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Optimum Location and Angle of Inclination of Cut-off to Control Exit Gradient and Uplift Pressure Head under Hydraulic Structures

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ABSTRACT

The work reported in this research presents numerical investigations on the effect of cut-off inclination angle on exit gradient and uplift pressure head under hydraulic structure and determines the optimum location and angle of inclination of cut-off. This problem is solved using the finite element method by using (ANSYS 11.0). It is concluded that using downstream cut-off inclined towards the downstream side with ϴ less than 120º is beneficial in increasing the safety factor against the piping phenomenon. The results are evaluated graphically in non-dimensional form.

KEYWORDS: Inclined cut-off, Optimum location, Hydraulic structures.

IINTRODUCTION

The foundation of any structure should be given greatest importance in analysis and design as compared with other parts of the structure, because failure in the foundation would destroy the whole structure.

Hydraulic structures such as dams, barrages, regulators, weirs, …etc. may either be founded on an impervious solid rock foundation or on a pervious foundation. Whenever such a structure is founded on a pervious foundation, the differential head formed by the structure acts on the foundation and generates seepage. The seepage flow exerts pressure on the structure and generates erosive forces which tend to pull soil particles with the flow. This causes the formation of irregular passages like pipes which move beneath the structure. This process is known as the piping phenomenon (Khassaf, 1998).

To design a safe hydraulic structure against seepage, the following two important points must be considered.

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(a) Safety against Uplift Pressure

The water seeping below the hydraulic structure exerts an uplift pressure on the floor. The uplift pressure is maximum at the point just downstream of the hydraulic structure, when water is full up on the upstream side and there is no water on the downstream side. If the thickness of floor is insufficient, its weight would be inadequate to resist the uplift pressure. This may ultimately lead to bursting of the floor, and thus failure of the hydraulic structure may occur.

(b) Safety against Piping

Exit gradient is usually considered as a measure of the effect of the piping phenomenon. Piping occurs if the exit hydraulic gradient at the downstream point approaches the critical hydraulic gradient. The exit gradient is said to be critical when the upward disturbing force on the grain is just equal to the submerged weight of the grain at the exit. Terzaghi defined I_{cr} as $I_{cr} = \frac{\gamma_{sub}}{\gamma_w}$ (Al-Senousi and Mohamed, 2008).

LITERATURE REVIEW

Limited literature is available for seepage through pervious medium beneath hydraulic structures with inclined cut-offs as a control device. Abbas (1994) used conformal transformation and gave a solution for seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and isotropic soil of infinite depth. Mohamed and Agiralioglu (2005) used a two-dimensional finite difference model to analyze steady state seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and anisotropic soil. Al-Senousi and Mohamed (2008) prepared a model to compute the piezometric head distribution under a hydraulic structure with inlclined cut-off for different flow conditions and soil characteristics.

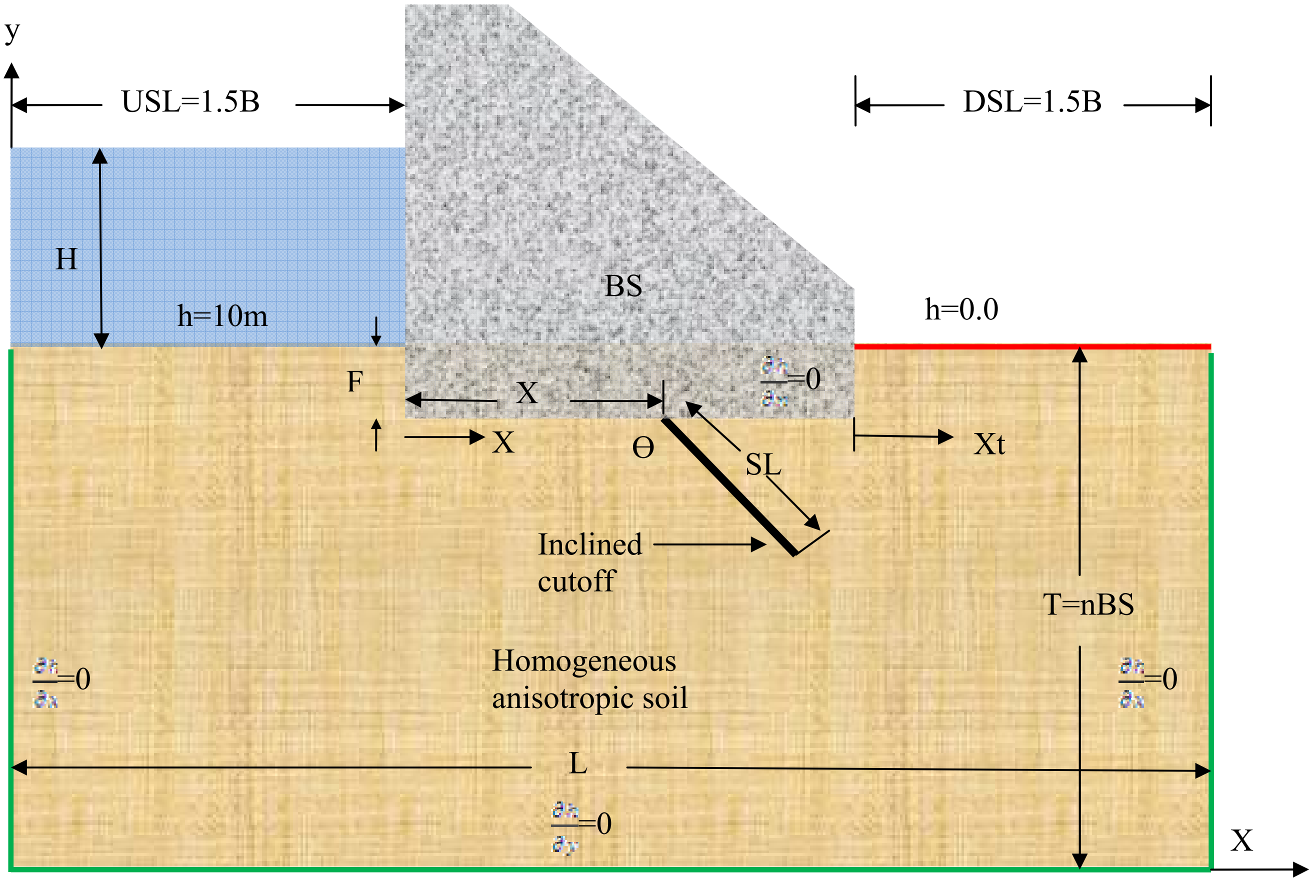

Figure 1: General case study and boundary conditions

Objectives of the Study

The main objectives of this work can be summarized by the following points:

1. Studying the effect of inclined cut-offs at different angles of inclination and different locations along the floor of a hydraulic structure on exit gradient and uplift pressure head.
2. Finding the optimum location and inclination angle of the cut-off.

