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Effect of Inherent Anisotropy on Shear Strength Following Crushing of Natural Aqaba Subgrade Sand

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ABSTRACT

Inherent anisotropy affects the overall shear strength of sand deposits. Soil inherent anisotropy was evaluated for pre-crushed Aqaba subgrade sand by deposition of soil grains onto an inclined surface. Crushing of Aqaba sand was induced by one-dimensional compression. Sand characteristic properties (mineralogical properties, grain size and crushing resistance strength) were determined by standard laboratory testing. Particle breakage factors and inter-particle void ratio were calculated from the initial and final gradations of the soil samples. Moreover, shear strength components for sand specimens were resolved. Inspection of the residual shear strength parameters showed an increase, where the amount of particle crushing increased regardless the level of the normal stress being applied. Furthermore, examining the effect of inherent anisotropy showed that a considerable amount of the dilation occurs when the particles tend to lie orthogonal to the horizontal plane regardless the extent of breakage.

KEYWORDS: Natural sand, Granular materials, Crushing, Breakage, Anisotropy, Fabric, Shear strength, Dilatancy, Direct shear.

INTRODUCTION

The variant of sand grading upon crushing tends to change irreversibly and uncontrollably the characteristics of the materials and will certainly influence their mechanical response. If the effect and evolution of grading are deemed to be important, researchers must include this aspect of the fabric of the soil as part of its state (Wood and Maeda, 2008). The particle breakage represents one of the most important factors that enable to produce an irreversible change to the material with the proportion of fine portion in the soil monotonically increasing.

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Particle breakage of soil grains occurs at different stress levels. It might occur due to extremely high stresses such as in the cases of pile driving through sand stratum, construction of high earthen fill dams, laying foundations of offshore gravity structures and/or constructing highway subgrades. Evidences of some of the above examples were reported by (Lobo-Guerrero and Vallejo, 2006; Wood, 2006; Bartake and Singh, 2007; Al-Hattamleh et al., 2009).

Stresses inside the granular medium are composed of multiple stress chains, which can lead to local high-stress concentrations within the system. Such high stress concentrations may lead to crushing of single particles even under relatively low stresses. If granules are broken into smaller particles due to the application of external force, the soil properties will change; its porosity and permeability decrease, the overall specific surface increases, the particle size distribution is modified and the load–deformation properties are altered. Moreover, crushing increases with increasing particle size and angularity. In contrast, it decreases with increasing density, coefficient of uniformity and mineral strength (Vesic and Clough, 1968; Hardin, 1985; Chuhan et al., 2003; Valdes and Koprulu, 2007). Changes in the original engineering properties due to granular crushing and fines' scouring could put the stability of structure in jeopardy and make it unsafe during its life of operation as the case of Bennet Dam in British Columbia, Canada (Wood, 2006). Therefore, understanding crushing in granular materials and its evolution is highly demanded.

Experimental works on crushing of granular materials point out many key factors associated with the occurrence of particle crushing. It has been established that grain crushing is influenced by: soil particle strength, topology such as angularity, physical properties of granular materials such as gradation, porosity and moisture content, induced stress level and anisotropy (Lee and Farhoomand, 1967; Hardin, 1985; Hagerty et al., 1993; Lade et al., 1996; McDowell and Bolton, 1998; Takei et al., 2001; Coop et al., 2004; Tarantino and Hyde, 2005; Lobo-Guerrero and Vallejo, 2005; Einav, 2007). Moreover, previous research has indicated that the angle of shearing resistance for granular materials which undergo crushing decreases as a consequence of particle crushing (Bolton, 1986; Feda, 2002). Coop et al. (2004) conducted a ring shear test on carbonate sand and reported that crushing of particles has taken place without a loss of residual angle of internal friction. In this regard, the impression is that crushable granular materials experience a reduction in the internal friction angle as a consequence of particle breakage prior to achieving a constant value of residual strength.

One of the most important factors that influence the crushing of a mass of granular materials is the crushing resistance of grains (Lee and Farhoomand, 1967). Coarse granitic sand particles with an average diameter of 2.8 mm experience breakage at a pressure equal to 2 MPa, while calcareous shells begin crushing at pressures ranging from 0.05 to 0.2 MPa (Lee and Farhoomand, 1967). Angular particles of freshly quarried materials undergo fragmentation under ordinary pressures (about 0.98 MPa) due to breakdown of sharp angularities (Ramamurthy et al., 1974).

Lade et al. (1996) found that if uniform sand is crushed, the resulting grain size distribution approaches that of a well-graded soil for large compressive loads. However, before the granular assembly reaches a well-graded particle distribution, the granular assembly will experience gradual changes in particle size depending on the level of progressive load being applied.

Crushing, Texture of Sand and Components of Shear Strength

The amounts of particles breakage could be evaluated from the grain size distribution curve using the method developed by Lade et al. (1996). In this method, crushing is evaluated by using the particle breakage factor named B_{10}. This parameter is based on the effective size (i.e., D_{10}) and can be obtained from:

$$B_{10} = \frac{D_{10i} - D_{10f}}{D_{10i}} \tag{1}$$

where B_{10} = particle breakage factor, D_{10f} = effective grain size of the final gradation and D_{10i} = effective grain size of the initial gradation.

An alternative definition of particles' breakage was also proposed by Einav (2007). Einav (2007) postulated that the grain size distribution will start from an initial grading and reach ultimately a final grading due to shearing and compression. The relative breakage index, B_r, is defined as an area ratio as:

$$Br = \frac{B_t}{B_p} \tag{2}$$

where B_p, 'the breakage potential', is defined by integrating the entire area confined between the initial and the final grain size distribution, whilst B_t between the current, at a given compression stress, and the initial grain size distribution when there are no applied shear or compression stresses.

Tarantino and Hyde (2005) demonstrated conducting simple direct shear tests on carbonate sand to study the effect of crushing on shear strength properties. The shear tests have been carried out on monogranular and fractal grain size distributions of crushable carbonate sand at vertical stresses ranging from 0.2 MPa to 1.4 MPa and horizontal displacements ranging from 0.5mm to 8mm. In order to compare particle breakage of specimens with different particle sizes, specimens were prepared with a sample height of about twenty times main grain diameter, D_{50}. Grain size distributions were measured before and after shearing. They established a link between grain crushing, shear strength and general mechanical behavior of sands. They found that the apparent critical state friction angle contained both frictional and clastic components. They concluded that the apparent critical state angle of friction increases as the rate of particle crushing normalized with respect to the normal force increases. Moreover, Wood and Maeda (2008) used discrete element method on two-dimensional assemblies of circular particles to analyze the effect of changing grading on the critical state conditions. They concluded that changing the grading leads the granular material to seek the asymptotic state appropriate to the current grading. However, experimental support is wanted from tests on real soils to support the above finding. Moreover, they pointed that other constituents such as soil fabric, which can be created by deposition, cannot be excluded when studying granular crushing.

