

## Probabilistic Durability Analysis of Reinforced Concrete Members with Corroded Reinforcing Steel

*Adamu Lawan*<sup>1)</sup>, *Mohammed Jibrin Kaura*<sup>2)\*</sup>, *Jaafar Abubakar Sadeeq*<sup>3)</sup> and *Ibrahim Aliyu*<sup>4)</sup>

<sup>1)</sup> King Fahd University of Petroleum and Minerals, KSA.

<sup>2), 3), 4)</sup> Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.

\* Corresponding Author. E-Mail: [jmkaura@abu.edu.ng](mailto:jmkaura@abu.edu.ng)

### ABSTRACT

The reliability of the structural performance of any given structure is affected by both in-service loading and material deterioration due to environmental attack. In this paper, probabilistic assessment of reinforced concrete members exposed to chlorine ingress was undertaken. A simply supported reinforced concrete slab was specifically used for the investigation. The mathematical models of capacity loss of reinforcing steel under corrosion, developed elsewhere, were incorporated in the analysis. The uncertainties in structural resistance and the applied loading were fully accommodated using probabilistic method. Limit state function for the flexural capacity of the slab was developed and evaluated using first order reliability method (FORM). The entire process was implemented through a developed program using MATLAB.

**KEYWORDS:** Corrosion, Chlorine ingress, Structural reliability, Uncertainty.

### INTRODUCTION

Reinforced concrete is one of the most important structural materials used in construction. It has excellent structural and durability performance. Nevertheless, there is evidence of early deterioration when located in marine environments. The most common cause of deterioration is corrosion of the reinforcement facilitated by chlorine ingress (Zaher and Samir, 2001; Torres-Acosta and Sagues, 2004). Corrosion is the chemical or electrochemical reaction between a material, usually a metal, and its environment that produces a deterioration of the material and its properties (Neville and Brooks, 1994). Reinforced concrete uses steel to provide the tensile properties that are needed in structural concrete. It prevents the failure of concrete structures which are

subjected to tensile and flexural stresses due to traffic, winds, imposed load and dead load. However, when reinforcement corrode, the formation of rust leads to the reduction of cross-sectional area of steel and the subsequent reduction of strength capacity.

Under normal environmental conditions, steel reinforcing bars embedded in reinforced concrete structures do not corrode. Usually, a thin film of iron oxide is present on the surface of the rebar when it is encased in the concrete. The high pH environment (approximately 13.5) associated with hydration of the Portland cement is usually sufficient to keep the protective film stable. However, sufficient concentration of chloride ions can lower the pH, and if moisture and oxygen are present, the rebar can begin to corrode.

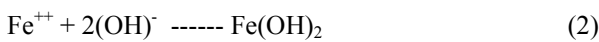
Chlorine ingress is a common phenomenon for concrete structures that are located in marine environment. Concretes are exposed to chloride by

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seawater. Chlorine-induced corrosion of reinforcing steel results in the reduction of the load carrying capacity of the structure. Significant distresses have been reported in concrete structures subjected to chloride-induced corrosion (Bordalle-Ruiz et al., 2007; Hyman and Hughes, 1983; Melchers, 1999; Liu and Weyers, 1998; Kranc et al., 2002; Torres-Acosta and Sagues, 2004).

Chloride-induced corrosion is schematically illustrated in Fig. 1 (Thoft-Christensen, 2000; Thoft-Christensen, 2002). An anodic region is established, where the passive film is broken down, whereby an electrochemical cell is formed. The passive surface is the chloride and the electrolyte is the pure water in the concrete. At the anode, the following reactions take place:



Chloride ions (Cl<sup>-</sup>) activate the unprotected surface and form an anode. The chemical reactions are:



It follows from (1) and (2) that two rust products; Fe(OH)<sub>2</sub> and Fe(OH)<sub>3</sub> are produced.

