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On The Buckling of Fiber-Bundle Type Beams

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ABSTRACT

Euler was the first who studied for engineering applications important problem of buckling arising in a simple, monolithic beam loaded axially by a concentrated load. As it has been shown from two solutions of the problem due to Timosenko theory, the elastic foundation increases the critical buckling load of the beam. Starting from the previous classical results, a mechanism to enhance the buckling strength of a cantilevered beam is investigated. In the place of a single, one-element, monolithic cross-section, the use of a bundle of more than one, similar or not, single cross-sections, placed with parallel axes, and staying in free (unilateral) contact along their adjacent boundaries, is proposed.

KEYWORDS: Fiber-bundle type beam, Composite beam, Buckling, Contact mechanics, External prestressing.

1. INTRODUCTION

The so-called Euler buckling load [1] (Tinoshenko,1934) can be calculated from the solution of the differential equation of equilibrium of an axially loaded beam in compression and is equal to:

$$p^{cr} = \frac{\pi^2 \cdot E \cdot J}{S_\kappa^{\ 2}} \tag{1}$$

where S_κ is the free buckling length of the beam.

In the case of a cantilevered beam $S_\kappa=\ell$ (where ℓ is the real length of the beam), Relation (1) holds for beams totally free to deform out of their central axis. In the initial investigation, the only hinges considered were placed at the two ends of the beam, suppressing the degrees of freedom in the horizontal plane (x, y). Later on, the buckling problem was extended to a cantilevered beam laying on an elastic foundation and bilaterally connected to it, so that both the beam and the elastic foundation share the same displacements. As it has been shown from two solutions of the problem due to Timosenko (1934; 1935) the elastic foundation increases the critical buckling load of the beam. Both solutions lead to the same relation:

$$p^{cr} = \frac{\pi^2 \cdot E \cdot J}{\ell^2}\left(n^2 + \frac{c\ell^4}{n^2 \cdot \pi^4 \cdot E \cdot J}\right). \tag{2}$$

Using the following definition:

$$m = \left(n^2 + \frac{c\ell^4}{n^2 \cdot \pi^4 \cdot E \cdot \zeta}\right) \tag{3}$$

relation (2) takes a form that is similar to the Euler

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relation (1), but having an additional factor m.

Factor m, as defined in relation (3), depends on the assumed elastic constant c of the elastic foundation (Winkler constant) as well as on the number of half-waves along the beam at the instant of buckling n. Therefore, the length of half wave is equal to $\frac{\ell}{n}$. As it is seen from equation, (2) for every n there exists a new critical load. One must find in every case the value of n which minimizes the value of p^{cr} in relation (2). For the case c=0, it is obvious that n=1, since in this case the elastic foundation does not exist and the beam, being cantilevered and free to buckle, has a buckling length equal to $S_\kappa=\ell$ and therefore, $p^{cr}=\frac{\pi^2 \cdot E \cdot J}{\ell^2}$ (Euler relation (1)). For a very small value of c with c>0, in relation (2) one has again n=1. As a consequence, for a very soft foundation the beam buckles without any intermediate saddle point. With increasing values of c, after a certain value of p^{cr}, for n=2 the beam admits in its center an inflation point and the length $\frac{\ell}{2}$ is called buckling half-length. The limiting value of c for which the change from state n=1 to state n=2 occurs, can be found from equation (2). The value of p^{cr} will become equal for n=1 and n=2 and therefore:

$$\left(1+\frac{c\ell^4}{\pi^4 \cdot E \cdot J}\right)=\left(4+\frac{c\ell^4}{4\pi^4 \cdot E \cdot J}\right) \tag{4}$$

That leads to:

$$c_{1.2}=\frac{4\pi^4 \cdot E \cdot \zeta}{\ell^4}. \tag{5}$$

If the value of c for the foundation of the beam is $c>c_{1,2}$ then n=2 and the beam will have two half-lengths along its length. As a generalization of relation (5), the transition from state n to state (n+1) is defined by

$$c_{n,\,n+1}=\frac{\pi^4 \cdot E \cdot J}{\ell^4} \cdot n^2 \cdot (n+1)^2. \tag{6}$$

As a consequence, for harder foundations, higher values for the critical load p^{cr} for the buckling of the beam resting on it appear, in comparison with the prediction of the Euler relation (1), which holds for a totally unsupported beam.

The increased critical load is due to the arising, transversal interaction forces between the beam in compression and the supporting soil, which can be assumed to be equal to $q(x)=-c.\upsilon(x)$ and prevent buckling. Here, c measures the elastic properties of the soil, which plays the role of the supporting medium for the beam. In all cases of relations (1) and (2), it is well-known that for $p<p^{cr}$ the equilibrium is stable. Nevertheless, for $p>p^{cr}$ the equilibrium becomes unstable and the deformation of the beam is self-excited up to the point of collapse.

Starting from the previous classical investigations, a mechanism to enhance the buckling strength of a cantilevered beam is investigated. In the place of a single, monolithic cross-section, it is proposed to use a bundle of more than one, similar or not, single cross-sections, placed with parallel axes and staying in free (unilateral) contact along their adjacent boundaries. In fact, a single cross-section is divided into a sum of several single cross-sections, so that the resulting bundle is statically equivalent to the initial cross-section (i.e., it has the same cross-sectional area F and the same moment of inertia J). It should be emphasized that the herein proposed bundle is different from the classical concept of multi-part cross-sections, where the different parts are placed in some distance between them with the help of steel bars, without direct contact, such that the total structural element has a higher moment of inertia, which in turn influences the critical load p^{cr} in the sense of Euler. In the herein proposed design, the bundle is tied by means of prestressed rods in the transversal direction (collars), which guarantee the interaction of the elements and the function of the total bundle as one composite structural element.

Every element of the bundle subjected to a compressive force will find support on the remaining, neighboring elements of the bundle. Therefore, its critical load against buckling will be increased, similarly to the beam supported on an elastic foundation, where the critical load increases proportionally to the stiffness of the foundation (Winkler's constant c). The same effect is repeated for all elements of the bundle. As a final result, the critical load of the whole bundle is higher than that of an equivalent, single-part cross-section (with the same area F and moment of inertia J).