Thus, soil fabric, which is composed of the particle packing arrangements and the number of sliding contacts, can be directly related to second order tensor called fabric tensor, F_{ij}. When grained soil is deposited with respect to a certain plane such as a horizontal plane, the fabric tensor designates a transversely isotropic case. In such case, the fabric tensor F_{ij} can be written explicitly in terms of an angle, α, where α is the depositional angle with respect to the horizontal plane as:

$$F_{ij} = \begin{bmatrix} \tan\alpha & 0 & 0 \\ 0 & \frac{1}{2}(1-\tan\alpha) & 0 \\ 0 & 0 & \frac{1}{2}(1-\tan\alpha) \end{bmatrix} \qquad (3)$$

where the value of the angle α varies between 0° and 90°. Note that $\alpha = 0^\circ$ corresponds to a fabric configuration where the particles lie parallel to the horizontal bedding plane, while $\alpha = 90^\circ$ implies a fabric configuration where the particles' long radii are oriented parallel to the vertical direction. Moreover, when $\alpha = 18^\circ 26'6''$, $F_{11} = F_{22} = F_{33}$, indicating a statistically isotropic particle orientation. Using $0^o \le \alpha \le 90^o$, the fabric configuration from a completely “horizontal” to a completely “vertical” orientation of particles can be characterized. This case of fabric measure solely represents the inherent anisotropy of the soil.

The shear resistance of pulverized sand comes from interparticle friction and dilation. The two basic parameters can be directly deduced from the direct shear box. The direct shear friction angle, ϕ', can be obtained from the boundary measurements of average horizontal shear stress, τ_{xy}, and from the average vertical normal stress, σ_{yy}, by the following equation:

$$\tan\phi' = \frac{\tau_{xy}}{\sigma_{yy}}. \qquad (4)$$

The dilation angle, ψ, is obtained from the ratio change of the increment of vertical strain, $\Delta\varepsilon_{yy}$, to the increment of horizontal shear strain, $\Delta\gamma_{xy}$, as:

$$\tan\psi = \frac{-\Delta\varepsilon_{yy}}{\Delta\gamma_{xy}} \qquad (5)$$

Moreover, at large displacement, the direct shear friction angle reaches a constant value called the residual friction angle, ϕ'_{res}, depending on the initial density of the sand. Davis (1968) derived an equation which linked ϕ' with the plane strain frictional angle, ϕ'_{ps}, describings the conditions of maximum stress obliquity in the soil as:

$$\tan\phi' = \frac{\cos\psi \sin\phi'_{ps}}{1-\sin\psi \sin\phi'_{ps}}. \qquad (6)$$

Moreover, at the critical state when the dilation angle is zero, ϕ'_{ps} becomes ϕ'_{crit} and

$$tan\,\phi'_{res} = sin\,\phi'_{crit}\,. \qquad (7)$$

Furthermore, Dietz (2000) uniquely linked ϕ' with ϕ'_{crit} through the dilation angle as:

$$tan\,\phi' = \frac{sin\,\phi'_{crit} + sin\,\psi}{cos\,\psi}\,. \qquad (8)$$

Substituting equation 7 in equation 8 yields:

$$tan\,\phi' = \frac{tan\,\phi'_{res} + sin\,\psi}{cos\,\psi}\,. \qquad (9)$$

Lee and Seed (1967) recognized that particle breakage becomes a crucial factor in the shear resistance once the confining pressure increases and the void ratio decreases. The peak friction angle, ϕ_p, customarily measured in laboratory tests, has many contributors. Guo and Su (2007) revised Rowe's (1962) shear resistance contribution in cohesionless soil. Figure 1 devised by Guo and Su (2007) shows the main contributors to shear strength of sand. The mobilized friction angle at the onset of dilation, ϕ_f, varies with particle packing arrangements and the number of sliding contacts. Moreover, ϕ_f varies with the level of confining pressure and initial density in the range of $\phi_\mu \leq \phi_f \leq \phi_{crit}$ with ϕ_μ being the interparticle friction angle associated with resistance to interparticle sliding, and ϕ_{crit} or ϕ_{cv} being the critical state friction angle.

The objective of this study is to investigate the factors affecting the crushing resistance of natural Aqaba subgrade sands from two different locations in Aqaba, Jordan. The main focus of this study is to separate the effects of particle breakage, depositional angle and fabric effect on the shear strength parameters for granular material subjected to direct shear test. The results from direct shear tests conducted on the soil samples will be presented and analyzed.

EXPERIMENTAL PROGRAM

Material

The material used for the tests was two sand type samples collected from Aqaba bay in Jordan. To differentiate the two types, they were designated as AS1 and AS2. The specific gravity values of AS1 and AS2 are 2.63 and 2.66. The particle shapes for both samples are subrounded. Moreover, maximum and minimum void ratio values were found to be 0.775 and 0.536 for AS1 and 0.926 and 0.494 for AS2, respectively.

Mineralogy

The shear resistances of the sands are mainly related to their mineralogical composition and textural characteristics, which vary considerably with ageing and ultimately lead to the final shape of the grains. Particle size and shape, therefore, reflect material composition, grain formation, transportation and depositional environments. Mechanical and chemical weathering determine the grain shape once it is released from the matrix of parent rocks.

X-ray diffraction analysis was made for the two samples AS1 and AS2. XRD analysis results for AS1 and AS2 are illustrated in Figures 2 and 3, respectively. The results indicate that the most dominant mineral is quartz with traces of Na-plagioclase (albite) and anorthoclase. For AS1, quartz is associated with albite in detrital sedimentary rocks; therefore, granite rock is the mother rock for AS1 sample (Dana, 1892). Moreover, for AS2, the presence of quartz with anorthoclase minerals indicates that the sample originates from high-temperature sodic volcanic and hypabyssal rocks (Dana, 1892). Thus, sample AS2 was weathered from ryholitic rocks.

Equipment and Procedures

The natural sand samples were prepared as follows. The sands were rained from a specific height to a standard compaction test mold of a size of 944cm^3. Thereafter, the sand was subjected to one-dimensional compression from a hydraulic jack to the desired pressure. The sand specimens were then sieved. These specimens were then subjected to direct shear tests using a standard laboratory shear box apparatus with an initial sample cross-section of 60mm x 40 mm.

