Fall 2013

"I had never really made sense of that before": Constructing Mathematical Content Knowledge and Teaching through Story Telling

Sara McCormick Davis

Gabriel T. Matney
Bowling Green State University, gmatney@bgsu.edu

"I had never really made sense of that before": Constructing Mathematical Content Knowledge and Teaching through Story Telling"

Sara McCormick Davis

Associate Professor of Early Childhood Teacher Education

University of Arkansas Fort Smith

sara.davis@uafs.edu

Phone: 479-788-7919

Gabriel Matney

Associate Professor Math Education

Bowling Green State University

Bowling Green, Ohio

gmatney@bgsu.edu

Abstract

How do teacher educators in two different fields help pre-service teachers bridge content and pedagogical knowledge? This paper explores the experience of using story-telling over two courses that are required in an early childhood teacher education program. The authors, from two different colleges, used reflection prompted by story-telling to collaboratively support the construction of content knowledge in both a math content class and in a math methods class. Students were able to reflect on their own math content knowledge by telling and retelling their understanding of math knowledge construction and that of children they worked with over the semester.

As Amanda approached her math classroom for the first time she slowed her pace and felt a deep sense of fear, dread, and loathing all wrapped up in the pit of her stomach. Her heart racing, she stepped across the threshold. With the first step into the room, she quickly glanced around to find an empty chair near the back. She immediately began making her way to the empty seat and noticed out of the corner of her eye that the professor was welcoming the students with a warm, gentle smile. Amanda nodded and tried to muster a smile that just didn't come out right, instead resembling more of snarl. Slightly embarrassed by this she quickly passed the professor, lowering her head and rolling her eyes up to the ceiling thinking to herself, "Oh geez did that just really happen, did I just make a face at my professor?" Trying to blend in she waited for math class to begin, with the deluge of symbols, note taking, and general lack of connected sense making that it had always been for her.

Her mathematics professor began to speak, "Welcome everyone. This course starts with who we are and ends with who we are becoming. In order to understand who 'we' are though we will need to be open with one another and be willing to deeply reflect and share. Your first assignment begins now and it may not be easy." Amanda held her breath and said to herself, "and here it comes." The teacher continued, "I want you to spend the next few minutes intensely reflecting on your first memory involving quantity. What were the context, issues, people, etc.? You don't need to write a lot down, just write enough that you can coherently re-tell the story to us."

Amanda and her classmates reflected and began to share. Her professor listened intently to their stories and sometimes expanded upon this shared experience to mention things noted from research on children's learning of quantity, the history of the development of mathematics, and the role language plays in understanding place value. Shrugging a little, she thought, "Hmmm,

who knew mathematics could be so connected to life?" Class ended and a journaling assignment was given as homework. "Wait! Is it already over!? How did it go so fast?" As Amanda gathered her things many ideas bounced around in her head. She continued this quiet introspection as she walked to the dorm. "Did I really talk out loud in math class? I had never considered how the way we say a number could affect understanding of place-value. Just thinking about that helped me understand place-value better. Are there other things like that that could have stopped me from understanding too?" Though she still wasn't sure that every class day would be like this first one, she harbored a hope that it would be. A hope that her own understandings of mathematics would deepen, her fear would finally subside, and that she would learn how to help children avoid the negativity about mathematics that she often felt.

The story above contains many smaller experiences often told by pre-service early childhood teachers. Feelings of anxiety about mathematics are certainly not new and we work feverishly to help those who will be teaching mathematics develop positive dispositions toward the discipline. In that spirit, this paper describes our findings from the story telling journey we have designed for our pre-service elementary teachers in order to help them bridge mathematics content knowledge and pedagogical knowledge.

Theoretical Perspective

During our collegial planning times we focus our efforts toward understanding how students' dispositions change as they actively engage in reasoning about their own learning of mathematics. We both hold constructivist beliefs about learning and define constructivism as a meaning-making theory. Piaget's (1965) writing has influenced both of us to think of constructivism as a way of explaining how knowledge is constructed through actions with the world and

trying to find meaning through those actions. Constructivist theory helps us understand why 30 different ways of knowing exist in a class of 30 students. Thus, we read our students' writing *expecting* to find different interpretations of information shared in class. Furthermore, we both reject reception of information as being the only way people learn. These beliefs mean that we both try to engage our students in constructing their own learning through problem *finding*, not just problem solving. Problem finding can be characterized as students actively engaged in recognizing their own cognitive dissonance and then working to re-organize their construction of knowledge, rather than being told that an answer is correct or that there is one procedure to follow (Richardson, 1997).

Many of our students have experienced mathematics as existing outside of themselves. Well-meaning teachers have presented mathematics information as knowledge to be memorized. However, when the teacher's perspective holds that mathematics is constructed through the building of quantitative relationships they then focus on the thinking process. Von Glasersfeld (1996) refers to the importance of the thinking process when he wrote, "…when we intend to stimulate and enhance a student's learning, we cannot afford to forget that knowledge does not exist outside a person's mind" (p.5). In other words, constructing a relationship happens in a different way for each person, the acquisition of knowledge is not something we collect like seashells.

