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Hall Current Effects on Free Convection Casson Fluid Flow in a Rotating System with Convective Boundary Conditions and Constant Heat Source

V. Sugunamma[1*] B. Ramadevi[1] J.V. Ramana Reddy[1] N. Sandeep[2]
1.Department of Mathematics, S V University, Tirupati, A.P, India
2.Fluid dynamics Division, VIT University, Vellore, India

Abstract
In this paper we investigated an unsteady free convection flow of casson fluid bounded by a moving vertical flat plate in a rotating system with convective boundary conditions. The governing equations are solved analytically by using perturbation technique. Finally the effects of various dimensionless parameters like inclined angle, Casson parameter, Heat source and Suction parameter on velocity, temperature, friction factor and local Nusselt number are discussed with the help of graphs and tables. Through this study, it is found that increasing values of casson parameter reduces the velocity and increase in inclined magnetic field or hall current parameter enhances the velocity profiles.
Keywords: Casson fluid, Rotation, inclined magnetic field, MHD, Heat source.

Introduction
Non-Newtonian fluids are more stable compared with Newtonian fluids. Hence it is necessary to study the behaviour of Non-Newtonian fluids like Jeffery fluid, Maxwell fluid, Viscoelastic fluid etc. However there is another non-Newtonian fluid called Casson fluid, which become more popular in recent years in the study of non-Newtonian fluids. Jelly, Tomato sauce, honey, soup and human blood are some examples for Casson fluid
Ghosh and Bhattacharjee [1] discussed the influence of inclined magnetic field on MHD flow in a rotating channel. The Boundary layer flow of nanofluid over a stretching sheet was reported by Makinde and Aziz [2].Sandeep and Sugunamma [3] reported the effects of inclined magnetic field on free convective flow over a vertical plate. Sandeep et al. [4] investigated the influence of thermal radiation on natural convective flow over a vertical plate. The effects of chemical reaction and thermal radiation on MHD flow past a moving vertical plate was discussed by Krishna et al. [5]. Saha et al. [6] studied the effects of Hall current on MHD natural convection flow over a permeable flat plate. Ramanareddy et al. [7] discussed Aligned magnetic field, chemical reaction and Radiation effects on unsteady dusty viscous flow. The flow characteristics of nanofluid in a rotating frame with heat transfer was reported by Das [8]. Sandeep and Sugunamma [9] analyzed the inclined Magnetic field and Radiation effects on MHD flow over an oscillating plate.

Raju et al. [10] heat and mass transfer in MHD Casson fluid over an exponentially stretching surface. The effect of nonlinear thermal radiation on MHD flow between rotating plates with homogeneous-heterogeneous reactions was studied by Ramanareddy et al. [11]. Shake and Karna [12] discussed the influence of radiation and chemical reaction on MHD boundary layer flow in a porous vertical surface and it is found that an increase in heat generation parameter or Eckert number reduces the nusselt number. The influence of non uniform heat source/sink parameters on the flow due to slendering stretching sheet was examined by Ramanareddy et al. [13] and it is found that increasing values of non uniform heat source/sink parameters raises the temperature profiles. Mahantesh et al. [14] investigated the heat and mass transfer effects on the flow of a nanofluid over a moving /stationary vertical plate. Recently, the effect of induced magnetic field on MHD stagnation point flow of a nanofluid past a stretching cylinder was studied by Sandeep and Sulochana [15].

With the help of the above mentioned studies, we make an attempt to study the effects of hall current, heat source on the flow of a casson fluid in rotating system.

