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Mass Transfer and Radiative Heat Transfer Flow of MHD Casson Fluid with Temperature Gradient Dependent Heat Sink and Internal Mass Diffusion in a Vertical Channel with Stretching Porous Walls

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Abstract

In the present paper an investigation of the flow, heat and mass transfer characteristics of a MHD Casson fluid in a parallel plate channel with stretching walls with respect to a uniform magnetic field and porous media is carried out. Similarity transformation technique is used to transform the system of partial differential equations into ordinary differential equations. The governing non-linear differential equations are solved numerically using Runge-Kutta method of fourth order via shooting technique. Graphs for the velocity, temperature and concentration are plotted to analyze the behavior of the flow with different physical parameters. The skin friction, Nusselt number and Sherwood numbers are computed and tabulated

Keywords: Casson Fluid, Porous Media, MHD, Heat and Mass Transfer, Stretching Walls

Nomenclature:

u and v	Velocity Components in x and y Directions
ρ	Density
p	Pressure
υ	Kinematic Viscosity
σ	Electrical Conductivity
B_0	Strength of the Magnetic Field
T	Temperature at the Plate
c_p	Specific Heat
q	Radiative Heat Flux
C	Concentration of Fluid
D	Mass Diffusivity
σ^*	Stephan-Boltzman Constant
k^*	Mean Absorption Coefficient
μ_B	Plastic Dynamic Viscosity of the Non-Newtonian Fluid
τ_y	Yield Stress of the Fluid
π	Product Component of Deformation Rate with Itself
e_{ij}	(i,j)th Component of Deformation Rate
π_c	Critical Value of π Based on Non-Newtonian Model.
β	non-Newtonian Casson Parameter
Q'	Temperature Gradient Dependent Heat Sink/Source Parameter
K	Chemical Reaction Parameter.
k_2	Permeability parameter
Re	Stretching Reynolds Number
Mn	Hartman Number
Sc	Schmidt Number
Pr	Prandtl Number
Sh	Sherwood Number
Nu	Nusselt Number

Introduction

Many researchers' interest is to know the varied applications of boundary layer flow behaviors of non-Newtonian fluids in natural sciences, engineering sciences and industry. The mutual coordination between motion of the fluid and magnetic field is required feature of the physical situation in the MHD fluid flow problems. Principles of MHD are required in the design of power generators, pumps, radar systems and flow meters. MHD parameter is one of the most important required parameter by which the cooling rate can be controlled. Besides non-Newtonian fluid have variety of applications in industry and engineering moreover in the extraction of crude oil from petroleum products.