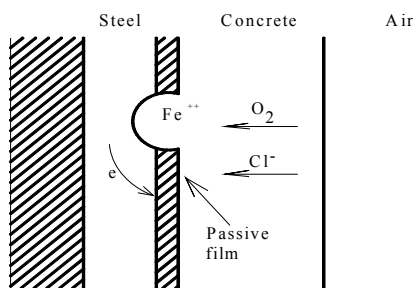


Figure (1): Chloride-induced corrosion

**METHODOLOGY**

**Corrosion Induced Reinforced Concrete Capacity Loss**

Mathematical model of structural capacity loss as a function of the loss of reinforcing steel cross-sectional area in reinforced concrete structures is well documented in literature (Cady and Weyers, 1984; DNV-OS-C50, 2012; Frangopol and Moses, 1994; Frangopol and Hendawi, 1994; Thoft-Christensen, 2000; Li, 2005; Vu and Stewart, 2005; Kupwade-Patil et al., 2012). The flexural strength of corroded reinforced concrete members depends mainly on the available area of rebars in the tension zone.

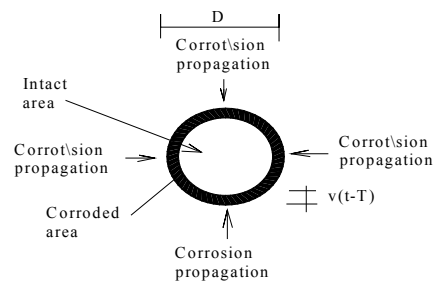


Figure (2): Uniform corrosion in a reinforcing steel

In this paper, rebar damage models based on the early work of (Frangopol and Hendawi, 1994; Lin and Frangopol, 1996) were used. In these models, direct satisfaction of both reserve (intact) and residual (damaged) reliability constraints is achieved. For a uniform corrosion as shown in Fig. 2, the total bending reinforcement area as function of time t is given by:

$$A_t(t) = \frac{n\pi D^2}{4} \tag{for } t \leq T_i \tag{6}$$

$$A_t(t) = \frac{n\pi[D-2v(t-T)]^2}{4} \tag{for } t > T_i \tag{7}$$

where D = diameter of reinforcing steel bar; n = number of bars; T<sub>i</sub> = time of corrosion initiation; and v = rate of corrosion. Factor 2 in equation 7 takes into

account the uniform corrosion propagation process from all sides at the level of rebar (Fig. 2).

Typical corrosion rates  $v$ , of steel in various environments have been reported in recent years. In 1989, Ting proposed that the average corrosion rate  $v$ , for passive steel in concrete attacked by chlorides is about 0.100 mm/year. From Mori and Ellingwood (1994), the typical corrosion rate,  $v$ , is a time-variant random variable described by a lognormal distribution with mean,  $\mu_v$  of 0.05 mm/year and coefficient of variation,  $V_v$  of 50%. Because the corrosion rate changes with the environment, no accurate data is available to predict the real corrosion rate. Based on the average corrosion rates reported in (Mori and Ellingwood, 1994; Ting, 1989), seven corrosion rates of 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 and 0.12 mm/year were used in the study to cover the most cases.

### NUMERICAL EXPERIMENT

The current provisions in the Eurocode 2 (2008) for the design of reinforced concrete structures are deterministic, using partial safety factors to accommodate uncertainties. However, the best way to accommodate uncertainties in structural analysis and design is to use structural reliability method (Melchers, 2001). Reliability-based analysis and design is a

probabilistic design process where loads and strengths of materials and sections represented by their known or postulated distributions, defined in terms of distribution type, mean and standard deviation are considered. Reliability analysis is used to investigate the influence of uncertainties on structural design.

In this study, the capacity loss of reinforced concrete in the tension zone of a simply supported reinforced concrete slab was considered. Limit state function was developed based on the recommendation of Eurocode 2 (2008). First order reliability method was used to evaluate the limit state function.

$$G(x) = \phi_r A_t(t) f_y [(h - a - A_t(t) \frac{f_y}{f_c}] - [0.125 \phi_c Q_k (\gamma_g \alpha + \gamma_q) L^2] \quad (8)$$

where  $\phi_r$  and  $\phi_c$  are the model uncertainties for resistance and load action, respectively;  $f_y$  is the characteristic strength of steel reinforcement;  $f_c$  is the characteristic cylinder strength of concrete;  $Q_k$  is the imposed load;  $\alpha$  is the dead to imposed load ratio;  $L$  is the span of the beam;  $h$  is the overall depth of the beam;  $a$  is the axis distance of the beam as shown in Fig. 3.

