

Modeling Inflation Rates using Periodogram and Fourier Series Analysis Methods: The Nigerian Case

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Abstract

This work considers the application of Periodogram and Fourier Series Analysis to model all-items monthly inflation rates in Nigeria from 2003 to 2011. The main objectives are to identify inflation cycles, fit a suitable model to the data and make forecasts of future values. To achieve these objectives, monthly all-items inflation rates for the period were obtained from the Central Bank of Nigeria (CBN) website. Periodogram and Fourier Series methods of analysis are used to analyze the data. Based on the analysis, it was found out that inflation cycle within the period was fifty one (51) months, which relates to the two government administrations within the period. Further, appropriate significant Fourier series model comprising the trend, seasonal and error components is fitted to the data and this model is further used to make a forecast of the inflation rates for thirteen months and this forecast compares favourably with the actual values for the thirteen months.

Keywords: Fourier Series analysis, periodogram, frequency domain, time series, forecasting.

JEL Subject Classification: E31, E37.

1.0 INTRODUCTION

Inflation is referred to as the persistent and appreciable rise in the general level of prices (Thinngan, 2002). It is not every rise in the price level that is termed inflation. Therefore, for a rise in the general price level to be considered inflation, such a rise must be enduring, constant and sustained. This rise should affect all commodities and should not be temporal.

Inflation rate has been regarded as one of the major economic indicators in any country. According to Ayinde et al (2010), inflation is undeniably one of the most leading and dynamic micro-economic issues confronting almost all economies of the world. Its dynamism has made it an imperative issue to be considered.

In view of its importance to the economic growth of a country, many researchers and economists have applied various time series models to forecast or model inflation rates of countries. These models include Autoregressive Integrated Moving Average (ARIMA) Models and Seasonal Autoregressive Integrated Moving Average (SARIMA) Models all in the time domain, but not much have been done using the Frequency domain models. Recently, the Central Bank of Nigeria Governor, Sanusi Lamido Sanusi, called on researchers to come up with a model that could be adopted to forecast inflation, in view of the recent economic meltdown all over the world; as this would further help in understanding more the current behaviour of inflation rates. Consequently, the researchers explore the frequency domain approach to model inflation rates as a time series, using Periodogram and Fourier Series Analysis methods, because of their simple way of modeling seasonality and eliminating significant peaks without re-estimating the model. We also seek to use this model to make forecast of future values.

Some econometric models have been used to model inflation rates, but they are restrictive in their theoretical formulations and often lack to incorporate the dynamic structure of the data and have tendencies to inflict improper restrictions and specifications on the structural variables (Saz, 2011). Odusanya and Atanda (2010) determined the dynamic and simultaneous interrelationship between inflation and its determinants – growth rate of Gross Domestic Product (GDP), growth rate of money supply (M_2), fiscal deficit, exchange rate (U.S dollar to Naira), importance and interest rates, using econometric time series model. Ayinde et al (2010) examined the factors affecting inflation in Nigeria using cointegration and descriptive statistics. They observed that there were variations in the trend pattern of inflation rates and some variables considered were significant in determining inflation in Nigeria. These variables include annual total import, annual consumer price index for food, annual agricultural output, interest rate, annual government expenditure, exchange rate and annual crude oil export. Stockton and Glassman (1987) conducted a comparative study on three different inflation processes namely rational expectations model, monetarist model and the expectation augmented Philips curve that are based on economic theory relationships that explain and form inflation. These processes were compared to one another, utilizing their out-of-sample forecast performance on an eight-quarter horizon and in addition a simple Autoregressive Integrated Moving Average (ARIMA) model was used as a bench mark to substantiate the theoretical validity of the econometric models. Their findings showed that the ARIMA model outperformed both the rational expectations model and the monetarist model and was found to perform just as good as the Philips curve in all specifications. They included that upon theoretical effort put to explain causes of inflation, a simple ARIMA model of inflation turned in such a respectable forecast performance relative to the theoretically based specifications.

A reason why simple time series models tend to outperform their theoretical counterparts lies in the

restrictive nature of econometric models with their improper restriction and specifications on structural variables. The absence of restriction in the ARIMA model gives it the necessary flexibility to capture dynamic properties and thus significant advantage in short-run forecasting (Saz, 2011). Encouraged by these empirical results on the superiority of ARIMA models, Saz (2011) applied Seasonal Autoregressive Integrated Moving Average (SARIMA) model to forecast the Turkish inflation.

Having known the advantages of using Fourier Series Analysis to model periodic data, Ekpenyong and Omekara (2007) modeled the mean monthly temperature of Uyo Metropolis using Fourier Series Method. Also Ekpenyong (2008) modeled rainfall data using Pseudo-additive Fourier series model, which he modified from Fourier Series Model. In the same vein, we seek to use Fourier Series Analysis and Periodogram analysis to investigate the properties of inflation rates in Nigeria and also model the rates so as to make good forecast of it.

2.2 METHOD OF ANALYSIS

The main methods of analysis of this work are Periodogram and Fourier Series Analysis. If the series is largely influenced by seasonality or periodicity, one can immediately inspect and guess the period or the frequency of the series; but if the period of the series cannot be guessed accurately, it calls for the construction of a Periodogram to determine the period or frequency of the series.

In this work, since inflation is affected by other factors, Periodogram analysis becomes necessary to determine the inflation cycle of the series for the period under study.

