

Two Stage Fuzzy Flow Shop Scheduling to Minimize Rental Cost with Job – Block Criteria

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Abstract

In this paper we consider two stage flow shop scheduling to minimize the total rental cost of machines for n-jobs in fuzzy environment. The processing time of jobs and setup time for machines are uncertain. The fuzzy triangular membership function is used to describe uncertain processing times and setup times. Further, the restriction of equivalent job-block on job processing is also considered. The objective of the paper is to develop a new heuristic algorithm to minimize the rental cost of machines which is simple and straight forward. A numerical illustration explaining the computational process of the proposed algorithm is also given.

Keywords: fuzzy processing time, fuzzy setup time, rental cost, average high ranking, utilization time and equivalent job –block

1. Introduction

Scheduling is the allocation of resources over time to perform a collection of tasks. The objective corresponding is to optimize one or several criteria's such as minimization of rental cost or maximization of profit (Baker 1974). In a general flow shop scheduling problem, n jobs are to be scheduled on m machines in order to optimize some measures of performance. All jobs have the same processing requirements so they need to be processed on all machines in same order. Two machine flow shop scheduling problem has been considered as a major sub problem due to its applications in real-life. For this several heuristics have been successfully applied to solve various parameters, but some approaches ignore the uncertainty or the complex nature of the real world. In real life situations, the processing times of jobs are not always exact due to incomplete knowledge or uncertain environment which implies the existence of various external sources and types of uncertainty. From this point of view the concept of fuzziness is used in the theory of scheduling. Fuzzy sets (Zadeh 1965) as a mathematical way of representing impreciseness or vagueness in everyday life induced the fuzzy set theory as the most frequently used theory in intelligent control. Fuzzy set theory because of its simplicity and similarity to human reasoning has numerous applications in various fields such as engineering, medicine, manufacturing and others. McCahon and Lee (1990) discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee (1996) addressed the formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Sanuja and Xueyan (2006) optimized the makespan in a two-machine flow shop problem in the presence of uncertainty and proved that their approach of using different fuzzy sets determined by α -cut of processing times is better than McCahon and Lee (1990). Some of the noteworthy approaches are due to Gupta *et al.* (2012, 2013), Martin and Roberto (2001), Singh, Sunita and Allawalia (2008), Yao and Lin (2002). An algorithm to minimize makespan (Johnson 1954) in two stage flow shop scheduling problem is the earliest work in scheduling. The setup time of various jobs on machines are considered to be negligible and therefore could be included in the jobs processing times. However, in some applications, setup has major impact on the performance measure considered for scheduling problem so they need to be considered separately. Further, there are many practical situations in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machines. In this regard, the rental situation (Bagga 1969) under specific rental policy specifies to take machines on rent in order to complete the assignments. For example, care giving techniques often require hi-tech, expensive medical equipment which are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. The concept of equivalent-job blocking (Maggu & Das 1977) in the theory of scheduling is significant in the sense to create a balance between the cost of providing priority in service to the customer and cost of giving services with non priority customers. The decision maker may decide how much to charge extra from the priority customer. Also, Singh T.P & Gupta D. (2006) and Sharma & Gupta (2011) associated probabilities with processing time in their studies to minimize the rental cost of machines.

In this paper, we consider a two stage flow shop with triangular fuzzy processing times and fuzzy setup times. The objective is to find a job sequence which minimizes the rental cost of machines with the job-block as restriction. During the comparison of fuzzy numbers, fuzzy ranking techniques are used in scheduling of jobs. Some applications in which an absolute ordering of fuzzy numbers is required, to deal with them defuzzification techniques are used. We considered a widely used defuzzification technique known as Yagers

first index (1981). The rest of paper is organized as follows: Section 2 describes the basics of fuzzy set theory. Section 3 gives the notations to be used throughout the paper. In section 4, problem is formulated. Section 5 deals with theorem for optimizing the problem. Section 6 describes the algorithm proposed to find the optimal sequence for minimizing the rental cost. In section 7, numerical illustration is given to support the proposed algorithm. The paper is concluded in section 8 followed by the references.

