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Flow and Heat Transfer of Casson Fluid due to Stretching Sheet with Convective Boundary Condition: An Analytical Solution

Mahantesh M. Nandeppanavar
Department of Studies and Research in Mathematics, Government College, Kalaburagi-585105, Karnataka, India

Abstract
In this Present paper an analytical solution is derived for a two dimensional boundary layer flow and heat transfer of a Casson fluid due to stretching sheet with convective boundary condition. Using closed form of analytical solution of the flow of non-Newtonian Casson fluid, the heat transfer equation is second order nonlinear differential equation with variable coefficients is solved analytically using confluent hypergeometrical series (Power Series) in terms of Kummer's function. The effects of various governing parameters on flow, heat transfer and wall temperature gradient are plotted and discussed.
Keywords: Casson fluid; flow analysis; Heat Transfer; Biot number, Hated Plate, Nusselt number

Nomenclature

b	stretching rate
Bi	Biot number
x	horizontal coordinate
y	vertical coordinate
u	horizontal velocity component
v	vertical velocity component
T	temperature
c_p	specific heat
f	dimensionless stream function
Pr	Prandtl number
l	Characteristic length
$'$	differentiation with respect to η

Greek symbols

η	similarity variable
θ	dimensionless temperature
k	thermal conductivity
μ	viscosity
υ	kinematic viscosity
ρ	density
α	thermal difusivity
β	Casson parameter

Subscripts

w	properties at the plate
∞	free stream condition

1. Introduction

The flow and Heat transfer of non-Newtonian fluids past a stretching sheet have been largely used in several manufacturing industrial engineering processes such as extrusion of molten polymers through the slit die for the production, processing of food stuffs and also in wire and fiber coating. On observing the literature Crane [1] has given the closed form of solution for steady two-dimensional flow incompressible viscous boundary layer flow generated by a stretching surface. Further Crane's works has been extended under various diverse physical aspects. Here we refer only some recent studies on flow heat and mass transfer of non-Newtonian Casson fluid over stretching surfaces. There are so many works available on studies of various fluid flows due to stretching surfaces. But there are few works on casson fluid and heat transfer on stretched surfaces which are solved analytically. Hayat et.al [2] Studied Soret and Dufour effects on MHD flow of casson fluid solution of governing equations are found by homotopy analysis method; here the only PST heating condition is used to analyze heat transfer characteristics. Nadeem et.al [3] studied the MHD flow of casson fluid due to an exponentially shrinking sheet where they used adomain decomposition method (with Pade'sapproximation)for obtaining the solution and they studied only flow analysis. Pramanik [4] studied flow of Casson fluid and Heat transfer past an exponentially porous stretching sheet with thermal radiation, to analyze Heat transfer characteristic author used constant surface temperature condition (CST) and used numerical method for obtaining solution. Bhattacharyya et.al [5] studied exact solution of casson fluid over a permeable stretching/shrinking sheet. But these authors ignored the heat transfer analysis which was very important.

Bhattacharyya [6] studied the MHD stagnation point flow of casson fluid and heat transfer over a stretching sheet with thermal radiation. The author used the constant surface temperature (CST) heating condition to analyze heat transfer and numerical method is used to solve the connected BVP's. Swati et.al [7] studied Casson fluid flow over an unsteady stretching surface. The authors used numerical Method used to obtain the solution. Quasim and Noreem [8] studied flow and heat transfer of casson fluid due to permeable shrinking sheet with viscous dissipation but the CST heating condition is used to analyse heat transfer analysis and Runge-Kutta numerical method is used to obtain solution of governing equations. On observing above, we may notice that

(i) Authors considered Casson fluid model to characterize non-Newtonian fluid behavior.
(ii) Hayat et .al [2] used analytic solution viz HAM and Bhattacharya [6] used closed form of analyse solution for flow of casson fluid only but in all other studies the numerical methods are used to solve the governing differential equations.
(iii) Most of the Authors used CST (constant surface temperature) condition to analyze heat transfer analysis. But no author used convective boundary condition to analyse Heat transfer analysis and solved analytically I power series method.

Whereas Haq et.al [9]studied the convective heat transfer of Casson fluid for nanofluid model and studied the heat transfer analysis for shrinking sheet problem. Hussain et.al [10] studied the flow of Casson nanofluid with viscous dissipation and convective heating boundary condition. Ramesh et.al[11] studied the heat transfer of dusty fluid with convective heating boundary condition. Rahaman et.al[12]studied the mixed convection boundary layer flow past vertically stretching sheet with convective heating condition. Ishak et.al [13] investigated the radiation effects of thermal boundary layer flow with convective heating condition. Rahaman [14] and Rahaman et.al[15] studied the radiative heat transfer in nanofluid with convective heating boundary conditions with variable fluid properties. Pantokratoras [16] investigated the effect of Grashof number on thermal boundary layer past vertical plate with convective heating boundary condition. Merkin et.al [17] investigated the mixed convection effects on the boundary layer flow over vertical plate in a porous medium in a constant convective boundary condition. Kameshwaran et.al [18] obtained the dual solutions of flow and heat transfer of casson fluid due to stretching or shrinking sheet. Makinde [19] studied the effects of variable viscosity on the thermal boundary layer over a permeable plate with radiation and convective surface boundary condition. Makinde and Aziz [21] studied boundary layer flow of nanofluid past a stretching sheet with convective boundary condition. Alsaedi et.al [22] studied the effects of heat generation/absorption on a stagnation point flow of a nanofluid over surface with convective boundary condition. Considering all above studies the no author is obtained analytical solution for the casson fluid flow and heat transfer analysis due to convective surface temperature. But Nandeppanavar [21] studied the flow and heat transfer analysis for two different heating conditions (PST and PHF) analatically.