In constructing the Periodogram, time series is viewed in the frequency-domain view point as one that consists of sine and cosine waves at different frequencies.

$$X_t = T_t + \sum_{i=1}^N [a_i \cos \omega_i t + b_i \sin \omega_i t] + Z_t \quad \text{--- (1)}$$

Where : T_t is the trend equation
 X_t is the series
 ω_i = the angular frequency measured in radians
 Z_t is the error term
 a_i, b_i are the coefficients

in short, the series consists of the trend, seasonal and error components. The series is first detrended and the ordinary least squares estimates of the parameters obtained on the detrended series as:

$$\hat{a}_i = \frac{2}{N} \sum_{i=1}^N \Delta X_t \cos \omega_i t \quad \text{--- (2)}$$

$$\hat{b}_i = \frac{2}{N} \sum_{i=1}^N \Delta X_t \sin \omega_i t \quad \text{--- (3)}$$

$$\text{Since } \sum_{i=1}^N \cos \omega_i t \cos \omega_j t = \begin{cases} 0, i \neq j \\ N/2, i=j \end{cases} \quad \text{--- (4)}$$

$$\sum_{i=1}^N \sin \omega_i t \cos \omega_j t = 0 \text{ for all } i \text{ and } j \quad \text{--- (5)}$$

$$\text{Also } \sum_{i=1}^N \sin \omega_i t \sin \omega_j t = \begin{cases} N/2, i=j \\ 0, \text{otherwise} \end{cases} \quad \text{--- (6)}$$

$$\text{And } \sum_{i=1}^N \sin \omega_i t = 0 \quad \text{--- (7)}$$

$$\sum_{i=1}^N \cos \omega_i t = 0 \quad \text{--- (8)}$$

for $1 < i < N/2$, where $\omega_i = 2\pi f_i$

The above results are obtained from complex variables as orthogonality and independent properties of Fourier Series or sinusoidal models. In case of time series with even number of observations $N = 2q$, $q = N/2$, the same

definitions are applicable except for

$$\hat{a}_q = \frac{1}{N} \sum_{i=0}^N (-1)^i \Delta X_t$$

$$\hat{b}_q = 0 \quad \text{--- (9)}$$

Therefore the Periodogram for a time series ΔX_t with $N = 2q + 1$ (odd number of observations) is defined as the function of intensities $I(f_i)$ at frequency f_i as:

$$I(f_i) = \frac{N}{2} [a_i^2 + b_i^2] \quad i = 1, 2, \dots, q \quad \text{--- (10)}$$

where f_i is the i th harmonic of frequency $\frac{1}{N}$ and $0 \leq f_i \leq 0.5$
 for even number of observations,

$$I(f_{0.5}) = Na_q^2 \quad \text{--- (11)}$$

The plot of the intensities against the frequencies (f_i) or periods $\left(\frac{1}{f_i}\right)$ is the Periodogram.

The Periodogram measures the amplitude of a time series for all possible frequencies and wavelengths. It can be interpreted as the amount of the total series sums of squares, that is explained by specific frequencies. The Period or frequency of the series is identified as that with the largest intensity, $I(f_i)$. The frequency would then be used as the Fourier frequency f_i to obtain the parameter estimates of the model.

The combination of methods of estimating the components of time series gives a general model of Fourier Series Analysis used in forecasting time series.

The general Fourier Series model is given by:

$$X_t = T_t + \alpha_1 \cos \omega t + \beta_1 \sin \omega t + \alpha_2 \cos 2\omega t + \beta_2 \sin 2\omega t + \dots + \alpha_k \cos k\omega t + \beta_k \sin k\omega t + Z_t \quad \text{--- (12)}$$

$$= T_t + \sum_{i=1}^k [\alpha_i \cos i\omega t + \beta_i \sin i\omega t] + Z_t \quad \text{--- (13)}$$

The estimated model for forecasting time series is given by:

$$\hat{X}_t = T_t + \sum_{i=1}^k [\hat{\alpha}_i \cos i\omega t + \hat{\beta}_i \sin i\omega t] + \hat{Z}_t \quad \text{--- (14)}$$

where \hat{X}_t = estimated values

\hat{T}_t = estimated trend equation

$\hat{\alpha}_i, \hat{\beta}_i (i=1, \dots, k)$ = parameter estimates

$\omega = 2\pi f_i$

$k =$ highest harmonic of ω

$\hat{Z}_t =$ estimate of the random error.

The highest harmonic, k in Fourier series analysis model is the number of observations per season divided by two (2) for an even number of observations and $(n - 1)/2$ for an odd number of observations (Priestly, 1981).

The trend is first isolated or removed by fitting the linear trend model or quadratic trend or even the overall mean of the data, using the method of least squares. The trend equation is fitted based on its significance in the model. Thus the trend equation includes:

$$\left. \begin{aligned} X_t &= a_0 + b_0 t && \text{Linear trend} \\ X_t &= a_0 + b_0 t + C_0 t^2 && \text{Quadratic trend} \\ \mu &= \frac{\sum X_t}{N} \end{aligned} \right\} \quad \text{--- (15)}$$

The trend, after estimating, is now removed from series and the detrend series used to estimate seasonal variation.

The sine and cosine functions in equation (2.14) give the estimated model for the seasonality of the model. It is given by:

$$\Delta \hat{X}_t = \sum_{i=1}^k \hat{\alpha}_i \text{Cos}i\omega t + \hat{\beta}_i \text{Sin}i\omega t \quad \text{---(16)}$$

The above equation is cast as a Multiple Linear Regression to obtain the estimates of α_i and β_i .