2. Basic Fuzzy Set Theory

2.1 Triangular Fuzzy Number

Triangular fuzzy number (TFN) is a fuzzy number (Figure 1) represented with three points as $\tilde{A} = (a_1, a_2, a_3)$, where a_1 and a_3 denote the lower and upper limits of support of a fuzzy set \tilde{A} . The membership value of the x denoted by $\mu_{\tilde{A}}(x), x \in R^+$, can be calculated according to the following formula.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \leq a_1 \\ \frac{x - a_1}{a_2 - a_1} & ; a_1 < x < a_2 \\ \frac{a_3 - x}{a_3 - a_2} & ; a_2 < x < a_3 \\ 0 & ; x \geq a_3 \end{cases}$$

2.2 Average High Ranking

The system characteristics are described by membership function; it preserves the fuzziness of input information. However, the designer would prefer one crisp value for one of the system characteristics rather than fuzzy set. In order to overcome this problem we defuzzify the fuzzy values of system characteristic by using the Yager's (1981) formula

$$\text{Crisp}(A) = h(A) = \frac{3a_2 + a_3 - a_1}{3}$$

2.3 Fuzzy Arithmetic operations

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

(i) $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_2})$

(ii) $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$ if the following condition is

satisfied $DP(\tilde{A}_1) \geq DP(\tilde{A}_2)$, where $DP(\tilde{A}_1) = \frac{\beta_{A_1} - m_{A_1}}{2}$ and $DP(\tilde{A}_2) = \frac{\beta_{A_2} - m_{A_2}}{2}$. Here, DP denotes

difference point of a Triangular fuzzy number (TFN).

Otherwise; $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - \beta_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - m_{A_2})$

(iii) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if $k > 0$.

(iv) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if $k < 0$.

3. Notation

S : Sequence of jobs 1,2,3,...,n

S_k : Sequence obtained by applying Johnson's procedure, $k = 1, 2, 3, \dots$

M_j : Machine $j, j = 1, 2$

i : Job index

M : Minimum elapsed time

A_{ij} : Fuzzy Processing time of the job i on machine M_j

S_{ij} : Fuzzy set up time of the job i on machine M_j

A'_{ij} : AHR of processing time of job i on machine M_j

S'_{ij} : AHR of set up time of job i on machine M_j

A''_{ij} : AHR of the fuzzy flow time of job i on machine M_j

β : Equivalent job for job – block
 C_j : Rental cost per unit time of machine M_j
 $t_{ij}(S_k)$: Completion time of job i for sequence S_k on machine M_j
 $R(S_k)$: Total rental cost for the sequence S_k of all machine
 U_k : Utilization time of machine M_2 for sequence S_k

3.1 Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required .i.e. the first machine will be taken on rent in the starting of the processing the jobs, second machine will be taken on rent at time when first job is completed on first machine and is ready for processing on second machine.

4. Problem Formulation

Let some job i ($i = 1, 2, \dots, n$) are to be processed on two machines M_j ($j = 1, 2$) under the specified rental policy P . Let A_{ij} be the fuzzy processing time of the job i on machine M_j and S_{ij} be the fuzzy setup time of the job i on machine M_j represented by triangular fuzzy numbers. Let A'_{ij} and S'_{ij} be the average high ranking (AHR) of processing time and setup time of the job i on machine M_j . Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines (shown in Table 1)

Mathematically, the problem is stated as

$$\text{Minimize } U_k(S_k) \text{ and Minimize } R(S_k) = t_{n1} \times C_1 + U_k(S_k) \times C_2$$

Subject to constraint: Rental Policy (P)