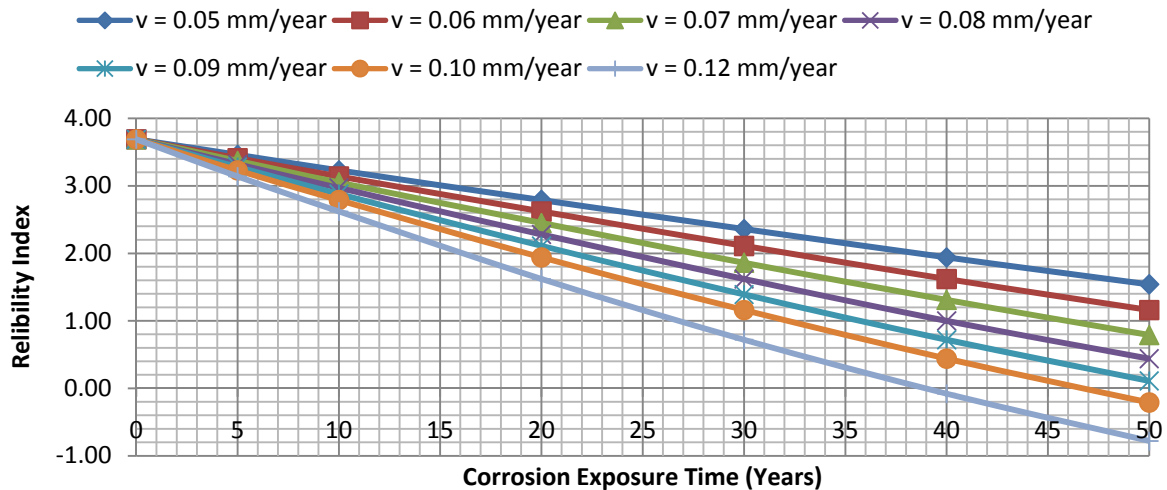


Figure (3): Reliability index against corrosion exposure time

**Table 1. Statistical models of the basic design variables**

S/No.	Design variable	Unit	Distribution model	Mean	Coefficient of variation
1	Resistance Model Uncertainty	-	Normal	1.0	0.05
2	Load Model Uncertainty	-	Normal	1.0	0.05
3	Steel Strength	N/mm <sup>2</sup>	Lognormal	500	0.01
4	Concrete Compressive Strength	N/mm <sup>2</sup>	Lognormal	30	0.05
5	Depth of Slab	mm	Normal	150	0.05
6	Axis Distance	mm	Normal	$C + \frac{D}{2}$	0.05
7	Imposed Load	kN/m <sup>2</sup>	Gumbel	2.0	0.40

(Source: Epaarachci et al., 2000)

First Order Reliability Method (FORM) requires the statistical parameters of each basic design variables. The statistical parameters include mean values, coefficient of variation and theoretical distribution model. The statistical parameters for the problem at hand are presented in Table 1.

The probability of failure of the slab is given by the following convolution integral (Ditlevsen and Madsen, 2005; Melchers, 1999).

$$P\{<0\} = \iint_{r<s} f_R(r)f_S(s) dr ds = \Phi(-\beta) \quad (9)$$

where  $\Phi(-\beta)$  = standard normal distribution for the variable  $\beta$ ;  $\beta$  = Hasofer-Lind reliability index which is a measure in standard deviation of the distance that the mean  $\mu(Z)$  is away from the safety-failure interface (Ditlevsen and Madsen, 2005; Melchers, 2001), given by:

$$\beta = \frac{\mu(Z)}{\sigma(Z)} \quad (10)$$

In this research, computation of safety indices was achieved through a developed MATLAB-based computer program.

## RESULTS AND DISCUSSION

Structural reliability analysis was implemented in

this study for the investigation of the reliability of reinforced concrete subjected to corrosion, initiated by chlorine ingress in sea water. First Order Reliability Method (FORM) was used. Reliability indices were generated using time-discretization strategy. The results are presented in Figures 6 to 10.