Mathematical formulation
Consider an unsteady three dimensional free convection flow of an electrically conducting incompressible casson fluid of ambient temperature T_∞ past a semi-infinite moving vertical plate with constant heat source and convective boundary conditions. The flow is assumed to be in the x - direction which is taken along the plate in the upward direction and z - axis is normal to it. Also it is assumed that the whole system is rotating with constant velocity Ω about z -axis. A uniform external magnetic field B_0 is taken in a direction which makes an angle α with the positive direction of z -axis. Furthermore the flow variables are functions of z and t only.
The governing equations of the flow are given by

$$\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+w\frac{\partial u}{\partial z}-2\Omega v\right)=\mu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial z^2}+\rho\beta_T g(T-T_\infty)+\frac{\sigma B_0^2(vm\cos\alpha-u)\cos^2\alpha}{1+m^2\cos^2\alpha}, \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+w\frac{\partial v}{\partial z}+2\Omega u\right)=\mu(1+\frac{1}{\beta})\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2(mu\cos\alpha+v)\cos^2\alpha}{1+m^2\cos^2\alpha}, \tag{3}$$

$$\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}-\frac{Q}{\rho c_p}(T-T_\infty), \tag{4}$$

Where u, v, w are velocity components along x, y, z -axis directions respectively, β is the casson parameter, ρ is the density of the fluid, μ is the viscosity of the fluid, β_T is the coefficient of thermal expansion of the fluid due to temperature difference, Ω is the rotation velocity of the system, α is the inclination of the magnetic field with the normal to the plate, m is the hall current parameter, g is the acceleration due to gravity, σ is the electrical conductivity of the fluid, Q is the temperature dependent volumetric rate of heat source and ρc_P is the heat capacitance of the nanofluid.

So the boundary conditions for this problem are given by

$u(z,t)=0, v(z,t)=0, T(z,t)=T_\infty$, for $t\leq 0$ and any z.

$$u(z,t)=U_r\left[1+\frac{\varepsilon}{2}\left(e^{\text{int}}+e^{-\text{int}}\right)\right], v(z,t)=0, -k_f\frac{\partial T}{\partial z}=h_f(T_w-T_\infty), \text{ for } t>0 \text{ and } z=0 \tag{5}$$

$$u(z,t)\to 0, v(z,t)\to 0, T(z,t)\to T_\infty, \text{ for } t>0 \text{ and } z\to\infty \tag{6}$$

Where U_r is the uniform velocity and ε is the small constant quantity.

We introduce the following non dimension variables into Eqs. (2)-(5)

$$u'=\frac{u}{U_r}, v'=\frac{v}{U_r}, z'=\frac{zU_r}{v}, t'=\frac{tU_r^2}{v}, n'=\frac{nv}{U_r^2}, \theta=\frac{(T-T_\infty)}{(T_w-T_\infty)}, \tag{7}$$

Substituting equation (7) in equations (2) – (4) yields the following dimensionless equations (after dropping the primes),

$$\left(\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial z}-Rv\right)=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial z^2}+\theta+\frac{M^2\cos^2\alpha(vm\cos\alpha-u)}{1+m^2\cos^2\alpha}, \tag{8}$$

$$\left(\frac{\partial v}{\partial t}-S\frac{\partial v}{\partial z}+Ru\right)=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 v}{\partial z^2}-\frac{M^2\cos^2\alpha(mu\cos\alpha+v)}{1+m^2\cos^2\alpha}, \tag{9}$$

$$\Pr\left(\frac{\partial\theta}{\partial t}-S\frac{\partial\theta}{\partial z}\right)=\frac{\partial^2\theta}{\partial z^2}-Q_h\theta, \tag{10}$$

Here $R=\frac{2\Omega v}{U_r^2}, S=\frac{w_0}{U_r}, M^2=\frac{\sigma B_0^2 v}{\rho U_r^2}, \Pr=\frac{\mu c_p}{k}, Q_h=\frac{Qv^2}{U_r^2 k}, U_r=\left[g\beta_T(T_w-T_\infty)\right]^{\frac{1}{3}}$ are rotational parameter, suction $(S>0)$ or injection $(S<0)$ parameter, magnetic field parameter, Prandtl number, heat source parameter and velocity characteristics respectively.

