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Stagnation Point Flow of Non-Newtonian Fluid and Heat Transfer over a Stretching/Shrinking Sheet in a Porous Medium

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Abstract
The Study of stagnation Point flow and heat transfer Phenomena is considered in this paper. We used suitable similarity transformation to reduce governing partial differential equations into ordinary differential equations. These equation are non-linear differential equations which are solved numerically using Runge Kutta Fourth order method with efficient shooting technique .The flow and temperature behaviour are analysed through graphs .The numerical values of skin friction coefficient and Local Nusselt are calculated, tabulated and discussed.
Keywords: Casson Fluid, Stagnation point flow, Heat transfer, Shrinking/streaching sheet, Skin Friction coefficient, Porous medium.

1. Introduction

Stagnation point flow generally describes the fluid motion near the stagnation region of a solid surface which exists in the case of fixed as well as moving body in a fluid. Stagnation point flow with various physical effects has greater importance, including the prediction of skin friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows; design of thrust bearings & radial diffusers, & drag reduction ,transpiration cooling & thermal oil recovery among others. The two dimensional flow of a fluid near stagnation point is a classical problem in fluid dynamics & its solution was first proposed by Hiemenz[1]

Porous materials such as sand & crushed rock underground are saturated with water which under the influence of local pressure gradients migrate & transport the liquid through the material. Such transport properties of fluid saturated porous materials are very important in the petroleum & geothermal industries Attia [2]; jat and chaudhary [3], pal and hiremath [4],Bhattacharyya & layek[5],singh and pathak[6],mukhopadhayay and layek[7], and Ram et al. [8], studied the boundary layer flow near the stagnation point of a stretching sheet through porous and non-porous boundaries under different physical situation .Very recently rakesh kumar and shilpa sood[9] studied the non-linear convection stagnation point heat transfer & MHD fluid flow in porous medium towards a permeable shrinking sheet.

On the other hand , the boundary layer flows of non-newtonian fluids caused by a stretching /shrinking sheet have vast application in several manufacturing processes such as extrusion of molten polymers through a slit die for the production of plastics sheets, hot rolling , wire & fiber coating, processing of food stuffs, metal spinning glass fibre & paper production . During such processes the rate of cooling has an important bearing on the properties of the final product . Hence , the quality of the final product depends on the rate of heat transfer.

Casson fluid is one of such non-newtonian fluids, which behaves like an elastic solid & for this fluid, a yield shear stress exists in the constitutive equations. Fredrickson(1964) investigated the steady flow of a casson fluid in a tube ; Nadeem et al [10] considered the flow analysis of casson fluid with uniform magnetic field under the influence of exponentially stretching sheet & obtained the solution by adomain decomposition method using the pade's approximation . Hayat et al [11] studied the soret & Doufour effects on , magneto hydrodynamic flow of casson fluid. Bhattacharya et al [12] studied the slip effects on parametric space and solution for the boundary layer flow due to non-porous stretching and shrinking sheet. Recently Nandeppanavar [13] studied the flow & heat transfer analysis of casson fluid due to a stretching sheet & given an analytic solution , Mahanta and Shaw [14] studied the mixed convection stagnation point flow of casson fluid with convective Boundary conditions .
In the present paper we discussed and investigated the stagnation point flow and heat transfer analysis of casson fluid over a stretching /shrinking sheet in a porous media .The governing partial differential equations are converted into the non-linear ordinary differential equations using suitable similarity transformations. The trsnsformed ODEs are then solved numerically using Rungekutta fourth order method with efficient numerical shooting method. Results of flow and heat transfer for the variation of the physical parameters are discussed elaborately.