Our students have experienced teaching and the learning of mathematics as a behaviorist approach where everything to be learned is decided by the teacher, who then carries out the plans and evaluates the students' ability to show that they can repeat information. They come into our courses already having what Anderson and Piazza (1996) term an "apprenticeship of observation" from the multitude of prior experiences over several years of schooling (Philipp, 2000). In

early childhood education the view of learning as being an outcome of maturation is a driving force behind theories of developmentally appropriate practice. Developmental readiness may explain specific behaviors, but does not offer a grand theory about learning. These two views have guided teachers' instruction almost exclusively in modern education (Fosnot, 1996).

"At the heart of constructivism is the idea that every process of interpretation is a construction based on the conceptions already held by the researcher (teacher) " (Duit, Tregust, and Mansfield, 1996, p. 18). For us, that means we make pedagogical decisions through our own concepts of teaching and learning. We understand that constructivist theory is a philosophical stance; the learner is constructing knowledge whether the teaching method is direct instruction or guided discovery. However, some pedagogical methods send the message that the person who knows *more* must transmit the knowledge to someone who knows *less*, thus reinforcing the idea that learning is simply receiving information (Chaille, 2008). Because we are concerned about both our students' construction of mathematics content and teaching, we want them to experience their own theory building and we want to make it explicit. Storytelling is one pedagogical tool we have found to engage students in meaningful constructions of mathematics and teaching.

Story Telling and Reflection

Many studies show that a teacher education program which promotes reflective thought and practice as one of its conceptual foundations can help those students, who tend to be more reflective, become highly reflective practitioners. (Britzman, 1991; Davis, 2005; Francis, 1995; Grimmett & Erickson, 1988; Kettle & Sellars, 1996; Nolan, 2008). Reflection strategies range from critiquing lessons to summarizing future plans. Sometimes journals are kept and responded to by instructors or peer reviewers. Many of these strategies become steps to go through as part

of class assignments rather than prompting critical thinking and learning. Our students sometimes complain when they are asked to "reflect", and view it as another assignment to do as quickly and efficiently as possible.

The question then becomes, how do we support our students becoming more reflective about mathematics learning and teaching? The research on reflection in teacher education has deepened over the last several years. We have chosen to use story telling as a powerful reflection tool because of the rich possibilities for learning. Personal memories of learning can prompt a new vision for what it means to learn and teach math (Gordon, 2009). Using story telling can help the learner construct new perspectives of teaching and learning. (Frost, 2010; Kenney, Shoffner & Norris, 2013).

Story telling as a vehicle for learning in education has proved valuable (Akinbode, 2011; Ross & Prior, 2012). As our students tell their math stories or their teaching math stories they become researchers into their own experiences. We help our students construct a deeper understanding of the role of the teacher by thinking about their own experiences in classrooms. The story telling process holds much value for this endeavor. As van Manen (1990) puts it, "The insight into the essence of a phenomenon involves a process or reflectively appropriating, of clarifying, and of making explicit the structure of meaning of the lived experience (p. 77)." Our questions and prompts are designed to help our students construct ideas about the connection between their own learning experiences in mathematics and their future teaching. "...story connects past experience with present concerns and future goals...(Jalongo, Isenburg, & Gerbracht, 1995, pg.xxi).

In traditional research there may be the attempt on the part of the researcher to interpret the researched, finding meaning out of context. However, in story telling or narrative research,

part of the purpose is for the participant to learn through their own experience. "Narrative, then, is not merely a precursor to revision and change in teachers' lives; in forcing us to compose, articulate, and reinterpret our lives, it can move us toward action" (Ritchie and Wilson, 2000, pg. 21).

Mathematics Content

As program developers and teachers of an early childhood program we wrestle with is how much math content knowledge is needed for teachers? It is not clear that simply requiring more math content classes will lead to better math understanding or instruction. (Hill & Ball, 2009; Ball, Thames, & Phelps, 2008). However, there is evidence that when teachers understand their students' math reasoning they can make informed decisions about teaching for mathematical growth and understanding. (Fuson, Kalchman, & Bransford, 2005; Kamii, 2000; Steinberg, Empson, & Carpenter, 2004). The focus of many research projects has been on the understanding of math thinking as a core principle in teaching math. (Brown, 2005; Stipek, Givvin, Salmon, & MacGyvers, 2001; Walshaw, 2013). Through storytelling we seek to bring children's mathematics reasoning to the front of our discussions about teaching mathematics.

Context of Teaching

We teach courses that are part of an undergraduate, early childhood teacher education program. Gabriel, in the college of science, technology, engineering, and mathematics, teaches mathematics content courses while Sara, in the college of education, teaches an early childhood math methods course. Gabriel teaches the two pre-requisite courses pre-service teachers must

take before Sara's math methods course. Hence, we teach the same group of students at different points in the program.