Figure 3 shows the relationship between safety index and corrosion exposure time, for various corrosion rate scenarios. The slab under investigation was first designed to meet a predefined target reliability index of 3.8, in accordance with the requirements of (Eurocode 0, 2002). At the target reliability, the corrosion exposure time is zero for all the considered values of reinforced concrete corrosion rates. This is an indication of the absence of corrosion.

As the corrosion initiates, (that is when corrosion initiation time  $T_i$ , in years is greater than zero), the corrosion exposure time ( $t - T_i$ ) also sets in.  $t$  is the concrete age in years after casting. The increase in corrosion exposure time then leads to a decrease in the capacity of the slab to resist applied loading. The extent to which the capacity is lost depends on the corrosion rate,  $v$ . As observed from the plot (Fig. 6), the capacity loss of reinforced concrete member exposed to corrosion decreases linearly with the corrosion exposure time. After twenty years from the onset of the corrosion, the reliability index (a probabilistic measure of the structural capacity) drops

from the target design value of 3.8 (when corrosion exposure time is zero) to 2.7, 2.6, 2.4, 2.3, 2.2, 2.0 and 1.6 at the respective corrosion rates of 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 and 0.12 mm/year. The respective capacity losses were therefore 29%, 32%, 37%, 40%, 43%, 47% and 58%.

In accordance with the Eurocode 0 (2002), the target reliability index of 3.8 corresponds to a 50 year design life. When the concrete member (reinforced concrete slab) is exposed to a 50 year service load, the member is expected to maintain its full capacity. However, if the member is exposed to time dependent corrosion, the reliability index gradually drops with time from the target value of 3.8 to 1.5, 1.15, 0.7, 0.1, -0.2 and -0.7 at the respective corrosion rates 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 and 0.12 mm/year. This implies that the structural element experiences total collapse after 50 years of exposure to corrosion if the corrosion rate exceeds 0.1mm/year. Therefore, after fifty years of exposure to corrosion, at the respective corrosion rates of 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 and 0.12 mm/year, the corresponding capacity losses are 60%, 70%, 82%, 97%, 99%, 100% and 100%.

In Fig. 4, the variation of slab reliability with the

coefficient of variation of load action is presented. The plot was for a corrosion rate of 0.05 mm/year. The capacity of the slab was checked when the corrosion initiation time was zero ( $T = 0$ ) and when the corrosion time was 50 years ( $T = 50$  years). At load action covariance of 5%, the safety index values of the slab are 6.8 and 11.4 at  $T = 0$  and  $T = 50$  years, respectively. For all cases, safety index displayed non-linear variation load action covariance. At high coefficient of variation of load action (say 25%), the reliability index for the slab dropped from 11.4 (when load action covariance was 5%) to 5.4 at  $T = 50$  years. Likewise, at  $T = 0$ , the reliability index dropped from 6.8 (at load action covariance of 5%) to 2.4. It is therefore clear from the plot that, in addition to corrosion, coefficient of variation of load action also has significant effect on the safety of reinforced concrete members. The effect of uncertainties in form of coefficient of variation can only be captured when reliability method is employed in structural reliability analysis and design. The current provision of the Eurocode 2 (2004), which is deterministic, is therefore conservative.

