

Analytical Solution for MHD Casson Fluid Flow Past a Porous Linearly Stretching Surface with Wall Mass Transfer

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Abstract
In this analysis, MHD Casson fluid flow past a porous linearly stretching surface with wall mass transfer is studied. Using similarity transformations, the governing equations are converted to an ordinary differential equation and then solved analytically. The fluid velocity and skin friction coefficient are obtained. Our analysis reveals that the effect of increasing Casson parameter and porosity parameter is to suppress the velocity field.
Keywords: Casson fluid; Stretching surface; MHD.

INTRODUCTION

The study of MHD porous flows is very important because in the influence of a magnetic field on the viscous flow of electrically conducting fluid applicable in many industrial process, such as polymer processing, paper production, hot rolling, purification of crude oil, glass manufacturing, geophysics, glass fiber production, wire drawing and many others.

In modern engineering, many materials are used whose flow characteristics are not explainable with the Newtonian fluid model. Hence non Newtonian fluid theory has become essential. The governing equations for non Newtonian fluid flows are highly nonlinear. Also, the industrial applications of non Newtonian fluid flow are increasing day by day. Among the many industrial non Newtonian fluids some fluids behave like elastic solids and for those fluids, a yield shear stress exists in the constitutive equations. Casson fluid is one of such non Newtonian fluids. So if the shear stress magnitude is greater than yield shear stress, then flow occurs. The flows of non Newtonian fluids are very important due to their industrial and technological applications. For example, if one uses a non Newtonian fluid as the coolant, the required pumping power might be greatly reduced.

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C.J. Toki [6]studied an analytical solution for boundary layer flows over a moving flat porous plate with viscous dissipation.KrishnenduBhattachary*et al.* [7] investigate analytic solution for MHD boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer.Swatimukopadyay*et al.*[8] discussed an exact solution for the flow of Casson fluid over a stretching surface with transpiration and heat transfer effects. Krishnendu Bhattachary*et al.* [9] showed an exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet. Swatimukhopadyay *et al.* [10] discussed boundary layer flow and heat transfer of a Casson fluid past a symmetric porous wedge with surface flux. Swatimukhopadyay [11] studied dual solutions in boundary layer flow of a moving fluid over a moving permeable surface in presence of prescribed surface temperature and thermal radiation. Swatimukhopadyay *et al.* [12] discussed mass transfer over an exponentially stretching porous sheet embedded in a stratified medium.

Swatimukhopadyay [13] studied analysis of boundary layer flow over a porous nonlinearly stretching sheet with partial slip at the boundary. Hitesh Kumar[14] reported heat transfer in MHD boundary layer flow through a porous medium, due to a non-isothermal stretching sheet, with suction, radiation and heat annihilation. Babu *et al.*[15] studied radiation effect on MHD heat and mass transfer flow over a shrinking sheet with mass suction. Swati Mukhopadyay *et al.*[16] investigated on MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation. Mahanta *et al.*[17] discussed on 3D Casson fluid flow past a porous linearly stretching sheet with convective boundary condition. Nadeem *et al.* [18] studied on MHD three dimensional Casson fluid flows past a porous linearly stretching sheet. Shaw *et al.* [19] discussed Homogeneous heterogeneous reactions in micropolar fluid flow from a permeable stretching or shrinking sheet in a porous medium. Raju *et al.*[20]studied Heat and Mass transfer in MHD Casson fluid over an exponentially permeable stretching surface.

The objective of present paper, the study MHD Casson fluid flow past a porous linearly stretching surface with wall mass transfer is investigated. The closed form analytic solutions of a transformed similarity equation are found for stretching sheet case. The graphs are presented and discussed for the physical parameters involved in the developed solutions.

MATHEMATICAL FORMULATION

Consider the flow of an incompressible viscous fluid past a porous linearly stretching sheet coinciding with the plane $y = 0$. The fluid flow is confined to $y > 0$. Two equal and opposite forces are applied along the x axis so that the wall is stretched with the origin fixed. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij}, \ \pi > \pi_c, \\ 2\left(\mu_B + p_y/\sqrt{2\pi_c}\right)e_{ij}, \ \pi < \pi_c, \end{cases} \quad (1)$$

where e_{ij} is the $(i, j)^{th}$ component of the deformation rate, π is the product of the component of deformation rate with itself, $\pi = e_{ij}e_{ij}$, π_c is a critical value of this product based on the non Newtonian model, μ_B is the plastic dynamic viscosity of the non Newtonian fluid and p_y is the yield stress of the fluid.