Hence in this paper we obtained the analytical solution for Casson fluid flow and heat transfer governing equations. The closed form of solution of flow is used to obtain the solution of heat transfer equation by using power series method (Confluent Hypergeometric Series) in terms of Kummer's function (Please See ref [23]).

2. Mathematical Formulation and Analytical Solution Flow analysis:

Consider the flow of an incompressible casson fluid past a stretching sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. Assuming the rheological equation of Casson fluid. Considering the rheological equation of stress transfer (τ) for an incompressible and isotropic flow of non-Newtonian Casson fluid can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}\,, \tag{2}$$

Where u and v are the velocity components of the fluid in x and y directions respectively and υ is kinematic viscosity and β is the Casson parameter (non-Newtonian parameter).

The boundary conditions for the problem are

$$\left.\begin{array}{ll} u_w(x)=bx,\ v=0, & y=0 \\ u\to 0, \quad as & y\to\infty \end{array}\right\} \tag{3}$$

with $b > 0$, the stretching rate. The Eqns. (1) and (2), subjected to the boundary condition (3), admit a self-similar solution in terms of the similarity function f and the similarity variable η defined by

$$u = b\,x\,f'(\eta),\quad v=-\sqrt{b\upsilon},\ \eta=\sqrt{\frac{b}{\upsilon}}\,y. \tag{4}$$

It can be easily verified that Eq. (1) is identically satisfied and substituting the above transformations in Eq. (2) we obtain

$$f'^2-f''f=\left(1+\frac{1}{\beta}\right)f'''. \tag{5}$$

Similarly the boundary conditions (3) can be written as:

$$\left.\begin{array}{lll} f'(\eta)=1, \quad f(\eta)=0 & at & \eta=0 \\ f'(\eta)\to 0, & as & \eta\to\infty \end{array}\right\}. \tag{6}$$

The exact solution of (5), satisfying the boundary conditions (6) is given by:

$$f=\sqrt{1+\frac{1}{\beta}}\left(1-e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right) \quad (\beta \text{ is positive}) \tag{7}$$

3. Solution of Governing Equations of Heat transfer analysis:

The Energy equations with boundary layer approximations can be written as:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where k is the thermal conductivity, ρ is the density of the fluid, C_p is the specific heat at constant pressure .

3.1 Convective temperature boundary condition:

It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of uniform temperature T_f which provides a heat transfer coefficient h_f. Under this assumption the thermal boundary conditions may be written as, ie

The CTBC (Convective temperature boundary condition) is:

$$\left.\begin{array}{lll} -k\dfrac{\partial T}{\partial y}=h_f\left(T_f-T\right) & at & y=0 \\ T\to T_\infty & as & y\to\infty \end{array}\right\}, \tag{9}$$

where T_∞ is the temperature of the fluid far away from the sheet (temperature of ambient cold fluid).

T is the uniform temperature on the top surface of the plate. Hence we have $T_f > T > T_\infty$.

Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}. \tag{10}$$

Where T_f the temperature of the sheet.

Using Eqn. (10), Eqs. (8) and (9) can be written as

$$\theta''+\Pr f\,\theta'=0, \tag{11}$$

$$\left.\begin{array}{lll}\theta'(\eta)=B_i\left(1-\theta(\eta)\right) & \text{at} & \eta=0,\\ \theta(\eta)\to 0 & \text{as} & \eta\to\infty.\end{array}\right\} \tag{12}$$

Where

$\Pr=\dfrac{\mu C_p}{k}$ is the Prandtl number.

$B_i=\dfrac{h}{k}\sqrt{\dfrac{\upsilon}{a}}$ *is the thermal Biot number*

The Biot number is the dimensionless parameter, it plays the fundamental role in conduction problems that involves surface convection effects. Biot number parameter provides the measure of temperature drop in the solid relative to the temperature difference between the surface and fluid.

For Bi <<1 shows the resistance to conduction within the solid is much less than the resistance to the convection across the fluid boundary layer, hence assumption of uniform temperature is reasonable. Whereas Bi>>1 says the temperature difference across the solid is much larger than that between surface and fluid.