5. Theorem

In processing a schedule $S = \{J_1, J_2, J_3, \dots, J_k, J_{k+1}, \dots, J_n\}$ of n jobs on two machines M_1 and M_2 in the order $M_1 M_2$ with no passing allowed, the job block (J_k, J_{k+1}) having processing times $\{A_{k,1}, A_{k,2}, A_{(k+1),1}, A_{(k+1),2}\}$ is equal to the single job β . The processing time of job block β on machine M_1 and M_2 denoted respectively by $A_{\beta,1}$ and $A_{\beta,2}$ are given by $A_{\beta,1} = A_{k,1} + A_{k+1,1} - \min\{A_{k,2}, A_{k+1,1}\}$ $A_{\beta,2} = A_{k,2} + A_{k+1,2} - \min\{A_{k,2}, A_{k+1,1}\}$.

Proof: Let $t_{k,l}$ denote the completion time of job k ($k = 1, 2, 3, \dots, n$) on machine l ($l = 1, 2$) for the sequence S of jobs.

Therefore, by definition, we have

$$t_{k,2} = \max(t_{k,1}, t_{k-1,2}) + A_{k,2} = \max(t_{k,1} + A_{k,2}, t_{k-1,2} + A_{k,2})$$

$$\begin{aligned} t_{k+1,2} &= \max\{t_{k+1,1}, t_{k,2}\} + A_{k+1,2} \\ &= \max\{t_{k+1,1}, t_{k,1} + A_{k,2}, t_{k-1,2} + A_{k,2}\} + A_{k+1,2} \\ &= \max\{t_{k+1,1} + A_{k+1,2}, t_{k,1} + A_{k,2} + A_{k+1,2}, t_{k-1,2} + A_{k,2} + A_{k+1,2}\} \end{aligned}$$

$$\text{Since, } t_{k+1,1} = t_{k,1} + A_{k+1,1}$$

$$t_{k+1,2} = \max\{t_{k,1} + A_{k+1,1} + A_{k+1,2}, t_{k,1} + A_{k,2} + A_{k+1,2}, t_{k-1,2} + A_{k,2} + A_{k+1,2}\}$$

$$\text{Also, } t_{k+2,2} = \max\{t_{k+2,1}, t_{k+1,2}\} + A_{k+2,2}$$

$$= \max\{t_{k+2,1}, t_{k,1} + A_{k+1,1} + A_{k+1,2}, t_{k,1} + A_{k,2} + A_{k+1,2}, t_{k-1,2} + A_{k,2} + A_{k+1,2}\} + A_{k+2,2}$$

$$\text{Since, } t_{k+2,1} = t_{k+1,1} + A_{k+2,1}$$

Therefore, we have

$$t_{k+2,2} = \max\left\{ \begin{array}{l} t_{k,1} + A_{k+1,1} + A_{k+2,1}, t_{k,1} + A_{k+1,1} + A_{k+1,2}, \\ t_{k,1} + A_{k,2} + A_{k+1,2}, t_{k-1,2} + A_{k,2} + A_{k+1,2} \end{array} \right\} + A_{k+2,2}$$

$$\text{Since, } \max\{t_{k,1} + A_{k+1,1} + A_{k+1,2}, t_{k,1} + A_{k,1} + A_{k+1,2}\} = t_{k,1} + \max\{A_{k+1,1}, A_{k,1}\} + A_{k+1,2}$$

Therefore, we have

$$t_{k+2,2} = \max\left\{ \begin{array}{l} t_{k,1} + A_{k+1,1} + A_{k+2,1}, t_{k,1} + \max\{A_{k+1,1}, A_{k,1}\} + A_{k+1,2} \\ t_{k-1,2} + A_{k,2} + A_{k+1,2} \end{array} \right\} + A_{k+2,2} \quad (1)$$

$$\text{Also, } t_{k+2,1} = t_{k-1,1} + A_{k,1} + A_{k+1,1} + A_{k+2,1} = t_{k,1} + A_{k+1,1} + A_{k+2,1} \quad (2)$$

