturbation to the configuration after each quasi-static step to destroy memory and training, and force the system to go to asymptotic state. The result is shown in the upper panel of Fig. 7, showing A_∞ alone in blue squares. Indeed A_∞ is finite and flat as a function of n. Using the measured value of A_∞ we get the power law scaling for $A_n - A_\infty$ as expected and as shown by the data with black rhombi. To underline the fact that this scaling law is governed by the memory during the training protocol we demonstrate the change that is caused by destroying the

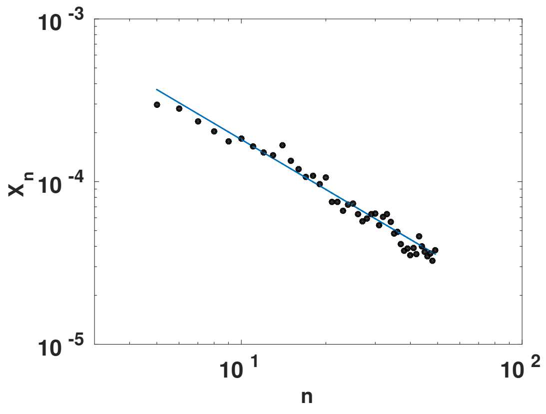

Fig. 5: Log-log plot of X_n vs. n. The black dots are the data, the blue line is the best fitting scaling law $y = 0.02x^{-1.02}$. The data corroborates Eq. (5).

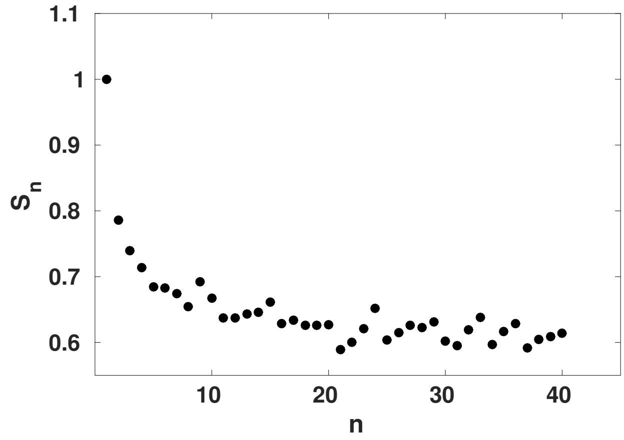

Fig. 6: The normalized frictional energy loss in each hysteresis loop. This energy loss drops to a stable value that is responsible for the asymptotic dissipation that is encoded by the area A_∞ in Eq.(1).

memory midway through the cycles; this is a further validation that the power law is indeed coming from training and memory formation. For the data shown in green triangles, there are no acoustic perturbations for $n = 1$ - 10; there we see the power-law behavior. From $n = 11$ - 50, we apply acoustic perturbations after each quasi-static step. The loop areas now fall drastically and become flat. Note that the magnitude is below A_∞ since we are plotting the difference $A_n - A_\infty$.

It should be commented that the simple scenario discussed in the Letter *requires* a subtle change in the shape of the hysteresis loops. The low order loops are increasing the volume fraction, such that the compression leg starts with at a lower value of Φ than the end of the decompression loop, see the upper panel in Fig. 8. This continues to be the case as long as the systems can be compactified further. The high order loops must begin and end at the same value of Φ_n, see the lower panel in Fig. 8. Thus for large value of n the hysteresis loops become repetitive, with an invariant trace in the $P - \Phi$ plane, even though they have frictional dissipation in the limit $n \to \infty$. This subtle change in the shape of the hysteresis loops allows the function $g(x)$ to have the fixed point at $x = 0$ around which the analytic expansion dictates the universality of the power law.

In conclusion, we presented and explained a universal scaling law in the context of the cyclic training of an amorphous assembly of frictional disks. The protocol exhibits a reduction in the dissipation per loop until the system reaches an asymptotic configuration with maximal volume fraction (for the maximal pressure chosen in the cyclic protocol). Once achieved, the system has a perfect memory of the stressed state even when it is completely decompressed to zero pressure. Interestingly enough, repeated compressions are not without dissipation, since shear always induces frictional slips. But the beginning and final volume fractions become invariant and the system repeats