Under these conditions the MHD boundary layer equations for the steady flow of Casson fluid past a porous linearly stretching sheet may be written in usual notation as

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = \nu\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{k}u \quad (3)$$

where u and υ are the velocity components in x and y directions respectively, x is the distance along the sheet, y is the distance perpendicular to the sheet, ν is the kinematic fluid viscosity, k is porous medium permeability, ρ is the fluid density, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the non Newtonian (Casson) parameter, σ is the electrical conductivity of the fluid and B_0 is the strength of magnetic field applied in the y direction, the induced magnetic field being neglected.

The boundary conditions are

$$\left.\begin{aligned} &u = U = cx, \ \upsilon = -\upsilon_w \ at \ y = 0 \\ &u \to 0 \ as \ y \to \infty \end{aligned}\right\} \quad (4)$$

Here $c > 0$ is the rate of stretching sheet and υ_w is the wall mass transfer velocity with $\upsilon_w > 0$ for mass suction and $\upsilon_w < 0$ for mass injection.

The following transformations are introduced

$$\psi = \sqrt{c\nu x}\, f(\eta) \text{ and } \eta = y\sqrt{c/\nu} \quad (5)$$

where ψ is the stream function defined in the usual notation as

$u = \dfrac{\partial \psi}{\partial y}, \upsilon = -\dfrac{\partial \psi}{\partial x}$ and η is the similarity variable.

Equation (2) is automatically satisfied, while equation (3) reduces to the following nonlinear self similar equation

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - f'^2 - (M + \lambda_1)f' = 0 \quad (6)$$

where primes denote differentiation with respect to η and $M = \dfrac{\sigma B_0^2}{\rho c}$ is the magnetic parameter $\lambda_1 = \dfrac{\nu}{ck}$ is the porosity parameter, f is a dimensionless stream function and f' is the dimensionless velocity.

The boundary conditions are described as follows

$$\left.\begin{aligned} &f(\eta) = S, \ f'(\eta) = 1 \ at \ \eta = 0 \\ &f'(\eta) \to 0 \ as \ \eta \to \infty \end{aligned}\right\} \quad (7)$$

Where $S = \dfrac{\upsilon_w}{(c\nu)^{\frac{1}{2}}}$ is wall mass transfer parameter with $S > 0$ (i.e. $\upsilon_w > 0$) for wall mass suction and $S < 0$ (i.e. $\upsilon_w < 0$) for wall mass injection.

SOLUTION OF THE PROBLEM

For MHD flow of Newtonian fluid, the exact solution is obtained by Chakrabarti and Gupta [1] (extended by Vajravelu and Rollins [2], Pop and Na [3] and Fang *et al.* [4]). Considering Ref. [5] we assume

$$f(\eta) = a + be^{-\lambda\eta} \tag{8}$$

be the solution of equation (6) subject to the boundary conditions

$$f(0) = S,\ f'(0) = 1,\ f'(\infty) = 0 \tag{9}$$

Substituting equation [8] into equation [6] and [9] we obtained

$$a = S + \frac{1}{\lambda},\ b = -\frac{1}{\lambda} \text{ and } \lambda = \frac{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}}{2\left(1+\frac{1}{\beta}\right)} \tag{10}$$

where a, b and λ are constants with $\lambda > 0$. So, the exact solution of equation (6) and subject to the boundary conditions is given by

$$f(\eta) = S + \frac{2\left(1+\frac{1}{\beta}\right)}{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}} - \frac{2\left(1+\frac{1}{\beta}\right)}{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}} \times \exp\left[-\frac{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}}{2\left(1+\frac{1}{\beta}\right)}\eta\right] \tag{11}$$

Hence $$f'(\eta) = \exp\left[-\frac{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}}{2\left(1+\frac{1}{\beta}\right)}\eta\right] \tag{12}$$

Skin friction coefficient

The physical quantity of skin friction coefficient which are defined as follows

$$f''(0) = -\lambda = -\frac{S + \sqrt{S^2 + 4\left(1+\frac{1}{\beta}\right)(1+M+\lambda_1)}}{2\left(1+\frac{1}{\beta}\right)} \tag{13}$$

RESULTS AND DISCUSSION

In this study we investigate MHD Casson fluid flow past a porous linearly stretching surface with wall mass transfer. The dimensionless parameters involved on the fluid velocity such as magnetic parameter M , Casson parameter β, suction/injection parameter S and porosity parameter λ_1.To illustrate the computed results, some figures are plotted and physical explanations are given.