Also the boundary conditions (5) and (6) become

$u=0, v=0, \theta=0$, for $t\leq 0$ and for any z.

$$u=\left\{1+\frac{\varepsilon}{2}(e^{\text{int}}+e^{-\text{int}})\right\}, v=0,, \theta'(z)=-a(1-\theta(z)) \text{ for } t>0 \text{ and } z=0, \tag{11}$$

$$u\to 0, v\to 0, \theta\to 0, \text{ for } t>0 \text{ as } z\to\infty, \tag{12}$$

Where $a=\dfrac{h_f v}{kU_r}$ is the convective parameter. We now simplify equations (6) and (9) by putting the fluid velocity in the complex form as $V=u+iv$ and we get

$$\left(\frac{\partial V}{\partial t}-S\frac{\partial V}{\partial z}+iRV\right)=A_1\frac{\partial^2 V}{\partial z^2}+\theta+\psi-M_1^2V, \tag{13}$$

The corresponding boundary conditions become

$V=0,\theta=0,$ for $t\le 0,$

$$V(z)=\left\{1+\frac{\varepsilon}{2}(e^{\text{int}}+e^{-\text{int}})\right\},\theta'(z)=-a(1-\theta(z)),\text{ at } z=0 \text{ and } t>0, \tag{14}$$

$$V\to 0,\theta\to 0,\text{ for } t>0 \text{ as } z\to\infty, \tag{15}$$

Solution of the problem

To obtain the solution of the system of partial differential equations (10) and (13) under the boundary conditions represented in (14)-(15), we express V and θ as

$$V(z,t)=V_0+\frac{\varepsilon}{2}\left[e^{\text{int}}V_1(z)+e^{-\text{int}}V_2(z)\right], \tag{16}$$

$$\theta(z,t)=\theta_0+\frac{\varepsilon}{2}\left[e^{\text{int}}\theta_1(z)+e^{-\text{int}}\theta_2(z)\right], \tag{17}$$

Substituting the above equations (16)-(17) in the equations (10) and (13), and equating the harmonic and non-harmonic terms and neglecting the higher order terms of ε^2, we get following equations.

The equations representing Temperature are

$$\theta_0''+A_2 pr\,S\,\theta_0'-Q_h\theta_0=0, \tag{18}$$

$$\theta_1''+A_2\theta_1'-A_3\theta_1=0, \tag{19}$$

$$\theta_2''+A_2\theta_2'-A_4\theta_2=0, \tag{20}$$

Along with the boundary conditions $\theta_0'=-a(1-\theta_0),\theta_1'=a\theta_1,\theta_2'=a\theta_2$ at $z=0$, (21)

$$\theta_0\to 0,\theta_1\to 0,\theta_2\to 0 \text{ as } z\to\infty. \tag{22}$$

Solving eqs. (18)- (20) with the help of boundary conditions in (21) & (22) , we get

The equations representing Temperature as

$$\theta_0=A_5e^{-B_1z},\ \theta_1=0,\ \theta_2=0\ , \tag{23}$$

Substituting the values of eqn. (23) in eqn. (17), we get the expression for temperature as

$$\theta=A_{10}e^{-B_1z}, \tag{24}$$

Now, substitute the expressions obtained for temperature given by (24) in (13), and using eqn. (16), we get the equations representing velocity as

$$A_1V_0''+SV_0'+A_6V_0+A_5e^{-B_1z}=0, \tag{25}$$

$$A_1V_1''+SV_1'-(A_6+in)V_1=0, \tag{26}$$

$$A_1V_2''+SV_2'-(A_6-in)V_2=0, \tag{27}$$

The corresponding boundary conditions are given by

$$V_0=V_1=V_2=1 \quad \text{at } z=0, \tag{28}$$

$$V_0\to 0,V_1\to 0,V_2\to 0 \text{ as } z\to\infty, \tag{29}$$

Where $V_0,\theta_0,V_1,\theta_1,V_2,\theta_2$, are functions of z only and prime denotes the differentiation with respect to z.