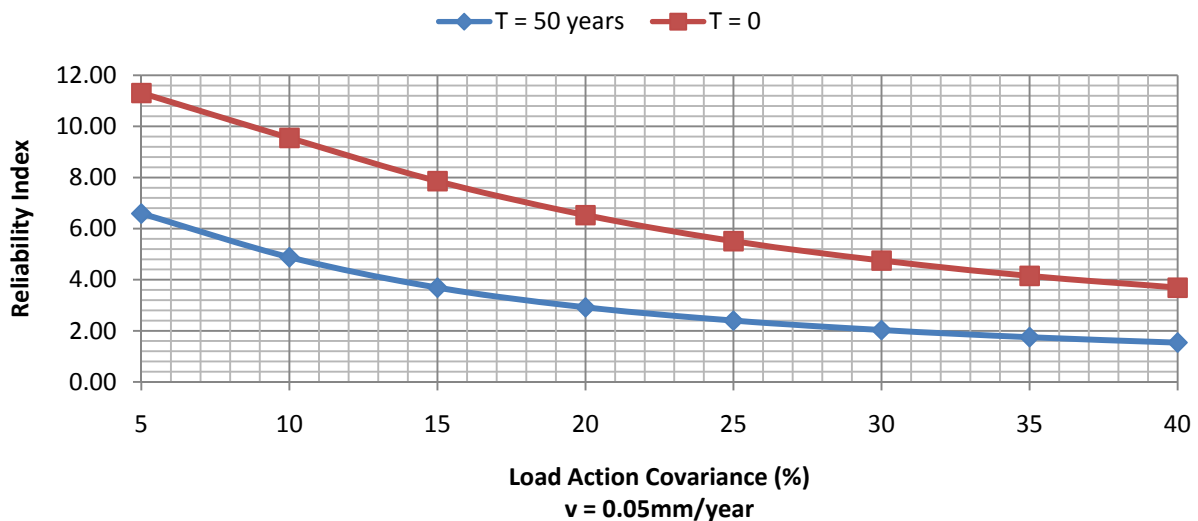
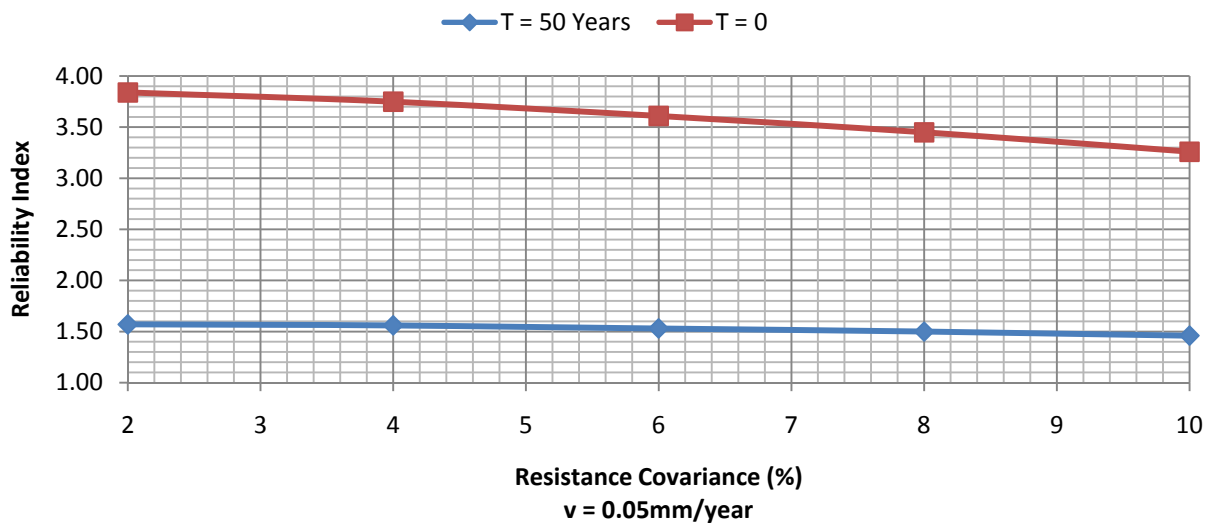


Figure (4): Reliability index against load action uncertainty



**Figure (5): Reliability index against resistance uncertainty**

Fig. 5 shows the variation of reliability index with the coefficient of variation of slab strength. When the corrosion initiation time was 0, the slab capacity was 3.8 at strength covariance of 5%. As the strength covariance increased up to 10%, the reliability index gradually dropped to 3.3 (at strength covariance of 10%). At 50 year corrosion exposure time, the change in strength covariance has negligible effect on the slab safety. However, large drop in slab safety was observed at 5% strength covariance when the corrosion initiation time changes from zero (safety index = 3.8) to 50 years (safety index = 1.55). This is a clear indication of time dependent corrosion effect on reinforced concrete structures. Once corrosion is initiated, it is only a matter of time, the entire load carrying capacity of structural member under load will be lost, causing serious threat to life and properties. The issue of reinforced concrete corrosion must therefore be taken with all seriousness, especially when the structure is located in marine environment, where chlorine is abundant.