Variation of velocity $f'(\eta)$ profiles for different values of Casson parameter β with mass suction and injection are shown in fig 1. and fig 2. We observe that the velocity is decrease with increase Casson parameter β.

Variation of velocity $f'(\eta)$ profiles for different values of magnetic parameter M with mass suction and injection are shown in fig 3. and fig 4. We observe that the velocity and boundary layer thickness is decrease due to an increase in the magnetic parameter M . This reduction is caused by the Lorentz force, a mechanical force arising due to the interaction of magnetic and electric fields for the motion of an electrically conducting fluid. The Lorentz force increases when magnetic M increases and consequently boundary layer thickness decreases.

Variation of velocity $f'(\eta)$ profiles for different values of porosity parameter λ_1 with mass suction and injection are shown in fig 5. and fig 6. We observe that the velocity and boundary layer thickness is decrease due to an increase in the porosity parameter λ_1 .

Variation of skin friction co efficient $f''(0)$ profiles against S for different values of Casson parameter β , magnetic parameter M and porosity parameter λ_1 are shown in fig 7-9. It is seen that for increasing values of β , M and λ_1 then the magnitude of $f''(0)$ is decreases. Here we observe that when the absence of porosity parameter λ_1 which is similar to the observations of Krishnendu Bhattacharyya *et al.* [7].

CONCLUSIONS

MHD Casson fluid flow past a porous linearly stretching surface with wall mass transfer is investigated analytically. Moreover, effects for various values of emerging parameters are discussed for velocity $f'(\eta)$. The main results of present analysis can be listed below.

1. Magnetic field, Casson fluid parameter and porosity parameter reduces on velocity profiles.
2. The magnitude of the skin friction coefficient decreases with an increase in Casson parameter β , magnetic parameter M and porosity parameter λ_1 respectively. Here we seen that when the absence of porosity parameter λ_1 which is similar to the observations of Krishnendu Bhattacharyya *et al* [7].