General Case Study

Finite element method was used to analyze the general case study shown in Fig.(1) using (ANSYS V.11.0) program.

Governing Differential Equation

The groundwater flow equation for the three-dimensional case can be expressed as:

$$\frac{\partial}{\partial x}\left[K_{\chi}\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_{y}\frac{\partial h}{\partial y}\right]+\left[\frac{\partial}{\partial z}\right]+\left[K_{z}\frac{\partial h}{\partial z}\right]+Q+C\frac{\partial h}{\partial t}=0 \quad (1)$$

where:

K_x , K_y and K_z = hydraulic conductivity in x, y and z directions, respectively.

Q = specified inflow or outflow.

h = piezometric head.

The x and z axes are in mutually perpendicular horizontal directions and the y axis is in the vertical direction.

Equation (1) is derived with the assumptions:

(a) Darcy's law is valid throughout the seepage domain.

(b) The soil is saturated.

(c) Both soil and water are incompressible.

With these assumptions of 2D steady state flow and setting Q equal to zero, equation (1) transforms to the form (Manna et al., 2003):

$$\frac{\partial}{\partial x}\left[K_{\chi}\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_{y}\frac{\partial h}{\partial y}\right]=0 \quad (2)$$

Boundary Conditions

The boundary conditions should be specified before starting the solution. For the steady state of a confined flow, the boundary conditions are defined as follows:

*** Reservoir Boundaries**

The height of the water above these boundaries has always a known value, so that the pressure on any point of these boundaries is also known; so, the pressure (p) at any point on these boundaries would be:

$$p = y_w H_o. \quad (3)$$

Therefore, the piezometric head distribution along the reservoir boundaries is constant; that is:

$$h = h_o = \frac{p}{\gamma_w} + z\ . \quad (4)$$

For this reason, all the reservoir boundaries are equipotential lines.

*** Impervious Boundary**

Impervious boundary has the perpendicular velocity function on the surface equal to zero ($\frac{\partial h}{\partial n} = 0$), so that the water cannot seep in that surface.

The symbol n represents the direction of the perpendicular line on the surface, and this condition can be expressed mathematically as follows:

$$\left(v_n = k_x\left(\frac{\partial h}{\partial x}\right)I_x + k_y\left(\frac{\partial h}{\partial y}\right)I_y = 0\right) \quad (5)$$

where (I_x and I_y) represent the cosine of the direction of the perpendicular velocity function on the surface with the directions (x and y), respectively, and the boundaries from this type represent a stream line with an affixed value for the stream function.

Finite-Elements Formulations of Seepage in Porous Medium

The basic idea of the finite element method is to discrete the problem domain to sub-domains or finite elements. These elements may

be one-, two- or three-dimensional and jointed to each other by nodes existing on element boundaries. The nodes are regarded as part of the element. After the discretization process, the behavior of the field variable on each element is represented approximately by a continuos function depending on nodal values of the field variable as follows:

$$H^e = \sum_{i=1}^{n} N_i H_i \quad (6)$$

where:

H^e = Approximate solution for piezometric head distribution in the element (e).

N = Shape function of the element (e).

H_i = Nodal values of head of the element (e).

n = Number of nodes in the element (e).

It is possible to write Equation (6) in matrix form as follows (Al-Senousi and Mohamed, 2008):

$$H^e = [N_i]\{H_i\} \quad (7)$$

where:

$[N_i]$ = Shape function matrix of element (e).

$\{H_i\}$ = Vector matrix of nodal values.

The approximate solution for head variation, H, over the whole domain is given as follows:

$$H = \sum_{e=1}^{n_e} H^e = \sum_{e=1}^{n_e}\sum_{i}^{n} N_i\ H_i \quad (8)$$

or

$$H = \sum_{e=1}^{n_e} [N_i]\{H_i\} \quad (9)$$

where (n_e) is the total number of elements in the problem domain.

The Galerkin Principle

The Galerkin principle is applied to derive the elements matrix. From equation (6):

$$H^e = \sum_{i=1}^{n} N_i\, H_i$$

where (H_i) is the value of the piezometric head in node (i).

For a two-dimensional flow, the general equation for seepage in porous media is:

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial H}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial H}{\partial y}\right] = 0 \quad (10)$$

Substituting Eq.(6) in Eq.(10) gives:

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial}{\partial x}\sum_{i=1}^{n} N_i\ H_i\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial}{\partial y}\sum_{i=1}^{n} N_i\ H_i\right] = R^e \neq 0 \quad (11)$$

Now, applying Galerkin principle and substituting Eq.(11) in Eq.(9) yield:

$$\sum_{1}^{ne}\left[\int_{A^e} N_j^e\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x}\sum_{i=1}^{n} N_i H_i\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y}\sum_{i=1}^{n} N_i H_i\right)\right] dA\right] = 0 \quad (12)$$

where:

$dA = dx\,.dy;$ (j=1, 2, …, n)

n = Number of nodes for each element.

To reduce continuity requirements for the shape function, (N), from (C^1-continuity) to (C^o-continuity), integration by parts with Green's theorem is applied to the second order derivatives terms, where (C^1) and (C^o) are the continuity for the shape function for the first and zero stage, respectively (Senda, 2003).