Fig. 6 displays the relationship between reliability index and characteristic strength of steel reinforcement at zero and 50 years after corrosion initiation. It is clear

from the plots that the reliability index increases linearly with the increase in the characteristic steel strength. When the steel characteristic strength is 500N/mm<sup>2</sup>, the reliability index is 3.7 before the onset of corrosion. However, after 50 years of corrosion, the reliability index changed to 1.5, indicating more than 50% capacity loss. It can also be observed that, at 50 years after corrosion initiation, the reliability index changed from 0 (at a characteristic strength of 250 N/mm<sup>2</sup>) to 1.5 (when the characteristic strength is 500 N/mm<sup>2</sup>). This implied that high strength steel increases the safety margin of reinforced concrete structures that are subjected to corrosion.

Fig. 8 is the plot of reliability index against dead to live load ratio, before the onset of corrosion and at 50 years after corrosion initiation. At a load ratio of 0.5, the reliability indices before the onset of corrosion (T = 0) and at 50 years after corrosion initiation are 7.0 and 4.0, respectively. As the load ratio increased, the reliability displayed a slight nonlinear reduction. At a load ratio of 2.0, the respective reliability indices are 2.8 and 0.9, respectively. Therefore, the capacity loss of reinforced concrete structures subjected to corrosion also depends on the value of dead to live load ratio.

The capacity loss is lower at very low load ratio and higher at high load ratio. Heavily loaded reinforced

concrete structures are therefore at high risk of collapse due to corrosion than lightly loaded structures.