The original value of Z_t is obtained by subtracting the trend and seasonal components from the original value (X_t). This gives the random error. To estimate Z_t , we first of all determine if the residual values are random. This can be done by assessing the autocorrelation function of the residual. If the residual or error component is not random, a first order autoregressive model can be fitted to the error values as:

$$Z_t = \phi Z_{t-1} + \mu_t \quad \text{--- (17)}$$

where Z_t = error term
 ϕ = the autoregressive coefficient
 μ_t = the purely random process (white noise)

The estimated equation is given as:

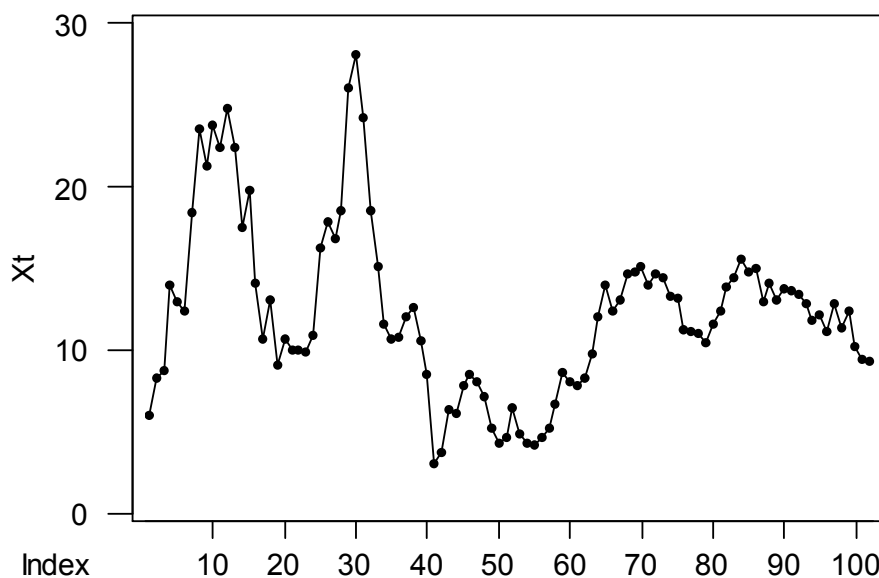
$$\hat{Z}_t = \hat{\phi} Z_{t-1} \quad \text{--- (18)}$$

These estimations are done using statistical packages such as MINITAB and SPSS/EXCEL.

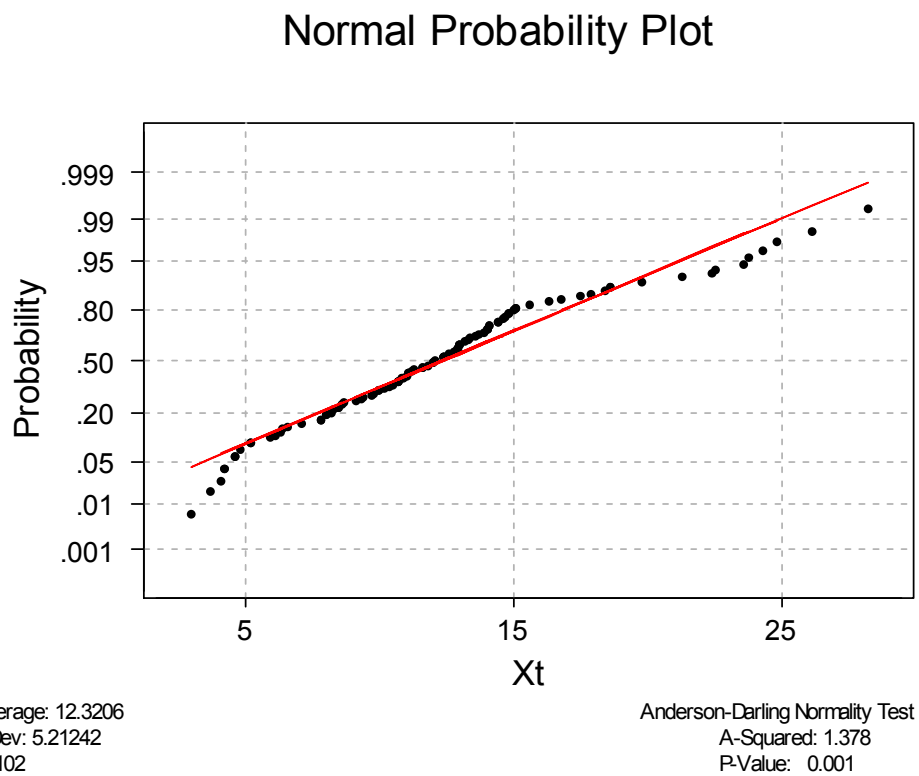
3.0 DATA ANALYSIS

A clear assessment of the graphical representation of original series shows that there is an existence of trend cyclical and seasonal variations, but the cycle or period of the series cannot be exactly ascertained (see figure I). In addition the normal probability plot of the series indicates that the series is not normal or stable. It is therefore important to transform the data for stability and normality. A square root transformation was therefore appropriate as shown in figure III below.