Substitute (7) in (11), we obtain

$$\theta''+\Pr\left(\sqrt{1+\frac{1}{\beta}}\left(1-e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\right)\theta'=0 \tag{13}$$

Since this is a second order differential equation with variable coefficients, to solve this differential equation (13), we introduce a new variable:

$$\xi=-\Pr\left(1+\frac{1}{\beta}\right)\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right). \tag{14}$$

Using (14) in (13), we obtain:

$$\xi\frac{d^2\theta}{d\xi^2}+\left(1-\left(\frac{\Pr}{\left(1+\frac{1}{\beta}\right)}\right)-\xi\right)\frac{d\theta}{d\xi}=0 \tag{15}$$

The boundary conditions (12) reduced to

$$\theta'\left(-\Pr\left(1+\frac{1}{\beta}\right)\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\right)=-B_i\left(1-\theta\left(-\Pr\left(1+\frac{1}{\beta}\right)\left(e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right)\right)\right);\quad \theta(\xi\to\infty)\to 0 \tag{16}$$

The solution of (15) in terms of confluent hypergeometric function is given by

$$\theta(\xi)=C_1\,\xi^{\Pr(1+\frac{1}{\beta})}M\left(\Pr\left(1+\frac{1}{\beta}\right)-2,\ \Pr\left(1+\frac{1}{\beta}\right)+1,\ \xi\right). \tag{17}$$

Making use of the boundary conditions (16) and re-writing the solution in variable η we get:

$$\theta(\eta) = \mathrm{C}_1\, e^{-\frac{\mathrm{Pr}}{\sqrt{1+\frac{1}{\beta}}}\eta}\, M\left(\mathrm{Pr}\left(1+\frac{1}{\beta}\right)-2,\ \mathrm{Pr}\left(1+\frac{1}{\beta}\right)+1,\ \ -\mathrm{Pr}\left(1+\frac{1}{\beta}\right)e^{-\frac{\eta}{\sqrt{1+\frac{1}{\beta}}}}\right) \tag{18}$$

Where

$$C_1 = \frac{1}{M\left(\left(1+\frac{1}{\beta}\right)\mathrm{Pr},\left(1+\frac{1}{\beta}\right)\mathrm{Pr}+1,-\left(1+\frac{1}{\beta}\right)\mathrm{Pr}\right)-\frac{1}{B_i}\left[\left(\frac{1}{1+\frac{1}{\beta}}\right)\left(\frac{\left(\left(1+\frac{1}{\beta}\right)\mathrm{Pr}\right)^2}{\left(1+\frac{1}{\beta}\right)\mathrm{Pr}+1}\right)M(b+1,b+2,-b)-\left(\left(\frac{1}{1+\frac{1}{\beta}}\right)b\right)M\left(\left(1+\frac{1}{\beta}\right)\mathrm{Pr},\left(1+\frac{1}{\beta}\right)\mathrm{Pr}+1,-\left(1+\frac{1}{\beta}\right)\mathrm{Pr}\right)\right]}$$

(19)

Where Kummer's function [ ] M is defined by:

$$M(\mathrm{a},\ \mathrm{b},\ \mathrm{z}) = 1+\sum_{n=1}^{\infty}\left(\frac{(a)_n\, z^n}{(b)_n\, n!}\right) \tag{20}$$

where

$$\left.\begin{aligned}(a)_n &= a(a+1)(a+2).....(a+n-1)\\ (\mathrm{b})_n &= b(\mathrm{b}+1)(\mathrm{b}+2).....(\mathrm{b}+n-1)\end{aligned}\right\} \tag{21}$$

The dimensionless wall temperature gradient $\theta'(0)$ is given by:

$$\theta'(0) = C_1\left[(-\lambda b)M(b,b+1;-b)+\left(\frac{b}{b+1}\right)(b\lambda)M(b+1,b+2-b)\right] \tag{22}$$

where

$$\left.\begin{aligned}b &= \left(1+\frac{1}{\beta}\right)\mathrm{Pr}\\ \lambda &= \left(\frac{1}{1+\frac{1}{\beta}}\right)\end{aligned}\right\} \tag{23}$$

4. Parameters of engineering interest:

4.1 Skin friction co-efficient:

$$\text{Skin friction co-efficient} = \frac{\tau_w}{\rho u_w^2(x)} \tag{24}$$

Where τ_w the shear tress or skin friction along the stretching is sheet and is defined as

$$\tau_w = \left(\mu_B + \frac{Py}{\sqrt{2\Pi}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{25}$$

Where

μ is the dynamic viscosity of fluid, μ_B is the plastic dynamic viscosity of the fluid, Py stress of the fluid, Π is product component of deformation rate with itself.

Thus we get the wall skin friction co-efficient as

$$Cf\sqrt{\mathrm{Re}_x} = \left(1+\frac{1}{\beta}\right)f''(0) \tag{26}$$

4.2 Nusselt Number (Heat transfer Coefficient):

The local nusselt number Nu_x is defined as:

$$\left. Nu_x = \frac{xq_x}{\alpha\left(T_w - T_\infty\right)} \right\} \tag{27}$$

Here

$$\left. q_x = \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} \right\} \tag{28}$$

$$\left. Nu_x = -\theta'(0)\sqrt{\mathrm{Re}_x} \right\} \tag{29}$$

$where\ \mathrm{Re}_x = \frac{u_w x}{\upsilon}$ is local Reynolds number , α is the thermal diffusivity

5. Results and Discussion:

Considering the closed form of analytical solution of flow of casson fluid, the heat transfer differential equation of a non-Newtonian fluid has been solved analytically using the power series method (in terms of confluent hypergeometric series, i.e Kummer's function). Geometry of considered problem is given by the Figure (1). The velocity distribution is presented in Figs 2. The temperature distribution is presented through the plots Fig. (2) to Fig.(7). The wall temperature gradient for different values of different parameters analyzed through the plots Fig.(8) to Fig.(13).

