Jordan Journal of Civil Engineering, Volume **6**, No. **1**, **2012**

Statistical Identification of a Prestressed Concrete Beam with Unbonded Tendons Using Modal Data

Hamid Al-Ani [1]***, Walid M. Hasan*** [2] ***and Moh'd El-Khatieb*** [3]

[1] Associate Professor, Department of Civil Engineering, Al-Zaytoona Private University of Jordan, Amman, Jordan,
E-mail: hamid_abdulsalam@hotmail.com
[2] Assistant Professor, Department of Civil Engineering, Al-Isra University, Amman, Jordan,
E-mail: walid.hasan@ iu.edu.jo
[3] Assistant Professor, Department of Civil Engineering, Al-Isra University, Amman, Jordan

ABSTRACT

An iterative statistical identification method, based on Bayesian approach, was used to identify the actual stiffness and prestressing force of a prestressed simply supported beam with unbonded curved tendons. A finite element model, with consistent mass matrix, was used as analytical model and the first three natural frequencies of the beam were used as experimental modal parameters. Because the procedure involves inversion of matrices, the ill-conditioning of the problem was also investigated. The aim of this paper is to identify a reliable model of a prestressed beam which represents very well the real structure by identifying the stiffness parameters and the prestressing force. This model can be used, then, as a reference model to detect damage or loss of prestressing force. It was seen that the accuracy of the identified parameters and the rate of convergence are highly influenced by the coefficients of variation assigned to the various parameters. The effect of the uncertainties associated with the physical and experimental parameters on the accuracy of the identification results was illustrated by some graphics and tables. Other graphics and tables show the utility of the improved statistical identification method to accelerate the convergence of the identified parameters.

KEYWORDS: Prestressed beam, Statistical identification, Model updating, Estimation methods, Finite element model.

INTRODUCTION

For a structure, an accurate mathematical model is necessary for the design of control system and integrity monitoring. The analytical modeling techniques, such as finite element method, although they have reached a high level of sophistication, are idealized and approximate real structures and, therefore, possess some uncertainties which depend on the analyst's intuition. The main sources of the uncertainties present in the analytical models are ascribed to inappropriate theoretical assumptions and inaccuracies in estimated material properties. The usefulness of the analytical solutions derived from these models is limited by the degree of the realistic representation of the mathematical models. To overcome these limitations, the analyst can use one of the identification methods in order to improve the existing finite element model of the structure by estimating some structural parameters using measured data of structural response to known excitation. In this regard, Model matrix optimization or updating procedures are defined as those techniques that use experimentally obtained modal data for modifying the model matrices of a finite element model in order to

Accepted for Publication on 13/11/2011.

construct a model that better fits the modal results (Farhat and Hemez, 1993; Marwala and Sibisi, 2005; Foster and Mottershead, 1990).

In their paper, Daghia et al. (2009) used the statistical identification method to update a finite element model of a Timoshenko beam-column on two-parameter elastic foundation. The identified parameters were the stiffness parameters, the moduli of subgrade reaction of the elastic support and the stiffness of the end restraints. Foster and Mottershead (1990) used a least-squares technique to estimate the mass, stiffness and damping parameters in the spatial model of a portal frame using a finite element model and incomplete experimental data. Torkamani and Ahmadi (1988) conducted a comparison between four diffrent structural identification methods for the identification of two frames using eigenfrequencies and eigenvectors. The importance of the prior analytical model on the rate of convergence of the estimated parameters was investigated. Yuen (2010) proposed an efficient model correction method to update the mass and stiffness matrices of a finite element model using modal measurements. The method does not require computation of the complete set of the eigenvalues and eigenvectors of the model.

A rather frequent aspect of the degrading of prestressed beams is the attainment of the cracking limit state during their life-time. The cracks may appear as a consequence of incorrect evaluation of the final prestressing load, or due to unpredicted actions in the design stage. This may cause corrosion of the prestressing steel with dangerous reduction of the structural safety.

The final value of the prestressing force is affected by many factors, like the creep and shrinkage deformation of concrete, the relaxation of the prestressing steel, the friction forces along the tendon and the settlement of the anchorage devices at the ends of the tendon (Nawy, 2003). Unger et al. (2005) used modal curvatures in combination with eigenfrequencies and mode shapes to update a finite element model of a prestressed concrete beam. An iterative sensitivity based approach was used in the updating procedure. The method was then applied to the assessment of a gradually damaged prestressed beam. In another paper, Unger et al. (2006) pointed out the importance of using gradually damaged beam to investigate changes in modal parameters. The results show that damage detection is difficult in the early damage state.

In this paper, an iterative statistical identification method was adopted to identify the stiffness parameters and prestressing force of a post-tensioned simply supported beam with unbonded tendons, using experimental modal data. The beam, whose geometric and mechanical characteristics are shown in Table 2, was discretized in 12 elements of equal length. A 24 D.O.F. finite element model with consistent mass matrix was adopted as analytical model of the beam. The mass matrix is assumed to be known for a high level of accuracy, while the stiffness parameters and prestressing force are approximately known and, thus, have to be improved using the first three natural frequencies of the actual beam. The influence of the uncertainties associated with the physical and experimental data on the rate and accuracy of the identified parameters was investigated.

Identification methods can be used to create reliable models for the scope of monitoring and damage assessment. Soyoz et al. (2010) presented a structural reliability estimation method incorporating identified parameters such as stiffness and damping values using seismic response measurements. The reliability estimation was performed for bridge models by two sets of parameters. The first set was obtained by simulated parameters; the second was obtained by updating the first set using Bayesian approach and vibration measurements. Strauss et al. (2010) presented a nondestructive damage detection approach based on measured structural response. The validity of the method was verified by laboratory specimens and real structures. The measured frequencies and their sensitivity to change in mechanical characteristics were used to predict the location and magnitude of the damage.