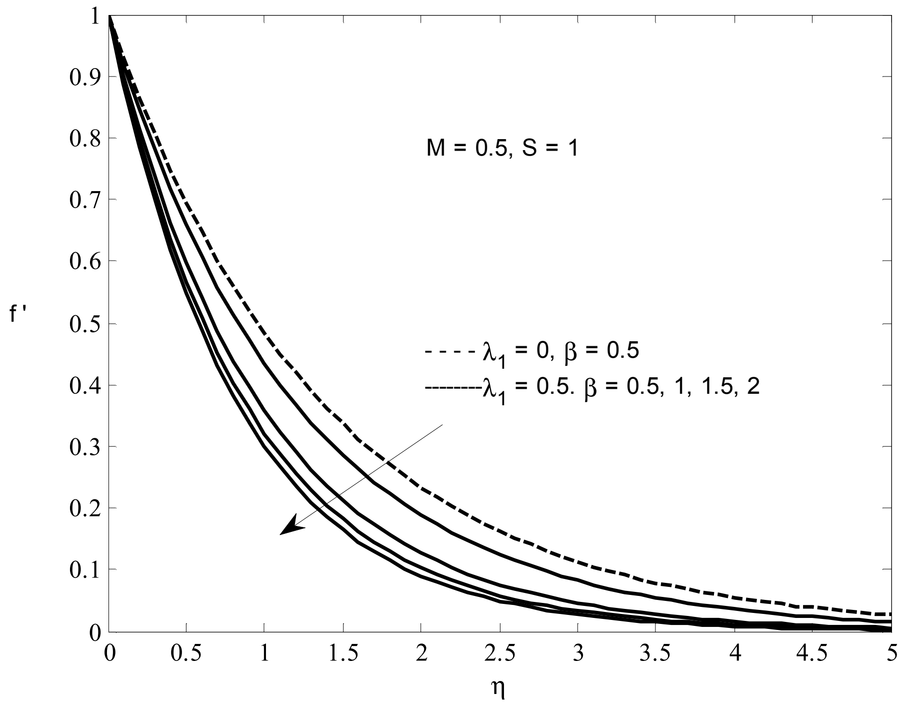

Fig 1.Velocity profiles for several values of β with mass suction

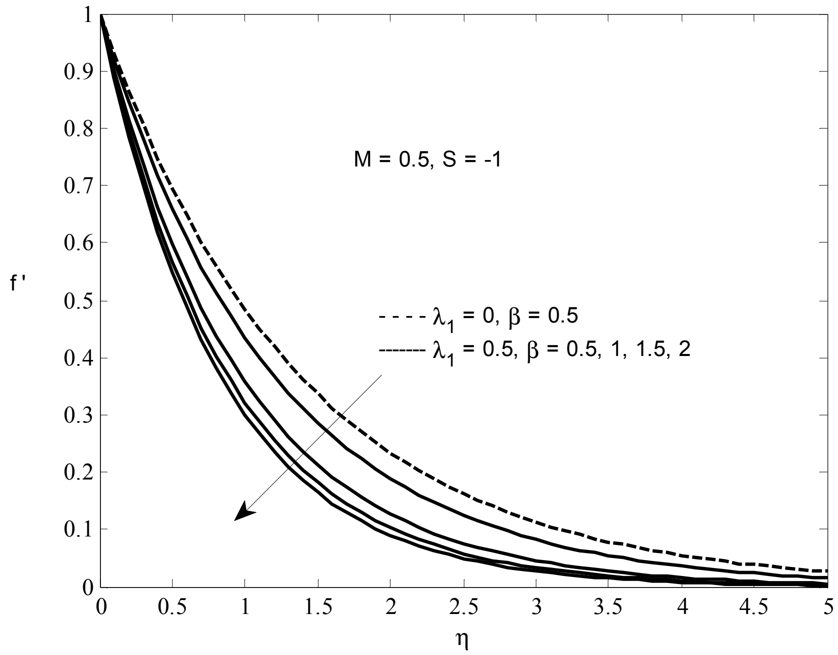

Fig 2.Velocity profiles for several values of β with mass injection

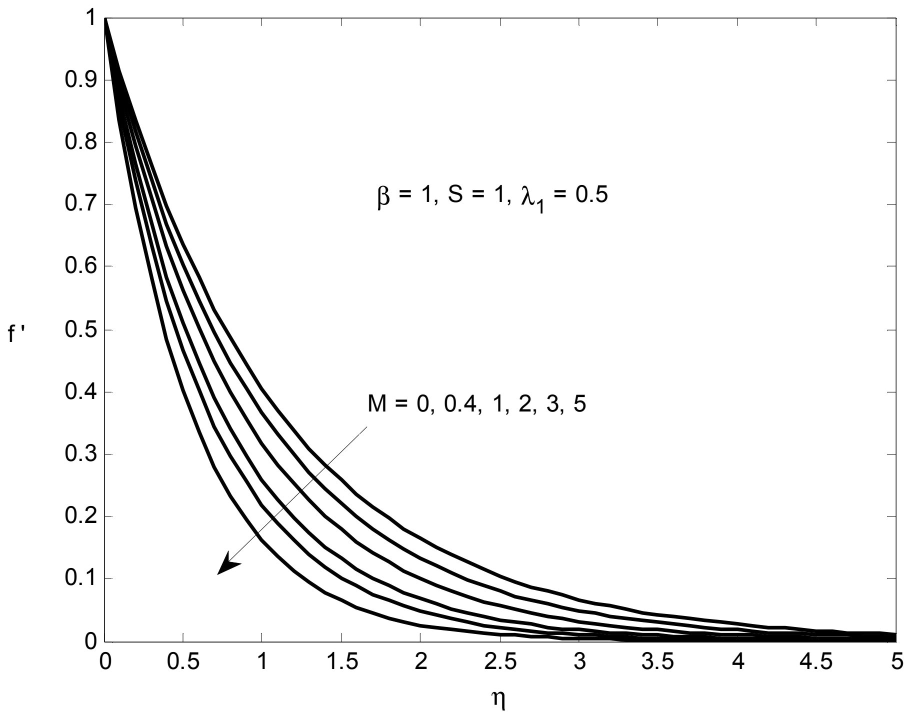