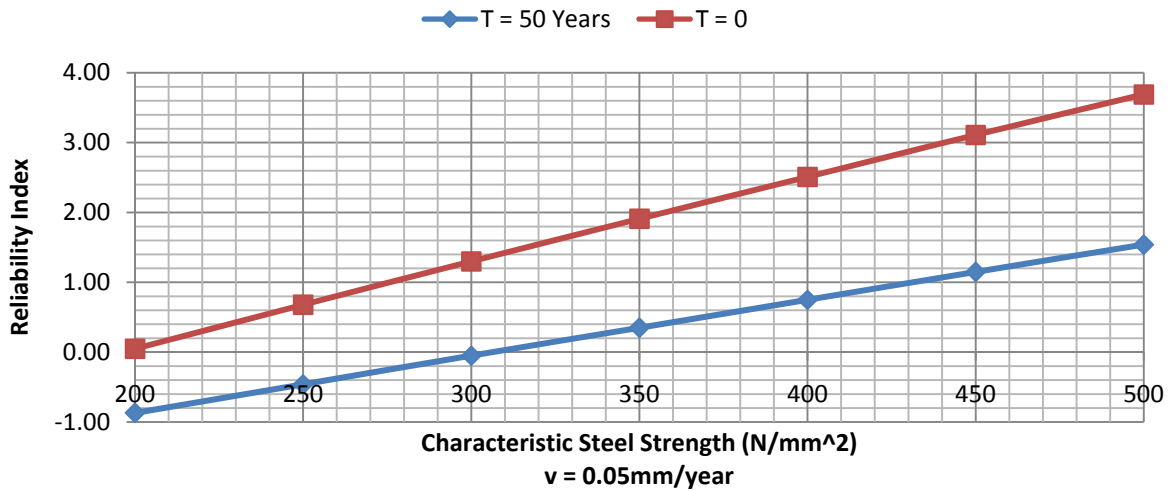


Figure (6): Reliability index against characteristic strength of steel

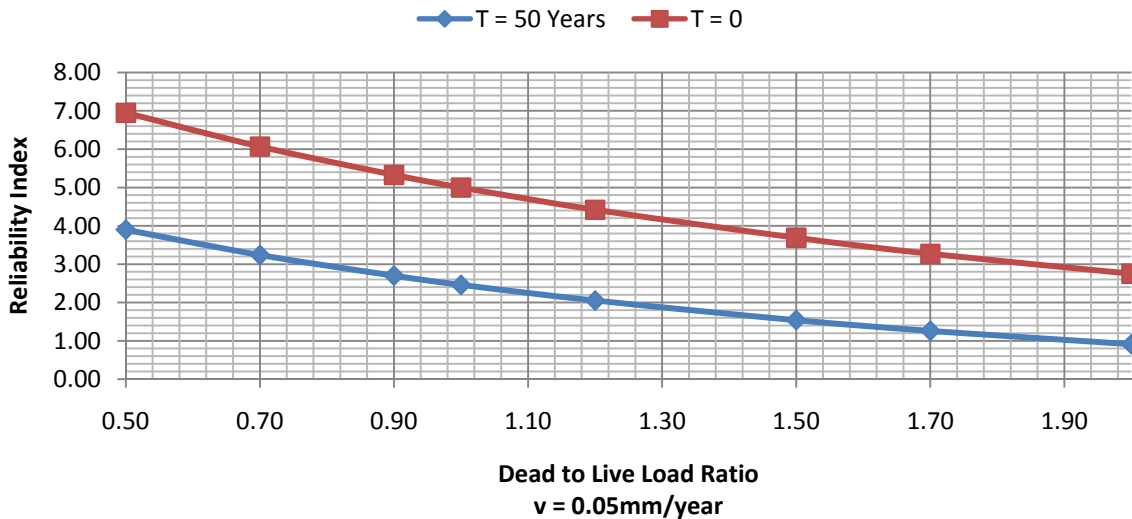


Figure (7): Reliability index against load ratio

**CONCLUSIONS AND RECOMMENDATIONS**

In this paper, the probabilistic method was used to assess reinforced concrete members subjected to corrosion due to chlorine ingress. Mathematical model

of strength capacity loss reported in literature was used in the limit state function for flexural capacity of the slab to accommodate the time dependent corrosion effect in the structural reliability analysis. Limit state function was evaluated and reliability indices were

generated using First Order Reliability Algorithms. The developed algorithm was coded using MATLAB programming language, and the process was fully automated. In the paper, the reliability of the slab before and after the onset of corrosion was checked. The study included the check of the effects of changes in the corrosion exposure time, coefficient of variation of load action and the structural resistance, characteristic steel strength and dead to live load ratio. In conclusion, it was clearly established that corrosion of reinforced concrete members subjected to chlorine ingress is a very serious durability problem that cannot be ignored. The problem was further aggravated by the existence of uncertainties due to random nature of design parameters that defined the applied loading as well as the structural resistance. The following recommendations are therefore made.

1. Since chlorine ingress into reinforced concrete

members in marine environment leads to destruction of the protective film of reinforcement, the tendency of chloride penetration into concrete should be addressed. Recent investigations confirmed that the use of nanosilica as admixture in concrete can inhibit chloride penetration.

2. Corrosion inhibitors such as fly-ash should be used in concrete mix to reduce the tendency of corrosion after the damage of the reinforcing steel protective film due to chlorine ingress.
3. Reliability based method should be integrated in the Eurocode 2 design criteria of reinforced concrete structures subjected to corrosion in order to fully accommodate the uncertainties in design parameters, since the partial safety concept is deterministic in nature, derived from the long time experience of building tradition.