Accordingly, the first term of Eq.(12) will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x}\sum_{1}^{n} N_i H_i\right) dA = \int_S N_j^e k_x \frac{\partial}{\partial x}\sum_{1}^{n} N_i H_i\, dy - \int_{A^e} \frac{\partial N_j^e}{\partial x} k_x \frac{\partial}{\partial x}\sum_{1}^{n} N_i\ H_i\ dA \quad (13)$$

The second term of Eq. (12) will be:

$$\int_{A^e} N_j^e \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y}\sum_{1}^{n} N_i H_i\right) dA = \int_S N_j^e k_y \frac{\partial}{\partial y}\sum_{1}^{n} N_i H_i\, dx - \int_{A^e} \frac{\partial N_j^e}{\partial y} k_y \frac{\partial}{\partial y}\sum_{1}^{n} N_i\ H_i\ dA \quad (14)$$

Substitution results in:

$$\sum_{1}^{n_e}\left[\int_{A^e} -\left(\frac{\partial N_j}{\partial x} k_x \frac{\partial}{\partial x}\sum_{1}^{n} N_i H_i + \frac{\partial N_j}{\partial y} k_y \frac{\partial}{\partial y}\sum_{1}^{n} N_i H_i\right) dA\right] + \int_S N_j k_n \frac{\partial}{\partial n}\sum_{1}^{n} N_i\ H_i\ ds = 0 \quad (15)$$

where ($S = S_1^e + S_2^e$) represents the surface boundaries of the element.

The boundary conditions are:

1: ($H=H_o$) (16)

on (S_1), which represents the reservoir boundaries; and

2: $$\left[k_x \frac{\partial H}{\partial x} L_x + k_y \frac{\partial H}{\partial y} L_y \right] = 0 \quad (17)$$

on (S_2), which represents the impermeable boundaries.

By applying the finite element method to Eq. (17), it becomes:

$$k_x \frac{\partial}{\partial x} \sum_{1}^{n} N_i\, H_i\, L_x + k_y \frac{\partial}{\partial y} \sum_{1}^{n} N_i\, H_i\, L_y = R^e = 0 \quad (18)$$

where (R^e) is the element boundary residual.

Using the Galerkin weighted residual method, Eq.(18) becomes:

$$\sum_{1}^{n_e} \left[\int_{S_2^e} \left(N_j\, k_x \frac{\partial}{\partial x} \sum_{1}^{n} N_i\, H_i\, L_x + N_j\, k_y \frac{\partial}{\partial y} \sum_{1}^{n} N_i\, H_i\, L_y \right) dS \right] = 0$$

(19)

where: $dx = L_x\, ds$ and $dy = L_y\, ds$.

Multiplying Eq.(15) by (-1) and adding it to Eq.(19) give:

$$\sum_{1}^{n_e} \left[\int_{A^e} \left(\frac{\partial N_j}{\partial x} k_x \frac{\partial}{\partial x} \sum_{1}^{n} N_i\, H_i + \frac{\partial N_j}{\partial y} k_y \frac{\partial}{\partial y} \sum_{1}^{n} N_i\, H_i \right) dx\, dy - \right.$$

$$\left. \int_{S_1^e} \left(N_j k_x \frac{\partial}{\partial x} \sum_{1}^{n} N_i\, H_i\, L_x + N_j\, k_y \frac{\partial}{\partial y} \sum_{1}^{n} N_i\, H_i\, L_y \right) ds \right] = 0$$

(20)

and in matrix form:

$$\sum_{1}^{n_e} \left[K^e \right] \{ H_i \} = 0 \quad (21)$$

where [K^e] represents the element matrix:

$$\left[K^e \right] = \int_{A^e} \left[B^e \right]^T \left[D^e \right] \left[B^e \right] dx\, dy = 0 \quad (22)$$

where:

$$\left[B^e \right] = \begin{bmatrix} \frac{\partial N_1^e}{\partial x} & \frac{\partial N_2^e}{\partial x} & \cdots & \frac{\partial N_n^e}{\partial x} \\ \frac{\partial N_1^e}{\partial y} & \frac{\partial N_2^e}{\partial y} & \cdots & \frac{\partial N_n^e}{\partial y} \end{bmatrix}$$

$$\left[D^e \right] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

and from assemblage:

$[K]\ \{H\} = 0$ (23)

where, [K] is the global matrix = $\sum \left[K^e \right]$

The assembled equation, Eq. (21), is solved using a frontal solution because of its efficiency in computer storage requirements. The main idea of the frontal solution is to assemble the equations and eliminate the variables at the same time (Al-Musawi, 2002).

ANALYSIS AND RESULTS

Hydraulic Structure with Inclined Downstream Cut-off

The best mesh of ANSYS program for exit gradient distribution along the downstream side of a hydraulic structure on an isotropic soil foundation is shown in Fig.(2).

The mesh of finite elements used in this analysis is shown in Fig.(3). Eight-node quadratic elements are used to describe the domain. The mesh contains 904 elements and 2280 nodes.

As shown in Fig. (4),when the cut-off is at the toe, high values for exit gradient are developed if the cut-off is inclined towards the upstream side (Θ is less than 90°), and the uplift head is greater than that of vertical cut-off as shown in Figures (6) and (7).

On the other hand, the exit gradient decreases as Θ increases towards the downstream (Θ≥90) as shown in Fig. (5) for a distance Xt/SL≈0.9 beyond the toe, then exit gradient and velocity start increasing slightly with increasing Θ. It is also clear that the maximum exit gradient decreases for Θ=120 and starts increasing for Θ=135. From Fig. (7), the uplift head decreases as Θ increases.