We both use a combination of journaling and class discussion to formatively assess our students' developing thought process. The information we used for this paper was derived from a careful reading of five journal assignments in Gabriel's classes and ten journal entries in Sara's classes. Our conversations about the meaning of our students' understanding of math and teaching math represent our effort over the past four years to find themes in their responses that would frame our responses to their needs as learners. As researchers we follow the narrative inquiry model in that we are constantly reading and rereading our students' work looking "for the patterns, narrative threads, tensions, and themes either within or across an individual's experience and in the social setting" (Clandinin & Connelly, 2000, p.132).

Participants

Over a two year period 241 pre-service early childhood teachers wrote personal journals about their mathematics learning experiences, both prior and present. For the purposes of this paper we have chosen to give voice to our findings through the story of one student, Amanda (pseudonym), as she progressed through the program. She attends a medium-sized southern undergraduate university. The vast majority of students tend to be like Amanda, who is Caucasian, young, female, the first person in her family to go to college, and has world experiences fairly limited to a small town lifestyle. Also like Amanda, her peers are representational of the majority of early childhood pre-service teachers in that many of them have chosen to major in early childhood because of negative feelings about math. (Dunphry, 2009; Gresham, 2007).

Establishing Storytelling as Meaningful Reflection

For this process to be real and meaningful for pre-service teachers we too authentically engage in story telling with them. Both of us share stories with our students about our own mathematics experiences, discuss how these reflections on past learning experiences contributed to our growth as teachers, and explain why our thinking changed because of this reflection. This process reveals much to all involved.

Sara's Story

When co-author, Sara applied story telling to her reflections, it allowed her to tap into her early school experiences to reflect on her own learning in math. In third and fourth grade her teacher tried to make math fun by providing drill type exercises in the form of games. She remembers holding a peanut shaped piece of paper that had an addition problem written on it. The teacher called on the students one at a time to tell the class what the problem was, say the correct answer and if correct, the students were allowed to feed the peanut to an elephant painted on poster board. There were certain things about this that Sara and her peers considered enjoyable; they got to get out of their seats, they got to show off their expertise in knowing one correct answer, and they got to admire the teacher's ability to paint a cartoonish elephant. But in the same class Sara experienced a sinking feeling when the teacher would say, "get out your math book, turn to page 24 and do all the even problems." Sara reflected, what did I learn about math? I learned that it was something to be avoided, something to manipulate or disguise. It was a chore. There were no conversations about what the context might be for addition in our lives. There was no recognition that math might be enjoyable in itself, that the process of problem finding could be the focus of a lesson; not the completion of 15 problems on a worksheet.

Sara's experiences were in the 1960's and might be relegated to the dusty history of education, the way things "used to be". But as the rest of this paper will discuss, our pre-service teachers' stories reveal that things have not changed in any substantial way, their stories of early math experiences are quite similar.

Gabriel's Story

Co-author Gabriel's reflections about his early math experiences took place in the late eighties and early nineties, but were essentially shaped by his teachers' beliefs about mathematics and learning too.

One of Gabriel's reflections focuses on how a single mathematical topic was approached over the course of his education. In middle school his teacher proclaimed that the area of a circle was πr^2. Gabriel's class knew that π was approximately 3.14 but had no idea where it came from or how it was related to circles. Out of curiosity Gabriel asked his teacher "How do we know that πr^2 is the area of a circle?" To which his teacher responded, "Because that is what gives us the correct answer." A few years later in geometry these same types of questions arose again for Gabriel and he sought to quench them by asking his teacher. "How do we know that πr^2 gives the area of a circle and why? What does π have to do with area anyway, isn't it the ratio of circumference to diameter? Other figures used side lengths, altitudes, and bases to find the area so the use of π seems so random?" To which his teacher replied, "I'm not sure why π helps us find the area but all you need to know for the homework and test is that it does."

In spite of Gabriel's success in his mathematics courses, he was not content with these unresolved questions. What was being expected as "mathematical knowledge" felt shallow and empty for him and, despite his teachers words, he felt that what was being taught was not all of

what he "needed to know" for himself. Without encouragement to pursue these authentically perturbing questions, he let it go and would not return to it until many years later.

We share these stories with our students to help them construct connections between the act of reflective thought and the reflective practice that can follow. This is the first step in using story telling throughout our three courses. We see ourselves as facilitating information, conceptual thinking, and logical thought and not simply transmitting knowledge.

Connecting Story to Learning Math

Through story telling we ask the students to draw from multiple personal experiences and their new daily course experiences of mathematics and teaching to bui-ld new understandings. As students move through the program in both their content and methods coursework they recursively reflect on these questions:

When have teachers expected me to truly understand mathematics?

What is the difference between understanding how to do a mathematical procedure and understanding why it makes sense?

What parts of K-4 mathematics do I know in a deep and connected way?