Now, let us define a sequence S' of jobs as

$$S' = \{J_1, J_2, J_3, \dots, J_{k-1}, \beta, J_{k+2}, \dots, J_n\}$$

$$\text{Where; } A_{\beta,1} = A_{k,1} + A_{k+1,1} - c \quad (3)$$

$$A_{\beta,2} = A_{k,2} + A_{k+1,2} - c; \text{ c is a constant.} \quad (4)$$

Let $t'_{k,l}$ denote the completion time of job k ($k=1, 2, 3, \dots, n$) on machine l ($l=1, 2$) for the sequence S' of jobs.

Therefore, by definition

$$t'_{\beta,2} = \max(t'_{\beta,1}, t'_{\beta-1,2}) + A_{\beta,2} = \max(C'_{\beta,1} + A_{\beta,2}, C'_{\beta-1,2} + A_{\beta,2})$$

$$\begin{aligned} t'_{k+1,2} &= \max\{t'_{k+1,1}, t'_{\beta,2}\} + A_{k+2,2} \\ &= \max\{t'_{k+2,1}, t'_{\beta,1} + A_{\beta,2}, t'_{\beta-1,2} + A_{\beta,2}\} + A_{k+2,2} \end{aligned} \quad (5)$$

$$\text{Since, } t'_{k+2,1} = t'_{k-1,1} + A_{\beta,1} + A_{k+2,1}$$

$$= t_{k-1,1} + (A_{k,1} + A_{k+1,1} - c) + A_{k+2,1}$$

$$= t_{k,1} + A_{k+1,1} - c + A_{k+2,1} \quad [\because t_{k,1} = t_{k-1,1} + A_{k,1}] \quad (6)$$

$$\text{Also, } t'_{\beta,1} = t'_{k-1,1} + A_{\beta,1} = t_{k-1,1} + A_{k,1} + A_{k+1,1} - c$$

$$= t_{k,1} + A_{k+1,1} - c \quad (7)$$

On combining the results (3), (4), (5), (6) and (7), we have

$$t'_{k+2,2} = \max\left\{ \begin{array}{l} t_{k,1} + A_{k+1,1} - c + A_{k+2,1}, t_{k,1} + A_{k+1,1} - c + A_{k,2} + A_{k+1,2} - c, \\ t_{k,1} + A_{k+1,1} - c + t_{k,2} + t_{k+1,2} - c \end{array} \right\} + A_{k+2,2} \quad (8)$$

$$\text{Let } c = \min\{A_{k+1,1}, A_{k,2}\}, \text{ then} \quad (9)$$

$$A_{k+1,1} - c + A_{k,2} = A_{k+1,1} - \min\{A_{k+1,1}, A_{k,2}\} + A_{k,2} = \max\{A_{k+1,1}, A_{k,2}\} \quad (10)$$

$$\text{Also, } t'_{k-1,2} = t_{k-1,2} \quad (11)$$

On combining results (8), (9), (10) and (11), we have

$$\begin{aligned} t'_{k+2,2} &= \max\left\{ \begin{array}{l} t_{k,1} + A_{k+1,1} + A_{k+2,1} - c, t_{k,1} + \max\{A_{k+1,1}, A_{k,2}\} \\ + A_{k+1,2} - c, t_{k-2,2} + A_{k,2} + A_{k+1,2} - c \end{array} \right\} + A_{k+2,2} \\ &= \max\left\{ \begin{array}{l} t_{k,1} + A_{k+1,1} + A_{k+2,1}, t_{k,1} + \max\{A_{k+1,1}, A_{k,2}\} \\ + A_{k+1,2}, t_{k-1,2} + A_{k,2} + A_{k+1,2} \end{array} \right\} + A_{k+2,2} - c \end{aligned} \quad (12)$$