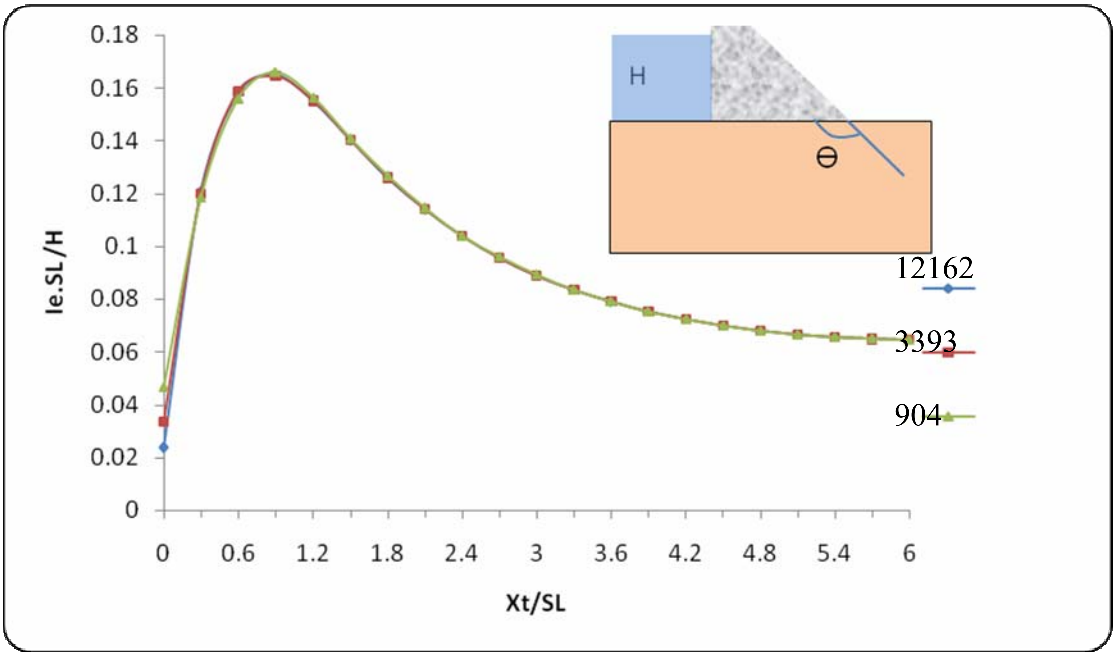

Figure 2: The best mesh of ANSYS program for exit gradient along the downstream side of a Hydraulic structure on isotropic soil