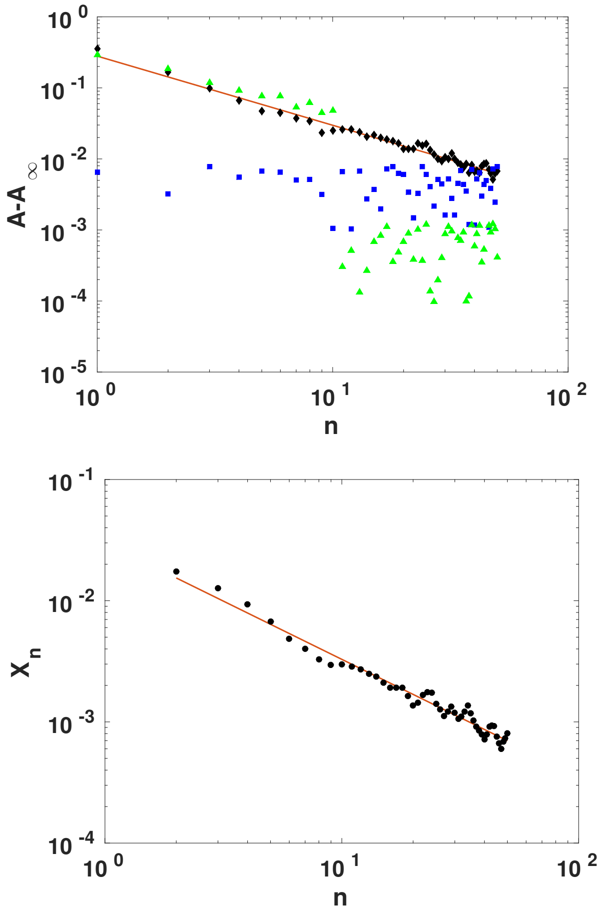

Fig. 7: Upper panel: The areas $A_n - A_\infty$ measured experimentally as a function of n, agreeing to Eq. (1) with $\theta \approx 0.97$; black rhombi. Blue squares: the estimate of A_∞ obtained by forcing the system to the asymptote by acoustic perturbations. Green triangles: the areas resulting from the destruction of memory after 10 regular loops. Lower panel: experimental results for X_n shown in log-log plot vs. n; the slope is approximately 0.96.

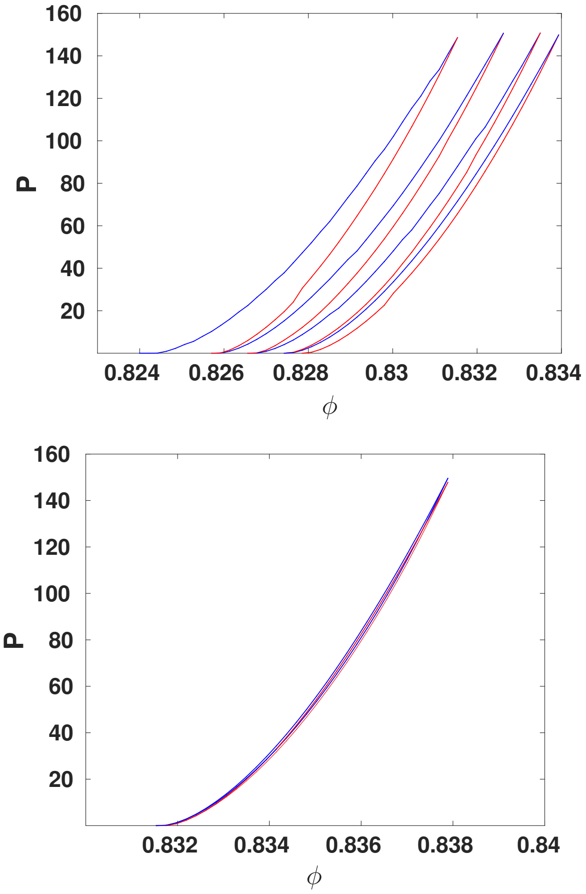

Fig. 8: Upper panel: examples of low order hysteresis loops in the $P-\Phi$ plane. The compression legs are blue and the decompression legs red. Lower panel: examples of higher order hysteresis loops in the $P-\Phi$ plane with the same color convention. The high order loops are no longer able to compactify the system further, and the compression leg begins at the same volume fraction where the decompression leg ends.

exactly the same hysteresis loop in the $P-\Phi$ space. The amount of dissipation in the cyclic loops is governed by the scaling law Eq. (1) which has a universal form with material dependent coefficients. The two terms in this equation were identified and related to the dissipation due to changes in the neighbor list and the frictional slips respectively.

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