This novel concept, being based on classical techniques, is proposed and studied in the present paper. The results have been compared to numerical and experimental investigations. It should be noted here that the outcome of this paper and the methodology proposed here for the study of the problem at hand, have interesting applications in fiber-reinforced composites and in nanocomposites, as can be seen from the recent investigations on compressive strength and buckling in these areas.

2. FORMULATION OF THE BUNDLE

In the case of an axially compressed beam resting on an elastic foundation, the axial force causes buckling. Without the presence of foundation, the critical load p^{cr} can be calculated by Euler's formula. The critical load is increased in the presence of the foundation, since the latter significantly participates into the deformation of the whole system (beam and foundation).

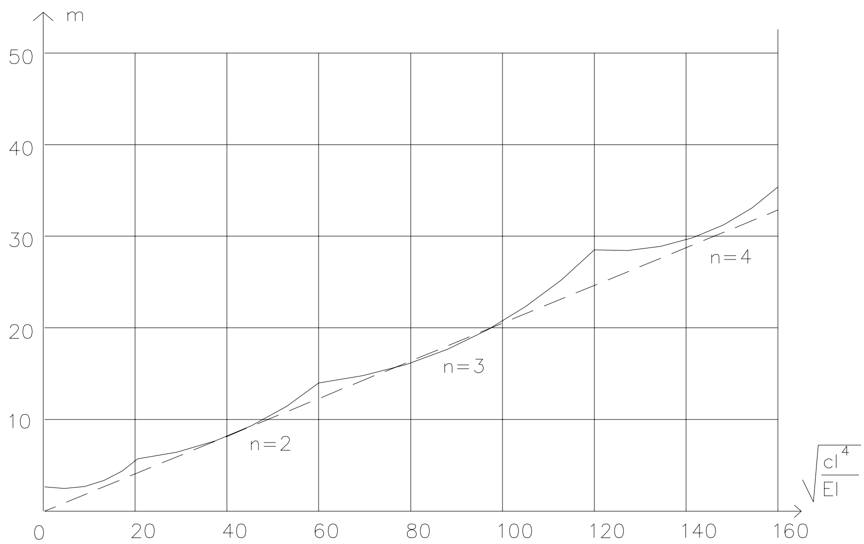

Fig. 1: Euler Buckling Parameter.

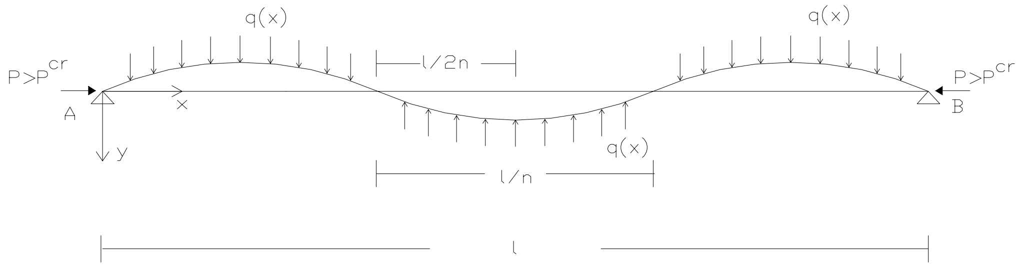

Fig. 2: Buckling of a Beam on Winkler Support.

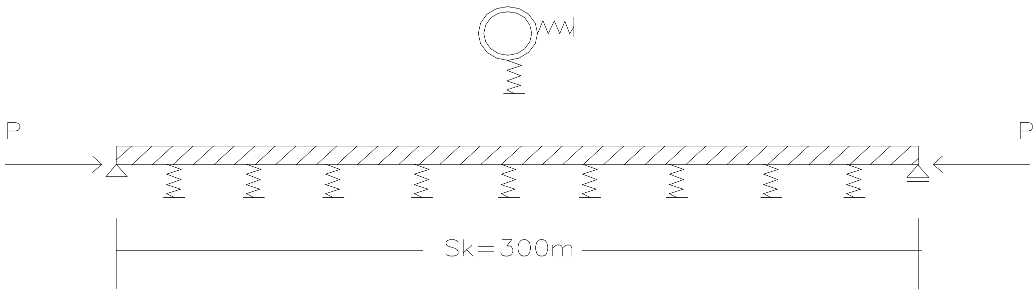

Fig. 3: Model of a Beam on Winkler Foundation (Springs).

The continuous soil (Winkler type foundation) with a given c>0 along the whole length of the compressed beam can be replaced by discrete springs placed in equal, small distances between them (equivalent spring constants concept). All springs have a constant c and are supported on a rigid support.

Using any available structural analysis software, the considered steel hollow beam, with cross-section equal to $F=521cm^2$, moment of inertia $J=915810cm^4$, external diameter D=120cm, internal diameter d=117.2cm and thickness of the wall equal to t=14mm is analyzed. The cantilever beam has a length equal to L=300.0m and the springs have been placed in distances α=25m, all having the same stiffness value

$$c = 2\ \text{t/m} = \frac{2000}{100} = 20\ \text{kg/cm} = 20\ \text{KN/m} = 0.20\ \text{KN/cm}\,.$$

The Euler critical load for the considered cantilevered beam with Sκ=300.0m and without the effect of the springs is $p^{cr} = \frac{\pi^2 \cdot E \cdot J}{5} = 210\text{kN}$. From the analysis of the aforementioned model, it comes out that p^{cr}=2500kN, which is 12 times higher than the previous value.

The same happens in the case of the bundle, as it has been described in the previous section. In this respect, two notions are involved: 1) the isolated element and 2) the whole bundle. The bundle is not merely the static sum of its elements. It has additional strength reserves against buckling in comparison to the critical load p^{cr} of a statically equivalent single element.