If there is something in K-4 mathematics I didn't know, why did I not know it?

What does it mean to be able "make sense of" and "justify" in K-4 mathematics?

How does one teach K-4 mathematics in a way that students make sense of it?

These reflective questions are continuously reconsidered as students revisit the mathematical content they are being asked to understand and explain. During part of their content learning they revisit arithmetic processes. In rethinking about multiplication they quickly come to realize that many of them only have one way of multiplying two numbers with two or more digits and that

very few of them have ever thought through why that memorized-process makes sense. We asked them to consider other ways of approaching it. After exploring, constructing, and justifying different ways to multiply, the pre-service teachers share ways they have made sense of these kinds of multiplication problems. Once the sharing of sense making has taken place we ask the pre-service teachers to reconsider the question, "What do you think the difference is between understanding how to do a mathematical procedure and understanding why it makes sense?" Amanda had already considered this question twice before with the operations of addition and subtraction. Her mathematical experiences with those two operations did not perturb her thinking about the question in the same way multiplication did. How to add and subtract remained the same as why the process worked for her until she experienced multiplication. Her story explains how her thinking is changing after experiencing multiplication tasks in our class.

> I am beginning to think there is a difference. I couldn't quite see it with addition and subtraction but with multiplication it seems as though there is a difference. For example, I was always able to "do" the process and in that sense I "understood" it but I never really thought about the values and where we place numbers and why placing them there worked. When we talk through the algorithm out loud for this problem 348 x 271 we never say 200 times 300 we just say 2 times 3 and then write down 6 in the 10000 place. I had never really made sense of that before. Of course there is the same thing with placing 1 zero in the second row and two in the third row. I can distinctly recall my teacher saying that we do that to "hold the place" but I had no idea at all what the heck that meant! Now I see that in both of those cases all you can ever have is zero because we are multiplying by groups of 10 and 100.

Through story telling Amanda draws upon her previous learning experiences and connects these to her recent re-visitation of the topic in which she has constructed new mathematical meanings. It is very common for pre-service teachers to feel cognitive perturbation throughout the process of story telling. There is an alignment between mathematical content and specific story telling prompts which dictate a particular order for the reflection questions. This seemingly rigid process is however actually very dynamic because it flows from the stories of the students and because the questions are asked recursively across the program. As Amanda's reflection above demonstrates, her sense making of mathematics is deepening upon her third time to encounter the same question. We offer specific reflection questions continuously throughout the program to prompt the pre-service teachers toward reflection and sharing about what happened in their previous learning experiences, describing the knowledge they gained from those experiences, reflecting on how previous topics in the course connect to current topics, and reflecting on whether or not they know the topic deeper or in a different way after our course explorations. From their reflective stories the pre-service teachers are able to construct new knowledge.

Mathematical Reasoning and Sense Making

In order for pre-service teachers to engage more fully in their future students' reasoning it is important for them to first have the experience of reasoning through the content themselves and reflect on the important connection between reasoning about mathematics and learning mathematics. We have found that through storytelling pre-service teachers convince themselves that being able to reason about mathematics is a vital part of learning mathematics. Near the beginning of the mathematical course of study for pre-service teachers we ask them to write and reflect on the following questions:

What is the nature of mathematics?

What is the nature of learning mathematics?

What is the nature of knowing mathematics?

Do you enjoy learning and thinking about mathematics?

And why does enjoyment impact learning?

While the answers to these questions are varied with multiple hues of meaning, one line of typical responses challenges mathematics teacher educators to help pre-service teachers understand the need for engaging with students' mathematical reasoning. In what follows, we discuss these pre-service teacher beliefs and how teacher educators can plan experiences that encourage them to *construct beliefs through reflection* that value engagement with students' mathematical reasoning.

Nature of mathematics.

When we ask pre-service teachers about the nature of mathematics we get a typical response exemplified by Amanda who said, "Mathematics is a list of rules and procedures about numbers and shapes." When we followed up with the question, "where did these rules and procedures come from?" Amanda answered, "They came from mathematicians many many years ago and most are too complicated to explain to children. Mathematicians proved them to be true a long time ago so we know they all work." As long as Amanda believes that the nature of mathematics is a list of procedures and that the reasons by which the procedures make sense are more than students can handle, she is unlikely to see the need for reasoning about mathematics. It would be difficult for her to believe that her future students need to reason about mathematics, or

orient her teaching to manifest a classroom ecology in which expectations of sense making about mathematical ideas are at the forefront.

Nature of learning and knowing.

Pre-service teachers who begin their course of study with these kinds of ideas about the nature of mathematics are also fairly consistent with one another in their beliefs about what it means to learn and know mathematics. Amanda says,

> When learning mathematics it is important for the teacher to go slow and show the steps in order. I have had bad mathematics teachers skip steps before and I was completely lost. If the teacher explains the steps in order and the student practices those steps enough, then they will learn. Rules really just have to be remembered but procedures you can practice a lot until you know it. This is harder to do with geometry though and that is why students have a harder time with it.