Fig 3.Velocity profiles for several values of M with mass suction

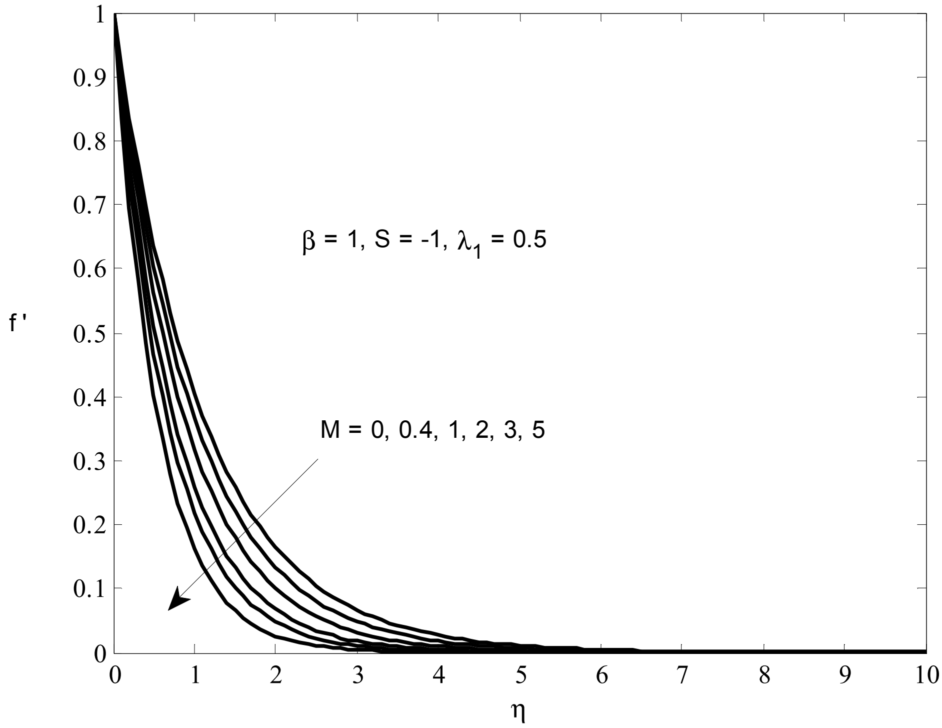

Fig 4.Velocity profiles for several values of M with mass injection

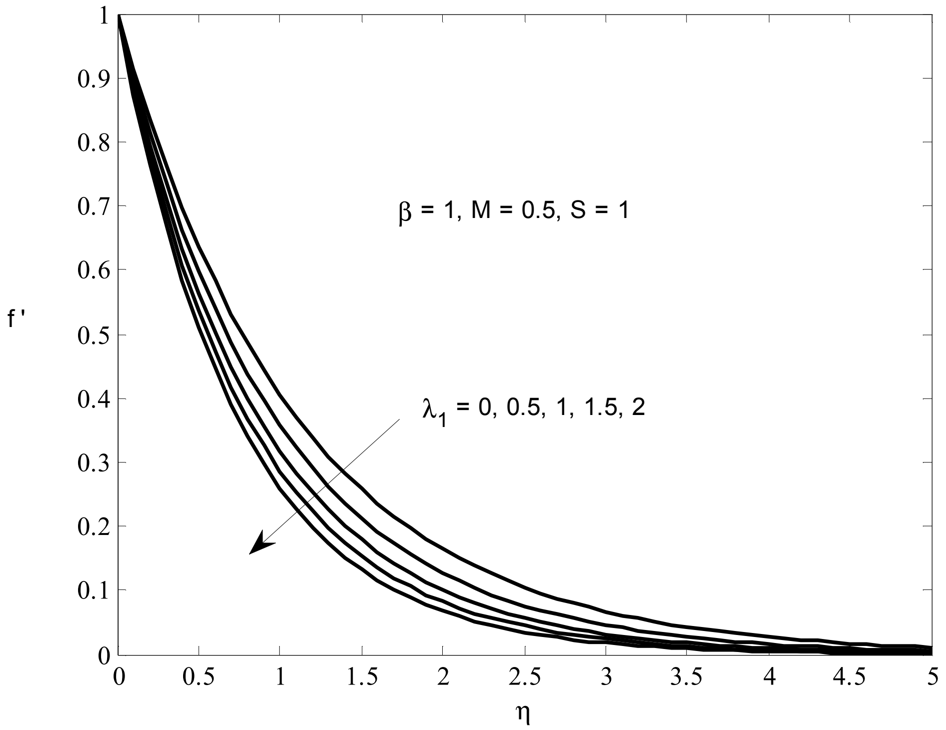

Fig 5.Velocity profiles for different values of λ_1 with mass suction