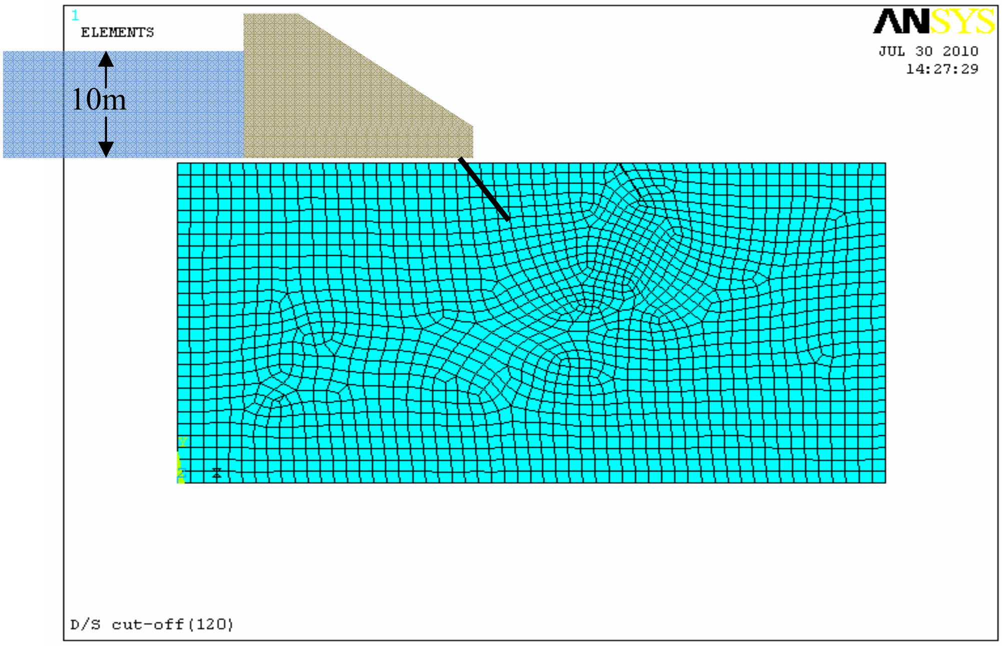

Figure 3: Finite element mesh

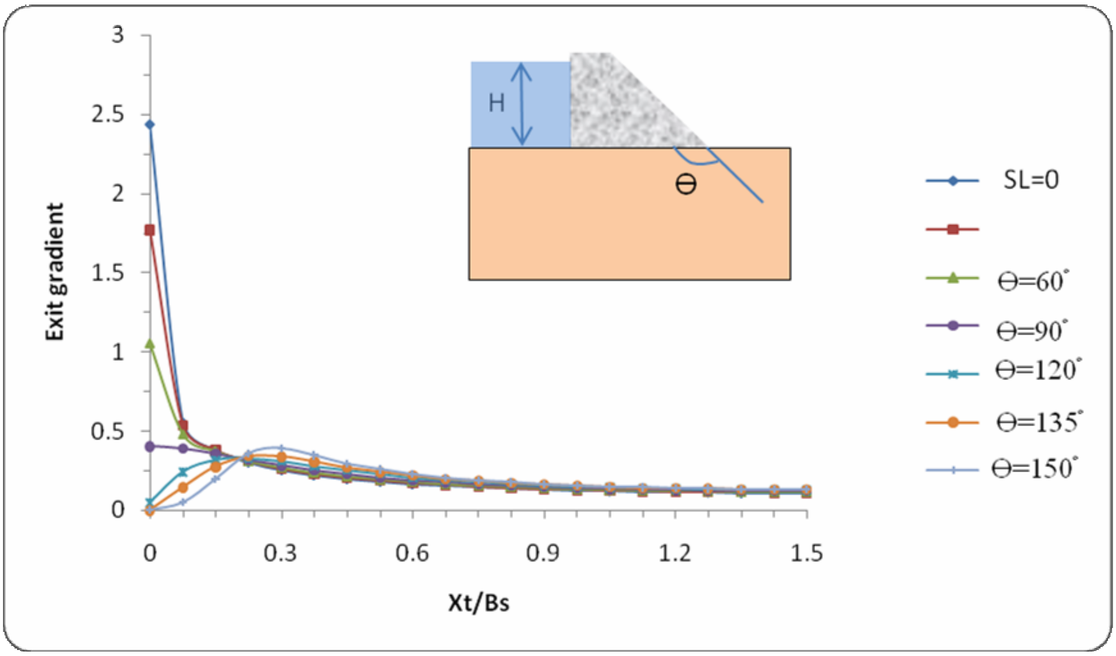

Figure 4: Variation of exit gradient for a hydraulic structure with (D/S) cut-off for different values of Θ

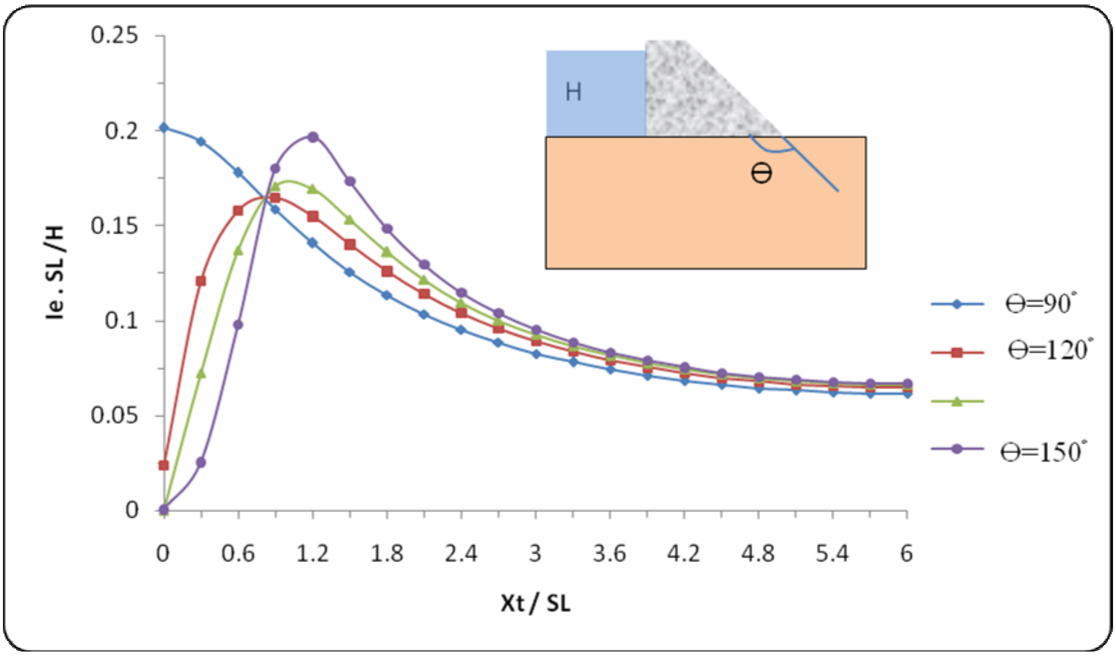

Figure 5: Variation of exit gradient for a hydraulic structure with (D/S) cut-off for different values of Θ (Θ ≥ 90)

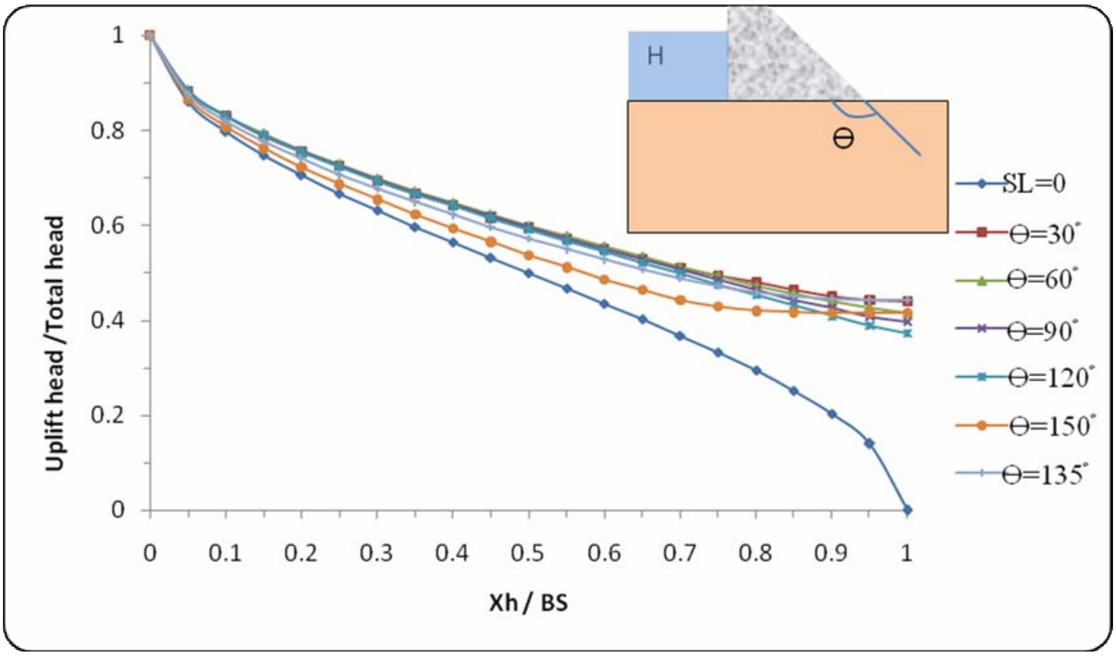

Figure 6: Variation of uplift head under a hydraulic structure with (D/S) cut-off for different values of Θ