Pre-service teachers like Amanda, who insist that "good" teaching of mathematics means the teacher would go slowly and show each step further indicates an acclimation to the belief that it is not the role of the student to reason or make sense of mathematics but rather learning mathematics is simply following procedure and practicing it until it is known. It is unlikely that Amanda and other pre-service teachers who share similar ideas will be willing to engage significantly with students' mathematical reasoning since the focus of learning is following teacher led procedure and practicing it until it is known. Amanda, like many of her peers, thinks of mathematics only as social knowledge; it can be transferred from the knower to the learner.

Enjoying thinking about mathematics.

When mathematics is relegated to declarative and procedural knowledge it comes as no surprise that most of our students do not find it enjoyable. Amanda says,

> Since you said to be honest when answering these questions I must say that if I were given the choice of doing math or something else I would nearly always choose something else. Don't get me wrong, I have always been able to do the math, I just would rather do something else.

Amanda is not alone in her sentiments about engaging with mathematics. At the beginning of the program, she claims that doing mathematics is not all that interesting. If Amanda and other pre-service teachers are unable to find positive elements of mathematics that make it exciting, they will likely be unable to engage students in ways that lead to the enjoyment of mathematical ideas. In our experience, we have found that learning to reason and make sense of an idea is a satisfying experience for pre-service teachers and students that can lead to a relevant enjoyment of mathematics. Many pre-service teachers have had very few experiences of this kind before entering their programs. By focusing the pre-service teachers on their own stories, a context for learning is created; one that weaves content with conceptual ideas about what it means to teach and learn.

Learning to reason mathematically.

The math content knowledge used and needed by teachers seems to be in a special niche. Math content by itself does not seem to support good math teaching, but a key to teaching math

is to be able to respond to another's math thought process (Cavey, Mahavier, Parker, & White, 2007). If teacher educators desire pre-service teachers to more fully engage with their future students' mathematical reasoning then we must address these kinds of beliefs in our programs of study. More to the point, teacher educators must provide multiple mathematical learning opportunities from which pre-service teachers can move through the mathematics curriculum as a "sense making" endeavor. As will be seen in the remainder of Amanda's story to follow, once they themselves find value in reasoning and sense making the way they see mathematics, learning, and knowing is transformed as well as revealing a more positive attitude for the enjoyment of mathematics. From these dispositions pre-service teachers can be prepared to engage more fully with their students' mathematical reasoning.

We have found that our beginning pre-service teachers have a wide range of mathematical content knowledge, confidence, and problem solving capacities. In order to challenge the pre-service teachers preexisting notions of mathematics learning and knowing and to build confidence in their own mathematical reasoning we start the program of study asking students to work out possible early number systems. This beginning time is very important to creating a classroom ecology in which pre-service teachers become comfortable with the expectation that they are the ones who must produce mathematics and explain their reasoning to others. Becoming comfortable explaining one's own mathematics reasoning to your peers is often no small matter for pre-service teachers to overcome. We asked students to reflect on the experience of justifying their mathematical reasoning to others. Amanda writes,

> When we were first told that we would have to explain our thinking I was very intimidated. Honestly I can only remember a handful of times that any teacher ever

> asked me to explain my thoughts and never in mathematics. I don't think that I would have been comfortable just getting up in front of everyone and showing my thoughts on the document camera except that you always allowed us to fully discuss our ideas in our group first. That group time was vital to building my confidence to share my ideas. If my group understood what I was thinking or if I understood one of their ideas it was easier to discuss with the whole class because I wasn't alone in my thinking. Also, you telling us "everyone always has the right to change their mind when presenting" helped make me comfortable enough to share. Now that we are at the end I realized that the sharing and discussion times of our course, when everyone was just hashing out the possibilities, was the most fun of all. I know that after we hashed it out in-group and then again as a class I felt so much more knowledgeable about what we were studying.

Most of our pre-service teachers mention these same elements in their reflections. They have generally had very little experience justifying themselves verbally and lack self-confidence in their own mathematical problem solving ideas. A classroom ecology that both expects and promotes openness for sharing ideas as well as the space to actively change one's mind on the matter at any time allows pre-service teachers to experience mathematical reasoning and communication for themselves.

As part of our commitment to developing pre-service teachers' mathematical reasoning we have also found it important to challenge them to seek out the mathematical knowledge that they have taken for granted. If they can "do" something, but are unsure "why" that process works, then our courses become a playground for reasoning about those mathematical ideas. Taking the

time to allow pre-service teachers to unpack (Young, 2002) these mathematical ideas has been of paramount importance to increasing their confidence in reasoning about mathematics. Amanda reflects,

> Working out why things are true in mathematics is the most difficult thing about this course right now. I never really thought about why the algorithms worked before. At first it was really difficult to explain why anything worked which is really an indictment on my education in mathematics up to this point since what we are doing is 5th grade or below. Now that we have thought about it as a class I can see not only why our traditional algorithms work but also why there are many other ways of doing it and that it all connects back to our choice of grouping in a base 10 way and the meaning of the operations. So long as I keep what things "mean" in mind then understanding why we can do things mathematically is much easier. Even though this process is hard I have to say it is the most rewarding. When we figure it out it is very exciting, and I didn't ever think math would be exciting. Through our class discussions about this I find that my mind has opened up to something completely new in teaching. Back when I didn't know why it worked my teaching would have been limited to simply telling the process. But now I realize it is so much more. Making the connections as to why something should work out mathematically is what learning mathematics truly is. I want to be a teacher that helps them understand mathematics and not just blindly do a process.