The identification method adopted in this paper is particularly useful as a nondestructive procedure for "in service" estimation of the loss of prestressing force following a hazardous event such as a strong-motion earthquake or vandalism action.

Description of Identification Algorithm

The major guidelines of the identification algorithm are highlighted here. For more information, the reader may refer to (Torkamani and Ahmadi, 1988). The equation of the transverse free vibration of n D.O.F. system can be written in matrix form as (Clough and Penzien, 2003):

$$\boldsymbol{M}\,\ddot{\boldsymbol{v}} + \boldsymbol{K}\,\boldsymbol{v} = \boldsymbol{0} \quad ; \tag{1}$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are $n \times n$ mass and stiffens matrices, while $\ddot{\boldsymbol{v}}$ and $\boldsymbol{v}$ are $n \times 1$ vectors of the system accelerations and displacements, respectively.

Indicating by ω and $\boldsymbol{\phi}$ the circular natural frequencies and the eigenvectors, respectively, the eigenvalue problem associated with (1) is given by:

$$\left(\boldsymbol{K} - \omega^2 \boldsymbol{M}\right)\boldsymbol{\phi} = \boldsymbol{0}\,. \tag{2}$$

Now, let

$$\boldsymbol{r} = \begin{pmatrix} r_1 & r_2 & \cdots & r_l \end{pmatrix}^T \tag{3}$$

be a vector of physical parameters of the system. Then, the mass and stiffness matrices are functions of $\boldsymbol{r}$, and thus, the modal parameters are functions of $\boldsymbol{r}$ as well.

The modal parameters of the system can be expanded in Taylor series:

$$\begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\phi} \end{pmatrix}^{i+1} = \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\phi} \end{pmatrix}^{i} + \boldsymbol{S}\left(\boldsymbol{r}^{i+1} - \boldsymbol{r}^{i}\right); \tag{4}$$

where

$$\boldsymbol{S} = \left[\partial\,\boldsymbol{\omega} / \partial\,\boldsymbol{r} \quad \partial\,\boldsymbol{\phi} / \partial\,\boldsymbol{r}\right] \tag{5}$$

is a sensitivity matrix of the modal parameters with respect to the physical parameters.

The partial derivatives in equation (5) can be calculated by the expressions (Wang et al., 1993):

$$\frac{\partial\,\omega_i}{\partial\,r_j} = \frac{1}{2\omega_i}\boldsymbol{\phi}^{(i)T}\left(\frac{\partial\,\boldsymbol{K}}{\partial\,r_j} - \omega_i^2\,\frac{\partial\,\boldsymbol{M}}{\partial\,r_j}\right)\boldsymbol{\phi}^{(i)}\,; \tag{6}$$

$$\frac{\partial\,\boldsymbol{\phi}^{(i)}}{\partial\,r_j} = -\left[\boldsymbol{K} - \omega_i^2\,\boldsymbol{M}\right]^{-1}\left[\frac{\partial\,\boldsymbol{K}}{\partial\,r_j} - 2\omega_i\frac{\partial\,\omega_i}{\partial\,r_j}\boldsymbol{M} - \omega_i^2\frac{\partial\,\boldsymbol{M}}{\partial\,r_j}\right]\boldsymbol{\phi}^{(i)}\,; \tag{7}$$

where i indicates the i^{th} measured modal parameter and j indicates the j^{th} parameter of vector $\boldsymbol{r}$ to be identified.

From (4), the iterative algorithm for the identification method can be written as:

$$\boldsymbol{r}^{i+1} = \boldsymbol{r}^{i} + \boldsymbol{H}\left(\begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\phi} \end{pmatrix}^{i+1} - \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{\phi} \end{pmatrix}^{i}\right) \tag{8}$$

where $\boldsymbol{H}$ is an estimator matrix, which depends on the adopted estimation method (Table 1).

In Table 1, $\boldsymbol{C}_{\varepsilon\varepsilon}$ and $\boldsymbol{D}_{\varepsilon\varepsilon} = \boldsymbol{C}_{\varepsilon\varepsilon}^{-1}$ are, respectively, the diagonal covariance and weighting matrices of errors on the measured modal parameters, while $\boldsymbol{C}_{rr}$ denotes the diagonal covariance matrix for errors on prior parameters and β is a coefficient which was introduced to improve the statistical identification method.

CASE STUDY

A prestressed simply supported beam with unbonded curved tendons, whose mechanical and geometric properties are illustrated in Figure 1 and Table 2, was considered. For the purpose of identification, a 24 D.O.F. finite element model with consistent mass matrix was adopted. The first three eigenfrequencies of the beam can be found by solving the eigenvalue problem given by equation (2).

The flexural rigidities k_i and the corresponding natural frequencies were obtained from an analytical model based on ideal assumptions that do not take into account the presence of cracks, voids,… etc. Furthermore, the prestressing force given in Table 2 is an approximation of the real prestressing force because the prestressing losses are difficult to quantify exactly.

For these reasons, the experimental frequencies measured on the actual structure would not match the analytical ones calculated from the finite element model. A reliable model of the beam that can be used as a reference model for control and monitoring of the beam in the future can be obtained by updating the finite element model using experimental data.