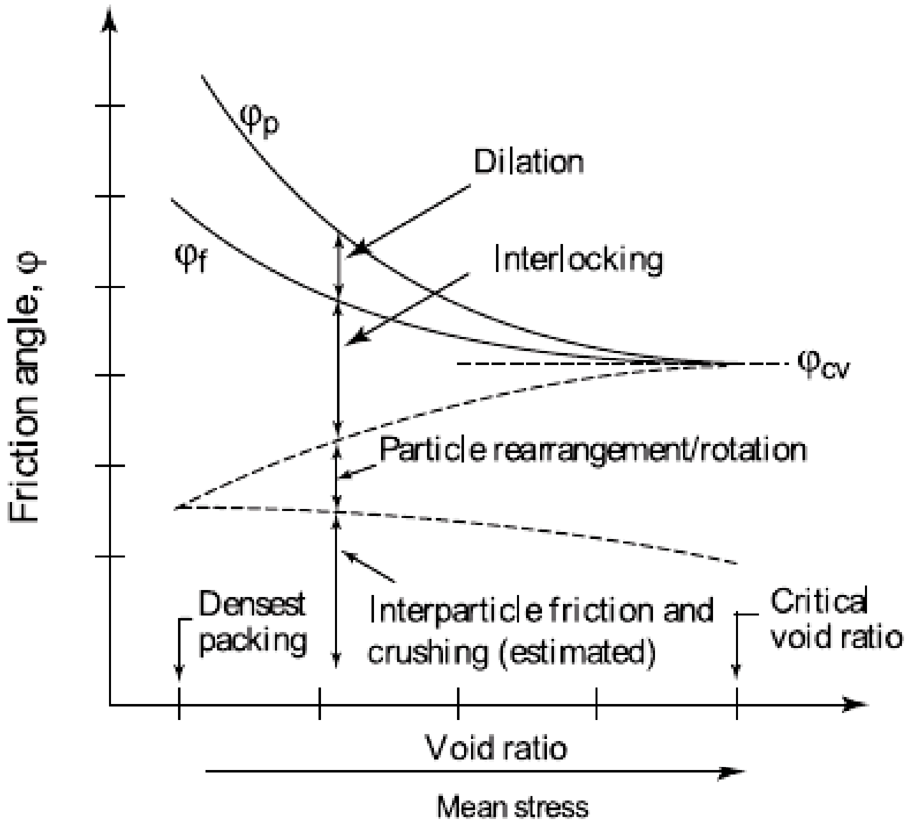

Figure 1: Contributions to shear resistance of granular materials

The sand samples were prepared by sieving a predetermined mass of sand over an open box-shaped metal grid inside the shear box and then slowly raising the grid. This technique assures that there is no segregation of the particle sizes during the deposition of sand. Moreover, to achieve different bedding angles, we followed (Al-Hattamleh et al., 2010) where the shear box was placed in an adjustable plate connected with a pin rotating around the horizontal plane through 90 degrees.

A series of shear tests was carried out, each at a constant vertical effective stress, σ_v, with values in 55kPa increments from 55kPa to 165kPa and total horizontal displacements until 12mm. Other sets of tests were also conducted after applying one-dimensional compression in order to quantify the amount of grains crushed due to vertical and horizontal shearing. This made it possible to differentiate crushing occurring during the horizontal shearing stage.

RESULTS AND DISCUSSION

Particle Breakage Quantification

AS1 samples were subjected to different levels of one-dimensional compression. Vertical stresses of 0 MPa, 5MPa, 10 MPa, 15 MPa and 20MPa were applied to the sand samples. Since the origin of sand was the granitic and ryholitic rocks, the chosen range of compressive stress induced on the sample to produce breakage was based on values reported earlier by Lee and Farhoomand (1967).

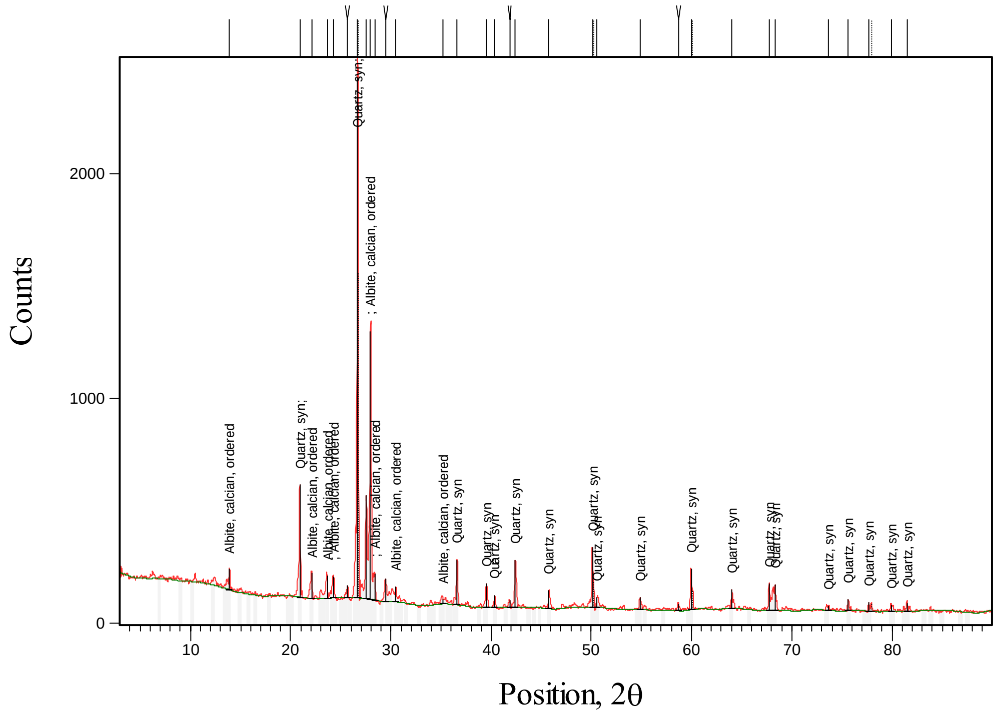

Figure 2: X-ray diffraction patterns for AS1 sand samples

The altering of grain size distribution due to one-dimensional compression is shown in Figure 4. The initial and final grain size distribution after compression are shown. It is obvious that vertical stresses induced by one-dimensional compression increase the amount of fine particles as a sequence of particle breakage increase. AS2 samples were also subjected to the same level of pressure. However, the variation of grain size distribution to this level of pressure was diminutive. Table 1 lists the progression of coefficient of uniformity, Cu, coefficient of curvature, Cc and the classification of the soil samples as a sequence of particle breakage due to the applied vertical stress. The data listed in Table 1 clearly indicate the increase of both Cu and Cc. However, the classification and the description of soil samples according to the unified soil classification system remain constant as poorly graded soil.