Fig1: shows the geometry of the considered problem, which shows the heated plate, flow direction etc.

Fig.2: shows the influence of Casson parameter β on velocity profile. We observe that the magnitude of velocity in the boundary layer decreases with an increase in the Casson fluid parameter β. It is noticed that when Casson parameter approaches infinity, the problem will reduce to a Newtonian case. Hence increasing value of Casson parameter β , decreases the velocity and boundary layer thickness.

Fig. 3:shows the effect of the Casson parameter β on the temperature profiles. The temperature and the thermal boundary layer thickness are increasing as decreasing function of β. Effect of casson parameter it leads to increase the temperature field, it also cause the thickening of the thermal boundary layer due to increase in the elastic stress parameter.

Fig.4, shows the effect of the Prandtl number Pr on temperature profile. On observing this plot we can conclude that the temperature and the thermal boundary layer thickness decrease as the Prandtl number increase.

Fig. (5), Fig.(6) and Fig.(7): shows the effect of the thermal Biot number Bi on the temperature profile, Fig.(5) is plotted for the different Smaller Biot number parameter (Bi< 0.09), Fig.(6) is plotted for the different Biot number (1<Bi<10) and Fig.(7) is plotted for the different higher values of Bi (10<Bi<1000). Temperature profile increases with increasing values of thermal Biot number

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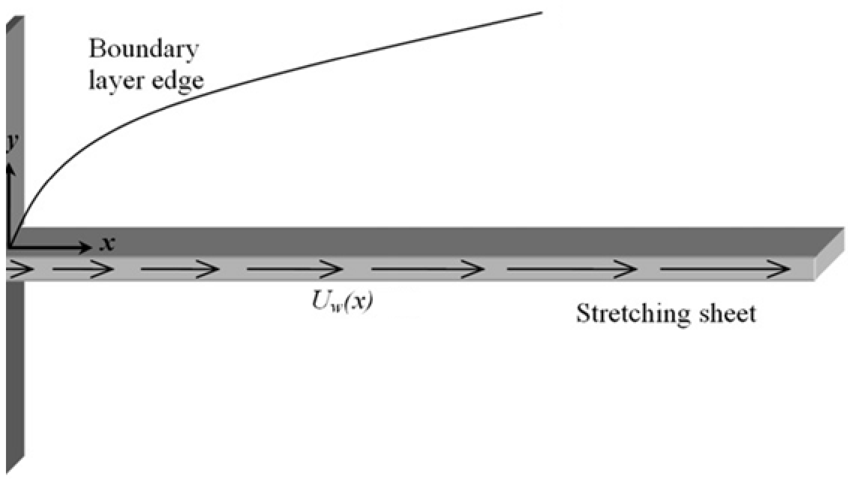