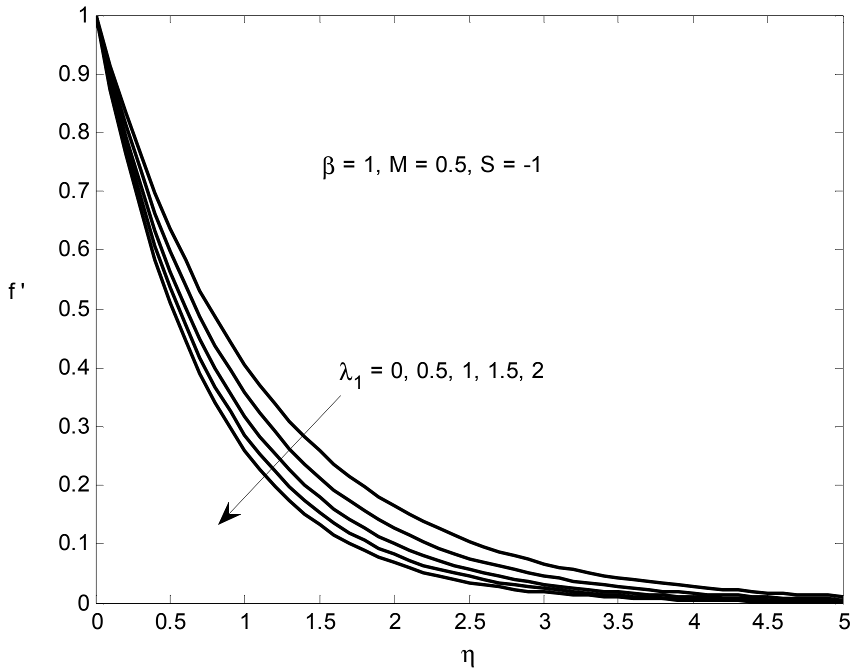

Fig 6.Velocity profiles for different values of λ_1 with mass injection

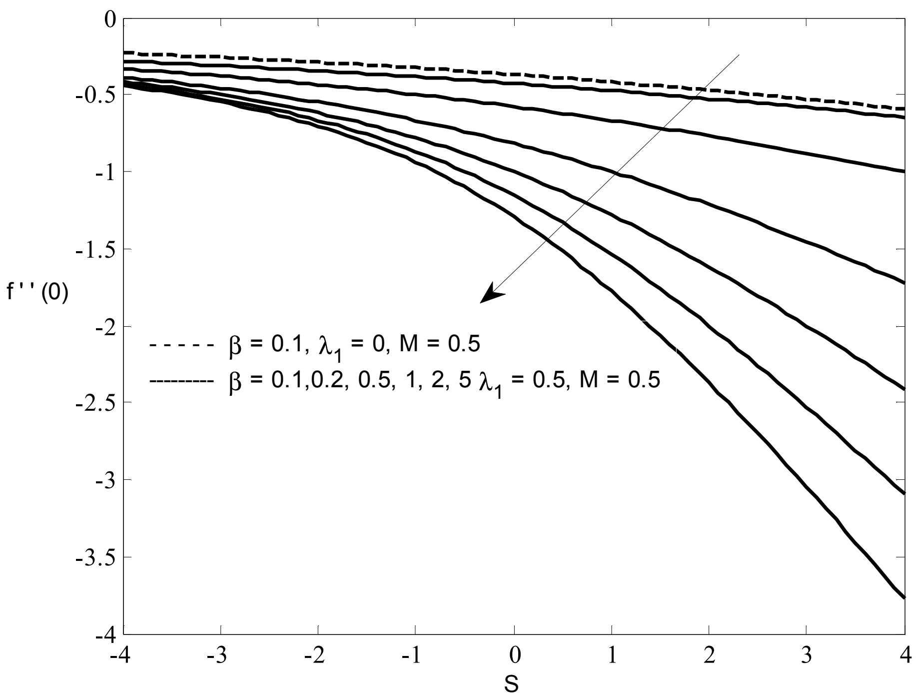

Fig 7.Skin friction coefficient versus S for several values of β .