Let us clarify this point by considering a two-element bundle. Two cantilever steel beams with a rectangular hollow cross-section of the type 225x225/9 with a cross-sectional area F=75.6cm^2, moment of inertia J=5800cm^4 and buckling length equal to S_κ=50.0m are considered. The central axes of the two beams are parallel, and contact develops along their adjacent sides. Contact is ensured by placing strong collars at every 1/8 of their length, i.e. every a=50/8=6.25m a collar is placed. For each one of the considered beams with S_κ=50.0m, the Euler critical load p^{cr} is equal to:

$$p^{cr} = \frac{\pi^2 \cdot E \cdot J}{S_K^{\ 2}} = \frac{3.14^2 \times 2.1 \times 10^6 \times 5800 \times 10^{-2}}{(5000)^2} = 48\,\mathrm{kN}$$

Let us further consider that only one of the two beams is loaded with a load equal to p^{cr}=48kN. Buckling is initiated. The axis of the beam is deflected and, due to the presence of the binding collars every α=6.25m, the second beam which is unloaded will follow the deformation with the same elastic deformation line. The arising reaction forces will resist the buckling of the first, loaded beam. As a result, its critical load p^{cr} will be increased. The transversal contact forces that arise between the two beams are relatively small. For a larger length of the two beams, the transversal forces required to maintain the same transversal deflection of the second beam at the point of p^{cr}, become lower. The maximum deflection is not larger than 2-2.5% of the total length. Let us consider the limit, such that a small increase of p^{cr} will lead to collapse. Therefore, although the support provided by the second beam to the first one is significant, the burden transferred to the unloaded beam through the contact forces is low. As a consequence, the second beam can undertake compressive load as well without significantly loosing its ability to support the first beam. Each one of the beams undertakes a portion of the total compressive load p of the bundle, while at the same time through the collar; it supports the other beam and causes an increase of the critical load p^{cr} of each one and of the total critical load of the two-member composite bundle. On the other hand, the elastic deformation energy of the first beam, which corresponds to a critical load p^{cr} for two beams, is not sufficient. Therefore, we must have an increase in the critical load p^{cr} due to the presence of the second column that provides the required support through the small, transversal interaction forces arising between the two columns.

The same effect appears in the case of a cantilevered, compressed beam on an elastic foundation that provides the elastic support and increases the critical load p^{cr} of the beam. The difference with the herein presented concept is that the elastic foundation lies outside the considered system, while in the bundled column concept every column is supported elastically in the second one; both of them are members of the same structural system subjected to a compressive loading.

3. COMPUTING THE CRITICAL LOAD OF THE BUNDLE

The problem becomes even more complicated if, instead of the two elements, a multi-element bundle is considered. Let us consider a bundle of nine hollow section elements placed symmetrically as shown in Figure 4, having adjacent sides in all possible buckling directions and a buckling length equal to S_κ=50.0m. The bundle is stabilized by means of collars placed every $\alpha = \frac{50}{15} = 3.33\,\mathrm{m}$ (see example 3), such that the distances between the center lines of the nine columns do not change, and every buckling-induced displacement of the bundle is common for all of its elements. In this case, every one of the nine elements of the bundle undertakes approximately the $\frac{1}{9}$ of the total loading p, that is applied on the bundle and the remaining 8 elements form the elastic support that, through the transversal forces arising at the places of the collars, increases the p^{cr} of the corresponding member. This holds for every element of the bundle. Therefore, the critical load of the bundle where the total moment of inertia of the bundle is calculated by means of the Steiner theorem is considerably higher than the value, which is calculated by the Euler theory (see Fig.5) for the statically equivalent single beam. In the case of the 9-element-bundle model, the buckling of each element is not free since it is surrounded by the elastic body of the remaining elements of the bundle. The elastic energy of each one element due to axial deformation, $\pi_\alpha = \int_0^\ell p(d_s - d_x)$, is equal to the sum $\pi_\alpha = \pi_1 + \pi_2$, where π_1 is its own elastic energy due to bending, $\pi_1 = \int_0^\ell \frac{M^2}{2EJ}dx$ and π_2 measures its own influence on the deformation energy of the neighboring elements of the bundle, that form its supporting environment. The same is repeated for all elements. Finally a coupled system of 9 similar differential equations has to be solved. This is done numerically with the use of the finite element method (Bathe,1987). In particular, a path-following (load-incremental) fully nonlinear finite element analysis, taking into account the unilateral contact interactions between the bundles, is performed. Although this method is able to follow the post-buckling behavior of the structure, in the present its use is restricted only to the investigation of the calculation of the buckling load.

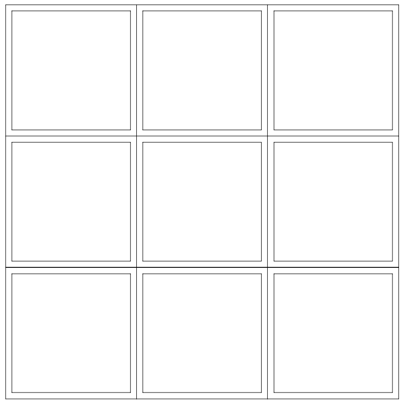

Fig. 4: Cross-section of the Proposed Element.

As a matter of fact, buckling is an evolving procedure. At the beginning, the system is at a stable equilibrium. By increasing the compressive load p, a transition through the neutral equilibrium and suddenly into the area of unstable equilibrium with collapse occurs. The latter corresponds to the critical load p^{cr}, that, as has been proved by Euler, is independent from the initial eccentricity from which the whole procedure is initiated. In the theoretical procedure of Euler, a small initial eccentricity has been assumed. The resulting critical load can be calculated to be equal to $p^{cr} = \frac{\pi^2 \cdot E \cdot J}{S_k^2}$. For completeness reasons, it is mentioned that if a known predefined, finite eccentricity exists, this should be taken into account in the calculation.