In this regard Bhattacharyya and Layek [1] have investigated the slip effect on diffusion of chemically reactive species in boundary layer flow over a vertical stretching sheet with suction or blowing. Awad et al. [2] have studied heat and mass transfer in unsteady rotating fluid flow with binary chemical reaction and activation energy. Boyd et al. [3] have discussed the analysis of the casson and carreau yasuda non-Newtonian blood models in steady and oscillatory flow using the lattice Boltzmann method. Makinde [4] has studied MHD mixed-convection interaction with thermal radiation and nth order chemical reaction past a vertical porous plate embedded in a porous medium. Misra et al. [5] have analyzed the flow and heat transfer of a MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall: some applications to haemodynamics. Ashraf et al. [6] have studied MHD non-Newtonian micro-polar fluid flow and heat transfer in a channel with stretching walls. Raftari and Vajravelu [7] have made use of homotopy analysis method to study MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall. Anjalidevi and Kandsamy [8] have investigated the effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. Hayat et al. [9] have discussed MHD flow and mass transfer of a Jeffery fluid over a nonlinear stretching surface. Shehzad et al. [10] have analyzed the effects of mass transfer on MHD flow of casson fluid with chemical reaction and suction. Shaw and Sibanda [11] have studied thermal instability in a non-Darcy porous medium saturated with a Nano fluid and with a convective boundary condition. Bhattacharyya et al. [12] have discussed the analytic solution for magneto hydrodynamic boundary layer flow of casson fluid over a stretching /shrinking sheet with wall mass transfer. Nadeem et al. [13] have studied MHD three dimensional casson fluid flows past a porous linearly stretching sheet. Kirubhashankar [14] has analyzed the casson fluid flow and heat transfer over an unsteady porous stretching surface. Hasanuzzaman 15] has made a study on similarity solution of unsteady combined free and forced convective laminar boundary layer flow about a vertical porous surface with suction and blowing. Kameswaran et al. [16] have examined a new algorithm for internal heat generation in Nano fluid flow due to a stretching sheet in a porous medium. Sibanda et al. [17] have studied the dual solutions of casson fluid flow over a stretching or shrinking sheet. Hayat et al. [18] have studied three dimensional stretched flow of Jeffrey fluid with variable thermal conductivity and thermal radiation. Islam et al. [19] have analyzed the MHD free convection and mass transfer flow with heat generation through an inclined plate. Rahman et al. [20] have examined the thermophoresis effect on MHD forced convection on a fluid over a continuous linear stretching sheet in presence of heat generation and power-law wall temperature. Khalid et al. [21] have studied unsteady MHD free convection flow of casson fluid past over an oscillating vertical plate embedded in a porous medium. Thiagarajan and Senthilkumar [22] have observed the DTM-Pade approximations of MHD boundary layer flow of a casson fluid over a shrinking sheet. Nadeem et al. [23] have made an investigation on MHD three dimensional casson fluid flows past a linearly stretching sheet. Rizwan et al. [24] have studied the convective heat transfer and MHD effects on casson Nano fluid flow over a shrinking sheet. Akbar [25] has examined the influence of magnetic field on peristaltic flow of a casson fluid in an asymmetric channel: application in crude oil refinement. Sinha and Shit [26] have studied electro magneto hydrodynamic flow of blood and heat transfer in a capillary with thermal radiation. Swati Mukhopadyaya [27] has investigated the effects of thermal radiation on casson fluid flow and heat transfer over an unsteady stretching surface subjected to suction/ blowing. Mukhopadyaya et al. [28] have investigated MHD boundary layer flow of casson fluid passing through an exponentially stretching permeable surface with thermal radiation. Khalid et al. [29] have analyzed the exact solutions for unsteady free convection flow of a casson Nano fluid past a linearly stretching sheet with convective boundary conditions. Nadeem et al. [30] have studied MHD three dimensional boundary layer flow of casson Nano fluid past a linearly stretching sheet with convective boundary conditions. Riwaz UI Haq et al. [31] have studied the convective heat transfer in MHD slip flow over a stretching surface in the presence of carbon Nano tubes. Khalid [32] has investigated the exact solutions for unsteady free convection flow of a casson fluid over an oscillating vertical plate with constant wall temperature.

Mathematical Model

Consider the two dimensional steady laminar flow of a casson fluid in a parallel plate channel with stretching walls in the presence of a transverse magnetic field. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The constitutive equation for the casson fluid can be written as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{\tau_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{\tau_y}{\sqrt{2\pi}}\right) e_{ij} & \pi < \pi_c \end{cases} \quad (1)$$

Using equation (1) the governing equations of the flow heat and mass transfer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{\rho}u - \frac{\upsilon}{k'}u \quad (3)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 v}{\partial y^2} \quad (4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho Cp}\frac{\partial q_r}{\partial y} + Q'\frac{\partial T}{\partial y} \quad (5)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1(c - c_2) \quad (6)$$

The boundary conditions are

$$u = bx, \quad v = 0, \quad T = T_1, \quad C = C_1, \quad y = 0$$
$$u = bx, \quad v = 0, \quad T = T_2, \quad C = C_2, \quad y = \infty \quad (7)$$

Where b >0 is for the stretch of the channel walls.

Using Roseland approximation the radiative heat flux can be taken as $q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$ (8)

Now assuming that the temperature difference within the flow is such that T^4 can be expressed as linear function of temperature .With Taylor series expansion about T_∞ and neglecting higher order terms we get

$$T^4 \approx 4T_\infty^3 T - T_\infty^4 \quad (9)$$

Now introduce the following similarity variables to convert the governing partial differential equations into ordinary differential equations

$$u = bxf'(\eta), \quad v = -abf(\eta) \quad \eta = \frac{y}{a} \quad 10(a)$$

$$\theta(\eta) = \frac{T - T_2}{T_1 - T_2} \quad \varphi(\eta) = \frac{C - C_2}{C_1 - C_2} \quad 10(b)$$