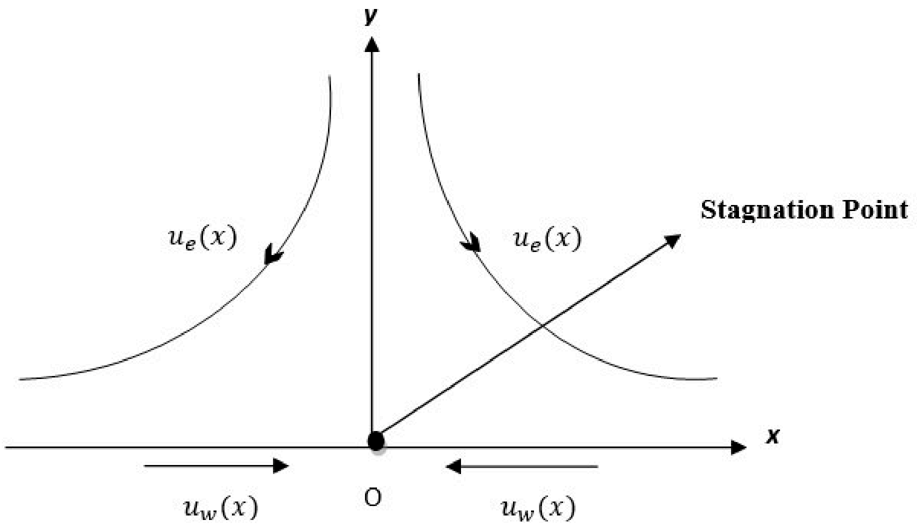
2. Flow analysis

Consider the steady two dimensional stagnation point flow of incompressible casson fluid induced by a stretching/shrinking sheet in porous media as shown in fig.1 .The cartesian co-ordinates x & y are taken with the origin O at the stagnation point , and are defined such that the x-axis is measured along the stretching /shrinking sheet and the y-axis is measured normal to it. It is assumed that the velocity of the external flow is given $u_e(x) = ax$, Where a >0 is the strength of the stagnation flow & the surface temperature T_w is a constant . It is also assumed that the velocity of stretching/shrinking sheet is given by $U_w(x) = bx$, where b is the stretching rate , with b>0 & b<0 are for stretching & shrinking case respectively.The boundary layer equations for the flow in the porous medium can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \left(1+\frac{1}{\beta}\right)\frac{\upsilon}{k_1}(u_e - u) \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$



Where u and v are the velocity components of the fluid in x and y directions respectively and υ is the kinematic viscosity , β is the casson parameter (Non-Newtonian parameter) and k_1 us the permeability of the porous medium.

The boundary conditions for the problem are,

$$u_w(x) = bx \qquad T = T_w \qquad \text{at } y = 0$$
$$u_e(x) = ax \qquad T = T_\infty \qquad \text{at } y \to \infty \quad (4)$$

To obtain the ODEs of (2) and (3) we introduce the following similarity variables.

$$\eta = \left(\frac{u_e x}{\alpha}\right)^{1/2}\frac{y}{x} \quad , \psi = (\alpha x u_e)^{\frac{1}{2}} f(\eta) \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

Where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ which identically satisfy Eq.(1) and substituting the above transformations Eq.(5) in Eqns. (2) and (3) we obtain ,

$$\left(1+\frac{1}{\beta}\right)[Prf''' + K(1-f')] + ff'' - f'^2 + 1 = 0 \quad (6)$$

$$\theta'' + f\theta' = 0 \quad (7)$$

Similarly the boundary conditions (4) reduces to

$$\left.\begin{aligned} f'(\eta) = \frac{b}{a} = c\,, f(\eta) = 0\,, \theta(\eta) = 0 \; at\; \eta = 0 \\ f'(\eta) \to 1, \theta(\infty) \to 0 \; as\; \eta \to \infty \end{aligned}\right\} \quad (8)$$

where primes denote the differentiation w.r.t η,$P_r = \frac{\upsilon}{\alpha}$is the Prandle number and $K = \frac{\upsilon}{a k_1}$ is the porosity parameter of the porous medium.