For the quantification of particle beeakage, we adopt the definition by Lade et al. (1996). The particle breakage factor's evolution as a function of compression stress is shown in Figure 5. It is obvious from this figure that there is an ultimate value for fragmentation of sand particles upon increasing the compression pressure on sand particles. Moreover, if the amount of fines' content (FC) is drastically increased, the soil strength may completely be governed by the contacts among the fines' (Thevanayagam, 1998; Thevanayagam and Mohan, 2000; Wang et al., 2007). In this case, the concept of intergranular void ratio, e_s, may be used to view the effect of crushing results. e_s is defined as (Wang et al., 2007):

$$e_s = \frac{e + FC/100}{e - FC/100} \qquad (10)$$

where e = global void ratio of the soil sample and

FC = percentage of the fines passing through number 200 (0.074 mm) sieve.

Figure 6 shows the effect of compression on e_s. It is clearly shown that the compression increases the intergranular void ratio exponentially.

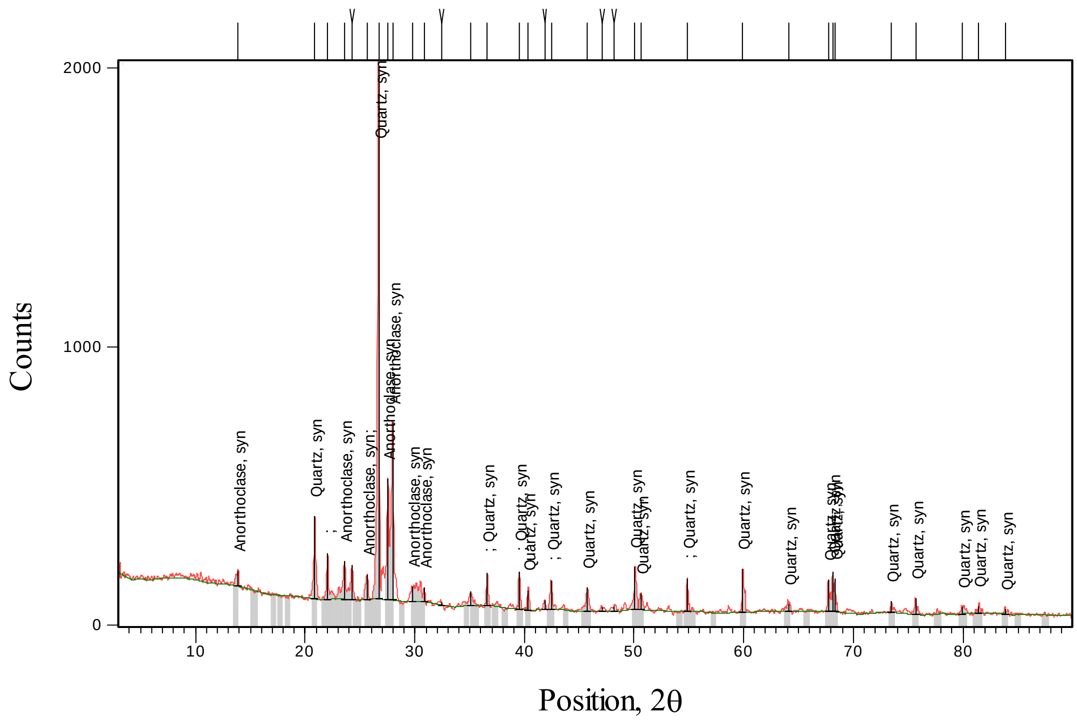

Figure 3: X-ray diffraction patterns for AS2 sand samples

Shear Stress Response to Particle Breakage

The response of stress-strain curve at particle breakage factor B_{10} = 0.582 for different bedding planes under a normal stress of σ_n = 165kPa is shown in Figure 7, whilst Figure 8 shows the volumetric change. It is clearly shown from these figures that in case of the bedding plane approach isotropic situation ($\alpha = 18^{\circ}\ 26'\ 6''$) and parallel to the direction of shearing, the sand has the larger peak strength and the less volumetric dilation. Hence, the bedding plane being parallel to the horizontal plane implies simply more contact normals oriented horizontally so that the specimen appears to be strong in this direction. Therefore, there is little potential for volume changes to occur. On the other hand, if the bedding plane ($\alpha = 30^{o}$), most contact normals are vertical and then the material appears to be overly weak.

The response of stress-strain curve at different particle breakage factors for bedding plane $\alpha = 0$ under normal stress of σ_n = 165kPa is shown in Figure 9, whilst Figure 10 shows the volumetric change. It is clearly is from these figures that in case of particle breakage factor (B_{10} = 0.6485) and parallel to the direction of shearing, the sand has the larger peak strength and the less volumetric dilation. Hence, the bedding plane being parallel to the horizontal plane implies simply more contact normals oriented horizontally so that the specimen appears to be strong in this direction. Therefore, there is little potential for volume changes to occur.

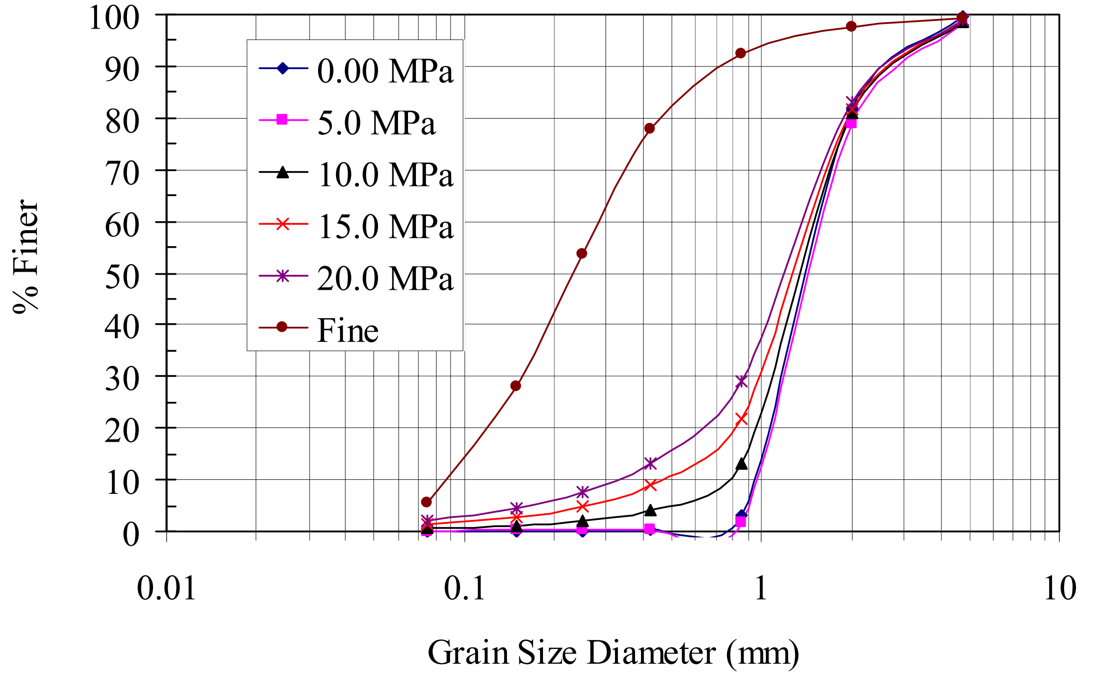

Figure 4: Grain size distribution before and after subjection to compressive loads