Table 1. Estimator H for different methods of parameter estimation

Estimation method	Estimator H	measured modal parameters	prior parameters
Least squares	$S^T(S^T S)^{-1}S^T$ (under-deter. sys.) $(S^T S)^{-1}S^T$ (over-deter. sys.)	deterministic	deterministic
Weighted least squares	$S^T(S^T S)^{-1}S^T$ (under-deter. sys.) $(S^T D_{\varepsilon\varepsilon} S)^{-1}S^T D_{\varepsilon\varepsilon}$ (over-deter. sys.)	stochastic	deterministic
Statistical identification method	$(C_{rr}^{-1} + S^T C_{\varepsilon\varepsilon}^{-1} S)^{-1}S^T C_{\varepsilon\varepsilon}^{-1}$ $C_{rr} S^T(S C_{rr} S^T + C_{\varepsilon\varepsilon})^{-1}$	stochastic	stochastic
Improved statistical identification method	$\beta^{-1} \mathbf{C}_{rr} \mathbf{S}^T(\beta^{-1}\mathbf{S}\mathbf{C}_{rr} \mathbf{S}^T + \mathbf{C}_{\varepsilon\varepsilon})^{-1}$	stochastic	stochastic

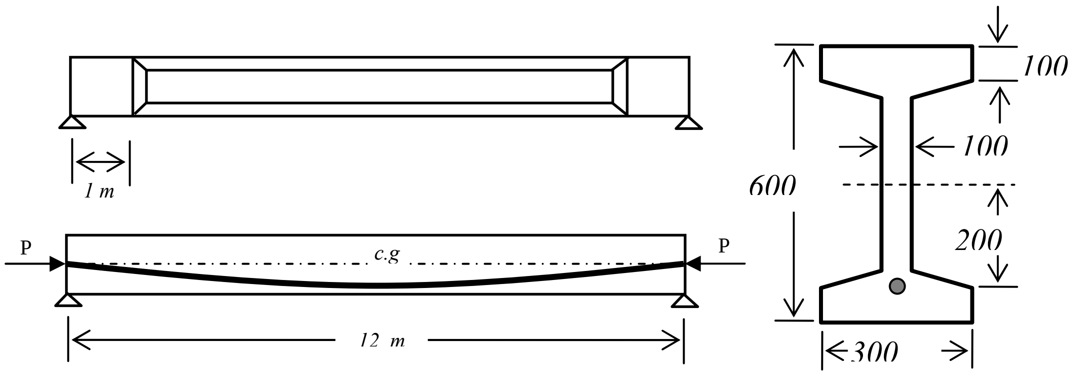

Figure 1: Prestressed beam-geometric properties

Using cubic interpolating functions, the mass matrix for the i^{th} element assume the aspect (Clough and Penzien, 2003):

$$M^{(i)} = \frac{ml}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ & 4l^2 & 13l & -3l^2 \\ & & 156 & -22l \\ & & & 4l^2 \end{bmatrix}; \qquad (9)$$

where m is the mass per unit length of the beam and l is the length of each element considered equal for all elements.

The stiffness and geometric-stiffness matrices have the form (Clough and Penzien, 2003):

Table 2. Geometric and mechanical properties of the beam

Beam length	L = 12 m
Finite element length	l_i = 1 m, $i = 1,2,3,...,12$
Beam transverse section dimensions	As shown in Fig. 1
Cross-sectional area of elements 1and12	A_i = 0.18 m^2, $i = 1\ and\ 12$
Cross-sectional area of other elements	A_i = 0.110 m^2, i = 2,3,...,11
Moment of inertia of elements 1and12	I_i = 0.0054 m^4, $i = 1\ and\ 12$
Moment of inertia of other elements	I_i = 0.0047 m^4, i = 2,3,...,11
R.C. specified compressive strength	$f_c' = 45\ N/mm^2$
Young's modulus	$E = 4700\sqrt{f_c'}\ \ N/mm^2 = 31.529 \times 10^6\ \ kN/m^2$
Mass density	$\rho = 2400\ \ kg/m^3$
Flex. Rigidity of elements 1 and 12	$k_i = (EI)^{(i)} = 170256\ \ kN.m^2$, $i = 1\ and\ 12$
Flex. Rigidity of elements 2, 3, ..., 11	$k_i = (EI)^{(i)} = 148186\ \ kN.m^2$, i = 2,3,...,11
Prestressing load	P = 600 kN

$$\boldsymbol{K}_s^{(i)} = \frac{(EI)^{(i)}}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ & & & 4l^2 \end{bmatrix},$$

$$\boldsymbol{K}_g^{(i)} = \frac{P}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ & 4l^2 & -3l & -l^2 \\ & & 36 & -3l \\ & & & 4l^2 \end{bmatrix}. \tag{10}$$

The stiffness matrix $\boldsymbol{K}$ in (1) reduces to:

$$\boldsymbol{K} = \boldsymbol{K}_s - \boldsymbol{K}_g\ . \tag{11}$$

Now, let

$$\boldsymbol{r} = (k_1\ k_2\ k_3\ k_4\ k_5\ k_6\ k_7\ k_8\ k_9\ k_{10}\ k_{11}\ k_{12}\ \ P)^T \tag{12}$$

be a vector of physical parameters to be identified, where $k_i = (EI)^{(i)}$ are expressed in $kN.m^2$ and P in kN

and let

$$\boldsymbol{r}^{(0)} = (170256 \quad 148186 \quad \cdots \quad 148186 \quad 170256 \quad 600)^T \tag{13}$$

be a vector of initial estimates for the components of $\boldsymbol{r}$ and the corresponding first three natural frequencies are $\omega_1 = 49.7$, $\omega_2 = 202.3$ and $\omega_3 = 453.6$ rad/sec.