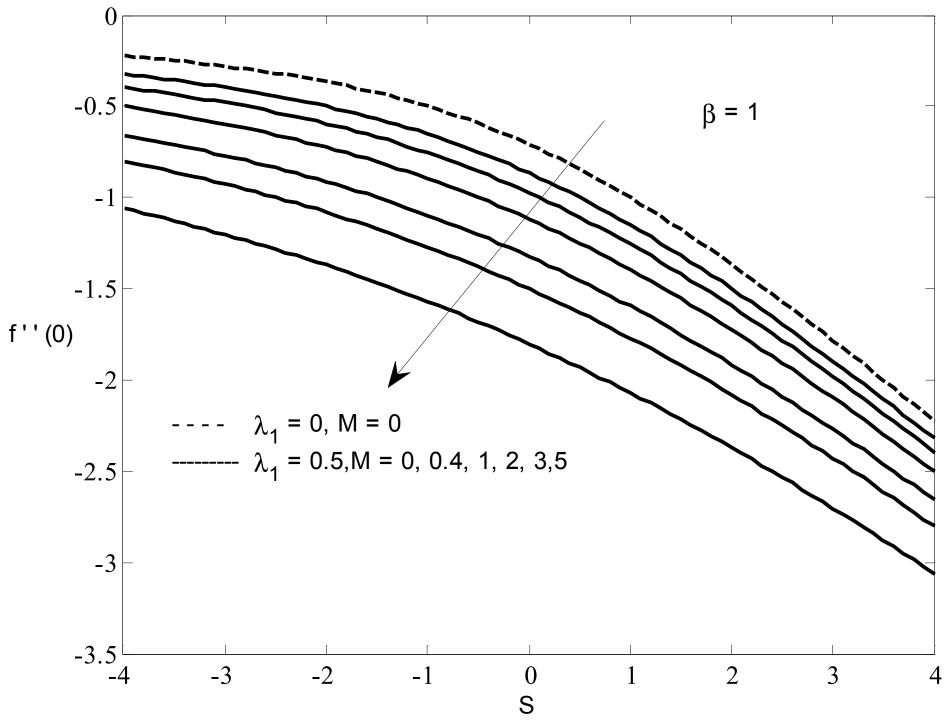

Fig 8.Skin friction coefficient versus S for several values of M

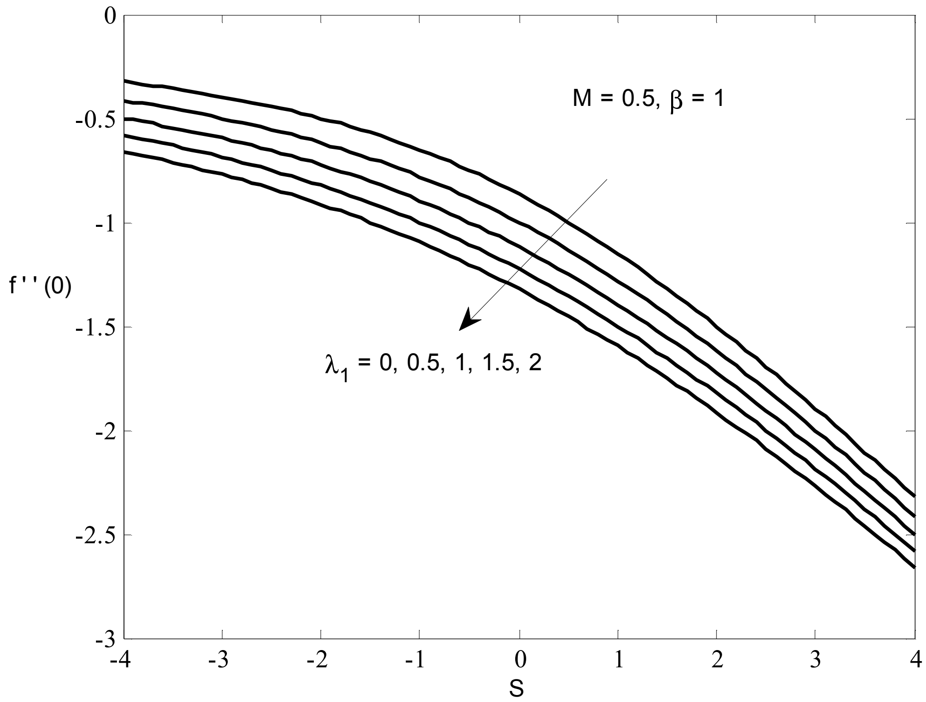

Fig 9.Skin friction coefficient versus S for several values of λ_1 .

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