Solving the equations from (25) - (27) under the boundary conditions (28) and (29), we obtain the expression for velocity as

$$V=\left((1+A_7)e^{-B_2z}-A_8e^{-B_1z}\right)+\left(\frac{\varepsilon}{2}\right)\left(e^{-B_3z}e^{\text{int}}+e^{-B_4z}e^{-\text{int}}\right), \tag{30}$$

The physical quantities of engineering interest are skin friction coefficient C_f and Nusselt number Nu, which are given below.

$$C_f = \left(-B_2(1+A_7)+B_1A_8\right)+\left(\frac{\varepsilon}{2}\right)\left(-B_3e^{\mathrm{int}}-B_4e^{-\mathrm{int}}\right), \frac{Nu}{\mathrm{Re}_x}=B_1A_5, \tag{31}$$

Where, $\mathrm{Re}_x = \dfrac{U_r x}{\nu}$ is the local Reynolds number.

Results and discussion

Finally the effects of various physical parameters like casson parameter, inclined angle, heat source and Hall current parameter involved in Eqs. (24) and (30), on velocity (V) and temperature(θ) profiles have been analyzed through graphs. Also tabular forms are presented to study the influence of these parameters on skin friction (C_f) and nusselt number (Nu_x). For the results we taken the values of physical parameters as $a = 0.2, \Pr = 7, S = 0.5$, $m = 0.5, Q_h = 1, R = 0.8$. These values have been kept in common except that varies values are represented in respective graphs and tables.

From Fig. 1, it is observed that increase in casson parameter (β) reduces the velocity profiles. But an opposite result can be observed from Fig. 2, which says that increase in hall parameter (m) enhances the velocity of the fluid. Fig.3 depicts the effect of inclined angle on velocity profiles. It is found that, an increase in aligned angle (α) causes for an increase in the velocity profiles. Fig. 4 illustrates the impact of Rotation parameter (R) on velocity field. It is found that increase in Rotational parameter slowdowns the motion of the fluid. Similar type of result was happened with suction parameter (S) on velocity profiles, which we can see in Fig. 5.