Fig. 1: Geometry of the considered Problem

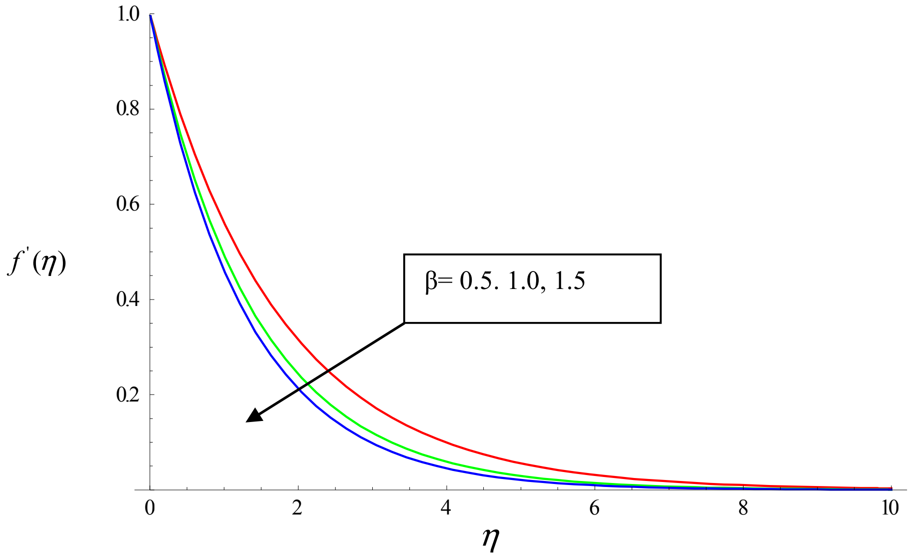

Fig 2: Velocity Profile for different values of β

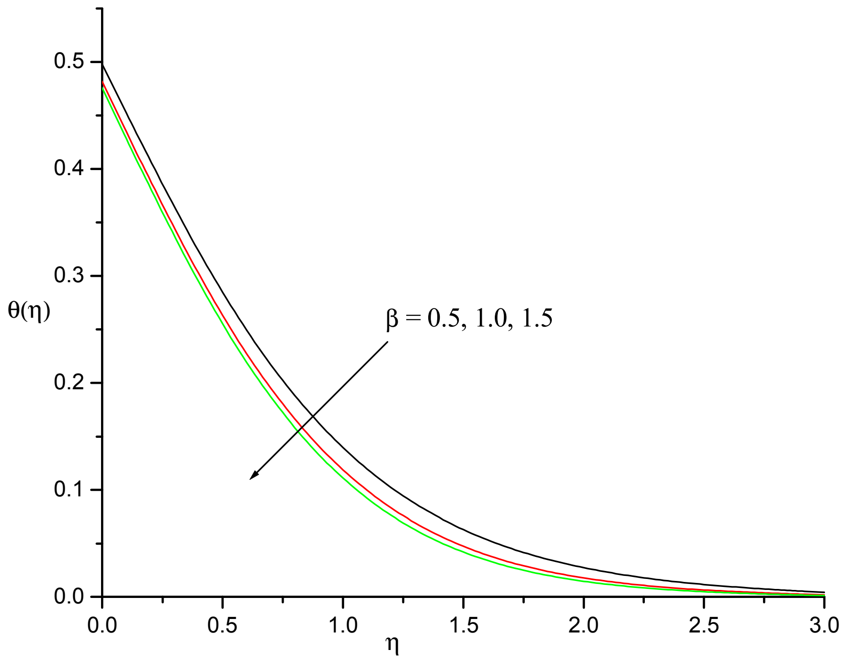

Fig. 3: Temperature Profile for different values of β when Pr=3.0 and Bi=1.0

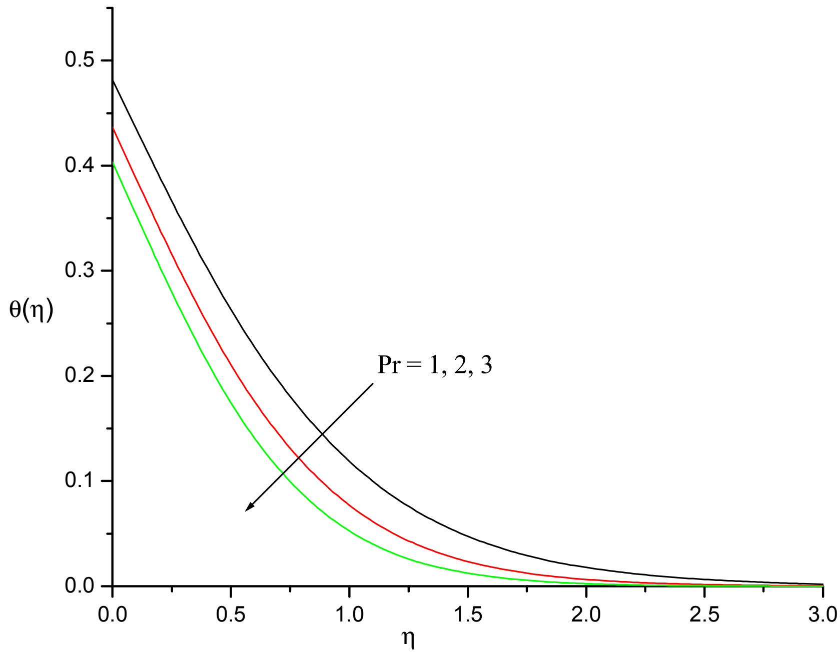

Fig. 4: Temperature Profile for different values of Pr when β =1.0 and Bi=1.0

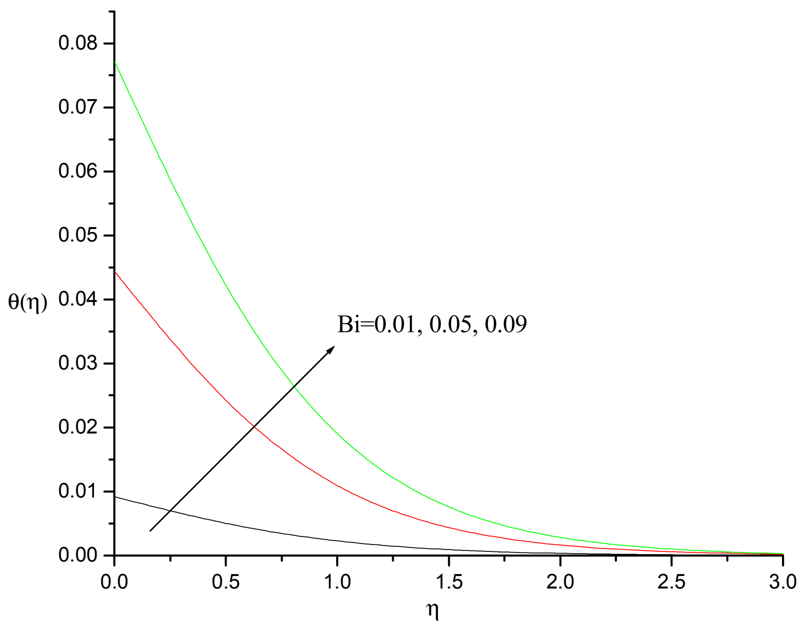

Fig. 5: Temperature Profile for different values of Bi (Bi<1.0) when β =1.0 and Pr =3.0