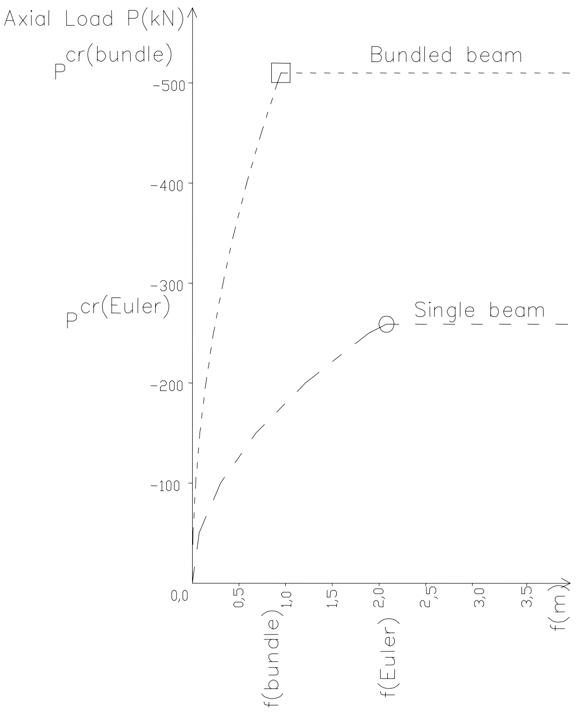

Fig. 5: Load–deformation Diagram in the Cases of
a) The Proposed Bundled Beam,
b) The Statically Equivalent Single Beam.

The triggering mechanism of buckling is considered to be a small initial eccentricity, which always exists in a real beam subjected to compressive loads. This holds for every element of the bundle. Nevertheless, the possibility that all these eccentricities are equal, both in direction and magnitude, for all elements of the bundle is, practically, equal to zero. Therefore, each one of the elements tends to buckle for a load greater than the critical one, in a different plane. In the very unrealistic situation that all elements will buckle in the same plane, there will be no lateral support of the type, which is exploited, in the proposed design. Therefore, we conclude that, at least at the first stages of buckling, the different elements of the bundle will tend to follow a buckling mode a different planes. Contact is realized between them and lateral support, with relatively small contact forces, appears. During evolution of buckling, in the post-buckling regime, the buckling modes will eventually coincide and buckling will appear at a critical load p^{cr} for the whole bundle. At this point all elements will have equal buckling elastic deformation lines. This is the global buckling mode of the bundle, as it has been previously defined.

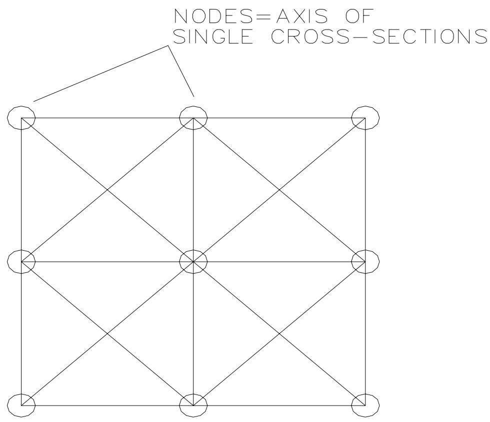

Fig. 6: Finite Element Model of the Proposed Structural Element (Cross-section).

A number of numerical results are in the sequel presented to demonstrate the validity of the proposed concept. The numerical solution of the examples has been carried out by applying the finite element method. The first practical problem, which must be solved, is the correct modeling of the collar. The technique followed takes rigid restrictions at the positions of the collars in order to enforce that the lateral displacements of the various beams are equal at the given point. This simple restriction enforces the whole bundle to work as an integral member with all previously mentioned unilateral interactions. Buckling eventually appears at a higher critical load, the one of the bundle.

4. NUMERICAL EXAMPLES

Example 1: Let us say: a multi-element (bundled beam) is composed by two (2) similar single elements of Quadrilateral Hollow Section QHS250x250/8 with $F = 77.44cm^2, J^{x=y} = 7567cm^4$, placed with parallel axes, having adjacent sides of the single beams in all possible buckling directions, where:

$$F_{bundle} = 2 \times 77.44 = 154.88cm^2 \text{ and}$$

$$J^{y}_{bundle} = 2 \times 7567 + 2 \times 77.44x12.5^2 = 39344cm^4.$$

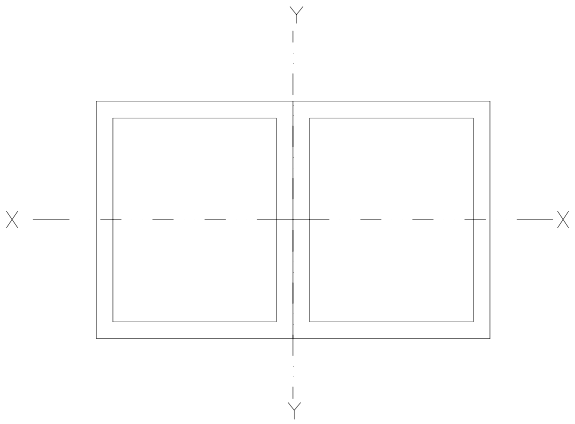

Fig. 7: A Two-element Bundle (Cross-section).

The buckling load for the herein proposed bundled beam is calculated by solving a finite element model of the beam by the available program Statik-3 (Statik-3, 1998), where the buckling length is $S_k = 10m$, $S_k = 20m$, $S_k = 30m$, $S_k = 50m$ and $S_k = 80m$. The distance between collars was also taken into account (cf. respectively a=2.0m, a=3.0m, a=4.0m, a=5.0m). In comparison, the statically equivalent single beam becomes with similar cross-section characteristics: $F_{Euler} = 154.88cm^2$ and $J^{y}_{Euler} = 39344cm^4$. The following form gives the buckling load of the statically equivalent single element:

$$p^{cr} = \frac{n^2 \cdot E \cdot (J = 39344)}{S_{\kappa}^2}$$

(Euler form) for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 1.