Eliminating pressure gradient from (2) and (3) and using (8) – 10(b), equations (2) – (6) are reduced to the following form

$$\left(1+\frac{1}{\beta}\right)f''' = \mathrm{Re}\{(f')^2 - ff''\} - (M_n^2 + k_2)f' \quad (11)$$

$$(1 + Nr)\theta'' + \mathrm{Re}\,\mathrm{Pr}\,\theta'(f + Q') = 0 \quad (12)$$

$$\varphi'' - Sc\,\mathrm{Re}(K\varphi - f\varphi') = 0 \quad (13)$$

The corresponding boundary conditions are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \varphi(0) = 1, \quad \text{as} \quad y = 0 \quad 14(a)$$
$$f(\infty) = 0, \quad \theta(\infty) = 0, \quad \varphi(\infty) = 0, \quad \text{as} \quad y = \infty \quad 14(b)$$

Where $\beta = \mu_B\frac{\sqrt{2\pi_c}}{Py}$, $\mathrm{Re} = \frac{a^2 b}{\upsilon}$, $Nr = \frac{16\sigma^* T_\infty^3}{3kk^*}$, $M_n^2 = \frac{\sigma}{\mu}B_0^2 a^2$, $\mathrm{Pr} = \frac{\mu c_p}{k}$,

$$k_2 = \frac{\upsilon}{k'} \qquad Sc = \frac{\upsilon}{D}, \qquad K = \frac{k_1}{b}$$

Skin Friction Coefficient
The skin friction that arise owing to the viscous drag in the vicinity of the plate is calculated as

$$C_f = \frac{\tau_w}{\mu \frac{bx}{a}} = f''(-1), \qquad \text{where } \tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=-a} = \mu \frac{bx}{a} f''(-1) \tag{15}$$

Heat Transfer Coefficient
The rate of heat transfer between the fluid and the walls is evaluated through the non-dimensional Nusselt number.
The Nusselt number is given by

$$Nu = \frac{q_w}{\frac{k}{a}(T_1 - T_2)} = \theta'(-1) \qquad \text{Where } q_w = k\left(\frac{\partial T}{\partial y}\right)_{y=-a} = \frac{k}{a}(T_1 - T_2)\theta'(-1) \tag{16}$$

Mass Transfer Coefficient
The rate of mass transfer coefficient between the fluid and the walls is derived and calculated in terms of Sherwood number which is given by

$$Sh = \frac{m_w}{\frac{D}{a}(C_1 - C_2)} = \varphi(-1), \text{ where } m_w = D\left(\frac{\partial C}{\partial y}\right)_{y=-a} = \frac{D}{a}(C_1 - C_2)\varphi(-1) \tag{17}$$

The governing equations (11) – (13) along with the boundary conditions 14(a) and 14(b) are solved numerically by using Runge-Kutta method of fourth order along with shooting technique.

Results and Discussion
Effect of Reynolds number Re on the velocity profile is shown in fig. 1. It has been clearly observed from graph that, the velocity decreases with increasing values of Reynolds number Re. In Fig. 2 the effect of non-Newtonian casson parameter β on the velocity profile is shown. It is noticed from the graph that thickness of the boundary layer decreases with increasing values of casson parameter β .Variation of velocity profile for different values of magnetic parameter Mn is shown in fig. 3. It is observed from the graph that, the velocity decreases with increasing values of magnetic parameter Mn. Graph of temperature profile for different values of temperature gradient parameter Q' is shown in fig. 4. It is noticed from the figure that, thickness of the boundary layer decreases with increasing values of temperature gradient. Variation of temperature profile for different values of Prandtl number Pr is shown in fig. 5. With increasing values of Prandtl number Pr the temperature profile decreases. Fig. 6 depicts the effect of temperature profile for different values of radiation parameter Nr. As we increase the values of radiation parameter Nr, the temperature profile increases. Fig. 7 illustrates the variation of temperature profile for different values of Reynolds number Re. It has been noticed from graph that, the temperature profile decreases with an increasing value of Reynolds number Re. Effect of concentration profile for different values of Reynolds number Re is shown in fig. 8. It is clearly observed from graph that, the thickness of boundary layer decreases with an increasing value of Reynolds number Re. Changes in concentration profile for different values of Schmidt number Sc is shown in fig. 9. Concentration profile decreases with an increasing value of Schmidt number Sc. Variation of concentration field for different values of chemical reaction parameter K is shown in fig.10. As we increase the chemical reaction parameter K the concentration profile decreases.