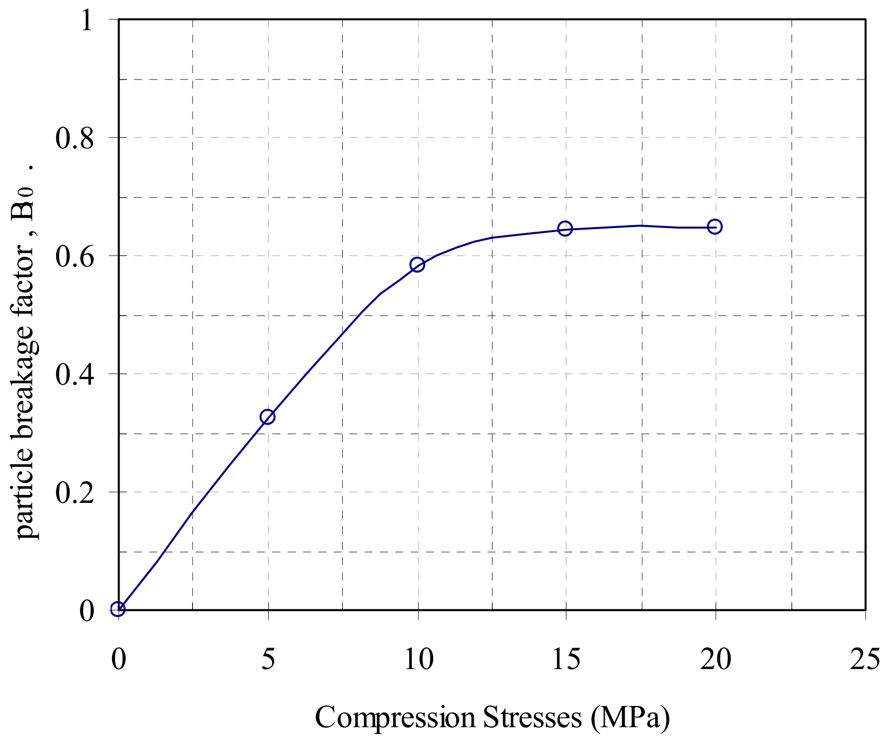

Figure 5: Progress of particle breakage due to compressive stress

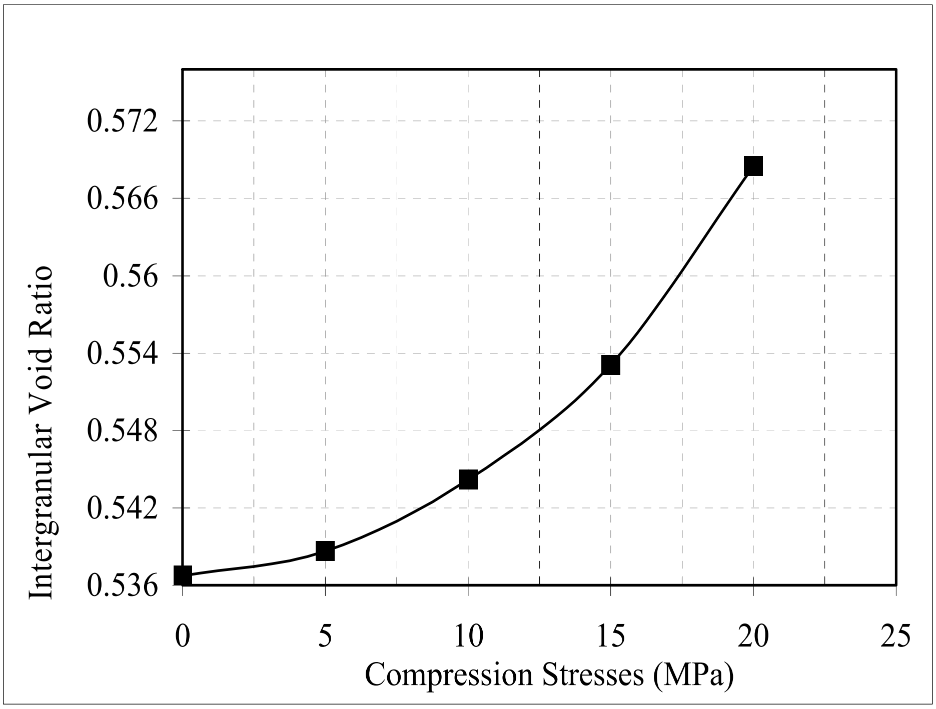

Figure 6: Effect of compressive stress on e_s

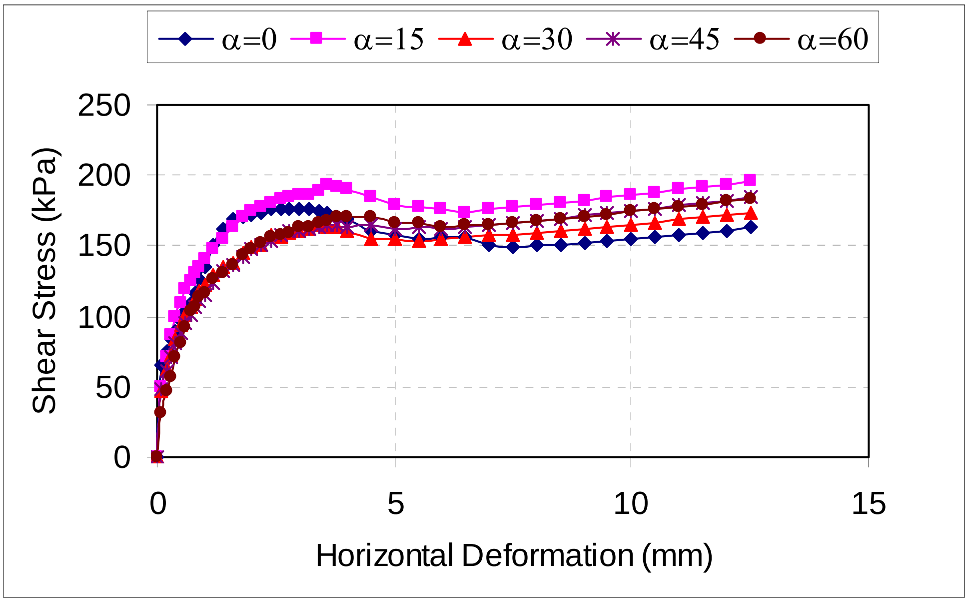

Figure 7: Effect of bedding plane (inherent anisotropy) on shear stress displacement curve [B_{10}=0.58; σ_n=165 kPa]