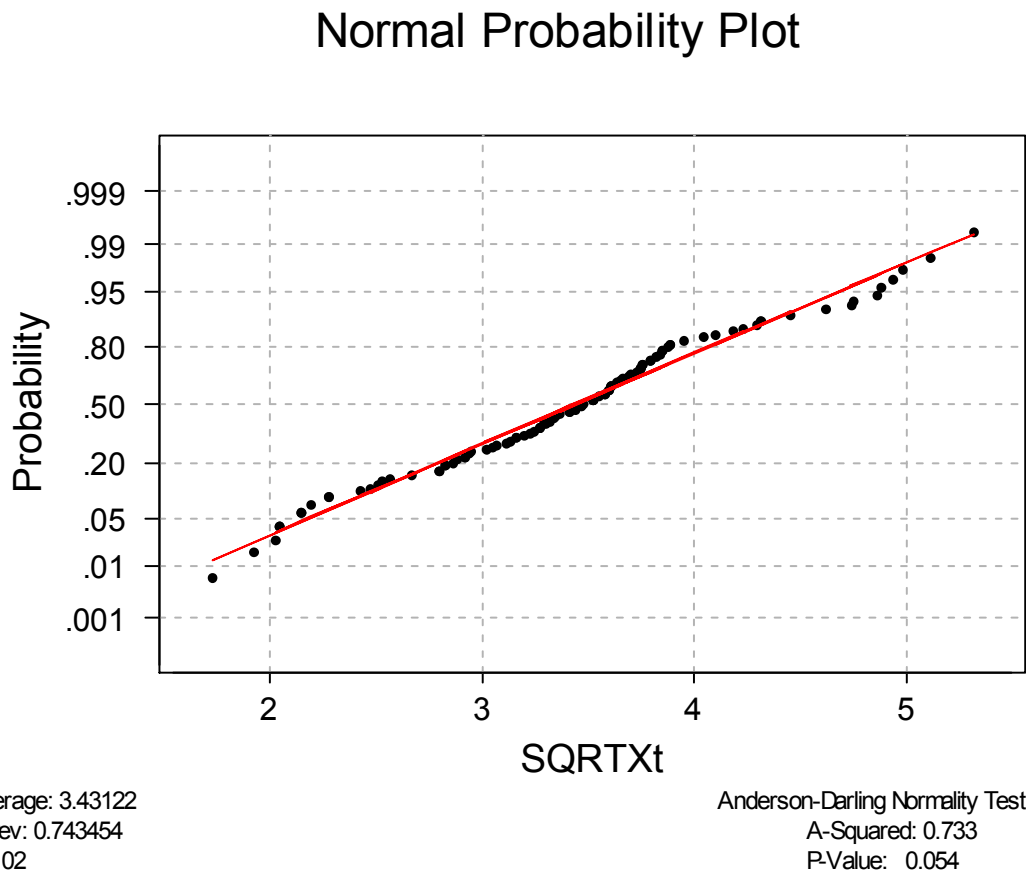
Figure I: Plot of Original Series



FigureII: Normal P-P Plot of the Series



FigureIII: Normal P-P Plot of the transformed Series



3.2 PERIODOGRAM ANALYSIS

The intensities $I(f_i)$ at various frequencies are obtained using equations (2), (3) and (10) as shown in

Table 6. The plots of these intensities against the frequencies and periods are given in the figures below.

FIGURE IV: Plot of Intensities against Frequency

PERIODOGRAM PLOT AGAINST FREQUENCY

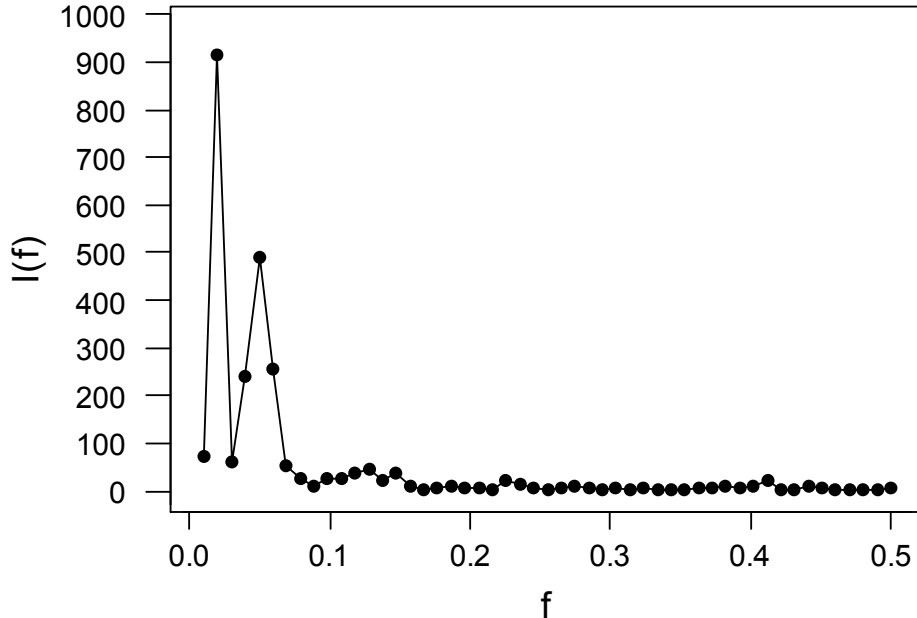
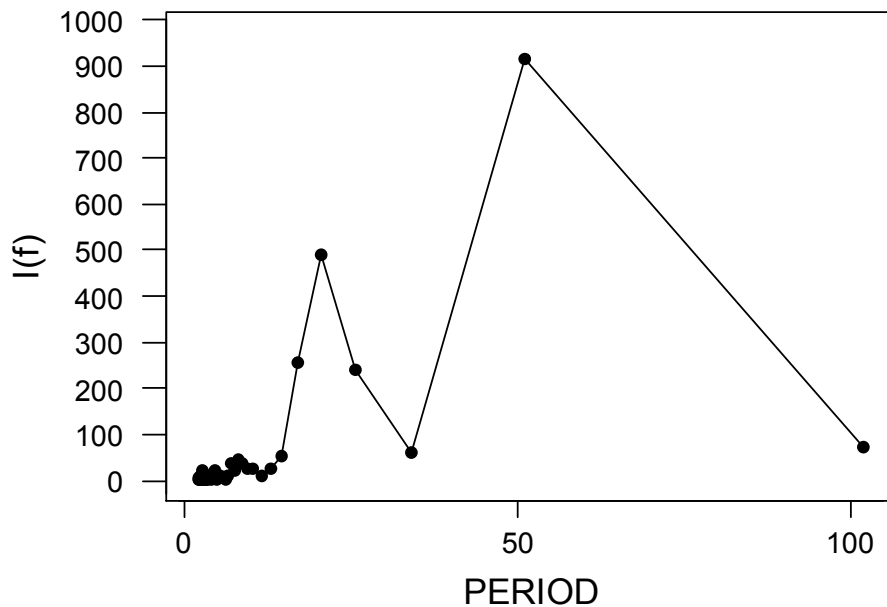


FIGURE V: Plot of Intensities against Periods

PERIODOGRAM PLOT AGAINST PERIOD



From figures I, II and III, it is observed that the frequency and period with the largest intensity are 0.0196 and 51 months respectively. Hence the period or cycle for the data is 51 months. These are now used to fit the Fourier series model.