3. Numerical method for solution

Eqs. (6) and (7) along with the boundary conditions (8) are converted into five first order differential equations as initial value problems . In this method it is necessary to choose a suitable finite value of $\eta \to \infty$ say η_∞. In order to integrate the IVPs the value for $f''(0)$ and $\theta'(0)$ are required , but no such values are given at the boundary.the suitable guess value for *f''(0)* and $\theta'(0)$ are chosen and then integration is carried out . then the calculated values for f' and θ at η_∞ are compared with the given boundary conditions $f'(\infty) = 1$ and $\theta(\infty) = 0$ and the estimated values *f''(0)* and $\theta'(0)$ are adjusted to give a better approximation for the solution . The values for *f''(0)* and $\theta'(0)$ are taken and by using the Fourth Order Classical Runge Kutta method the problem is solved . the above procedure is repeated until the asymptotically converged results within a tollerence level 10^{-5} are obtained and the step size chosen is $\Delta\eta = 0.001$

4. Results And Discussions

The numerical computations have been carried out using above described shooting method for several values of physical parameters such as the velocity ratio parameter c , porosity parameter K, casson parameter β and the Prandle number Pr.then acquired results are presented in graphs Fig.2-Fig.12.to study the velocity and temperature fields.Fig.2 and Fig.3 shows the influence of the casson parameter β on the velocity and temperature profiles . we observe that the magnitude of velocity in the boundary layer decreases with an increase in the casson fluid parameter β . It is noticed that when casson parameter approaches infinity the problem will reduce to a Newtonian case. Hence increasing the value of casson parameter β , decreases the velocity and boundary layer thickness. The dimensionless temperature increases as decreasing function of casson parameter β . Fig.4 and Fig.5 represent the effects of porosity parameter K on the dimensionless velocity and temperature profiles. It is observed that the velocity increases and the temperature decreases as the porosity parameter increases. Fig.6 and Fig.7 represents the influence of the Prandle number Pr on the dimensionless velocity and temperature profiles. And it is observed that an increase in Pr decreases the velocity and temperature profiles.Fig.8 represents the influence of the velocity ratio parameter c on the velocity profile. And here it is observed that the velocity increases with increasing magnitude of c. Fig.9 and Fig. 10 represents the variation of the skin friction coefficient and the temperature profile gradient with parameter c for β =0.5,1.0,1.5. It is observed that with increasing values of casson parameter β the skin friction coefficient and the temperature profile gradient decreases . Fig.11 and Fig.12 represents the variation in skin friction coefficient and the temperature profile gradient with c for K=0.5,1.0,1.5. Here it is observed that as the porosity of the porous medium increases the skin friction coefficient also increases but the temperature gradient profile decreases.

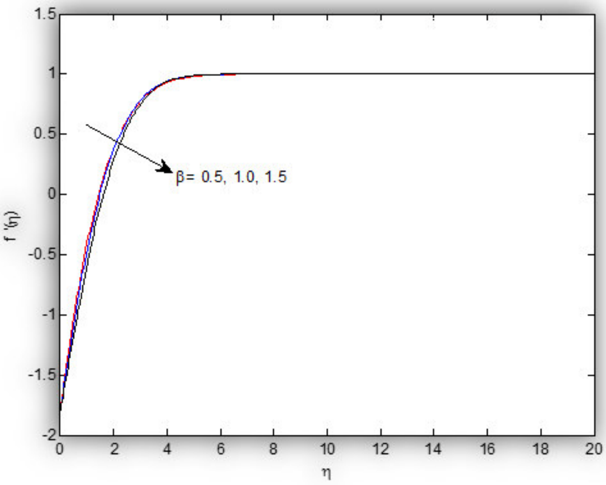


Fig 2. Variation of velocity profile for various values of β when Pr=1.0, K=0.5, C=-1.8