Amanda has begun to make a turn from relegating mathematics as a series of ever more complex discrete algorithms into mathematics as a connected set of ideas that are meant to be understood and reasoned about. She has begun to see that there are actually many logical ways of doing mathematical operations or solving mathematics problems and that these multifarious ways connect. In making these connections herself, she has come to value their importance in understanding mathematics.

Amanda furthermore points to another important result in the aim to support pre-service teachers in engagement in student's mathematical reasoning. She has connected the importance of reasoning about mathematics and wishes to apply it to her own practice. To support this aim our program courses focus attention toward understanding others ideas and gives K-8 curriculum tasks to the pre-service teachers that ask students to reason about mathematical ideas. Continually asking the pre-service teachers to make connections between their own learning stories and their future pedagogical practice by engaging them with K-8 curriculum materials helps develop their sense of mathematical knowledge and their confidence as teachers of mathematics (Lloyd, 2006).

Connecting Story to Math Pedagogy

When early childhood pre-service teachers have completed the prerequisite math content classes they can then take a math methods class that focuses on the math pedagogical decisions early childhood teachers make in prekindergarten through fourth grade classroom. Again, story telling is used to help them construct their ideas about mathematics reasoning and mathematics teaching.

They are required to keep a journal of their work with one child over the course of the semester. They are expected to describe the context for the mathematics they are doing with the child and to analyze the child's thinking. Since we know they have had the experience of thinking about their own thinking we know we can help them make connections between their own content learning and that of the child's content learning. They reflect on their notes about the experience with the child and are given metacognitive prompts to aid in their analysis. The prompts include:

"I was surprised by..."

"I thought...but now I think..."

I didn't understand why..."

I expected...because..."

I told the child that...but he/she...."

I am going to try..."

We want them to remember what the struggle to understand was like and what it took to construct their own understanding. However; the image of the teacher as "teller" is difficult to overcome. Student teachers have had many years of experience in classrooms and come into teacher education having constructed images of teachers as having all the answers. It is very important to provoke thought about the role of the teacher. Discussion in class revolves around the following points:

Why is "telling" not necessarily "teaching"?

How did you learn what you know? What does it mean, "to know"?

How do you know the child understands?

Why is listening the most important part of teaching?

These questions are visited on a regular basis throughout the semester, using story telling. We model listening by quieting our own teacher voice and ask our pre-service teachers to think about why we do this. We challenge them to think about pedagogical actions that might perturb a student's thinking as well as what might stop a student's thinking. Within this vein of pedagogical reasoning the pre-service teachers reflect on the irony of being too quick to jump in and correct a student's thinking instead of allowing them to continue to think about it.

Amanda's math journal work with a young child showed a deeper reflective relationship with both the child's learning and the content to be taught. Her notes in her journal reflected a dawning understanding of the connection between her own content knowledge and the challenge to teach.

> I showed Christopher the National Virtual Library of Math Manipulatives web site at nvlm.usa.edu We looked around at the site for a little bit. When he spotted the geoboard section, that was what he wanted to do. I watched him make a 5-peg triangle. The next part was to complete the left half of a Christmas tree type shape that was given. I wondered how could you know how to do this the same on each side? I watched him manipulate the "rubber band" on the pegs. One was wrong and I thought about showing him, but I held my tongue and just said look closely at both sides. I noticed he was looking detail-by-detail at one side and then the other. "Oh," he said. The next row, he began then said, "No," and moved what he had done saying, "They move by one each time." He had noticed a pattern. I had not really thought about it being a pattern. I just saw it but it

was not aware of thinking of it in that way, but he did. I must be aware of patterns more to be able to help students see them as well.

Christopher: What is 4 x ____= 32?

Me: Use these foam shapes to decide.

I gave him some different shaped foam cutouts. My first thought was to guide him into how to figure out the answer. I wanted to say something about counting out 32 and divide them into the number of groups you need. I thought - NO. He will learn more if I just let him work on his own. Of course, he did figure it out and I'm sure the answer will stay with him longer, as well as, how he thought about it will help him in working other problems. If I had told him my thought about how to do it, he would not have made the connection for himself and most likely would have trouble doing the same problem again. I am continually holding myself back. I want to jump in and explain how "I" would do it. Even though my way is a legitimate way of thinking, it may not be the way a child would think of it and I may prohibit the child from discovering an important connection.