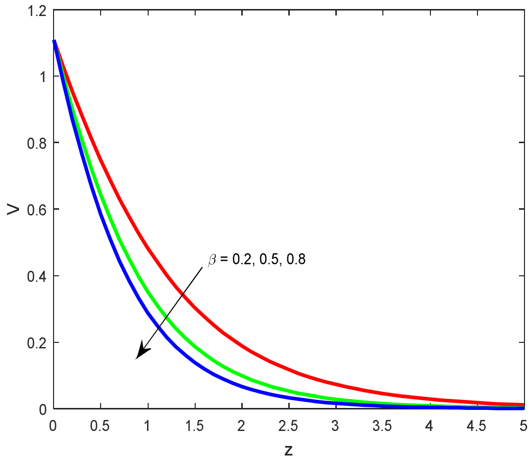

Fig.1 Variation of velocity with β

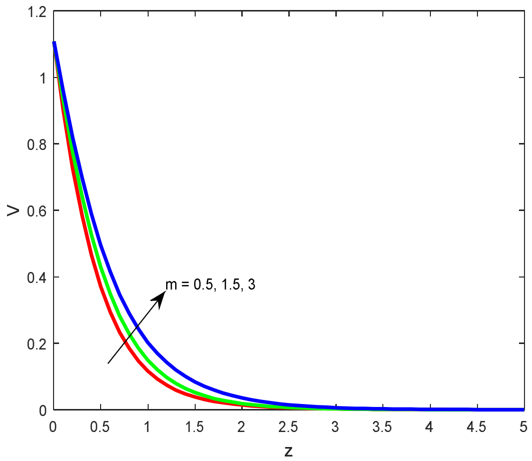

Fig.2 Variation of velocity with m

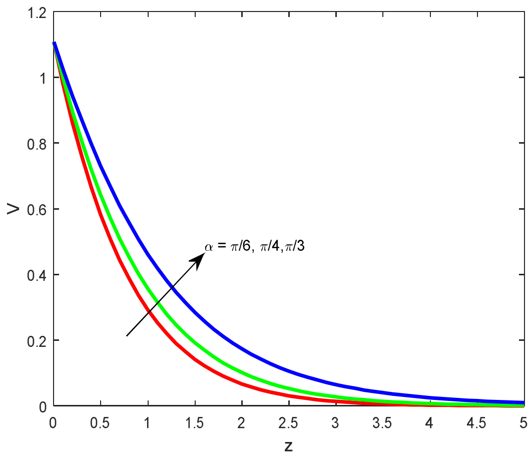

Fig.3 Variation of velocity with α

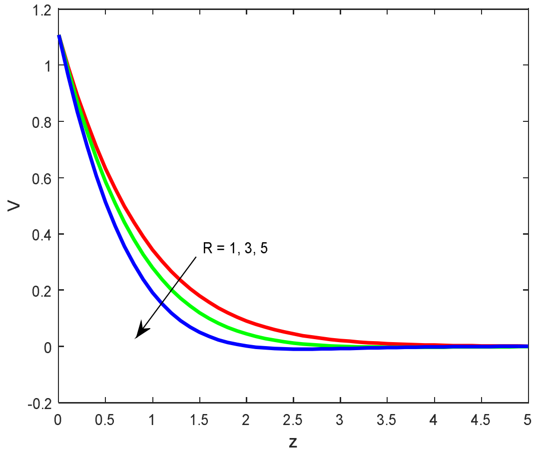

Fig.4 Variation of velocity with R

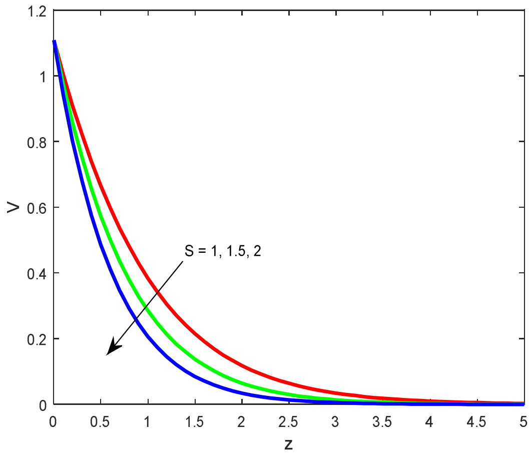

Fig.5 Variation of velocity with S

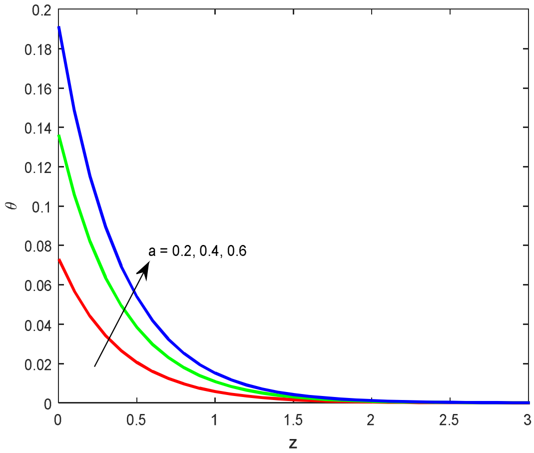

Fig.6 Variation of temperature with a

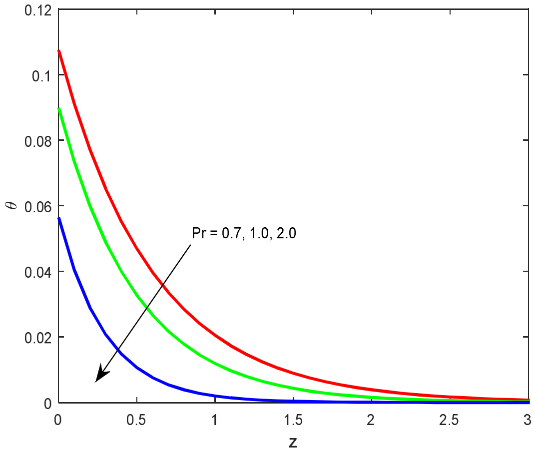

Fig.7 Variation of temperature with $\Pr$

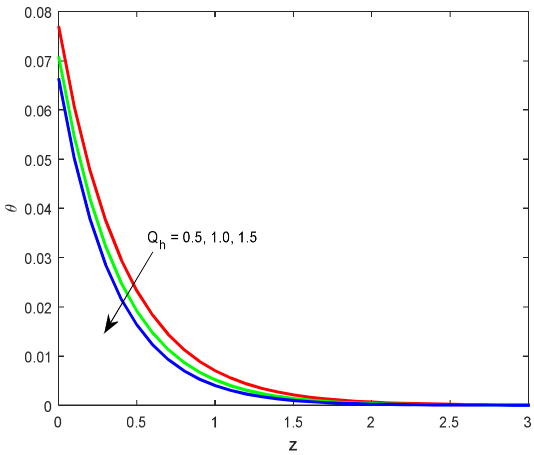

Fig.8 Variation of temperature with Q_h

Figs. (6)-(8) illustrate the effects of some physical parameters of the flow on temperature profiles. From Fig. 6 it is found that convection parameter (a) raises the temperature of the fluid. It may happen due to the fact that increase in convection parameter generates the heat to the fluid. Further, it is found from Figs. 7 and 8 that increase in either Prandtl number ($\Pr$) or in Heat source parameter (Q_h) reduces the temperature profiles. The reason behind this is increase in the value of these parameters causes for thinner boundary layer thickness.

Table 1 and Table 2 denote the influence of various physical parameters involved in the flow on Skin friction and nusselt number respectively. It is observed from Table 1 that, Friction factor increases with an increase in Magnetic field parameter or for Positive increment in the values of Suction parameter. Also it is seen from Table