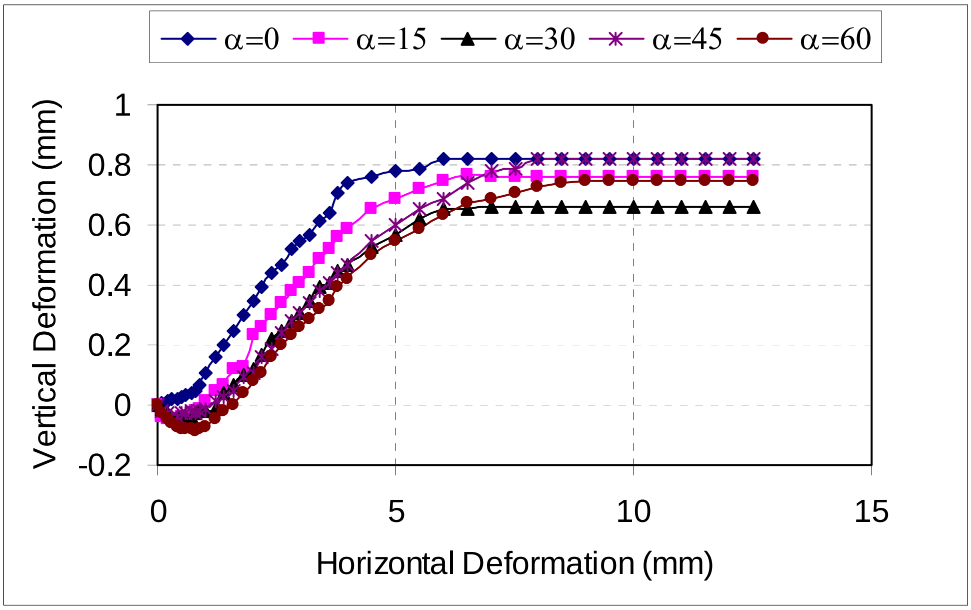

Figure 8: Effect of bedding plane (inherent anisotropy) on volumetric deformation displacement curve [B_{10}=0.58; σ_n=165 kPa]

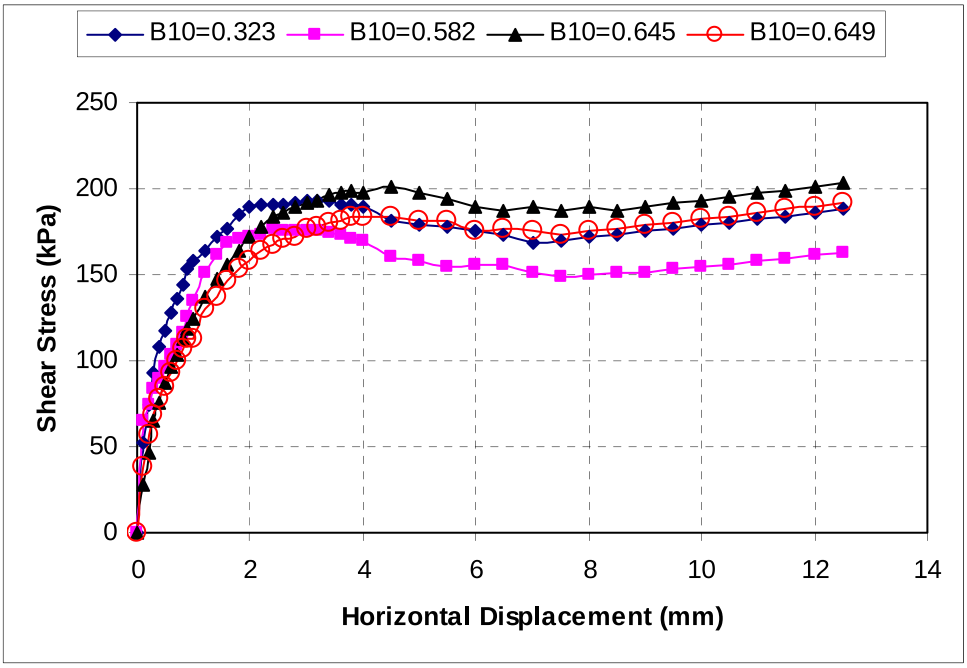

Figure 9: Effect of inherent anisotropy and particle breakage on shear stress displacement curve [σ_n=165 kPa]

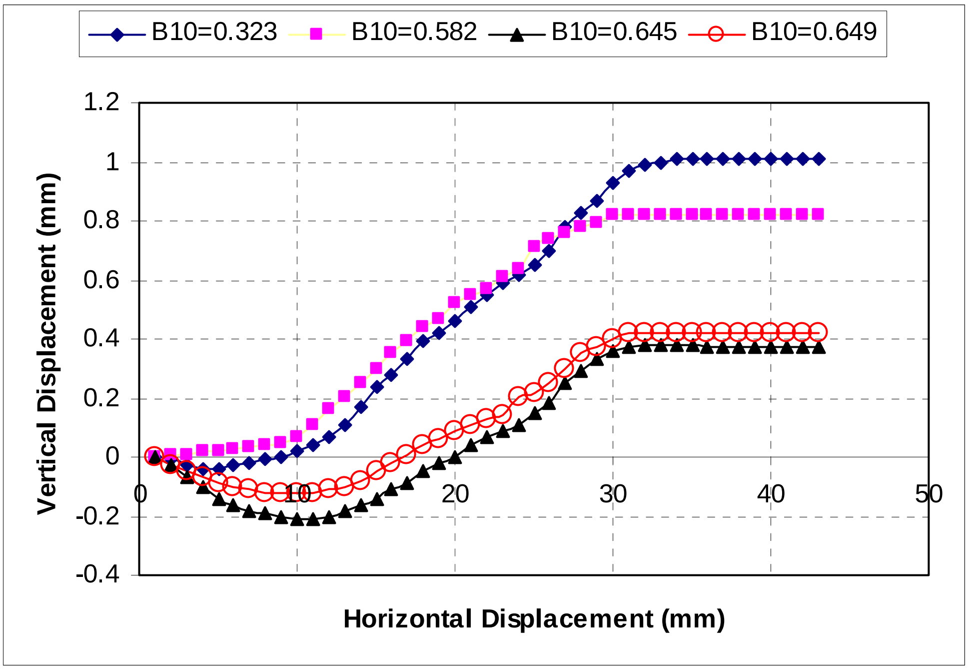

Figure 10: Effect of inherent anisotropy and particle breakage on volumetric deformation displacement curve [σ_n=165 kPa]