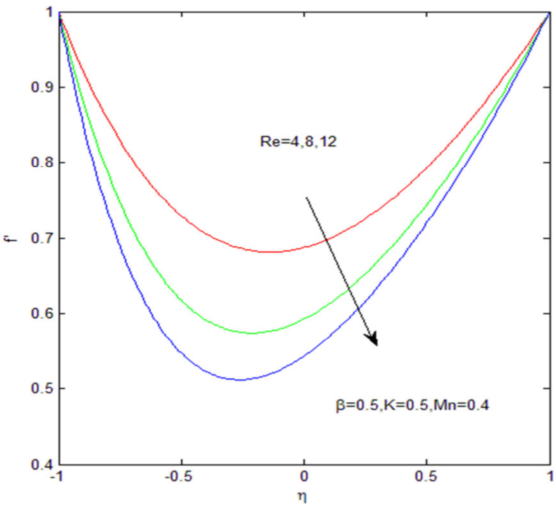

Fig.1 Velocity Profiles for Different Values of Reynolds Number Re

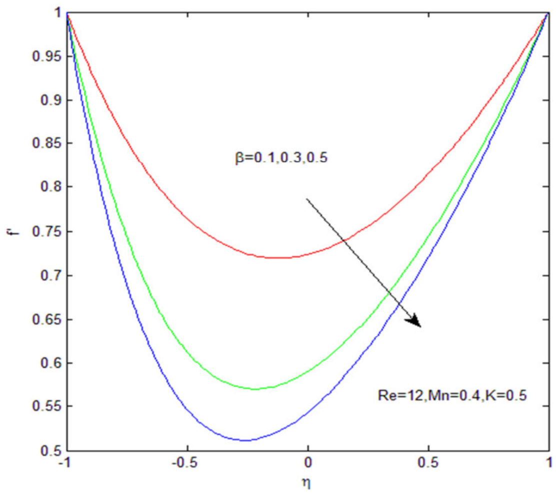

Fig.2 Velocity Profiles for Different Values of Casson Parameter β

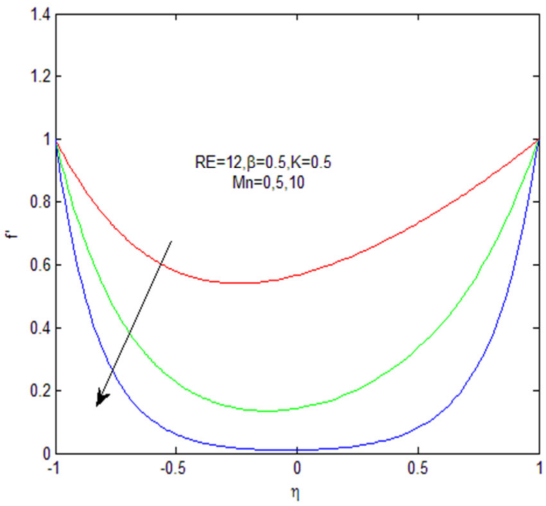

Fig.3 Velocity Profiles for Different Values of Magnetic Parameter Mn

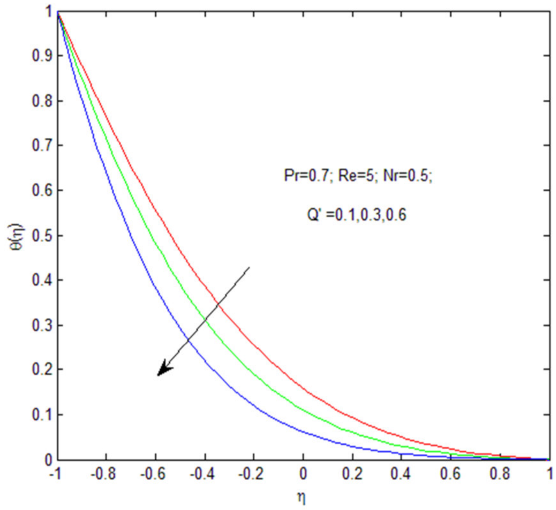

Fig.4. Temperature Profiles for Different Values of Temperature Gradient Heat Sink Parameter