3.3 FITTING THE GENERAL FOURIER SERIES MODEL

The trend (T_t) is estimated as:

$$T_t = a_0 + b_0 t \quad \text{and its analysis is given in table 1.}$$

The regression equation is

$$T_t = 3.6529 + 0.0043t$$

From table 1, it is seen that the parameter estimate a_0 is significant in the trend equation at 5% level of significance. Therefore $T_t = 3.6529$.

The seasonal component is then estimated from the detrended series as:

$$k = \frac{(n-1)}{2} = \frac{51-1}{2} = 50/2 = 25$$

$$\text{and } \omega = 2 \times \pi \times 0.019608$$

$$\therefore X_t = \sum_{i=1}^{25} [\alpha_i \text{Cos}i\omega t + \beta_i \text{Sin}i\omega t]$$

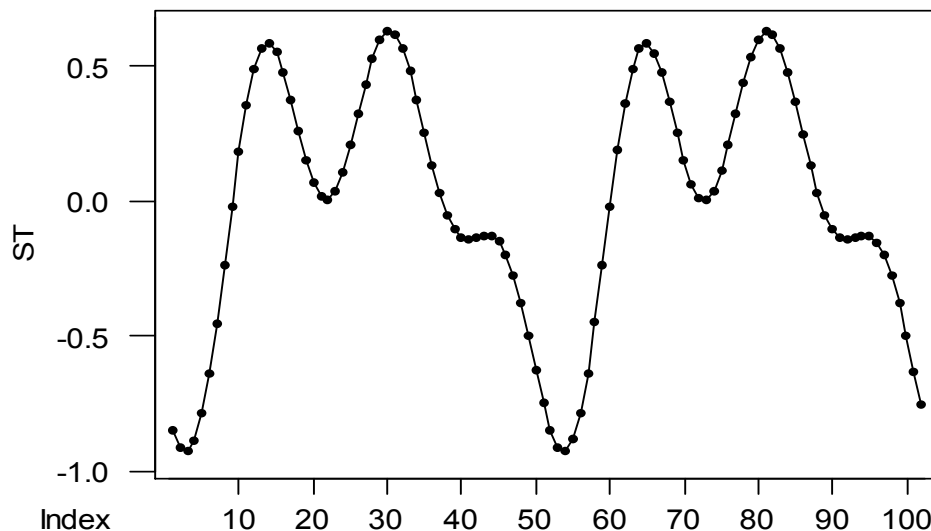
The parameters α_i^s and β_i^s are obtained by method of least squares as shown in the table 2 .

From table 2, it is observed that the parameter estimates that are significant in the model are: $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\beta}_3$.
 Therefore, the estimated seasonal model is

$$SQRT\Delta\hat{X}_t = \hat{\alpha}_1 \text{Cos}\omega t + \hat{\alpha}_2 \text{Cos}2\omega t + \hat{\beta}_3 \text{Sin}3\omega t$$

$$SQRT\Delta\hat{X}_t = -0.5071\text{Cos}\omega t - 0.2432\text{Cos}2\omega t - 0.3088\text{Sin}3\omega t \quad \dots(20)$$

Figure VI: Plot of Estimated Seasonal Component



In assessing the autocorrelation and partial autocorrelation function of the error component, it was found out that the error was not random. The behaviour of the autocorrelation and partial autocorrelation function suggest an autoregressive model of order one. Table 3 shows the test for significance of the model. From table 3, the parameter estimate in the error component is significant in the model.

$$\text{Hence } \hat{Z}_t = 0.9030Z_{t-1} \quad \dots(21)$$

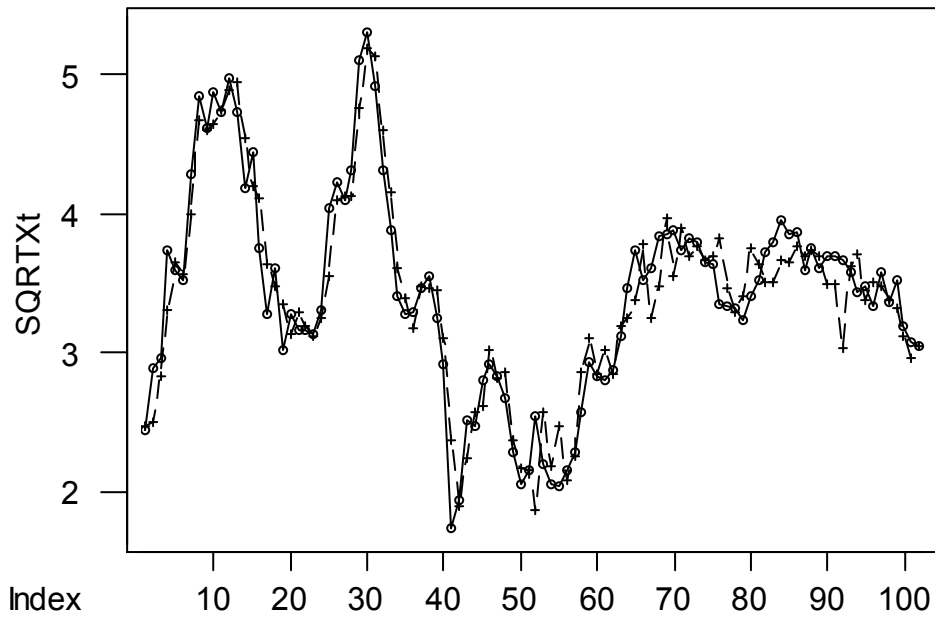
The general model for the series, which consists of the estimated trend, seasonal and error component is given as:

$$SQRT\hat{X}_t = 3.6529 - 0.5071\text{Cos}\omega t - 0.2432\text{Cos}\omega t - 0.3088\text{Sin}3\omega t + 0.903Z_{t-1} \quad \dots(22)$$

The model is now used to estimate inflation rates. (See Appendix I).