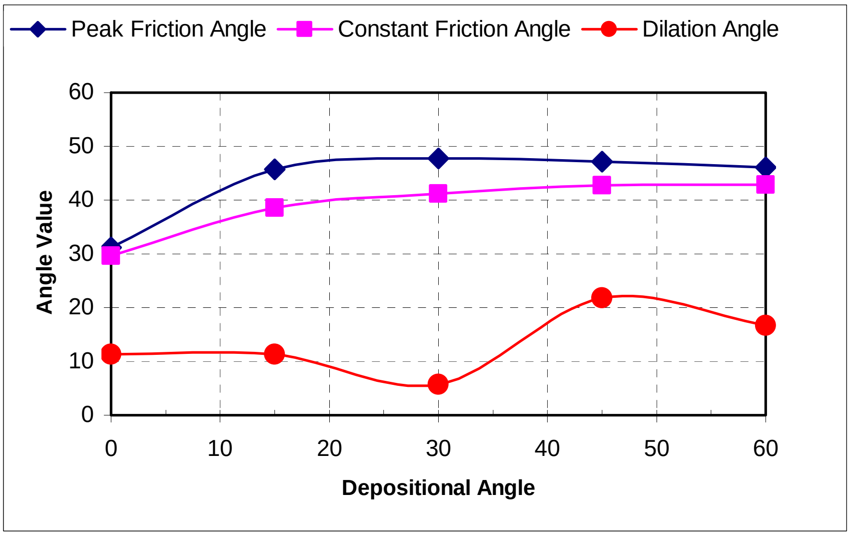

Figure 11: Shear resistance components for AS1

Contribution to Shear Resistance of Granular Materials

Figure 11 shows the evolution of the different components of shear resistance for AS1 samples with increasing the bedding plane. The lower value for the dilation component is at bedding plane α=30^{o}. On the contrary to the dilation angle, peak friction is at a higher value compared with other bedding planes. This is due to the fact that when the bedding plane is near to 20° particles are in statistically isotropic orientation. Therefore, the interlocking between the particles will be higher and this will lead to a higher peak frictional angle. In addition to the effect of inherent anisotropy, bedding plane has a profound effect on peak friction, dilation and residual angle.

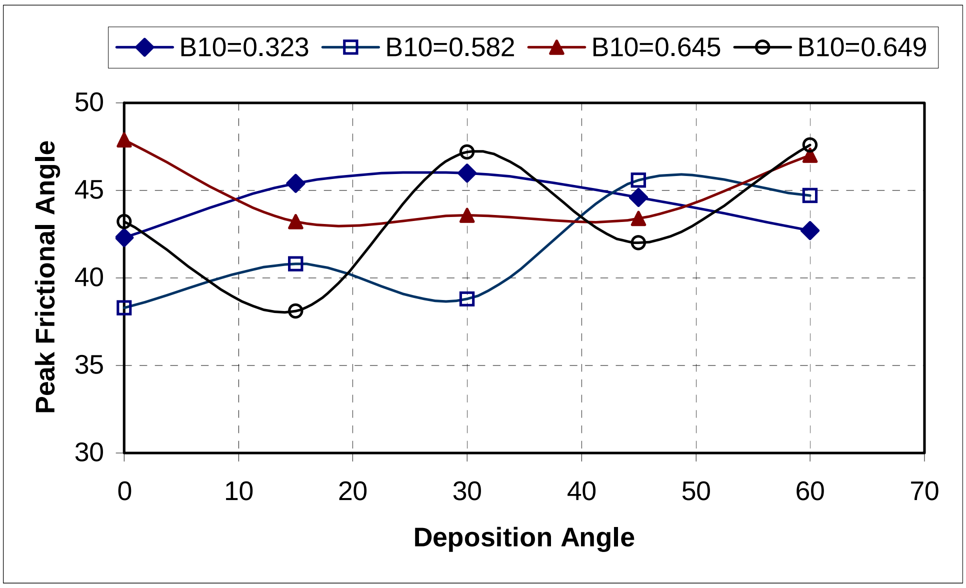

Figure 12: Effect of inherent anisotropy and particle breakage on peak friction angle for AS2

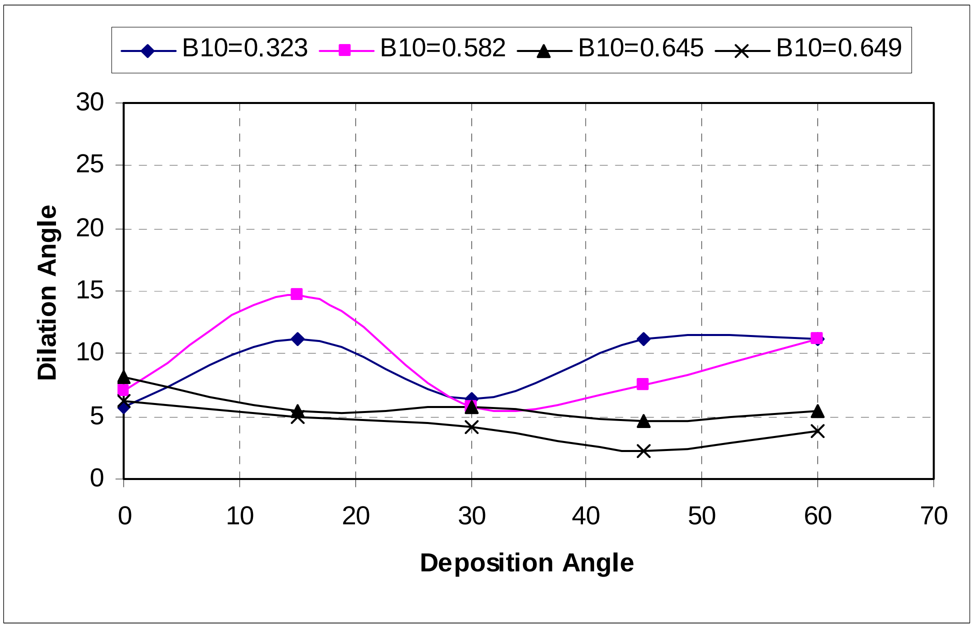

Figure 13: Effect of inherent anisotropy and particle breakage on dilation angle for AS2