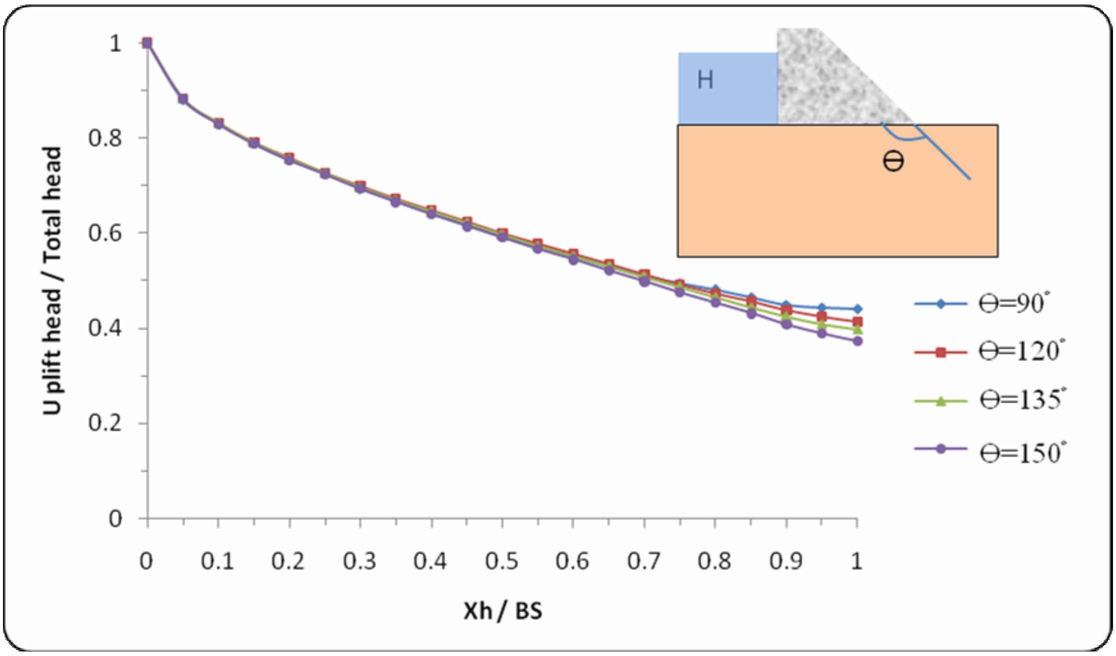

Figure 7: Variation of uplift head under a hydraulic structure with (D/S) cut-off for different values of Θ (Θ ≥ 90)

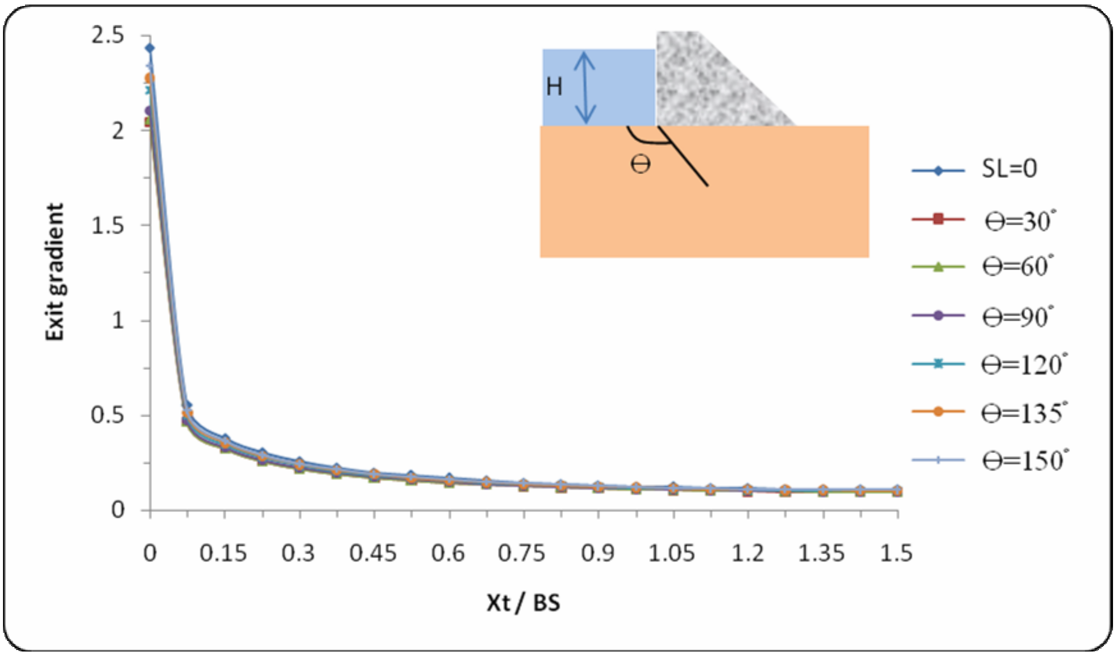

Figure 8: Variation of exit gradient along a downstream hydraulic structure with inclined (U/S) cut-off for different values of Θ