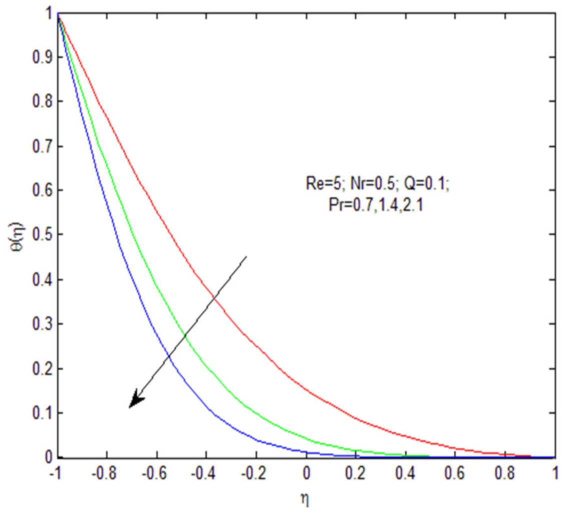

Fig.5 Temperature Profiles for Different Values of Prandtl Number Pr

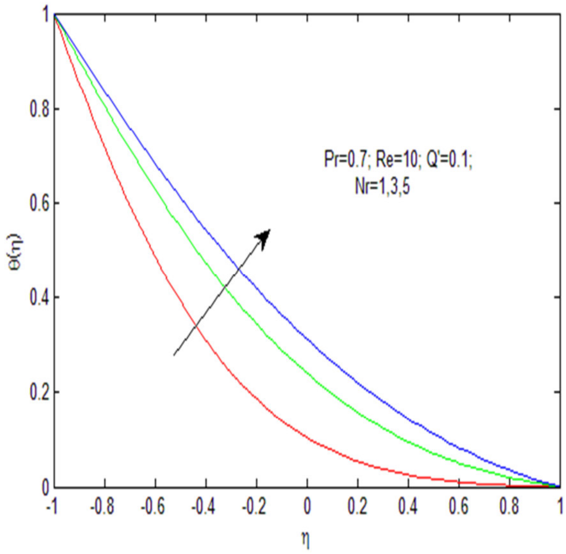

Fig.6, Temperature Profiles for Different Values of Radiation Parameter Nr

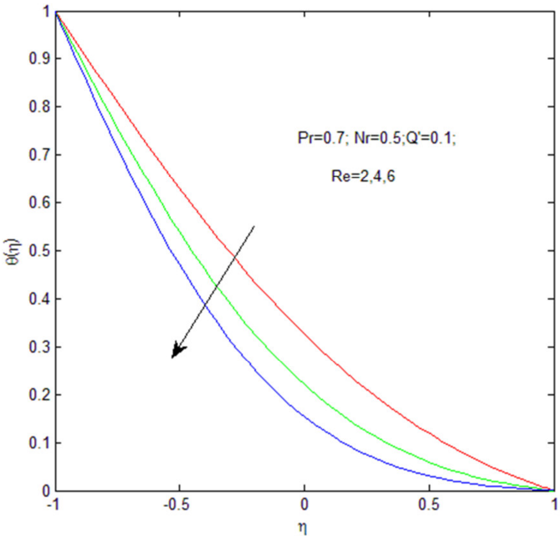

Fig.7. Temperature Profiles for Different Values of Reynolds Number Re

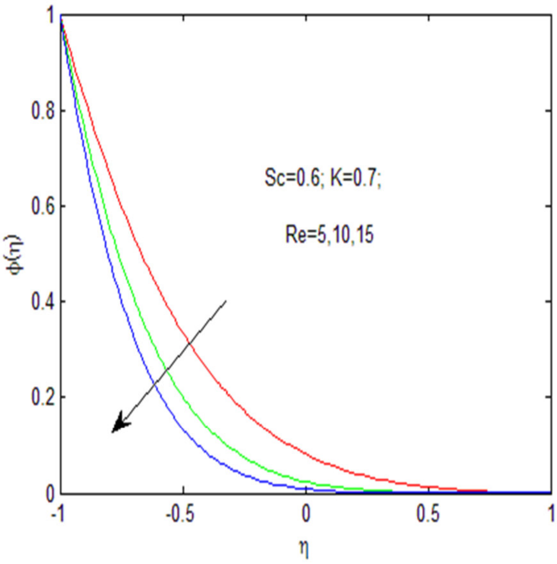

Fig.8, Concentration Profiles for Different Values of Reynolds Number Re

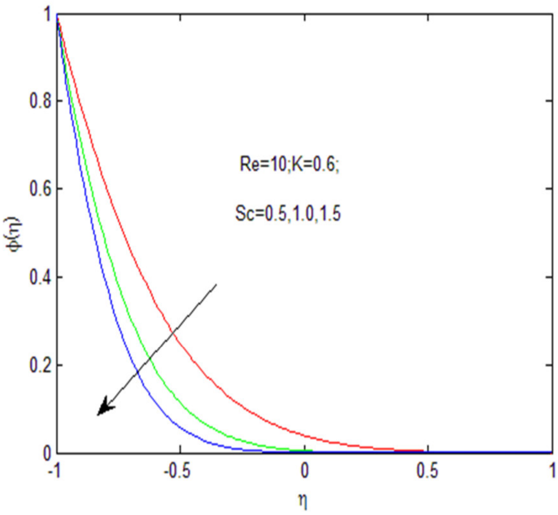

Fig.9, Concentration Profiles for Different Values of Schmidt Number Sc

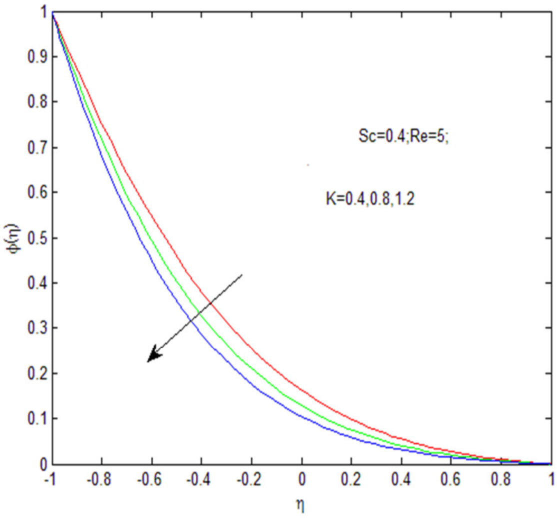

Fig.10 Concentration Profiles for Different Values of Chemical Reaction Parameter K

Table 1. The values of skin friction, Nusselt number and Sherwood number on the lower wall for different parameters.

Re	β	M	Nr	Pr	Sc	K	Q	K2	f''(-1)	θ'(-1)	φ'(-1)
5 10 15	0.6	0.5	0.1	0.7	0.5	0.6	0.1	0.1	-1.156 -1.7577 -2.2223	-1.4677 -2.1593 -2.746	-1.6288 -2.2773 -2.7754
10	0.1 0.3 0.5	0.5	0.1	0.7	0.5	0.6	0.1	0.1	-0.6864 -1.2997 -1.6362	-2.3235 -2.2273 -2.1769	-2.3695 -2.3143 -2.2868