## REFERENCES

- Bordallo-Ruiz, A., McNally, C., Caprani, C. C., and O'Brien, E. J. (2007). "The structural reliability of bridges subject to time-dependent deterioration". Proceedings of the Eleventh International Conference on Civil, Structural and Environmental Engineering Computing.
- Cady, P. D., and Weyers, R. E. (1984). "Deterioration rates of concrete bridge decks". *J. Trans. Engr., ASCE*, 110 (1), 34-44.
- Dario, C., Kamyab, Z. H., and Karin, L. (2013). "Severely corroded RC with cover cracking". *J. Struct. Eng.*, 139 (2), 221-232.
- Ditlevsen, O., and Madsen, H. O. (2005). "Structural Reliability Methods". Internet Edition.
- DNV-OS-C50. (2010). "Offshore concrete structures". <http://www.dnv.com> retrieved on Monday, 10<sup>th</sup> August, 2012.
- Epaarachci, D., Stewart, M., and Rosowsky, D. (2000). "Structural reliability of multistory buildings during construction". *Journal of Structural Engineering*, 128 (2), 205-213.
- Eurocode 0. (2002). "Eurocode-basis of structural design". British Standards. CEN, Brussels.
- Eurocode 2. (2008). "Design of reinforced concrete structures, part 1-1, general rules and rules for buildings". CEN, Brussels.
- Frangopol, D. M., and Hendawi, S. (1994). "Incorporation of corrosion effects in reliability-based optimization of composite hybrid plate girders". *Struct. Safety*, 16 (1,2), 145-169.
- Frangopol, D. M., and Moses, F. (1994). "Reliability-based structural optimization. Advances in design optimization". H. Adeli, ed., Chapman and Hall, Ltd., London, England, 492-570.
- Hyman, W. A., and Hughes, D. J. (1983). "Computer model for life- cycle cost analysis of statewide bridge repair and replacement needs". *Transp. Res. Rec.*, 899, Nat. Acad. of Sci., Washington, D.C., 52-61.
- Kranc, C., Sagüés A. A., and Presuel-Moreno, F. I. (2002). "Decreased corrosion initiation time of steel in concrete due to reinforcing bar obstruction of diffusional flow". *ACI Materials Journal*, 99, 51.



- Kupwade-Patil, K., Cardenas, H., Gordon, K., and Lee, L. (2012). "Corrosion mitigation in RC beams via nanoparticles treatment". *ACI Materials Journal*, 109 (6), 617-626.
- Li, C. Q. (2005). "Life cycle modeling of corrosion affected concrete structures-initiation". *Journal of Materials in Civil Engineering*, 15 (6), 594-601.
- Lin, K. Y., and Frangopol, D. M. (1996). "Reliability-based optimum design of reinforced concrete girders". *Struct. Safety*, 18 (213), 239-258.
- Liu, Y., and Weyers, R. E. (1998). "Modelling the time-to-corrosion cracking of the cover concrete in chloride contaminated reinforced concrete structures". *ACI Materials Journal*, 95, 675-681.
- Melchers, R. E. (1999). "Structural reliability analysis and prediction". 2<sup>nd</sup> edition. John Wiley, Chichester.
- Melchers, R. E. (2001). "Assessment of existing structures – approaches and research needs". *Journal of Structural Engineering*, 127 (4), 406-411.
- Mori, Y., and Ellingwood, B. R. (1994). "Maintaining reliability of concrete structures I: Role of inspection/repair". *J. Struct. Engr., ASCE*, 120 (3), 824-845.
- Neville, A. M., and Brooks, J. J. (1994). "Concrete technology". Longman Scientific and Technical, England, 1994.
- Thoft-Christensen, P. (2000). "Stochastic modeling of the crack initiation time for reinforced concrete structures". 2000 Structures Congress, Philadelphia, May 8-10.
- Thoft-Christensen, P. (2002). "Deterioration of concrete structures". First International Conference on Bridge Safety and Management. Barcelona 14-15 July, 1-8.
- Ting, S. C. (1989). "The effects of corrosion on the reliability of concrete bridge girders". PhD Thesis, Dept. of Civ. Engr., Univ. of Michigan, Ann Arbor, Mich.
- Torres-Acosta, A., and Sagues, A. A. (2004). "Concrete cracking by localized steel corrosion-geometric effects". *ACI Materials Journal*, 101, 501.
- Vu, K. A. T., and Stewart, M. G. (2005). "Predicting the likelihood and extent of reinforced concrete corrosion induced cracking. *Journal of Structural Engineering*. ASCE, 131, 1681-189.
- Zaher, K., and Samir, S. (2001). "Permeability and chloride penetration of concrete subjected to Gaza seawater exposures". *Journal of the Islamic University of Gaza*, 9 (2), 67-84.