Suppose that the flexural rigidities k_i in the analytical model were overestimated by 10% and the prestressing force was underestimated by 5%. Therefore, the actual values of the flexural rigidities will be given in the following vector:

$$\boldsymbol{r}_{actual} = (153000 \quad 133000 \quad \cdots \quad 133000 \quad 153000 \quad 630)^T \tag{14}$$

The corresponding first three simulated experimental frequencies are $\omega_1 = 46.8$, $\omega_2 = 191.5$ and $\omega_3 = 429.8$ rad/sec.

Using the statistical and improved statistical identification methods, whose algorithm is expressed by (8) and the last two rows of Table 1, the identification procedure was performed to update the initial estimates of the components of $\boldsymbol{r}$ using the above experimental natural frequencies.

As the estimation algorithm involves the inversion of the matrix $(\boldsymbol{S}\,\boldsymbol{C}_{rr}\,\boldsymbol{S}^T + \boldsymbol{C}_{\varepsilon\varepsilon})$, it is interesting to study the condition of the problem. This can be achieved by calculating the condition number of the above matrix. The condition number is the ratio of the largest singular value to the smallest one using the singular-value decomposition method.

The sensitivity matrix $\boldsymbol{S}$ was calculated using the partial derivatives of ω_1, ω_2 *and* ω_3 with respect to the components of $\boldsymbol{r}$ and the following matrix was obtained:

$$\boldsymbol{S} = \begin{bmatrix} 5.5082E-10 & 4.7992E-9 & 1.1794E-8 & 1.9854E-8 & 2.6827E-8 & 3.0852E-8 & \ldots & -2.7609E-6 \\ 8.6709E-9 & 6.2810E-8 & 1.1198E-7 & 1.1087E-7 & 6.0397E-8 & 1.0426E-8 & \ldots & -2.6612E-6 \\ 4.3338E-8 & 2.3154E-7 & 2.1893E-7 & 4.6760E-7 & 4.9313E-8 & 2.1788E-7 & \ldots & -2.6147E-6 \end{bmatrix}.$$

Table 3. Identified parameters and errors on estimation for different uncertainties

Coefficiens of variations	No. of Iterations	$k_1 = k_{12}$ kN-m^2	$k_2 = k_{11}$ kN-m^2	$k_3 = k_{10}$ kN-m^2	$k_4 = k_9$ kN-m^2	$k_5 = k_8$ kN-m^2	$k_6 = k_7$ kN-m^2	P kN
a = 10% b = c = d = 0.1% $\beta = 1$	462961	167490	135330	130270	133210	134440	131420	618
		8.6%	1.7%	-2.05%	0.16%	1.1%	-1.2%	-1.9%
a = 10% b = c = d = 1% $\beta = 1$	4634	167440	135150	130070	133120	134350	131200	618
		8.6%	1.6%	-2.2%	0.09%	1%	-1.5%	-1.9%
a = 10% b = c = d = 10% $\beta = 1$	51	167400	134980	129980	133230	134410	131040	618
		8.6%	1.5%	-2.3%	0.17%	1%	-1.5%	-1.9%
a = 10%, $\beta = 1$ b = 2.5% c = 1%, d = 1.3%	3860	154030	136070	131090	133620	134790	132130	630
		0.7%	2.25%	-1.44%	0.46%	1.33%	-0.7%	0%

Table 4. Influence of coefficient β on the rate of convergence of parameters

Coefficiens of variations	**No. of Iterations**					
	$\beta = 1$	$\beta = 0.5$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.01$	$\beta = 0.001$
a = 10% b = c = d = 0.1%	462961	231483	46300	23152	4634	467
a_1 = 1%, a_2 = 5%, a_3 = 10% b = 2.5%, c = 1%, d = 1.3%	11255	5629	1128	565	115	15

A preliminary investigation was carried out to illustrate the influence of the uncertainties (coefficients of variation) associated with the various parameters, on the accuracy of the identification results. These uncertainties express the confidence of the analyst on the various parameters.

For simplicity, we will denote with (a) the coefficients of variation of the experimental frequencies ω_1, ω_2 *and* ω_3, respectively, with (b) the coefficients of variation of the flexural rigidities k_1 and k_{12}, with (c) the coefficients of variation of the flexural rigidities of k_2, k_3,…, k_{11}, and, finally, with (d) the coefficient of variation of the prestressing force *P*.