Amanda's ability to integrate her reflection into her work with a child was apparent in her journal entries. As teacher educators we have to keep in mind that not all of our students will be so highly reflective. It is a goal for us to provide reflection as a way to develop knowledge for teaching, but we also know that non-reflective students will struggle with this expectation (LaBoskey, V. K., 1994).

As teacher educators our focus is on giving the pre-service teachers as many possible chances to make sense of mathematics and connect that reasoning to pedagogy, as possible. Pre-service teachers can be supported to engage more fully with students' mathematical reasoning by

revisiting the beginnings of our number system, holding firm to the expectation of justifying ones reasoning and communicating it to others, giving ample time to rehash through why processes are true and why other processes are equally valid, and reflecting on the continuous interplay between ones learning and ones teaching. Story telling serves as the bridge between the prerequisite mathematics content classes to the early childhood math methods class. We want our students to have the mathematics content and the pedagogy modeled both as something experienced and as something to promote in their future classrooms.

Coming full circle at the end of the program we asked Amanda to reflect on her pre-service teacher learning and what she thought about mathematics, learning, knowing, and enjoyment. Amanda reflects,

> Before starting this program I really thought that I knew mathematics well. I mean, I listened to my school teachers and pretty much learned the rules and procedures they taught me. Whenever I forgot something I just passed it off as having not practicing enough. If I did not remember it I was stuck though, because I had not really made sense of it. I would have to either look it up or go ask my teacher what it was. As I was going through the program however I began to see the power of actually thinking through why the mathematical ideas were true. If I forgot something I now would start to play with the idea, and since I had already made sense of it once, I knew I could do it again. It sounds strange but sometimes forgetting it was a good thing, because the second time I considered it I found a new way to make sense of it. Explaining my new way to the class made me feel intelligent. I have a different kind of confidence now. Before I was confident in the process someone told me to do but now I am confident in my own ability to

> understand and think through why mathematics makes sense. It is fun to think about mathematics and think about others ideas. I sometimes wonder who I would be if my teachers expected me to make sense of mathematics instead of allowing me to skate by just memorizing what they told me. Now that I have tasted this sweet apple of mathematical sense making I want to give it to my students, so that they too can enjoy mathematics.

We have focused on Amanda,for this particular paper, but she represents many of our students. Through story telling our students come to know more about their own learning, but they also know more about the perspective of the child's learning. "...teaching transcends knowledge and skills to also include how the teacher frames the world and then approaches and communicates with others" (Mueller, J.J.; Wisneski, D.B.; & File, N. p. 74). The story telling our students participate in supports their development in becoming teachers who can facilitate a deeper communication with their future students.

Final Remarks and Future Plans

We know our students assimilate and accommodate new information in ways that are highly personal. We would like go into the field with our students to find out if their new view of math teaching and learning is going to stand up to the realities of teaching in primary classrooms. Are they going to be able to follow through on their quest to understand their students' math thought process? How does the context impact their ability to use their new concepts of teaching and learning math? These questions, and others, guide our teaching decisions.

From our narrative inquiry with these pre-service teachers we believe that to help them bridge content and pedagogical knowledge, teacher educators should invoke students stories and

allow those stories to provide a powerful dialogue about what it means to teach and learn mathematics. Without such dialogue, we would not have researched our own practice, we would not have focused on our student's construction of knowledge, and we would not have valued how their construction of knowledge could impact our own knowledge. We would encourage our colleagues to look for opportunities to share common concerns, questions, and interests with others. Ours happened to revolve around the content of math, but these commonalities may be found in any content area.

References

Akinbode, A. (2013). Teaching as lived experience: The value of exploring the hidden and emotional side of teaching through reflective narratives. *Studying Teacher Education*, 9(1), 62-73. http://dx.doi.org/10.1080/17425964.2013.771574.

Anderson, D., & Piazza, J. (1996). Changing beliefs: Teaching and learning mathematics in constructivist pre-service classrooms. *Action in Teacher Education, 18(2)*, 51-62.

Ball, D. L., Thames, M.H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407

Britzman, D.P. (1991). *Practice makes practice: A critical study of learning to teach*. SUNY: Albany, NY.

Brown, E. T. (2005). The influence of teachers' efficacy and beliefs regarding mathematics instruction in the early childhood classroom. *Journal of Early Childhood Teacher Education, 26*(3), 239-257.

Cavey, L. O., Mahavier, W. T., Parker, G. E. & White, A. (2007). Math nerds and mathematical knowledge for teaching. *The Constructivitst, 18* (1)

Chaille, C. (2008). Constructivisim across the curriculum in early childhood class rooms: Big ideas as inspiration. Pearson.

Clandinin, D.J. & Connelly, F. M. (2000). Narrative inquiry: Experience and story in Qualitative research. Jossey-Bass. San Francisco: CA.