2 that increases in Suction parameter, Heat source parameter or convective parameter helps to enhance the heat transfer rate.

Conclusions

The conclusions of the present study are as follows.

- Increasing values of Casson parameter or Rotation parameter slowdowns the fluid motion.
- Velocity increases with an increase in the values of hall current parameter or inclined angle.
- Suction parameter enhances the heat transfer rate.
- Increasing values of Suction/hall current parameter increases the friction factor.

Table 1: Effects of different parameters on skin-friction (C_f)

M	m	S	Q_h	a	C_f
0.5	0.5	1	0.5	0.2	**2.2439**
1.0	0.5	1	0.5	0.2	**2.4193**
1.5	0.5	1	0.5	0.2	**2.6901**
0.5	**1**	1	0.5	0.2	**2.2417**
0.5	**2**	1	0.5	0.2	**2.2262**
0.5	**3**	1	0.5	0.2	**2.2143**
0.5	0.5	**0.5**	0.5	0.2	**1.5166**
0.5	0.5	**1.0**	0.5	0.2	**2.2439**
0.5	0.5	**1.5**	0.5	0.2	**3.3742**
0.5	0.5	1	**0.2**	0.2	**2.2438**
0.5	0.5	1	**0.6**	0.2	**2.2439**
0.5	0.5	1	**1.0**	0.2	**2.2439**
0.5	0.5	1	0.5	**0.2**	**2.2439**
0.5	0.5	1	0.5	**0.4**	**2.2107**
0.5	0.5	1	0.5	**0.6**	**2.1791**

Table 2: Effects of different parameters on Nusselt number (Nu)

S	Q_h	Pr	a	Nu
0.2	0.5	7	0.3	**0.4005**
0.4	0.5	7	0.3	**0.4244**
0.6	0.5	7	0.3	**0.4345**
0.2	**0.3**	7	0.3	**0.3980**
0.2	**0.6**	7	0.3	**0.4015**
0.2	**1.0**	7	0.3	**0.4049**
0.2	0.5	**0.7**	0.3	**0.3220**
0.2	0.5	**1**	0.3	**0.3296**
0.2	0.5	**3**	0.3	**0.3656**
0.2	0.5	7	**0.2**	**0.2786**
0.2	0.5	7	**0.4**	**0.5125**
0.2	0.5	7	**0.6**	**0.7116**

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