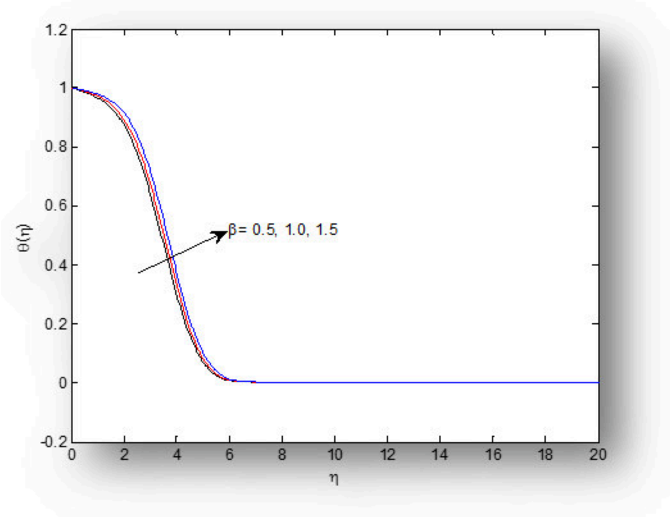


Fig .3 Variation of temperature profile for various values of β when Pr=1.0, K=0.5, C=-1.8

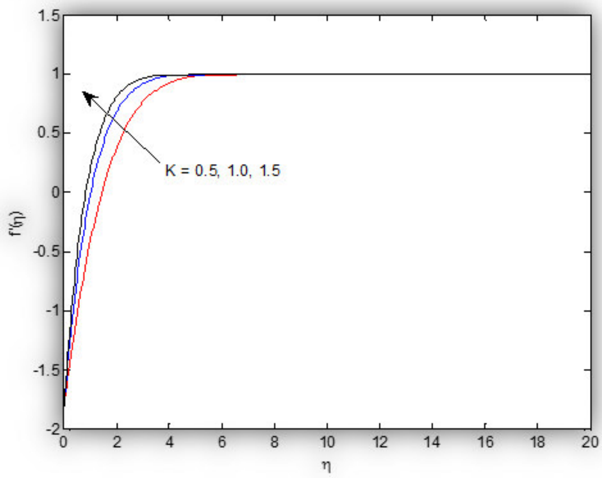


Fig .4 Variation of velocity profile for various values of K when Pr=1.0, β =0.5, C=-1.8

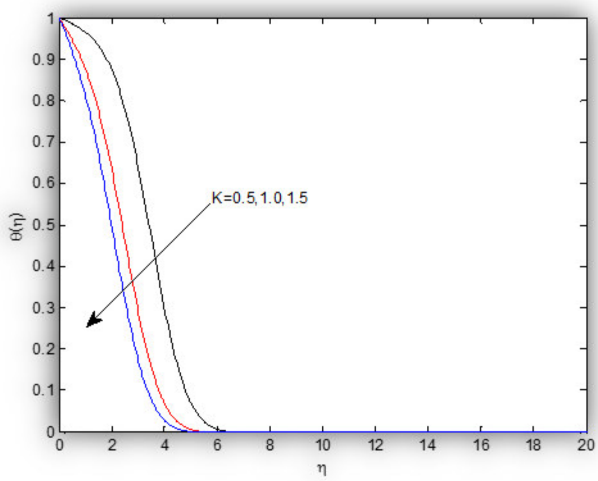


Fig 5. Variation of temperature profile for various values of K when Pr=1.0, β =0.5, C=-1.8

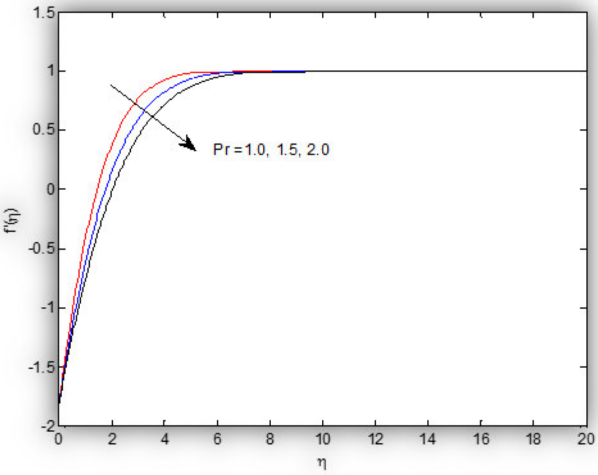


Fig 6. Variation of velocity profile for various values of Pr when K=0.5, β=0.5, C=-1.8