Davis, S.M. (2005). Developing reflective practice in pre-service student teachers: What does art have to do with it? *Teacher Development*, *9* (1), 19.

Dunphy, E. (2009). Early childhood mathematics teaching: challenges, difficulties and priorities of teachers of young children in primary schools in Ireland. *International Journal of Early Years Education, 17*(1), 3-16

Francis, D. (1995). The reflective journal: A window to pre-service teachers' practical knowledge. *Teaching and Teacher Education,* 11, 229-241.

Frost, J. (2010). Looking through the lens of teacher's life: The power of prototypical stories in Understanding teachers' instructional decisions in mathematics. *Teaching and Teacher Education*. 26 (2010) 225-233. doi: 10.1016/j.tate.2009.03.020

Fosnot, C.T., (Ed.), (1996). Constructivism: Theory, perspectives, and practice. Teachers College.

Fuson, K.C., Kalchman, M. & Bransford, J.D. (2005) Mathematical understanding: An introduction. In M. S. Donovan & J.D. Brandford (Eds.), How students learn: Mathematics in the classroom (217-240). National Academies Press.

Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. *Early Childhood Education Journal. 35*(2), 181-188.

Gordon, M. (2009). Toward a pragmatic discourse of constructivism: Reflections on lessons from practice. *Educational Studies,* 45: 39-58. Doi: 10.1080/00131940802516894.

Grimmett, P. P. & Erickson, G. L. (Eds.). (1988). Reflection in teacher education. NewYork: Teachers College Press.

Hill, H., & Ball, D. (2009). The curious -- and crucial -- case of mathematical knowledge for teaching. *Phi Delta Kappan, 91*(2), 68-71

Kamii, C.(October, 2000). Teachers need more knowledge about how children learn mathematics. *Teaching Children Mathematics. Retrieved from* http://www.nctm.org/resources/content.aspx?id=1760

Kenney, R., Shoffner, M. & Norris, D. (2013). Reflecting to learn mathematics: Supporting pre-service teachers' pedalogical content knowledge with reflection on writing prompts in mathematics education. *Reflective Practice: International and Multidiscplinary Perspectives,* 14(6), 787-800. Doi.10.1080/146223943.2013.836082

Jalongo, M.R. & Isenberg, J. P., & Gerbracht, G.(1995). Teacher's stories: From personal narrative to professional insight. San Francisco: Jossey-Bass

Kettle, B. & Sellars, N. (1996). The development of student teachers' practical theory of teaching. *Teaching and Teacher Education, 12*, 1-24.

LaBoskey, V.K. (1994). Development of reflective practice: A study of pre-service teachers. New York: Teachers College Press.

Lloyd, G. M. (2006). Using K-12 mathematics curriculum materials in teacher education: Rationale, strategies, and pre-service teachers' experiences. In K. Lynch-Davis & R. L. Rider (Eds.), *The work of mathematics teacher educators* (pp. 11 28). San Diego CA: Association of Mathematics Teacher Educators.

Mueller, J.J., Wisneski, D.B. & File, N. (2010). Grappling with big ideas: The process of program improvement. *Journal of Early Childhood Teacher Education, 31*, (1), 71-85.

Nolan, A. (2008). Encouraging the reflection process in undergraduate teachers using guided reflection. *Australian Journal of Early Childhood. 33* (1), 31-36.

Piaget, J. (1965). The moral judgment of the child. New York: Free Press.

Philipp, R.A. (2000). Mathematics teachers' beliefs and affect. In F. Lester (Eds.), *Second handbook of research on mathematics teaching and learning*. United States: Information Age Pub Inc.

Richardson, V. Ed. (1997). Constructivist teacher education: Building a world of new understandings. RoutledgeFalmer. Philadelphia: PA.

Ritchie, J.S. & Wilson, D.E. (2000). Teacher narrative as critical inquiry: Rewriting the script. Teachers College.

Ross, V. & Prior, J. (2012). Curriculum: The inside story. *Curriculum and Teaching Dialogue*.14, (1-2), pg. 101.

Steinberg, R.M., Empson, S.B. & Carpenter, T.P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education, 7,* (3), 237-267.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213-226. doi:10.1016/S0742-051X(00)00052-4

Treagust, D. F., Duit, R., & Fraser, B. J. edt (1996). Improving teaching and learning in science and mathematics. Teachers College Press.

Van Manen, M. (1990). Researching lived experience: Human science for an action sensitive pedagogy. The State University of New York, London: Ontario.

van Wyk, M. M. (2011). Student teachers' personal stories—Identity, social class and learning : A life history approach. *Loyola Journal Of Social Sciences*, *25*(2), 141-161.

Walshaw, M. (2013). Explorations into pedagogy within mathematics classrooms: Insights from contemporary inquiries. *Curriculum Inquiry*, 43 (1), 71-94. doi: 10.1111/curi. 1200

Young, E. (2002). Unpacking mathematical content through problem solving. *Dissertation Abstracts International A, 63*(08), 2819.