Example 2: Here, the multi-element (bundled beam) is composed of 4 similar single elements of Quadrilateral Hollow Section QHS250x250/10 with $F = 96.00cm^2$, $J^{x=y} = 9233cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where: $F_{bundle} = 4 \times 96.00 = 384.00cm^2$ and

$$J^{x=y}_{bundle} = 4 \times 9232 + 4 \times 96 \times 12.5^2 = 96928cm^4.$$

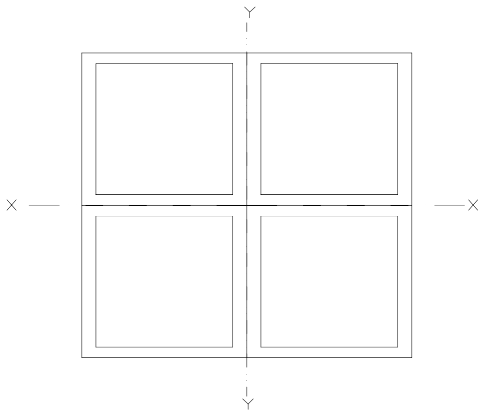

Fig. 8: A Four-element Bundle (Cross-section).

The buckling load for the here proposed bundled beam is calculated by solving a finite element model of the beam by the available program Statik-3 for several cases of buckling length and distance between collars a(m). In comparison, the statically equivalent single element becomes with similar cross-section characteristics: $F_{Euler} = 384.00cm^2$ and $J_{Euler}^{x=y} = 96928cm^4$. The following form gives the buckling load of the statically equivalent single element: $p_{Euler}^{cr} = \frac{n^2 \cdot E \cdot (J = 96928)}{S_k^2}$, for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table2.

Example 3: Let us consider now a multi-element (bundled beam) composed of nine (9) similar single elements of Quadrilateral Hollow Section QHS130x130/5 with $F = 25.00cm^2$, $J^{x=y} = 652.08cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where:

$$F_{bundle} = 9 \times 25{,}00 = 225{,}00cm^2 \text{ and}$$

$$J_{bundle}^{x=y} = 9 \times 652.08 + 23 \times 25 \times 13^2 = 31218cm^4.$$

Table 1: Example 1 - Results.

$S_k(m)$	$P_{Euler}^{cr}(kN)$	$P_{Bundle}^{cr}(kN)$	**Collars per a(m)**	$P_{bundle}^{cr} / P_{Euler}^{cr}$
10,00	8140,00	13280,00	2,0	1,63
20,00	2030,00	3580,00	2,0	1,76
30,00	905,00	1590,00	3,0	1,75
50,00	325,70	589,00	5,0	1,80
80,00	127,20	224,00	4,0	1,76

Table 2: Example 2 - Results.

$S_k(m)$	$P_{Euler}^{cr}(kN)$	$P_{Bundle}^{cr}(kN)$	Collars per a(m)	$P_{bundle}^{cr} / P_{Euler}^{cr}$
10,00	20060,00	30480,00	3,3	1,51
20,00	5020,00	9120,00	4,0	1,81
30,00	2230,00	4420,00	3,0	1,98
50,00	802,70	1590,00	5,0	1,98
80,00	313,50	630,00	5,0	2,00
100,00	200,60	398,0 (407)	10,0 (5,0)	1,98(2,02)

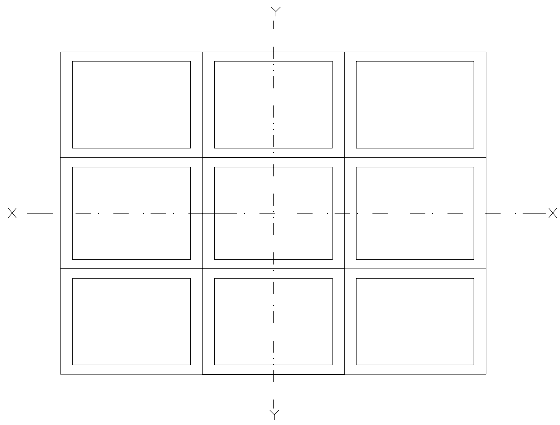

Fig. 9: A Nine-element Bundle (Cross-section).

The buckling load for the proposed bundled beam results from solving a finite element model of the beam by the available program Statik-3 for several cases of buckling length and distance between collars a(m). In comparison, the statically equivalent single element results with similar cross-section characteristics: $F_{Euler} = 225.00 cm^2$ and $J_{\text{Euler}}^{\text{x=y}} = 31218 cm^4$. The following form gives the critical buckling load of the statically equivalent single element:

$$p_{\text{Euler}}^{\text{cr}} = \frac{n^2 \cdot E \cdot (J = 31218)}{S_\kappa^2}$$

for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 3.

The form of the considered statically equivalent single beam is obtained by solving one of the following systems I or II. The difference between the two systems is the form of the cross-section, Quadrilateral Hollow Section (system I) or Circular Hollow section (system II). This procedure is used to determine the statically equivalent single element.

$$\left.\begin{aligned} t_1^2 - t_2^2 &= 225\,cm^2 \\ \frac{t_1^2 - t_2^2}{12} &= 31218\,cm^4 \end{aligned}\right\} \text{ where: } \begin{aligned} t_1 &= 30.74\text{ cm and} \\ t_2 &= 26.83\text{ cm} \end{aligned} \qquad \text{(I)}$$

$$\left.\begin{aligned} 0.049 \cdot \left(D^4 - d^4\right) &= 31218\text{cm}^4 \\ \frac{\pi}{4} \cdot \left(D^2 - d^2\right) &= 225\text{cm}^2 \end{aligned}\right\} \text{ where: } \begin{aligned} D &= 35.45\text{cm and} \\ d &= 31.16\text{cm} \end{aligned} \qquad \text{(II)}$$

The buckling load of the bundled beam at any one of the above 3 examples becomes 1.7 to 2 times bigger than the statically equivalent single beam.