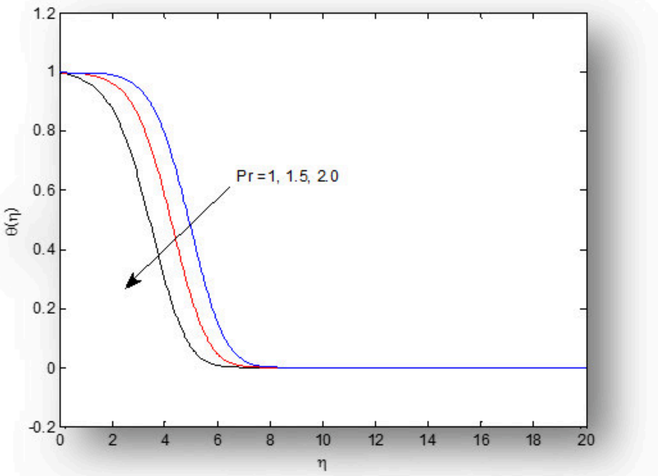


Fig 7. Variation of temperature profile for various values of Pr when K=0.5, β=0.5, C=-1.8

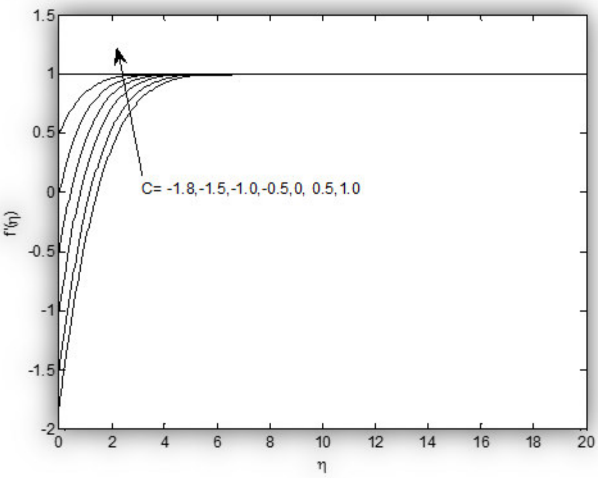


Fig 8. Variation of velocity profile for various values of C when K=0.5, Pr=1.0, β=0.5, C=-1.8

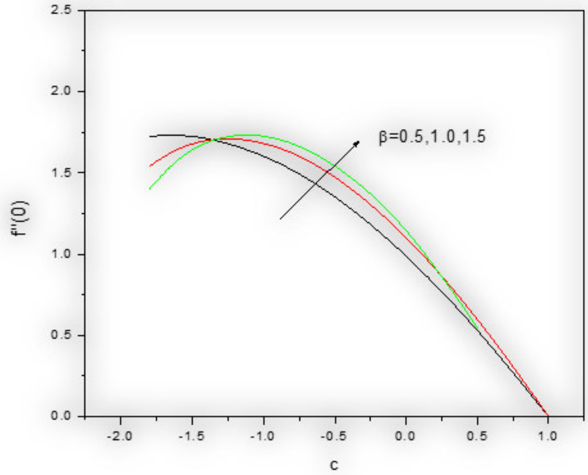


Fig 9.Variation of skin friction coefficient f''(0) with c for β=0.5,1.0,1.5 and Pr=1.