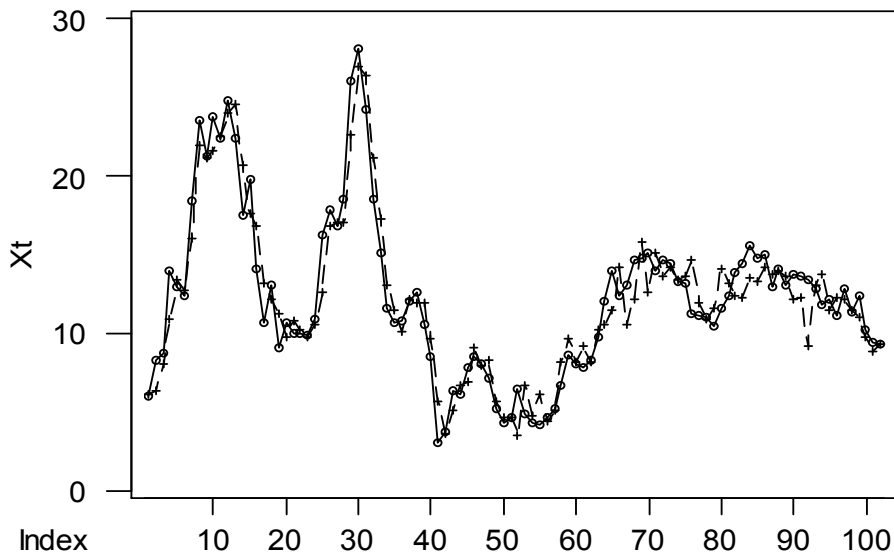
The plots of the original estimated series of both the actual and transformed values are shown in figures VII and VIII below:

Figure VII: Plot of Actuals and Estimates of the Transformed Series



ACTUALS IN CIRCLE
 ESTIMATES IN PLUS

Figure VIII: Plot of Actuals and Estimates of the Original Series



ACTUALS IN CIRCLE
 ESTIMATES IN PLUS

The above plots show that the model fit well to the data.

3.4 FORECAST FOR THE FUTURE RATES

A forecast from September 2011 to September 2012 was made using the estimated general model and the forecast is shown in the table below:

4.0 DISCUSSION

The periodogram analysis reveals that there exist both short term and long term cycles within the period under study. The long term cycle is 51 months while the short term cycle is approximately 20 months. This is

known by checking the second largest intensity in Appendix III. This goes a long way to buttress the fact that inflation is influenced by cyclical or periodic variation. It can also be deduced from the periodogram the relationship between the inflation cycle and the various government administrations within the period. Under this period of study, it is known that two administrations existed, namely Obasanjo's and Yaradua/Jonathan's administrations. The inflation cycle relates to these two administrations. The first fifty one months represents the administration of President Olusegun Obasanjo, while the second cycle represents Yaradua/Jonathan's administration.

From the foregoing, it can be deduced that economic policies, activities, implementation of policies, budgets, etc of government administration influence inflation rates in Nigeria. Most economists have confirmed that an expansionary budgetary provision among other factors helps to increase inflation rates. A clear look at the original plot of the series shows that during Obasanjo's regime, there were higher inflation rates than the Yaradua/Jonathan's administration. This may be as a result of incessant supplementary budgets raised and the inability of the administration to follow budgetary provisions in the implementation of the budget. But the latter administration records lower inflation rates, which may be as a result of reduced passage of supplementary budgets and implementation of some reforms in the economic sectors.

5.0 CONCLUSION

The periodogram analysis has identified a major inflation rate cycle for the period under study to be fifty one (51) months and the former series analysis has established a former series model for the all-items inflation rates to be

$$SQRT\hat{X}_t = 3.6529 - 0.5071\cos \omega t - 0.2432\cos \omega t - 0.3088\sin 3 \omega t + 0.903Z_{t-1}$$

This model is used to make good forecast of inflation rates. Therefore Fourier series models can also be used to model inflation rates because of its advantage of identifying inflation cycles in addition to establishing a suitable model for the series.

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Table 1: Test for Significance of the Parameter Estimates in the Trend Model

Predictor	Coef	StDev	T	P
Constant	3.6529	0.1469	24.87	0.000
t	-0.004304	0.002475	-1.74	0.085

S = 0.7361 R-Sq = 2.9% R-Sq(adj) = 2.0%

Table 2: Test for Significance of the Parameter Estimates in the Seasonal Component