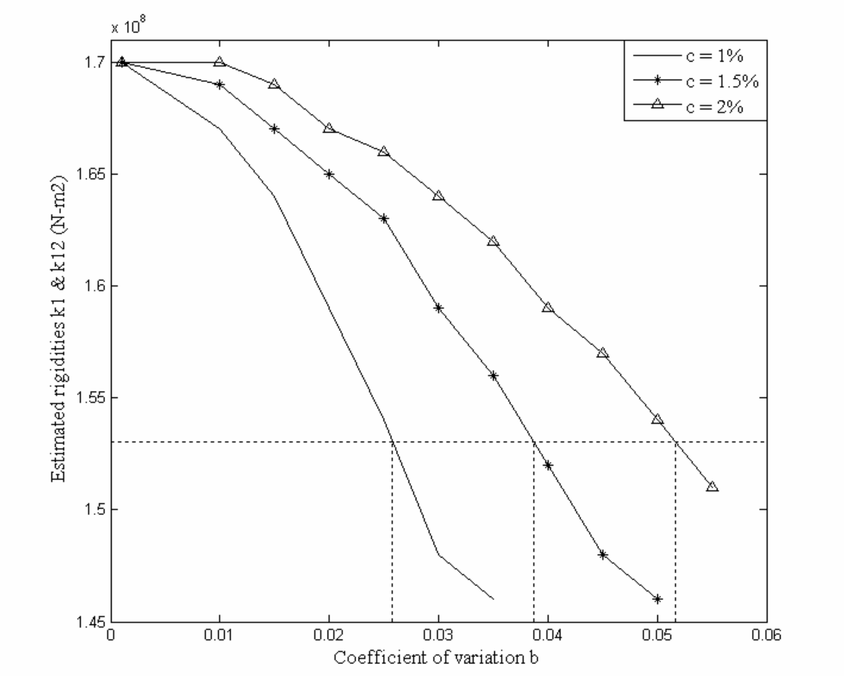

Figure 2a: Convergence of k_1 and k_{12} for various uncertainties

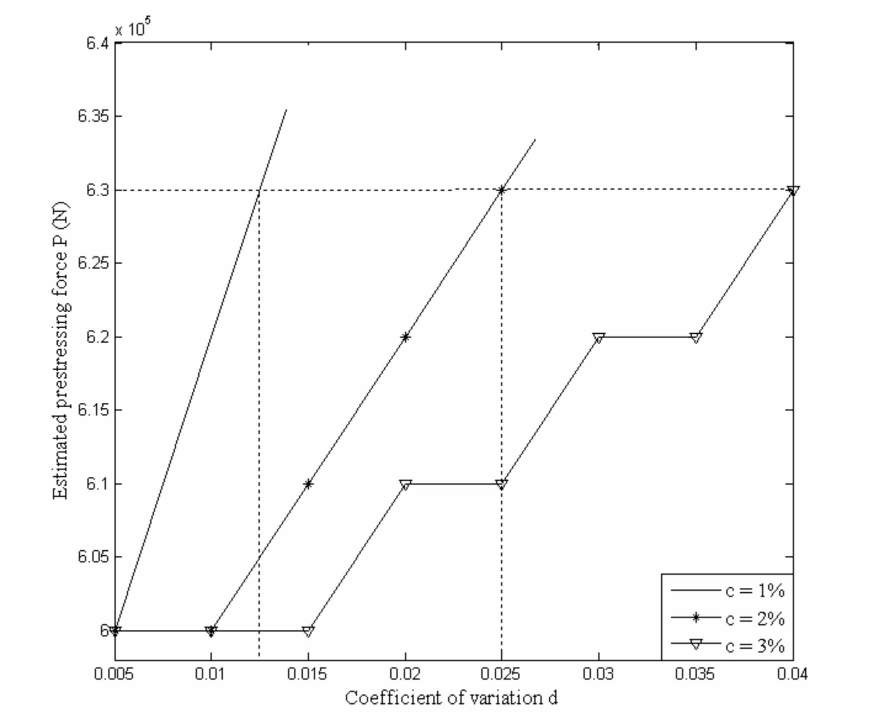

Figure 2b: Convergence of P for various uncertainties

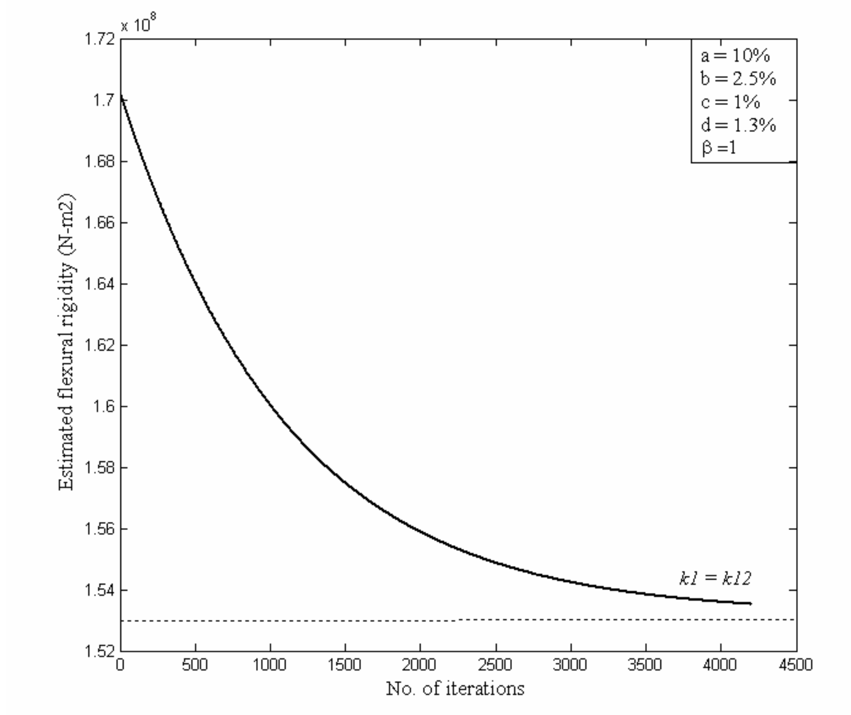

Figure 3a: Convergence of k_1 and k_{12}

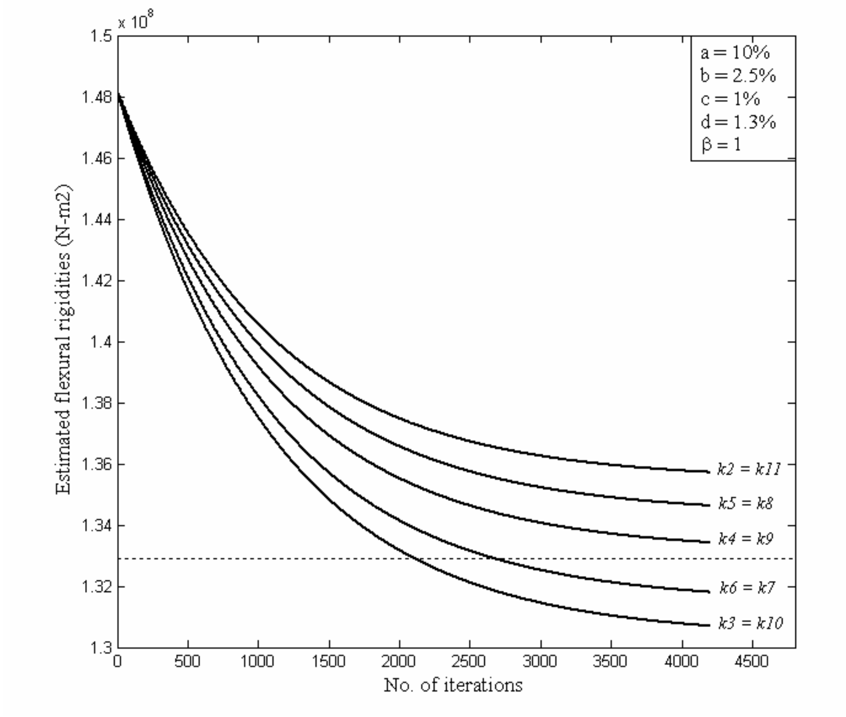

Figure 3b: Convergence of k_2 , ... , k_{11}

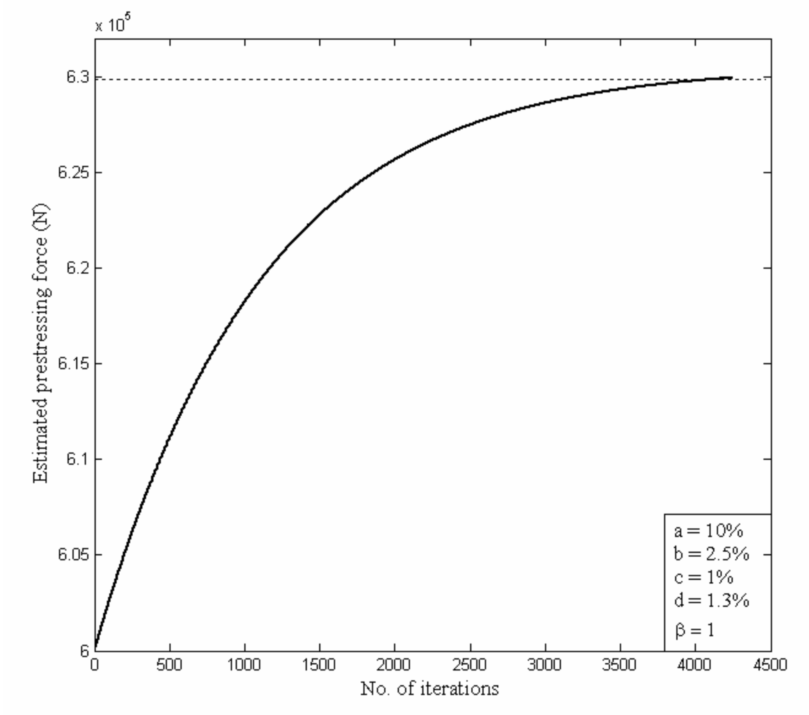

Figure 3c: Convergence of prestressing force P

From the sensitivity matrix ***S***, it can be seen that the eigenfrequencies are not equally sensitive to each parameter. This will affect differently the convergence of each parameter. In fact, if the uncertainty is the same for all physical parameters (for example b = c = d = 0.1%, 1%, 10%), it can be seen from Table 3 that poor estimations were obtained for k_1, k_{12} and P and good estimates are obtained for $k_2, ..., k_{11}$ for any value of the coefficients of variation of the physical parameters and for a = 10%. It can also be seen that the rate of convergence is very slow for the uncertainty of 0.1%, but it increases drastically as the uncertainty increases to 1% and 10%. The problem is not well conditioned as the condition number equals 86.

The rate of convergence can be drastically increased using the improved statistical identification method with the introduction of the coefficient β. The influence of coefficient β on the rate of convergence of the physical parameters is illustrated in Table 4. It can be seen that as the value of the coefficient β decreases, the rate of convergence increases very rapidly.

Good estimations for all identified parameters can be obtained assigning different uncertainties to the physical parameters. From Figures 2a and 2b, it can be seen that maintaining the same value of the coefficient (b) and varying the two coefficients (c) and (d), there is always a value of (c) and (d) for which good estimations can be obtained for all parameters. By inspection, these values are estimated as $c \cong 2.5\,b$ and $d \cong 1.3\,b$.

Figures 3a, 3b and 3c illustrate the convergence of the identified parameters assuming that $a = 10\%$, $b = 2.5\%$, $c = 1\%$ and $d = 1.3\%$. It can be seen that good estimates are achieved after nearly 3860 iterations with errors within 2.5% (see also Table 3). More accurate estimates could be obtained if different uncertainties on the rigidities of the internal elements were assigned. Also, in this case the condition number equals 84.

Once again, the improved statistical identification method can be used to accelerate the convergence of the identified parameters. Figures 4a, 4b and 4c illustrate the influence of the coefficient β on the rate of convergence of k_1 as a representative of the external elements, k_4 as a representative of the internal elements, and P. It can be seen that as β decreases, the rate of convergence increases rapidly.