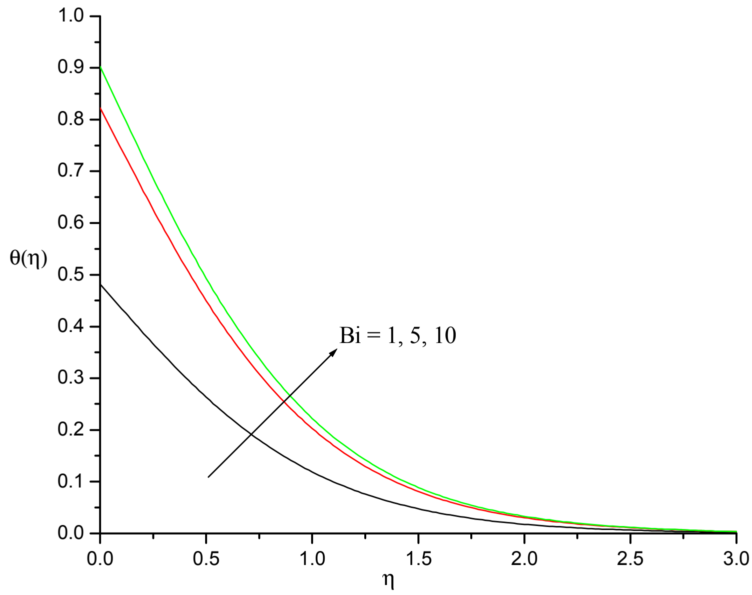

Fig. 6: Temperature Profile for different values of Bi (Bi>.9) when β =1.0 and Pr =3.0

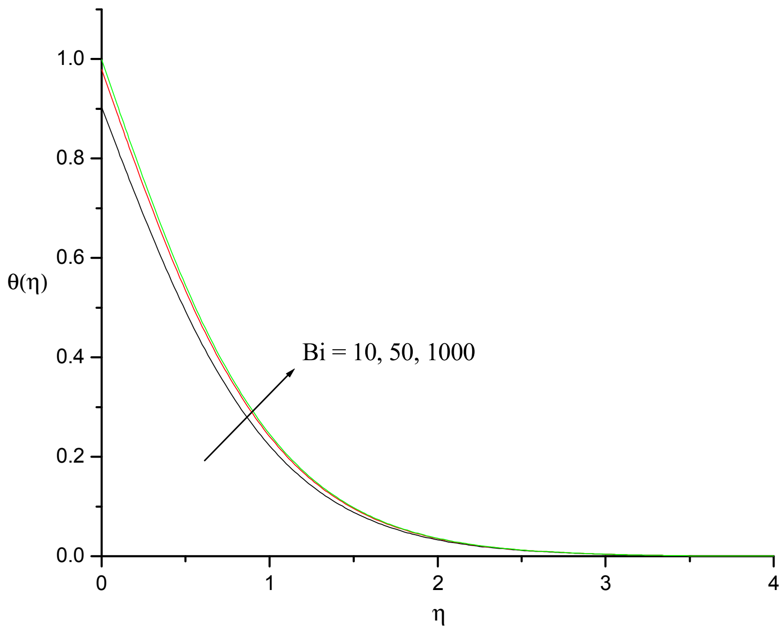

Fig. 7: Temperature Profile for different values of Bi (Bi>5) when β =1.0 and Pr =3.0