Predictor	Coef	StDev	T	P
Noconstant				
coswt	-0.5071	0.1041	-4.87	0.000
sinwt	0.0387	0.1041	0.37	0.712
cos2wt	-0.2432	0.1041	-2.34	0.023
sin2wt	0.1524	0.1041	1.46	0.149
cos3wt	-0.0323	0.1041	-0.31	0.758
sin3wt	-0.3088	0.1041	-2.97	0.005
cos4wt	-0.0326	0.1041	-0.31	0.756
sin4wt	-0.0813	0.1041	-0.78	0.438
cos5wt	0.0120	0.1041	0.12	0.909
sin5wt	-0.0951	0.1041	-0.91	0.365
cos6wt	-0.1057	0.1041	-1.01	0.315
sin6wt	-0.0322	0.1041	-0.31	0.759
cos7wt	0.0163	0.1041	0.16	0.876
sin7wt	0.0834	0.1041	0.80	0.427
cos8wt	0.0563	0.1041	0.54	0.591
sin8wt	0.0132	0.1041	0.13	0.900
cos9wt	-0.0449	0.1041	-0.43	0.668
sin9wt	0.0050	0.1041	0.05	0.962
cos10wt	-0.0113	0.1041	-0.11	0.914
sin10wt	-0.0420	0.1041	-0.40	0.688
cos11wt	-0.0230	0.1041	-0.22	0.826
sin11wt	-0.0119	0.1041	-0.11	0.910
cos12wt	0.0714	0.1041	0.69	0.496
sin12wt	0.0119	0.1041	0.11	0.910
cos13wt	0.0230	0.1041	0.22	0.826
sin13wt	-0.0025	0.1041	-0.02	0.981
cos14wt	0.0529	0.1041	0.51	0.613
sin14wt	0.0183	0.1041	0.18	0.861
cos15wt	-0.0212	0.1041	-0.20	0.839
sin15wt	0.0095	0.1041	0.09	0.928
cos16wt	-0.0235	0.1041	-0.23	0.822
sin16wt	0.0108	0.1041	0.10	0.918
cos17wt	0.0021	0.1041	0.02	0.984
sin17wt	0.0165	0.1041	0.16	0.875
cos18wt	0.0004	0.1041	0.00	0.997
sin18wt	0.0164	0.1041	0.16	0.875
cos19wt	0.0192	0.1041	0.18	0.855
sin19wt	-0.0428	0.1041	-0.41	0.683
cos20wt	0.0309	0.1041	0.30	0.768
sin20wt	-0.0322	0.1041	-0.31	0.759
cos21wt	0.0041	0.1042	0.04	0.969
sin21wt	0.0088	0.1041	0.08	0.933
cos22wt	-0.0011	0.1042	-0.01	0.992
sin22wt	-0.0039	0.1041	-0.04	0.970
cos23wt	0.0248	0.1042	0.24	0.813
sin23wt	-0.0205	0.1041	-0.20	0.845
cos24wt	-0.0086	0.1042	-0.08	0.935
sin24wt	-0.0243	0.1040	-0.23	0.816
cos25wt	0.0184	0.1045	0.18	0.861
sin25wt	-0.0052	0.1038	-0.05	0.961

S = 0.7436

Table 3: Test for Significance of Parameter Estimate I the Error Component

Final Estimates of Parameters				
Type	Coef	StDev	T	
AR 1	0.9030	0.0441	20.49	

Number of observations: 102

Table 4: Forecasts for Future Periods

FORCAST	ACTUALS	t
9.2881	10.3	103
9.1516	10.5	104
9.2528	10.5	105
9.3567	10.3	106
9.436	12.6	107
9.4507	11.9	108
10.5905	12.1	109
11.6472	12.9	110
12.1333	12.7	111
12.7911	12.9	112
13.3627	12.8	113
12.3368	11.7	114
10.7698	11.3	115

Table 5: Actuals and Estimates of Nigeria All-items Inflation Rates(2003-20011)

t	ESTSQRTXt	ESTXt	Xt	SQRTXt
1	2.45894	6.0464	5.9	2.42899
2	2.49951	6.2476	8.3	2.88097
3	2.82162	7.9615	8.7	2.94958
4	3.29974	10.8883	14	3.74166
5	3.6537	13.3495	12.9	3.59166
6	3.56475	12.7074	12.4	3.52136
7	3.99915	15.9932	18.4	4.28952
8	4.68373	21.9373	23.6	4.85798
9	4.60276	21.1854	21.3	4.61519
10	4.65238	21.6446	23.8	4.87852
11	4.75027	22.5651	22.4	4.73286
12	4.90114	24.0212	24.8	4.97996
13	4.95641	24.566	22.5	4.74342
14	4.55181	20.719	17.5	4.1833
15	4.1966	17.6114	19.8	4.44972
16	4.108	16.8757	14.1	3.755
17	3.63175	13.1896	10.7	3.27109
18	3.48247	12.1276	13	3.60555
19	3.3535	11.246	9.1	3.01662
20	3.12435	9.7616	10.7	3.27109
21	3.28923	10.819	10	3.16228
22	3.19264	10.193	10	3.16228
23	3.11341	9.6933	9.8	3.1305
24	3.24506	10.5304	10.9	3.30151
25	3.55351	12.6274	16.3	4.03733
26	4.1022	16.828	17.9	4.23084
27	4.12568	17.0213	16.8	4.09878

28	4.13136	17.0681	18.6	4.31277
29	4.75936	22.6515	26.1	5.10882
30	5.19483	26.9863	28.2	5.31037
31	5.14782	26.5001	24.3	4.9295
32	4.6063	21.218	18.6	4.31277
33	4.16304	17.3309	15.1	3.88587
34	3.60814	13.0187	11.6	3.40588
35	3.38407	11.4519	10.7	3.27109
36	3.17771	10.0978	10.8	3.28634
37	3.48145	12.1205	12	3.4641
38	3.45813	11.9586	12.6	3.54965
39	3.45463	11.9345	10.5	3.24037
40	3.10574	9.6456	8.5	2.91548
41	2.357	5.5554	3	1.73205
42	1.8921	3.5801	3.7	1.92354
43	2.2347	4.9939	6.3	2.50998
44	2.56834	6.5964	6.1	2.46982
45	2.61611	6.844	7.8	2.79285
46	3.01449	9.0871	8.5	2.91548
47	2.80696	7.879	8	2.82843
48	2.86217	8.192	7.1	2.66458
49	2.36306	5.5841	5.2	2.28035
50	2.15326	4.6365	4.2	2.04939
51	2.15144	4.6287	4.6	2.14476
52	1.86214	3.4676	6.4	2.52982
53	2.56943	6.602	4.8	2.19089
54	2.17886	4.7474	4.2	2.04939
55	2.46632	6.0827	4.1	2.02485
56	2.07906	4.3225	4.6	2.14476
57	2.23993	5.0173	5.2	2.28035
58	2.86022	8.1808	6.6	2.56905
59	3.10472	9.6393	8.6	2.93258
60	2.84175	8.0755	8	2.82843
61	3.01905	9.1147	7.8	2.79285
62	2.84466	8.0921	8.2	2.86356
63	3.19439	10.2041	9.7	3.11448
64	3.2467	10.5411	12	3.4641
65	3.37694	11.4037	14	3.74166
66	3.77535	14.2533	12.4	3.52136
67	3.24661	10.5405	13	3.60555
68	3.47907	12.104	14.7	3.83406
69	3.96803	15.7453	14.8	3.84708
70	3.54911	12.5962	15.1	3.88587
71	3.89074	15.1378	14	3.74166
72	3.69084	13.6223	14.6	3.82099
73	3.76744	14.1936	14.4	3.79473