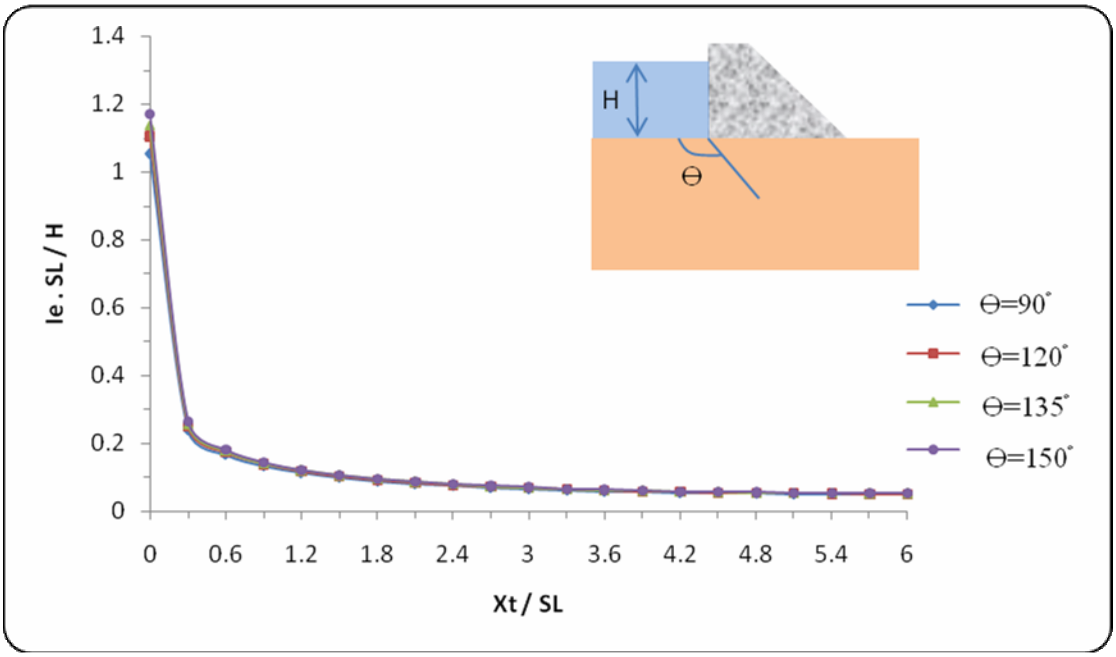

Figure 9: Variation of exit gradient along a downstream hydraulic structure with inclined (U/S) cut-off for different values of Θ (Θ ≥ 90)

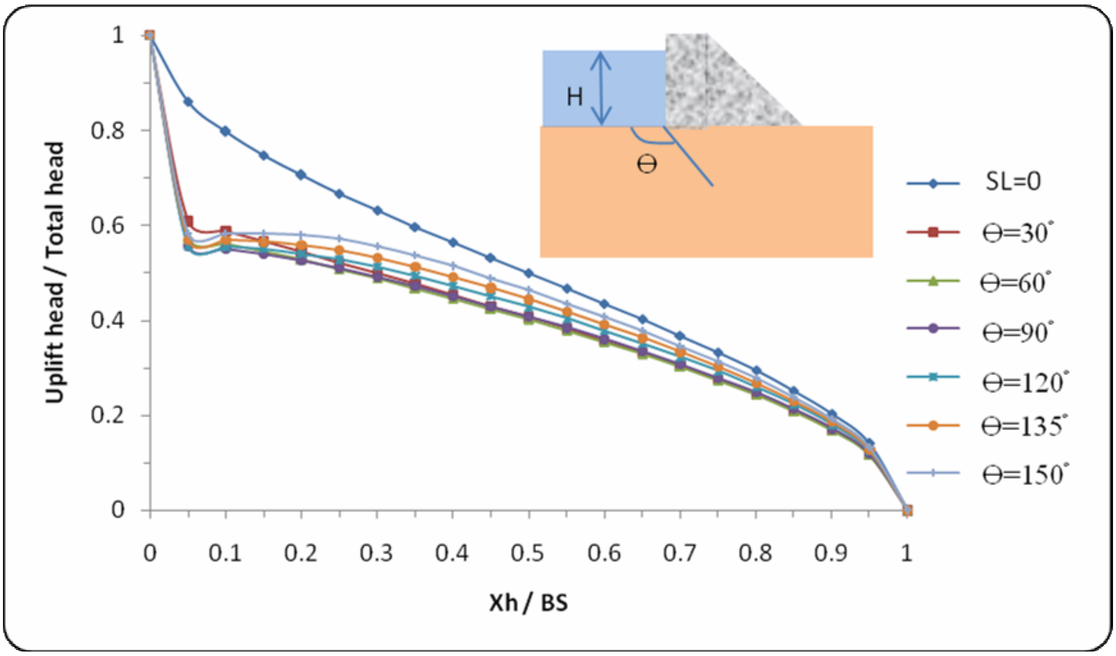

Figure 10: Variation of uplift head under a hydraulic structure with (U/S) cut-off for different values of Θ

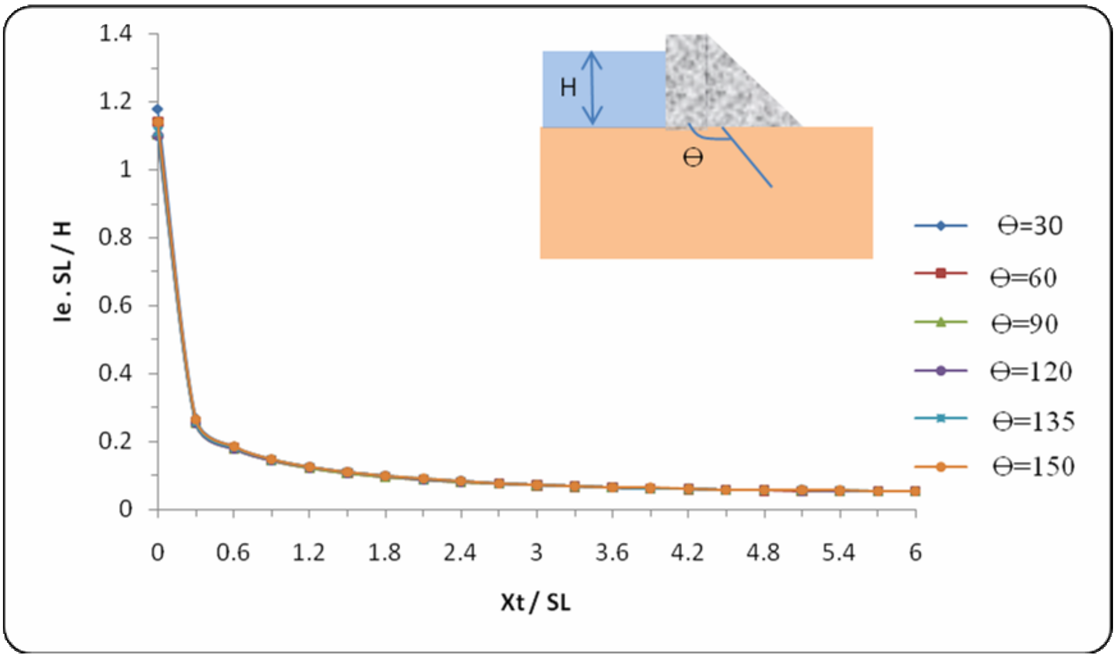

Figure 11: Variation of exit gradient along the downstream side of a hydraulic structure with mid cut-off for different values of Θ