Example 4: Let us consider a multi element (bundled beam) composed of sixteen (16) single elements of Quadrilateral Hollow Section QHS100x100/3 with $F = 11.60 cm^2$, $J^{x=y} = 183 cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where: $F_{bundle} = 16 \times 11.60 = 185.60 cm^2$ and

$$J_{\text{bundle}}^{\text{x=y}} = 16 \times 183 + 2 \times 4 \times 11.6 \times 15^2 + 2 \times 4 \times 11.6 \times 5^2 = 26128\text{cm}^4$$

In comparison, the statically equivalent single element results with similar cross-section characteristics: $F_{Euler} = 185.60 cm^2$ and $J_{\text{Euler}}^{\text{x=y}} = 26128\text{cm}^4$.

Table 3: Example 3 - Results.

$S_k(m)$	$P^{cr}_{Euler}(kN)$	$P^{cr}_{Bundle}(kN)$	Collars per a(m)	$P^{cr}_{bundle}/P^{cr}_{Euler}$
15,00	2870,00	5410,00	1,5	1,88
20,00	1610,00	3050,00	2,0	1,89
25,00	1034,00	1950,00	2,5	1,89
30,00	718,20	1380,00	2,5	1,92
50,00	258,50	510,00	3,33	1,97
80,00	100,90	200,00	5,0	1,98

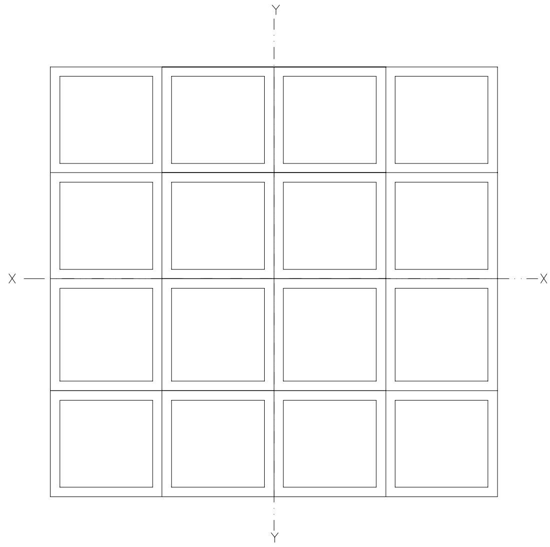

Fig.10: A Sixteen-element Bundle (Cross-section).

The following form gives the buckling load of the statically equivalent single element:

$$p^{cr}_{Euler} = \frac{n^2 \cdot E \cdot (J = 26128)}{S^2_K}$$

for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 4.

Example 5: Let us consider an orthogonal but not rectangular form of cross section of a multi- element (bundled beam) composed of twelve (12) single elements of Quadrilateral Hollow Section QHS140x140/6 with $F = 31.20cm^2$, $J^{x=y} = 920cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where:

$$F_{bundle} = 12 \times 31.20 = 374.40cm^2,$$

$$J^{y}_{bundle} = 12 \times 920 + 2 \times 3 \times 31.2 \times 7^2 - 2 \times 3 \times 31.2 \times 21^2$$
$$= 102768cm^4$$

In comparison, the statically equivalent single element results with similar cross-section characteristics: $F_{Euler} = 374.40cm^2$, $J^x_{Euler} = 51961cm^4$, $J^y_{Euler} = 102768cm^4$.

The following form gives the buckling load of the statically equivalent single element:

$$p^{cr} = \frac{n^2 \cdot E \cdot (J = 26128)}{S_\kappa^2}$$ for $J = J^x$ or J^y and for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 5.

Example 6a: Let us consider now a multi-element (bundled beam) composed of nine (9) single elements of Quadrilateral Full Section 10x10 with $F = 100.0cm^2$, $J^{x=y} = 833.3cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where $F_{bundle} = 9 \times 100.0 = 900.0cm^2$,

$J^{x=y}_{bundle} = 9 \times 833 \times 2 \times 3 \times 100 \times 10^2 = 67500cm^4$.

In comparison, the statically equivalent single beam results with similar cross-section characteristics: $F_{Euler} = 900.0cm^2$ and $J^{x=y}_{Euler} = 67500cm^4$. The following form gives the buckling load of the statically equivalent single element:

$$p^{cr} = \frac{n^2 \cdot E \cdot (J = 67500)}{S_\kappa^2}$$, for $J = J^{x=y}$ and for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 6.

Example 6b: In this example, the same single cross sections as in Example 6a are used, but the bundled beam is composed of (9) single elements placed with double distance between the parallel axes as in Example 6a. The already known collar at suitable positions is formed here by the use of an external tight zone and an internal hollow steel part with nine quadrilateral holes of dimensions 10cmx10cm for each one. Thus, the distance between the parallel axes of the single cross sections becomes 20cm. Both results and the ratio between the buckling loads of each case are presented in Table 7.

Table 4: Example 4 - Results.

$S_k(m)$	$P^{cr}_{Euler}(kN)$	$P^{cr}_{Bundle}(kN)$	Collars per a(m)	$P^{cr}_{bundle} / P^{cr}_{Euler}$
15,00	2400,00	4280,00	1,5	1,78
20,00	1350,00	2410,00	2,0	1,785
25,00	865,00	1540,00	2,5	1,78
30,00	601,00	1110,00	2,5	1,85
50,00	216,00	410,00	3,33	1,95
80,00	84,50	160,00	5,0	1,90

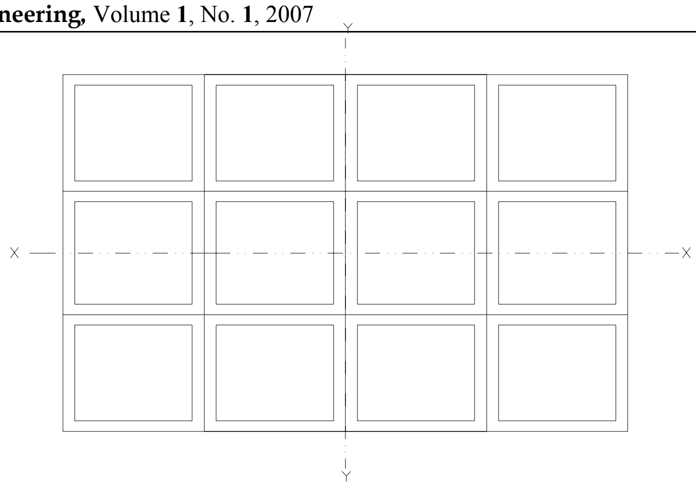

Fig. 11: A Rectangular, Twelve-element Bundle (Cross-section).