From (1) and (12), we have

$$t'_{k+2,2} = t_{k+2,2} - c \quad (13)$$

From (2) and (6), we conclude that

$$t'_{k+2,1} = t_{k+2,1} - c \quad (14)$$

From results (13) and (14), we observe that the replacement of job-block (J_k, J_{k+1}) in a sequence S of by a job β decreases the completion times of the later job J_{k+l} on both the machines by a constant c in S' , i.e. if T and T' be the completion times of sequence S and S' , then we have $T = T' - c$, i.e. the completion times on both the machines are changed by a value which is independent of the particular sequence S . Hence, the substitution does not change the relative merit of different sequences. Hence, job block β is an equivalent job for job block (J_k, J_{k+1}) .

6. Algorithm

The following algorithm is proposed for nx2 flow shop scheduling to minimize the rental cost of machines when fuzzy processing time of jobs and setup time are considered involving equivalent job-block.

Step 1: Find average high Ranking (AHR) of the fuzzy processing time and setup time of various jobs on different machines.

Step 2: Calculate the AHR of fuzzy flow time for the two machines M_1 and M_2 as follows

$$A''_{i1} = A'_{i1} - S'_{i2} \text{ and } A''_{i2} = A'_{i2} - S'_{i1} \quad \forall i.$$

Step 3: Take equivalent job $\beta(k, m)$ and calculate the processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ on the guide lines of Maggu and Das (1977) as $A'_{\beta 1} = A''_{k1} + A''_{m1} - \min(A''_{m1}, A''_{k2})$, $A'_{\beta 2} = A''_{k2} + A''_{m2} - \min(A''_{m1}, A''_{k2})$.

Step 4: Define a new reduced problem with the processing times A''_{i1} and A''_{i2} as defined in step 2 and jobs (k, m) are replaced by single equivalent job β with processing time $A''_{\beta 1}$ and $A''_{\beta 2}$ as defined in step 3.

Step 5: Using Johnson's technique (1954) obtain all the sequences S_k having minimum elapsed time M . Let these be S_1, S_2, \dots .

Step 6: Observe the processing time of first job of S_1 on the first machine M_1 . Let it be α .

Step 7: Obtain all the jobs having processing time on M_1 greater than α . Put these job one by one in the first position of the sequence S_1 in the same order. Let these sequences be $S_2, S_3, S_4, \dots, S_r$.

Step 8: Prepare in-out flow table only for those sequence S_p ($p=1, 2, \dots, r$) which have job block $\beta(k, m)$ and evaluate total elapsed time of last job of each sequence i.e. $t_{n1}(S_p)$ and $t_{n2}(S_p)$ on machine M_1 and M_2 respectively.

Step 9: Evaluate completion time $t_{11}(S_p)$ of each of above selected sequence S_p on machine M_1 .

Step 10: Calculate utilization time U_p of machine M_2 for each of above selected sequence S_p as:

$$U_p = t_{n2}(S_p) - t_{11}(S_p) \text{ for } p=1, 2, 3, \dots, r.$$

Step 11: Find $\text{Min} \{U_p\}$, $p=1, 2, \dots, r$. let it be corresponding to $p = m$, then S_m is the optimal sequence for minimum rental cost.

$$\text{Minimum rental cost} = t_{n1}(S_m) \times C_1 + U_m \times C_2$$

7. Numerical Illustration

Consider 5 jobs, 2 machine flow shop scheduling problem with processing time and setup time represented by triangular fuzzy numbers (Shown in Table 2). The rental costs per unit time for machines M_1 & M_2 are 10 and 8 units respectively, and jobs (2, 4) are to be processed as an equivalent group job. The objective is to obtain an optimal sequence of jobs to minimize the total rental cost of machines.

Solution: The AHR of processing times and setup time of jobs on machine M_1 and M_2 as per the step 1 are shown in Table 3.

The AHR of fuzzy flowtime as per the step 2 are given in Table 4.