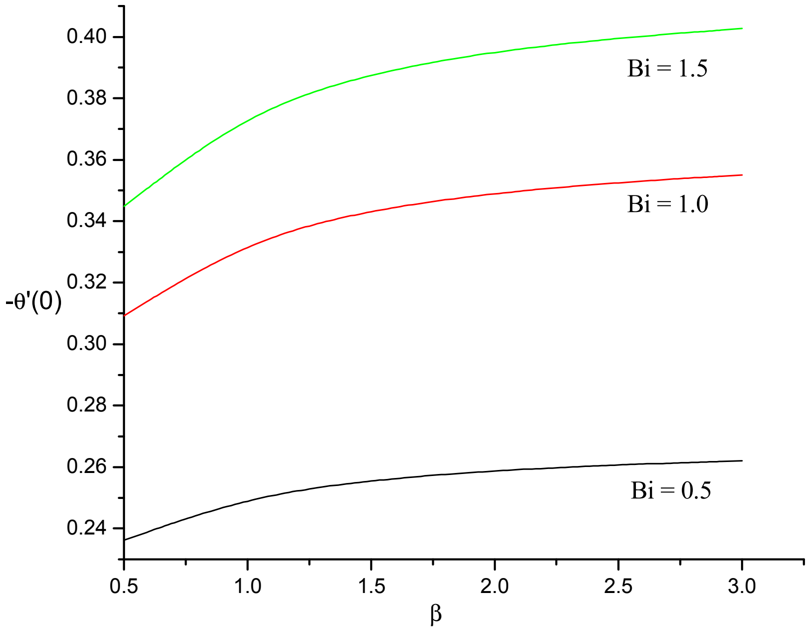

Fig.8: Wall temperature Profile for different values of Bi Vs β when Pr =1.0

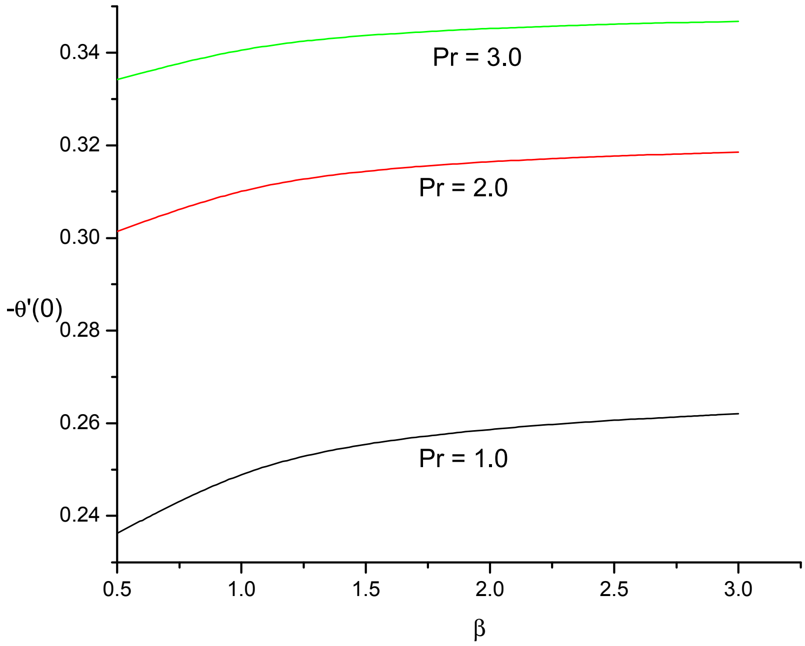

Fig.9: Wall temperature Profile for different values of Pr Vs β when Bi =0.5

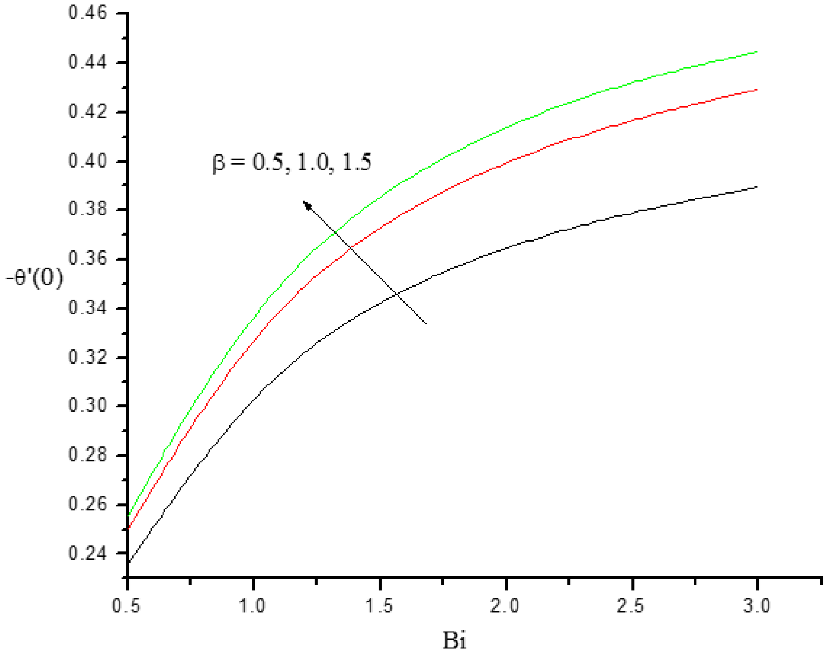

Fig. 10: Wall temperature Profile for different values of β Vs Bi when Pr =1.0