Table 5: Example 5 - Results.

$S_k(m)$	$P^{cr(x)}_{Euler}(kN)$	$P^{cr(y)}_{Euler}(kN)$	$P^{cr(x)}_{bundle}(kN)$	$P^{cr(y)}_{bundle}(kN)$	Collar a/m	$P^{cr}_{bundle} / P^{cr}_{Euler}(x)$	$P^{cr}_{bundle} / P^{cr}_{Euler}(y)$
15,00	5510,00	9450,00	7070.00	10260,00	3,00	1,28	1,08
20,00	3100,00	5318,00	5530.00	9040/14620	5,0/2,5	1,78	1,70/2,54
25,00	1980,00	3400,00	4357,00	5230,00	3,12	2,20	1,54
50,00	4960,00	850,00	1440,00	1400,00	6,25	2,20	1,65
80,00	194,00	332,00	600,00	560,00	8,0	1,70	1,686

Table 6: Example 6a - Results.

$S_k(m)$	$P^{cr}_{Euler}(t)$	Collars per a(m)	$P^{cr}_{Bundle}(kN)$	$P^{cr}_{bundle} / P^{cr}_{Euler}$
20	3490,00	2	5317,00	m=1,52
		1	6700,00	m=1,92
		0,5	7010,00	m=2,01
30	1550,00	3	2355,00	m=1,52
		1,5	2975,00	m=1,92
		5,75	3120,00	m=2,01
50	559,00	2,5	854,00	m=1,52
		1,25	1070,00	m=1,92
		0,625	1120,00	m=2,01

Table 7: Example 6b - Results.

$S_k(m)$	$P_{Euler}^{cr}(kN)$	Collars per (m)	$P_{Bundle}^{cr}(kN)$	$P_{bundle}^{cr} / P_{Euler}^{cr}$
20	3490,00	2	8580,00	m=2,45
		1	21310,00	m=6,10
		0,5	24690,00	m=7,07
30	1550,00	3	3855,00	m=2,48
		1,5	9585,00	m=6,17
		0,75	11040,00	m=7,11
50	559,00	5,0	1398,00	m=2,5
		2,5	3485,00	m=6,23
		1,25	3990,00	m=7,14
		0,625	4140,00	m=7,47

The buckling load of the bundled beam of example 6b becomes 1.6 to 3.7 times greater than the corresponding one of example 6a. It is clear that for the example 6b $J_{bundle}^{x=y}(6b) = 9 \times 833 + 2 \times 3 \times 100 \times 20^2 = 247497 cm^4$ and for the corresponding example 6a, $J_{bundle}^{x=y}(6a) = 67500 cm^4$. Especially for example 6b, the buckling load p_{bundle}^{cr}(6b) is seven times bigger than the buckling load $p_{Euler}^{cr}(6a)$ of the statically equivalent single beam of example 6a. The same conclusion is received by multiplying the **m** column of the example 6a results' table by the ratio

$$\frac{J(6b)}{J(6a)} = \frac{247497}{67500} = 3.66 .$$

Example 7a: Let us consider, in comparison with example 6a, a multi-element (bundled beam) composed of nine (9) single elements of Quadrilateral Hollow Section 100x100/6 with $F = 21.2 cm^2$, $J^{x=y} = 303 cm^4$, placed with parallel axes having adjacent sides of the single beams in all possible buckling directions, where $F_{bundle} = 9 \times 21.2 = 90.8 cm^2$,

$$J_{bundle}^{x=y} = 9 \times 303 + 2 \times 3 \times 21.2 \times 10^2 = 15447 cm^4,$$

and the distance between them is 10cm. The following form gives the buckling load of the statically equivalent single beam:

$$p^{cr} = \frac{n^2 \cdot E \cdot (J = 67500)}{S_K^2},$$

for $J = J^{x=y}$ and for any case of S_k. Both results and the ratio between the buckling loads of each case are presented in Table 8.

Example 7b: In this example, the same single cross sections are used as in Example 7a, but the bundled beam is composed of (9) single elements placed with double distance between the parallel axes as in Example 6a. The already known collar at suitable positions is formed here by the use of an external tight zone and an internal hollow steel part with nine quadrilateral holes of dimensions 10cmx10cm for each one. Thus, the distance between the parallel axes of the single cross sections becomes 20cm. Both results and the ratio between the buckling loads of each case are presented in Table 9.

The buckling load of the bundled beam of example 7b becomes 2 to 3.4 times greater than the corresponding one of example 7a. It is clear that for example 7b obtained

$J^{x=y}_{bundle}(7b) = 9\times 303 + 2\times 3\times 21.2\times 20^2 = 53607 cm^4$

and for the corresponding example 7a $J^{x=y}_{bundle}(7a) = 15447 cm^4$. Especially for example 7b, the buckling load p^{cr}_{bundle}(7b) is seven times greater than the buckling load $p^{cr}_{Euler}(7a)$ of the statically equivalent single beam of example 7a. The same conclusion is received by multiplying the **m** column of example 7a results' table by the ratio

$$\frac{J(7b)}{J(7a)} = \frac{53607}{15447} = 3.47 .$$

Example 8: Finally, let as consider a large scale example of a multi-element (bundled beam) composed of ninety six (96) single elements of Quadrilateral Hollow Section 265x265/9, where $F = 90 cm^2$, $J^{x=y} = 9720 cm^4$ with 4 horizontal lines of 24 single beams each and 24 columns of 4 single beams each. The composed multi-element (bundled beam) has a rectangular cross section with external dimensions $b\times h = 6.36\times 1.06 m$ and an area

$$F^{part}_{bundle} = 96\times 90.0 = 8640 cm^2 .$$

The total length of the bundle is $L=200.0m$ and is stabilized by suitable collars placed every $\alpha = \frac{200}{20} = 10.0\,\mathrm{m}$, having adjacent sides of the single beams in all possible buckling directions. The multi-element (bundled beam) is used in this example as a bridge deck mainly forced by compress load and supported by cable elements, see (Bisbos, 2003; Nitsiotas, 1985; Michalopoulos et al., 2005; Nikolaidis, 2003). Here, the x-x and y-y axes of the cross section of the beam (see Fig.13) are identical with the Y-Y and Z-Z axis of the bridge respectively. It is noticed that the buckling length here is S_κ=200.0m. In comparison, the statically equivalent single beam results with similar cross-section characteristics: $F_{Euler} = 4\times 90 = 360.0 cm^2$ and

$$\mathrm{J}^{\mathrm{x=y}}_{\mathrm{Euler}} = 4\times 9720 + 2\times 2\times 90\times 13.25^2 = 102082\mathrm{cm}^4 .$$