As per step 3 and 4, the processing times of equivalent job block $\beta = (2, 4)$ by using Maggu and Das (1977) criteria and the reduced problem (Table 5) is given by

$$A''_{\beta 1} = 36/3 + 33/3 - 21/3 = 48/3 \text{ and } A''_{\beta 2} = 21/3 + 30/3 - 21/3 = 30/3$$

Using Johnson's two machines algorithm in step 5, the optimal sequence is

$$S_1 = 5 - 3 - \beta - 1 = 5 - 3 - 2 - 4 - 1$$

As per step 6, the processing time of first job on $S_1 = 15/3$, i.e. $\alpha = 15/3$.

The other optimal sequences for minimizing rental cost as per step 7 are

$$S_2 = 3 - 5 - 2 - 4 - 1, S_3 = 2 - 5 - 3 - 4 - 1, S_4 = 4 - 5 - 3 - 2 - 1, S_5 = 1 - 5 - 3 - 2 - 4$$

The in-out flow tables for sequences S_1, S_2 and S_5 having job block (2, 4) are as shown in Table 6, 7 and 8.

For $S_1 = 5 - 3 - 2 - 4 - 1$

$$\text{Total time elapsed on machine } M_1 = t_{n1}(S_1) = (77, 86, 95)$$

$$\text{Total time elapsed on machine } M_2 = t_{n2}(S_1) = (89, 99, 109)$$

$$\text{Utilization time of second machine } M_2 = U_1 = (89, 99, 109) - (11, 12, 13) = (78, 87, 96)$$

For $S_2 = 3 - 5 - 2 - 4 - 1$

$$\text{Total time elapsed on machine } M_1 = t_{n1}(S_2) = (77, 86, 95)$$

$$\text{Total time elapsed on machine } M_2 = t_{n2}(S_2) = (91, 101, 111)$$

$$\text{Utilization time of second machine } M_2 = U_2 = (91, 101, 111) - (13, 14, 15) = (78, 87, 96)$$

For $S_5 = 1 - 5 - 3 - 2 - 4$

$$\text{Total time elapsed on machine } M_1 = t_{n1}(S_5) = (73, 82, 91)$$

$$\text{Total time elapsed on machine } M_2 = t_{n2}(S_5) = (88, 98, 108)$$

$$\text{Utilization time of second machine } M_2 = U_5 = (88, 98, 108) - (10, 11, 12) = (78, 87, 96)$$

The total minimum utilization of machine M_1 is (73, 82, 91) units and minimum utilization of M_2 is (78, 87, 96) units with defuzzified value as 93 units for the sequence S_5 . Therefore, the optimal sequence is $S_5 = 1 - 5 - 3 - 2 - 4$ and the minimum rental cost is $(73, 82, 91) \times 10 + (78, 87, 96) \times 8$

$$= (739, 820, 910) + (624, 696, 768) = (1354, 1516, 1678) \text{ units with defuzzified value}$$

as 1624 units.

8. Conclusion

In the past, the processing time for each job was usually assumed to be exactly known, but the processing times may vary dynamically due to human factors or operating faults. Therefore, the concept of fuzzy processing time

is introduced in processing of jobs and the setup time for machines to deal with uncertainty and vagueness in real life situations. This paper deals with the minimization of rental cost for two stage flow shop scheduling in fuzzy environment with equivalent job-block as restriction. The present work can further be extended by taking trapezoidal fuzzy numbers, considering weighted jobs and by introducing the concept of breakdown of machines etc.