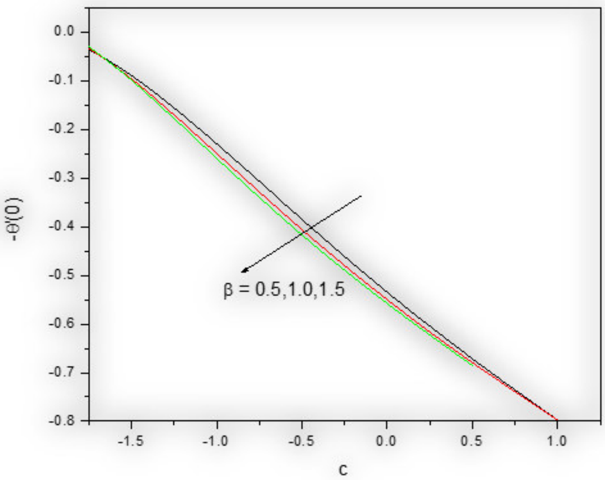


Fig .10Variation of temperature gradient profile $\theta'(0)$ with c for β=0.5,1.0,1.5 and Pr=1.

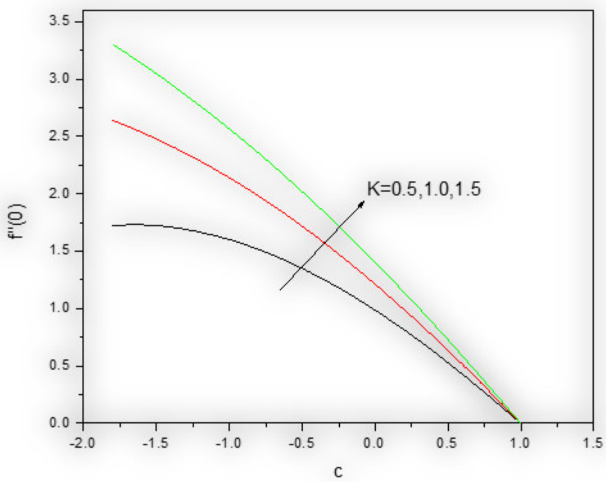


Fig .11Variation of skin friction coefficient f''(0) with c for k=0.5,1.0,1.5 and Pr=1.

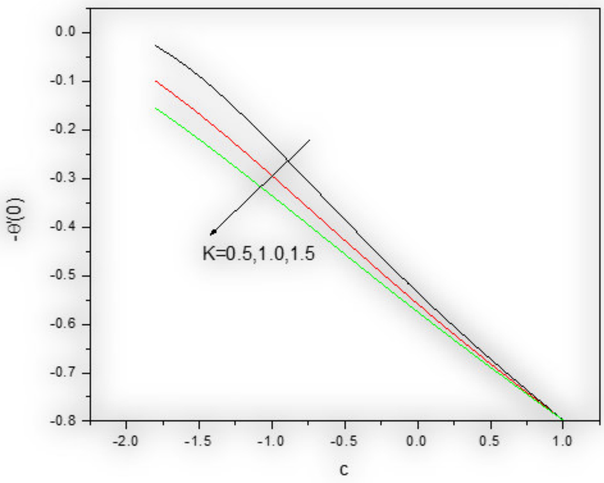


Fig .12Variation of temperature profile $\theta'(0)$ with c for K=0.5,1.0,1.5 and Pr=1.

Conclusion

The main points of the present study are listed below.

- The numerical solution for flow and heat transfer are obtained.
- The effects of casson fluid parameter β and porosity parameter K on flow and temperature are quite opposite.
- Skin friction decreases with an increase in the casson parameter
- The velocity and thermal boundary layer thickness decreases with increasing prandle number.

Nomenclature:

a,b,c	Constants
f	dimensionless stream function
f'	dimensionless velocity
k	thermal conductivity
K	permeability parameter
K_1	permeability of the porous medium
T	fluid temperature
T_w	surface temperature
T_∞	ambient temperature
u,v	velocity components along x and y directions respectively
x,y	Cartesian ao-ordinates along the surface normal to it.respectively

Greek letters:

$\propto$	thermal diffusivity
β	Non-newtonian/casson parameter
η	similarity variable
υ	kinematic viscosity
θ	dimensionless temperature
ψ	stream function

Subscripts:

w	condition at the surface
∞	condition away from the surface

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