Four same basic parts of 4x4 elements model the bundled beam of the structure. In this analysis, by using the part of 4x4 elements obtained for the statically equivalent single beam,

$$\mathrm{J}^{\mathrm{y\text{-}y}}_{\mathrm{Euler}} = 2\times 12\times 102082 + 2\times 12\times 360\times 26.5^2 = 8.577\times 10^6\,\mathrm{cm}^4 ,$$

$\mathrm{J}^{\mathrm{z-z}}_{\mathrm{Euler}} = 292.26\times 10^6\,\mathrm{cm}^4$ and finally $\mathrm{p}^{\mathrm{cr}}_{\mathrm{Z\text{-}Z}} = \dfrac{\pi^2\cdot \mathrm{E}\cdot \mathrm{J}^{\mathrm{z\text{-}z}}_{\mathrm{total}}}{\mathrm{S}^2_{\kappa}} = 151100\mathrm{kN}$. It is an ideal buckling load because the real buckling load corresponds to a multi composed cross section, and that is because it is impossible to construct an ideal (compact) cross section of this scale of dimensions. In comparison, the buckling load for the bundled beam here results by solving a finite element model of the beam, where a) If the distance between collars is a=10m, the buckling load is $\mathrm{p}^{\mathrm{cr}}_{\mathrm{Z\text{-}Z}}(bundle) = 169200\mathrm{kN}$ and b) if the distance between collars is a=5m, the buckling load is $\mathrm{p}^{\mathrm{cr}}_{\mathrm{Z\text{-}Z}}(bundle) = 240690\mathrm{kN}$, and in this case the ratio between the bundled beam and the statically equivalent Euler beam is $\frac{240690}{60448} = 3.98$.

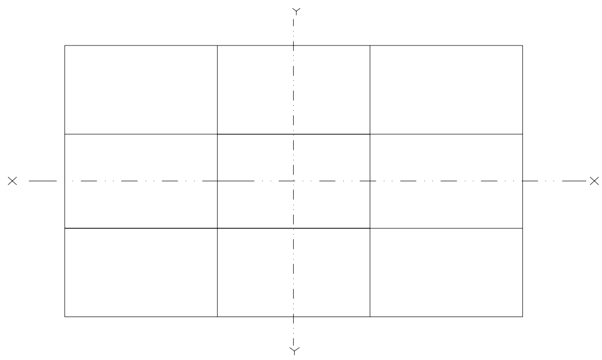

Fig. 12: A nine-element Bundle (Cross-section).

Table 8: Example 7a - Results.

$S_k(m)$	$P^{cr}_{Euler}(kN)$	Collars per a(m)	$P^{cr}_{Bundle}(kN)$	$P^{cr}_{bundle}/P^{cr}_{Euler}$
20	799,00	2	1419,00	m=1,77
		1	1658,00	m=2,07
		0,5	1719,00	m=2,15
30	355,00	3	628,00	m=1,77
		1,5	735,00	m=2,07
		0,75	764,00	m=2,15
50	128,00	5	227,00	m=1,77
		2,5	265,00	m=2,07
		1,25	275,00	m=2,15

Table 9: Example 7b - Results.

$S_k(m)$	$P^{cr}_{Euler}(kN)$	Collars per a(m)	$P^{cr}_{Bundle}(kN)$	$P^{cr}_{bundle}/P^{cr}_{Euler}$
20	799,00	2	3087,00	m=3,86
		1	5349,00	m=6,69
		0,5	5838,00	m=7,3
30	355,00	3	1386,00	m=3,9
		1,5	2395,00	m=6,7
		0,75	2603,00	m=7,33
50	128,00	5	503,00	m=3,93
		2,5	867,00	m=6,77
		1,25	942,00	m=7,36

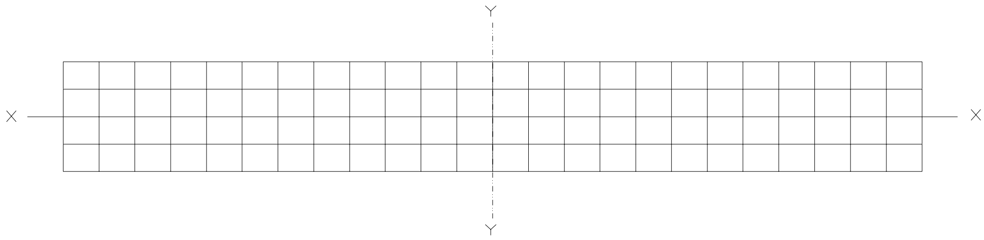

Fig. 13: A Bundled Beam with Several Elements.

5. CONCLUSIONS

A composite bundled structure with a complicated cross section by the use of a number of single elements and collars can be easily designed and constructed having the advantage of low weight and strong resistance against buckling. Such a beam would have a variety of practical applications in engineering structures, especially on bridge design. This research work has started since 1992 by the research group of the late Prof. P. D. Panagiotopoulos (1993) with theoretical research and laboratory tests on analytical and computational models of multi-element beams. For the future, extensive investigation on the comparison between the above-mentioned numerical models and the laboratory tests on real scale specimens has been already scheduled.

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