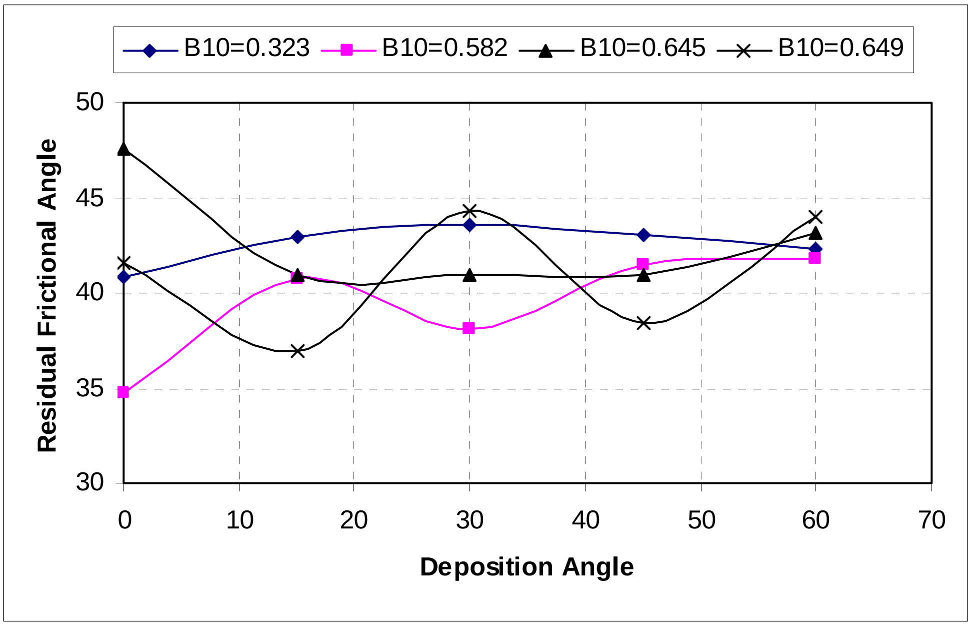

Figure 14: Effect of inherent anisotropy and particle breakage on residual frictional angle for AS2

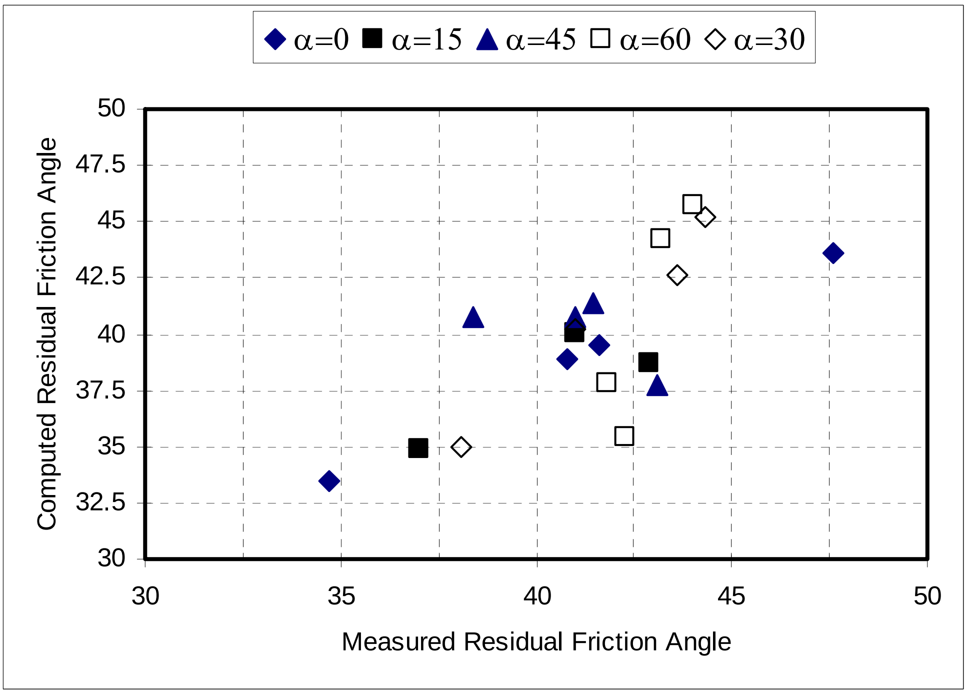

Figure 15: Effect of inherent anisotropy on residual frictional angle regardless the amount of particle breakage for AS2

The effects of bedding plane and particle breakage on the peak friction, dilation and residual angle for AS2 are shown in Figures 12, 13 and 14. All the samples, regardless the amount of particle breakage, have the same value of dilation angle when the depositional angle $\alpha=30^o$. Moreover, both peak and residual angles approach the asymptotic values with increasing the bedding plane angle. Furthermore, the particle breakage showed a higher effect on the residual angle compared with other components of the shear resistance. This is due to the fact that when more breakage occurs, more abrasion to the particles occurs, leading to less asperities of the sand particles, less interlocking and more particle arrangement and rotation.

Table 1. Progression of Cu, Cc and the classification of the soil samples

	D_{10}	D_{30}	D_{60}	Cu	Cc	USCS	Description
fine	0.087	0.15	0.29	3.33	0.89	SP	Fine Sand
20	0.33	0.85	1.4	4.24	1.56	SP	Medium Fine Sand
15	0.49	0.98	1.45	2.96	1.35	SP	Medium Fine Sand
10	0.79	1.2	1.5	1.90	1.22	SP	Medium Fine Sand
5	0.93	1.3	1.55	1.67	1.17	SP	Medium Fine Sand
0	0.94	1.35	1.6	1.70	1.21	SP	Medium Fine Sand

Lastly, the measured residual frictional angle was drawn against the value obtained from equation 9. Figure 15 depicts the quality of measured residual frictional angle taking into consideration the effect of inherent anisotropy regardless the amount of particle breakage for AS2.

CONCLUSIONS

Particle breakage and inherent anisotropy for natural Aqaba subgrade sand were experimentally investigated. The natural sand specimens were subjected to one-dimensional compression to induce breakage. The grain size distributions of the specimens were reported before and after the application of the stresses. Thereafter, the sand shear strength parameters were assessed using direct simple shear. Inspection of the residual shear strength parameters; i.e. the residual angle of friction, showed an increase as the amount of particle crushing increases regardless the level of normal stress being applied. Moreover, examining the effect of angle of deposition and inherent anisotropy showed that a considerable amount of dilation occurs at a higher deposition angle regardless the extent of breakage reported.

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