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## On the Number of Nilpotent Conjugacy Classes of the Symmetric Inverse Transformation Semigroup

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### On The Number of Nilpotent Conjugacy Classes of the Symmetric Inverse Transformation Semigroup

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#### ABSTRACT

From the conjugacy classes in the Symmetric inverse transformation semigroup, we obtained its nilpotent conjugacy classes. A general expression was obtained for the number of nilpotent conjugacy classes in the Symmetric inverse transformation semigroup.

Keywords: Nilpotent conjugacy classes, Symmetric inverse transformation semigroup

#### **1. PRELIMINARIES**

Let  $X_n = \{1, 2, ..., n\}$ . Then a (partial) transformation  $\alpha$ : **Domain**  $\alpha \subseteq X_n \to Im\alpha$  is said to be full or total if **Dom**  $\alpha = X_n$ , otherwise it is called strictly partial.

The set of all partial transformation on n-object forms a semigroup under the usual composition of functions. It is denoted by  $R_{12}$ , when it is strictly partial,  $T_{12}$  when it is full or total and  $l_{22}$  when it is partial 1-1(or the symmetric inverse).

An element  $\alpha \in S$  is nilpotent  $(\alpha^n = 0)$  for some  $n \ge 0$ . A property of nilpotent element among others is  $x\alpha \ne x$ ,  $\forall x \in X_n$ , where  $\alpha$  is nilpotent.

Let  $\alpha, \beta \in I_n$ , then chart  $\alpha$  is conjugate to chart  $\beta$  if and only if  $\alpha$  and  $\beta$  have the same path structure

#### 2. NILPOTENT CONJUGACY CLASSES IN THE SYMMETRIC INVERSE TRANSFORMATION SEMIGROUP

The following nilpotent conjugacy classes are arranged according to the number of their images in any number of  $I_{n}$ .

In	Number of images	Conjugacy classes
When $n = 1$	No image	(1]

Total nilpotent conjugacy classes = 1

In	Number of Image	Conjugacy classes
When $n = 2$	No Image	(1](2]
	1 Image	(12]

Total nilpotent conjugacy classes = 2

Number of Image	Conjugacy classes
No Image	(1](2](3]
1 Image	(12](3]
2 Images	(123]
	No Image 1 Image

Total nilpotent conjugacy classes = 3

In	Number of Image	Conjugacy classes
When n = 4	No Image	(1](2](3](4]
	1 Image	(12](3](4]
	2 Images	(12](34], (123](4]
	3 Images	(1234]

Total nilpotent conjugacy classes = 5

In	Number of Image	Conjugacy classes
When $n = 5$	No Image	(1](2](3](4](5]
	1 Image	(12](3](4](5]
	2 Images	(12](34](5],
		(123](4](5]
	3 Images	(123](45],(1234](5]
	4 Images	(12345]
		-

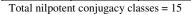
Total nilpotent conjugacy classes = 7



In	Number of Image	Conjugacy classes
When $n = 6$	No Image	(1](2](3](4](5](6]
	1 Images	(12](3](4](5](6]
	2 Images	(12](34](5](6],
		(123](4](5](6]
	3 Images	(12](34](56],
		(123](45](6],
		(1234](5](6]
	4 Images	(123](456],(1234](56],
		(12345](6]
	5 Images	(123456]

Total nilpotent conjugacy classes = 11

In	Number of	Conjugacy classes
	Image	
When n = 7	No Image	(1](2](3](4](5](6](7]
	1 Image	(12](3](4](5](6](7]
	2 Images	(12](34](5](6](7],
		(123](4](5](6](7]
	3 Images	(12](34](56](7],
	5 mages	(123](45](6](7],
		(1234](5](6](7]
		(123](45](67],
	4 Images	(123](456](7],
		(1234](56](7],
		(12345](6](7]
		(1234](567],
	5 Images	(12345](67],
		(123456](7]
	6 Images	(1234567]



#### 3. RESULTS

From the enumeration above, a summary of the sequence of the number of nilpotent conjugacy classes of  $I_{\mathfrak{M}}$  is listed below

1, 2, 3, 5, 7, 11, 15, ... where n = 1, 2, ...Let a(n) be the number of nilpotent conjugacy classes in  $I_{n}$ 

a(n) is the number of partitions of n(the partition numbers)which is generally given as

$$a(n) = \frac{1}{n} \sum_{k=0}^{n-1} [S(n-k)a(k)],$$
 where

a(0) = 1 and S(k) is the sum of divisors of k.

For sum of divisors of n, for example, S(8) = 1 + 2 + 4 + 8 = 15.

For higher values of k, we use the formula

$$S(k) = \prod_{i=1}^{m} \frac{p_i^{r_i t + 1}}{p_i - 1} : k = p_1^{r_2} p_2^{r_2} ... p_m^{r_m}$$

where the ps are distinct primes.

$$\alpha(4) = \frac{1}{4}(7 \times 1 + 4 \times 1 + 3 \times 2 + 1 \times 3) = \frac{1}{4}(20) = 5$$

#### 4. CONCLUSION

It has been shown that the number of nilpotent conjugacy classes in  $I_n$  for  $n \ge 1$ , can be calculated using the formula:

$$a(n) = \frac{1}{n} \sum_{k=0}^{n-1} [S(n-k)a(k)],$$
 where

$$a(0) = 1$$
 and  $S(k)$  is the sum of divisors of k.

For higher values of k, we use the formula

$$S(k) = \prod_{i=1}^{m} \frac{p_i^{r_i+1}}{p_i-1} : k = p_1^{r_i} p_2^{r_2} ... p_m^{r_m}$$

where the ps are distinct primes.

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