10	0.6	0 5 10	0.1	0.7	0.5	0.6	0.1	0.1	-1.7273 -3.6539 -6.4517	-2.1648 -1.8093 -1.4992	-2.2802 -2.1198 -2.004
10	0.6	0.5	0.1 0.3 0.6	0.7	0.5	0.6	0.1	0.1	-1.7577 -1.7577 -1.7577	-2.1593 -1.9389 -1.6991	-2.2773 -2.2773 -2.2773
10	0.6	0.5	0.1	0.7 1.4 2.1	0.5	0.6	0.1	0.1		-2.1593 **-3.4119** -4.4933	
10	0.6	0.5	0.1	0.7	0 0.3 0.6	0.6	0.1	0.1		-2.1593 -2.1593 -2.1593	
10	0.6	0.5	0.1	0.7	0.5	0.5 1 1.5	0.1	0.1			-2.1658 -2.6794 -21.59
10	0.6	0.5	0.1	0.7	0.5	0.6	0.1 0.3 0.6	0.1			-2.2773 -2.2773 -2.2773

Conclusions

An analysis of MHD flow of Casson fluid in vertical channel and the associated problem of heat and mass transfer have been investigated with internal mass diffusion and radiative temperature gradient dependent heat sink/source .

The velocity decreases with an increasing values of Re, Mn, β and velocity attains same position with an increasing the values of Pr, Nr, Sc and K.

The temperature decreases with an increasing values of Re, Pr, and temperature increases with an increasing values of Nr, Mn, Q', β and attains uniform thickness of the boundary layer for Sc and K.

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