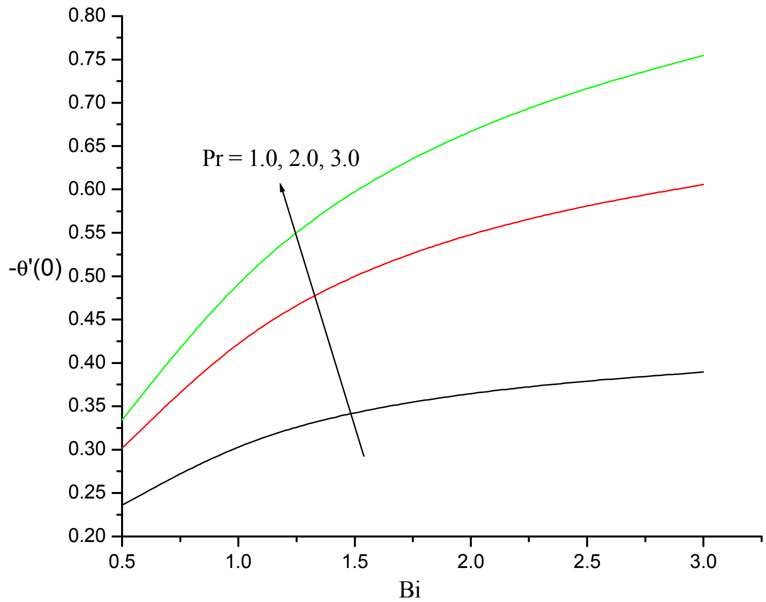

Fig. 11: Wall temperature Profile for different values of Pr Vs Bi when β =0.5

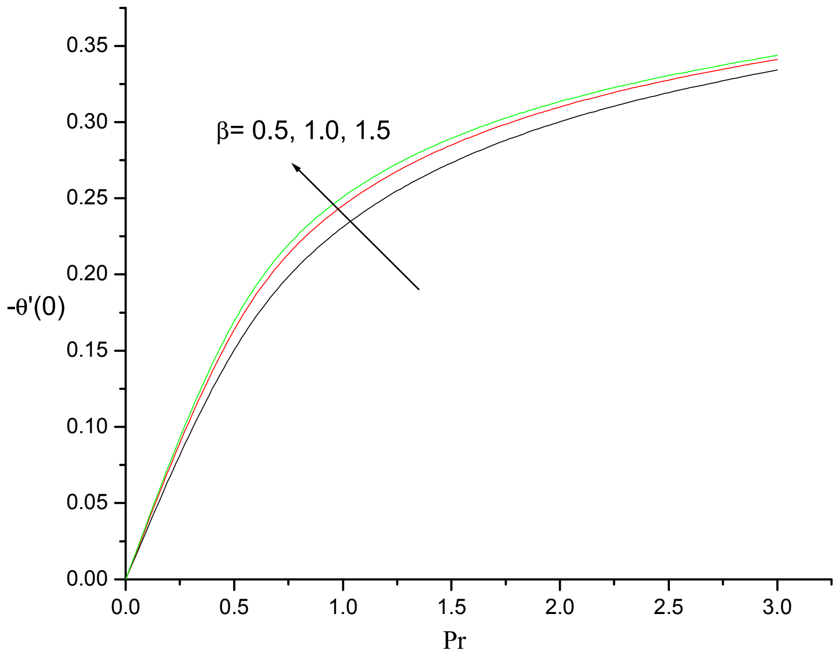

Fig. 12: Wall temperature Profile for different values of β Vs Pr when Bi=0.5

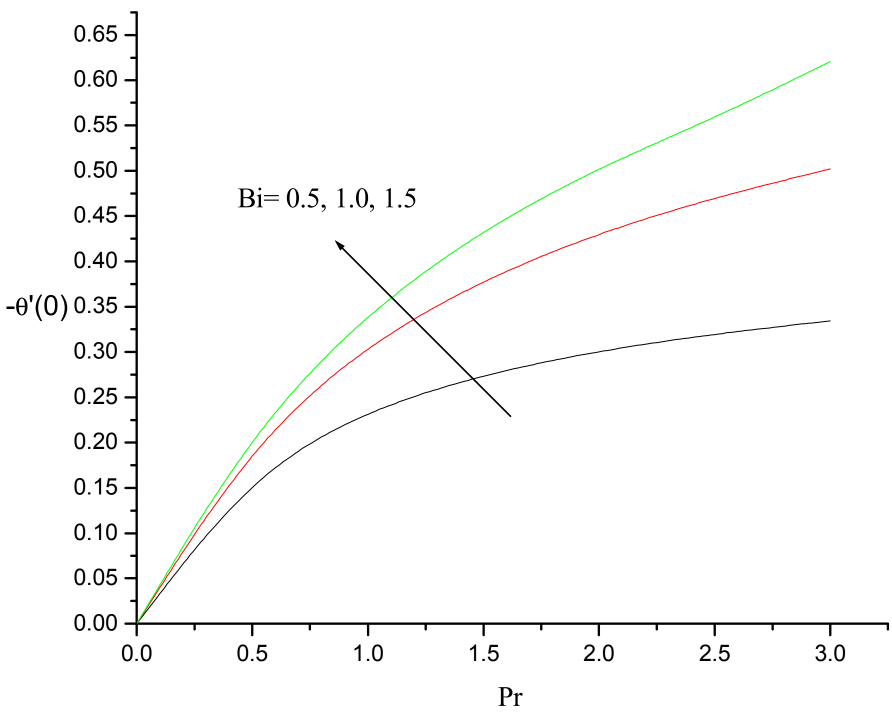

Fig. 13: Wall temperature Profile for different values of Bi Vs Pr when β = 0.5