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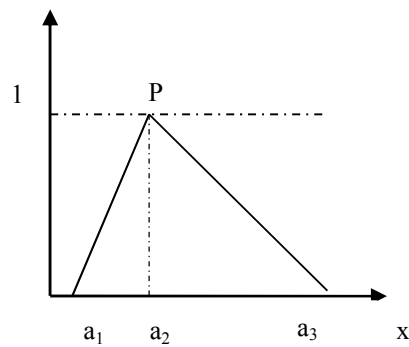


Figure 1. Triangular fuzzy number

Table 1. The mathematical model of the problem in matrix form

Jobs	Machine M_1		Machine M_2	
i	A_{i1}	S_{i1}	A_{i2}	S_{i2}
1	A_{11}	S_{11}	A_{12}	S_{12}
2	A_{21}	S_{21}	A_{22}	S_{22}
3	A_{31}	S_{31}	A_{32}	S_{32}
-	-	-	-	-
m	A_{m1}	S_{m1}	A_{m2}	S_{m2}
-	-	-	-	-
n	A_{n1}	S_{n1}	A_{n2}	S_{n2}

Table 2. The machines with fuzzy processing time and fuzzy setup time

Jobs	Machine M_1		Machine M_2	
I	A_{i1}	S_{i1}	A_{i2}	S_{i2}
1	(10,11,12)	(1,2,3)	(7,8,9)	(1,2,3)
2	(14,15,16)	(3,4,5)	(10,11,12)	(2,3,4)
3	(13,14,15)	(3,4,5)	(14,15,16)	(2,3,4)
4	(16,17,18)	(5,6,7)	(15,16,17)	(5,6,7)
5	(11,12,13)	(2,3,4)	(17,18,19)	(6,7,8)

Table 3. The AHR of fuzzy processing time and fuzzy setup time

Jobs	Machine M_1		Machine M_2	
i	A'_{i1}	S'_{i1}	A'_{i2}	S'_{i2}
1	35/3	8/3	(7,8,9)	(1,2,3)
2	47/3	14/3	(10,11,12)	(2,3,4)
3	44/3	14/3	(14,15,16)	(2,3,4)
4	53/3	20/3	(15,16,17)	(5,6,7)
5	38/3	11/3	(17,18,19)	(6,7,8)

Table 4. The AHR of fuzzy flow time

Jobs	Machine M_1	Machine M_2
i	A''_{i1}	A''_{i2}
1	27/3	18/3
2	36/3	21/3
3	30/3	33/3
4	33/3	30/3
5	15/3	45/3

Table 5. Reduced problem

Jobs	Machine M_1	Machine M_2
i	A''	A''
1	27/3	18/3
β	48/3	30/3
3	30/3	33/3
5	15/3	45/3

Table 6. In-Out table for sequence S_1

Jobs	Machine M_1	Machine M_2
i	IN – OUT	IN – OUT
5	(0,0,0) – (11,12,13)	(11,12,13) – (28,30,32)
3	(13,15,17) – (26,29,32)	(34,37,40) – (48,52,56)
2	(29,33,37) – (43,48,53)	(50,55,60) – (60,66,72)
4	(46,52,58) – (62,69,76)	(62,69,76) – (77,85,93)
1	(67,75,83) – (77,86,95)	(82,91,100) – (89,99,109)

Table 7. In-Out table for sequence S_2

Jobs	Machine M_1	Machine M_2
i	IN – OUT	IN – OUT
3	(0,0,0) – (13,14,15)	(13,14,15) – (27,29,31)
5	(16,18,20) – (27,30,33)	(29,32,35) – (46,50,54)
2	(29,33,37) – (43,48,53)	(52,57,62) – (62,68,74)
4	(46,52,58) – (62,69,76)	(64,71,78) – (79,87,95)
1	(67,75,83) – (77,86,95)	(84,93,102) – (91,101,111)

Table 8. In-Out table for sequence S_3

Jobs	Machine M_1	Machine M_2
i	IN – OUT	IN – OUT
1	(0,0,0) – (10,11,12)	(10,11,12) – (17,19,21)
5	(11,13,15) – (22,25,28)	(22,25,28) – (39,43,47)
3	(24,28,32) – (37,42,47)	(45,50,55) – (59,65,71)
2	(40,46,52) – (54,61,68)	(61,68,75) – (71,79,87)
4	(57,65,73) – (73,82,91)	(73,82,91) – (88,98,108)

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