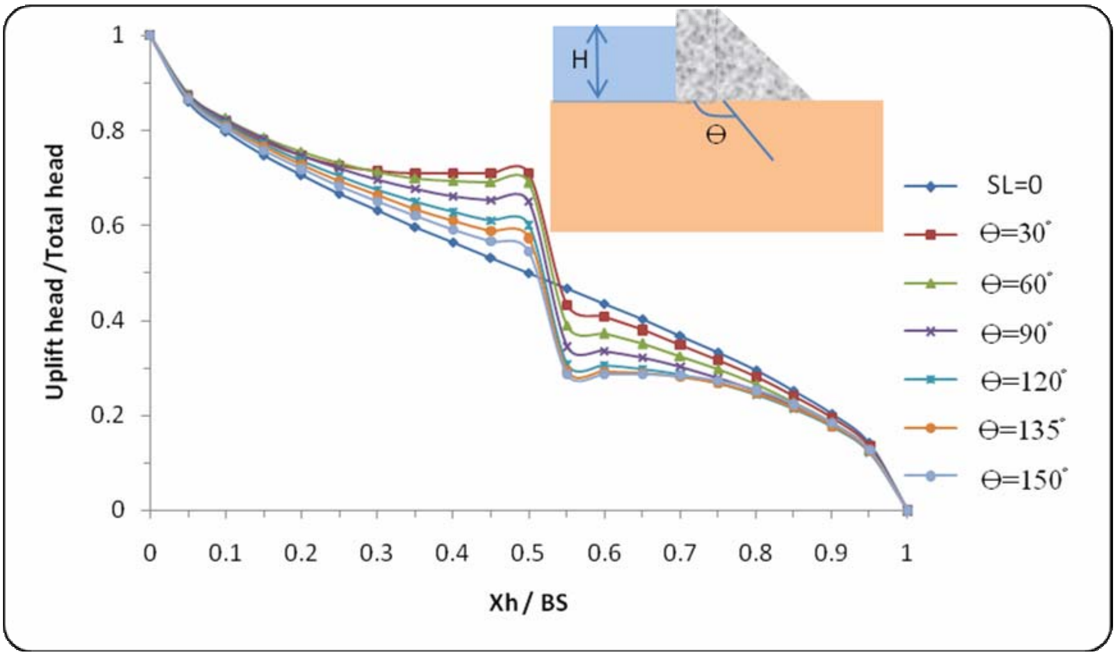

Figure 12: Variation of uplift head under a hydraulic structure with mid cut-off for different values of Θ

From these figures, it is concluded that using downstream cut-off inclined towards the downstream side with Θ less than 120° is beneficial in increasing the safety factor against the piping phenomenon. The exit gradient before a point where (Xt/SL ≈ 0.9) downstream from the hydraulic structure toe decreases with increasing cut-off inclination and this behavior is reversed beyond the point where (Xt/SL ≈ 0.9) so that the danger of undermining will be shifted further downstream from the toe of the hydraulic structure.

The effect of downstream cut-off inclination angle in reducing uplift pressure head under hydraulic structure is very small compared to its effect on exit gradient.

Hydraulic Structure with Upstream Cut-off

Figures (8) and (9) illustrate the effect of cut-off inclination angle on exit gradient distribution along the downstream side of a hydraulic structure with upstream inclined cut-off. It can be seen that high values for exit gradient and velocity are developed if the cut-off is inclined towards the upstream side (Θ is less than 90°) or towards the downstream side (Θ is more than 90°). These figures also show that the upstream cut-off inclination angle has no effect on exit gradient and exit velocity.

The effect of cut-off inclination angle on uplift pressure head is represented in Figure (10). It can be seen that it has a very small effect; so it can be neglected.

It can be concluded that placing an inclined cut-off at the hydraulic structure heel is not recommended under any angle of inclination.

Hydraulic Structure with Cut-off at the Mid Distance of the Hydraulic Structure Base

From Fig. (11), when the cut-off is at the mid distance of the hydraulic structure base, high values for exit gradient and velocity are developed when the cut-off is inclined towards the upstream side (Θ is less than 90°) or towards the downstream side (Θ is more than 90°), and the uplift head is greater than that of vertical cut-off and decreases as the inclination angle increases as shown in Fig. (12).

It can be concluded that when the exit gradient along the downstream side of the hydraulic structure is considered the major factor in the design of the hydraulic structure, the optimum location of the cut-off is at the toe of the hydraulic structure with an inclination angle of 120°. Placing an inclined cut-off at the hydraulic structure heel is not recommended under any angle of inclination. If the uplift head is considered the major factor, the optimum location of the cut-off is at the heel of the hydraulic structure with an inclination angle of 90°.

CONCLUSIONS

The following conclusions are drawn from the present study based on the results discussed above:

1. Using downstream cut-off inclined towards the downstream side with Θ less than 120° is beneficial in increasing the safety factor against the piping phenomenon. The exit gradient before a point where (Xt/SL ≈ 0.9) downstream from the hydraulic structure toe decreases with increasing the cut-off inclination, and this behavior is reversed beyond the point where (Xt/SL ≈ 0.9), so that the danger of undermining will be shifted further downstream from the toe of the hydraulic structure.
2. The effect of downstream cut-off inclination angle in reducing uplift pressure head under the hydraulic structure is very small compared to its effect on exit gradient.
3. Placing an inclined cut-off at the hydraulic structure heel is not recommended under any angle of inclination.

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