74	3.66179	13.4087	13.3	3.64692
75	3.69333	13.6407	13.2	3.63318
76	3.83035	14.6716	11.2	3.34664
77	3.45736	11.9534	11.1	3.33167
78	3.29035	10.8264	11	3.31662
79	3.40812	11.6153	10.4	3.2249
80	3.75526	14.102	11.6	3.40588
81	3.63406	13.2064	12.4	3.52136
82	3.50865	12.3106	13.9	3.72827
83	3.50096	12.2567	14.4	3.79473
84	3.67314	13.4919	15.6	3.94968
85	3.64484	13.2849	14.8	3.84708
86	3.76182	14.1513	15	3.87298
87	3.70015	13.6911	12.9	3.59166
88	3.7377	13.9704	14.1	3.755
89	3.6965	13.6641	13	3.60555
90	3.48558	12.1493	13.7	3.70135
91	3.4968	12.2276	13.6	3.68782
92	3.03385	9.2043	13.4	3.6606
93	3.61712	13.0835	12.8	3.57771
94	3.70553	13.7309	11.8	3.43511
95	3.38216	11.439	12.1	3.47851
96	3.50707	12.2995	11.1	3.33167
97	3.47876	12.1018	12.8	3.57771
98	3.38236	11.4403	11.3	3.36155
99	3.32326	11.0441	12.4	3.52136
100	3.11168	9.6825	10.2	3.19374
101	2.96192	8.773	9.4	3.06594
102	3.04434	9.268	9.3	3.04959

Table 6: ESTIMATES, FREQUENCIES, PERIOD AND INTENSITIES

a	b	a**2	b**2	I(f)	f	PERIOD
8.4864	0.663	72.019	0.44	72.459	0.009804	102
-30.243	1.6932	914.639	2.867	917.506	0.019608	51
7.5888	0.0204	57.59	0	57.59	0.029412	34
-13.5048	7.6398	182.38	58.367	240.746	0.039216	25.5
-21.828	3.8556	476.462	14.866	491.327	0.04902	20.4
-2.142	-15.8406	4.588	250.925	255.5	0.058824	17
3.5598	-6.0486	12.672	36.586	49.258	0.068627	14.571
-1.9482	-4.2228	3.795	17.832	21.628	0.078431	12.75
-2.652	-0.0204	7.033	0	7.034	0.088235	11.333
0.4284	-4.9062	0.184	24.071	24.254	0.098039	10.2
4.4778	1.581	20.051	2.5	22.55	0.107843	9.273
-5.5182	-1.6932	30.451	2.867	33.317	0.117647	8.5
0.5916	-6.4056	0.35	41.032	41.382	0.127451	7.846
0.7344	4.2126	0.539	17.746	18.285	0.137255	7.286

-4.4472	-3.927	19.778	15.421	35.199	0.147059	6.8
2.7948	0.6324	7.811	0.4	8.211	0.156863	6.375
-0.9996	-0.153	0.999	0.023	1.023	0.166667	6
-2.346	0.2142	5.504	0.046	5.55	0.176471	5.667
0.459	-2.8458	0.211	8.099	8.309	0.186275	5.368
-0.6222	-2.1828	0.387	4.765	5.152	0.196078	5.1
1.8462	0.0306	3.408	0.001	3.409	0.205882	4.857
-1.2138	-0.6426	1.473	0.413	1.886	0.215686	4.636
-0.0102	-4.5186	0	20.418	20.418	0.22549	4.435
3.6006	0.5712	12.964	0.326	13.291	0.235294	4.25
0.6426	-1.3158	0.413	1.731	2.144	0.245098	4.08
1.1424	-0.1632	1.305	0.027	1.332	0.254902	3.923
1.377	-0.9282	1.896	0.862	2.758	0.264706	3.778
2.6724	0.8976	7.142	0.806	7.947	0.27451	3.643
0.3774	2.346	0.142	5.504	5.646	0.284314	3.517
-1.1016	0.4488	1.214	0.201	1.415	0.294118	3.4
0.8772	-1.122	0.769	1.259	2.028	0.303922	3.29
-1.224	0.5202	1.498	0.271	1.769	0.313725	3.188
1.224	-1.0914	1.498	1.191	2.689	0.323529	3.091
0.0918	0.8058	0.008	0.649	0.658	0.333333	3
-0.0816	-0.2958	0.007	0.087	0.094	0.343137	2.914
0	0.8058	0	0.649	0.649	0.352941	2.833
-1.9278	-0.6732	3.716	0.453	4.17	0.362745	2.757
0.9588	-2.2134	0.919	4.899	5.818	0.372549	2.684
0.8058	-2.3052	0.649	5.314	5.963	0.382353	2.615
1.5606	-1.6728	2.435	2.798	5.234	0.392157	2.55
2.8764	0.9792	8.274	0.959	9.233	0.401961	2.488
4.2126	0.4182	17.746	0.175	17.921	0.411765	2.429
0.6528	-0.7038	0.426	0.495	0.921	0.421569	2.372
-0.0612	-0.2346	0.004	0.055	0.059	0.431373	2.318
1.734	-2.346	3.007	5.504	8.51	0.441176	2.267
1.2546	-1.0812	1.574	1.169	2.743	0.45098	2.217
0.2448	-0.7548	0.06	0.57	0.63	0.460784	2.17
-0.4488	-1.275	0.201	1.626	1.827	0.470588	2.125
0.7038	-0.255	0.495	0.065	0.56	0.480392	2.082
0.9282	-0.306	0.862	0.094	0.955	0.490196	2.04
1.7442	-0.0112	3.042	0	3.042	0.5	2