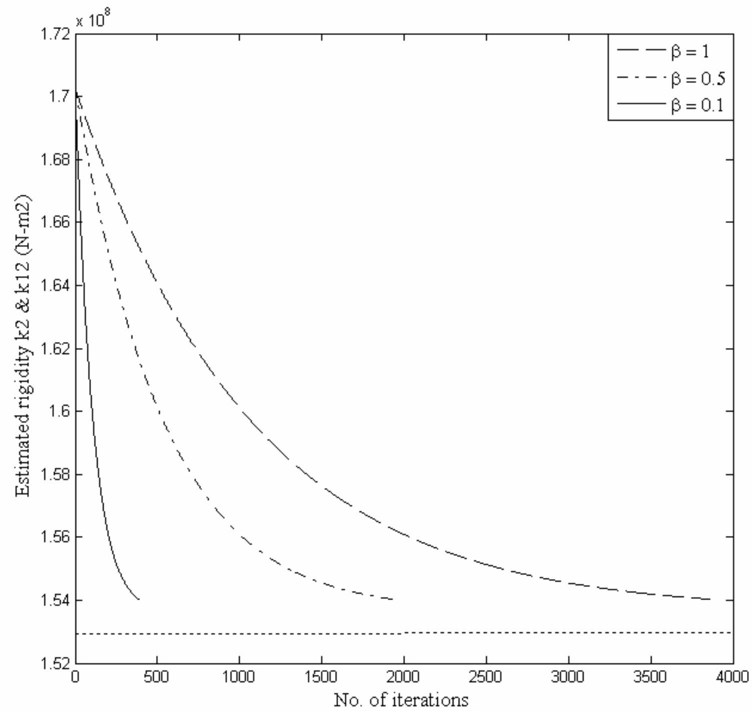

Figure 4a: Influence of coefficient β on the convergence of k_1 and k_{12}

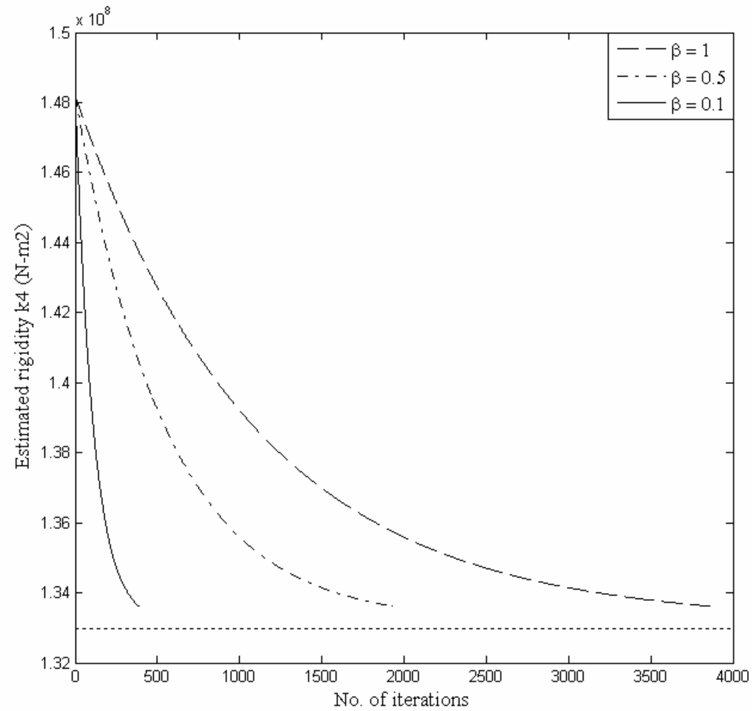

Figure 4b: Influence of coefficient β on the convergence of k_4

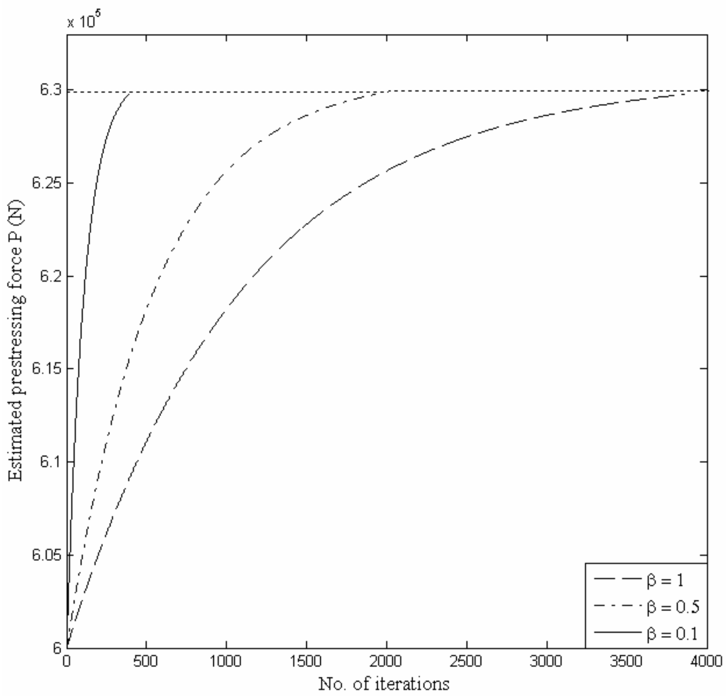

Figure 4c: Influence of coefficient β on the convergence of P

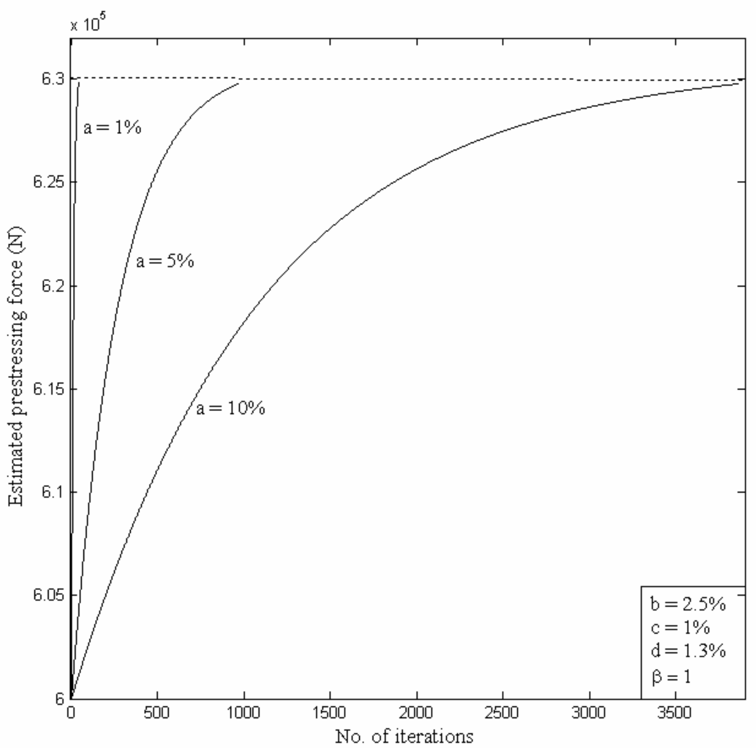

Figure 5: Influence of the uncertainty on experimental data on the rate of convergence of P

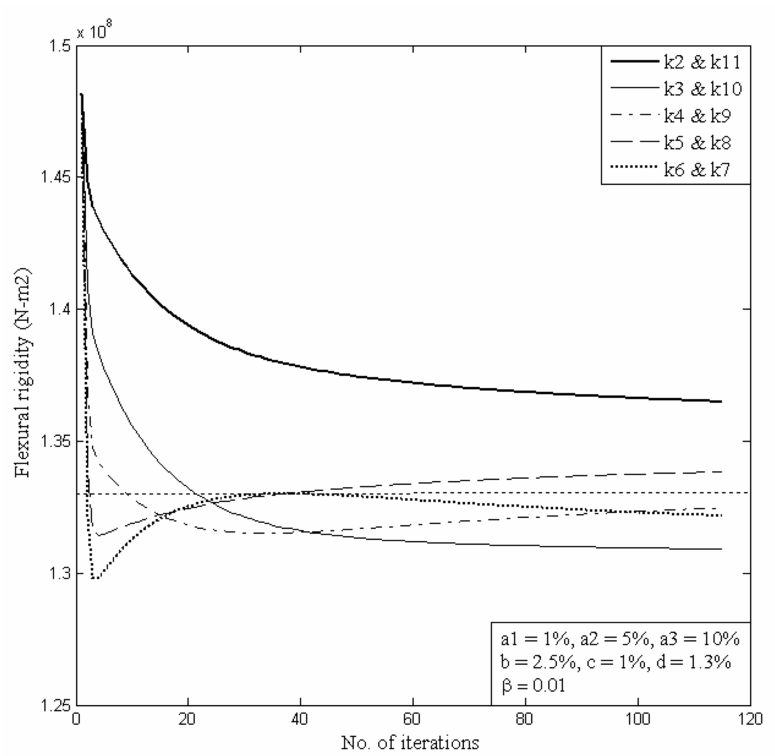

Figure 6: Convergence of k_1, k_2,…, k_{11} for different uncertainty on each eigenfrequency

The effect of changing the uncertainties on the experimental frequencies was also investigated. It was seen that as the uncertainty on the experimental data decreases (maintaining the same uncertainty for all three frequencies), the rate of convergence of the identified parameters increases vary rapidly (see Figure 5 for the convergence of the prestressing force *P*). The condition number does not change from the previous cases.

Until now, equal uncertainties were assigned to the three measured natural frequencies. Now, consider the case where each natural frequency has a different uncertainty. Figure 6 shows the convergence of the identified rigidities $k_2, k_3, ..., k_{11}$ assigning a different uncertainty to each frequency (a_1 = 1%, a_2 = 5%, a_3 = 10%), maintaining the same uncertainties on the physical parameters; i.e., b = 2.5%, c = 1%, d = 1.3%. The convergence in this case is completely different from the previous cases. The convergence is very slow and, therefore, we used the improved statistical method with $\beta = 0.01$ to accelerate the convergence. The rate of convergence is very rapid at the first iterations and becomes very slow in the subsequent iterations. In this case, the problem becomes badly conditioned as the condition number equals 8102.

Concluding Remarks

1. The statistical identification method can be used to identify the physical parameters of the prestressed beam, but its convergence depends greatly on the uncertainties on the various parameters.
2. Because the natural frequencies are less sensitive to the rigidities of the external elements and the prestressing force, good estimates for these parameters can be obtained assigning larger coefficients of variation to these parameters than those assigned to the parameters of the internal elements.
3. Good estimates were obtained for any uncertainty on the physical and experimental parameters.
4. The improved statistical identification method can

accelerate the rate of convergence as the coefficient *β* decreases.

5. Changing the uncertainties associated with the experimental data had little effect on the accuracy of the estimations, but the rate of convergence was highly affected.

Acknowledgements

The authors are most grateful to Dr. Rami Al-Hmouz from the Department of Electrical Engineering at Al-Isra University for the help in implementing MatLab programs.

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