

Nonlinear Thermal Radiation and Chemical Reaction Effects on MHD 3D Casson Fluid Flow in Porous Medium

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Abstract

In this study, we analyzed the influence of nonlinear thermal radiation and viscous dissipation on three-dimensional MHD Casson fluid flow over a stretching surface in the presence of chemical reaction. The transformed governing equations are solved numerically using Runge-Kutta based shooting technique. The influence of non-dimensional parameters on velocity, temperature and concentration profiles along with the friction factor, local Nusselt and Sherwood numbers are discussed with the help of graphs and tables. It is found that an increase in the nonlinear thermal radiation parameter enhances the temperature profiles of the flow. Chemical reaction parameter have tendency to enhance the mass transfer rate.

Keywords: MHD, Casson fluid, Radiation, Chemical Reaction, Dissipation.

1. Introduction

The study of Non-Newtonian fluids has attracted much attention because of their extensive variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. In this category of non-Newtonian fluids Casson fluid has distinct features. This model is cast off by fuel engineers in the description for adhesive slurry and is improved for forecasting high shear rate viscosities when only low and transitional shear rate data are accessible. The Magnetohydrodynamic boundary layer flow and heat transfer of a fluid with variable viscosity through a porous medium towards a stretching sheet by taking in to effects of viscous dissipation in presence of heat sources was analyzed by Dessie and Kishan (2014). The Magnetohydrodynamic boundary layer flow of a Casson fluid over an exponentially shrinking sheet has been investigated by Nadeem et al. (2012). The Magnetohydrodynamic Casson fluid flow in porous medium through linear stretching sheet was discussed by Nadeem et al. (2013). The influence of aligned magnetic field and thermal radiation on the flow of nanofluid passing exponentially stretching sheet in porous medium was investigated by Sulochana et al. (2015).

The study of triple diffusive boundary layer flow of a nanofluid over a nonlinear stretching sheet was studied by Goyal and Rama Bhargava (2014). An unsteady mixed convection boundary layer flow over an impermeable stretching sheet in a porous medium in the presence of chemical reaction was examined by Nayak et al. (2014). MHD Casson fluid flow and heat transfer with second-order slip velocity over a permeable stretching sheet in the presence of internal heat generation/absorption was discussed by Ahmed (2015). The heat transfer characteristics of the flow over a stretching sheet the presence of magneticfield and thermal radiation was analyzed by Anjali Devi and Prakash (2015). The effects of thermal radiation on the flow of an incompressible viscous electrically conducting fluid over an unsteady stretching sheet embedded in a porous medium in the presence of heat source or sink was studied by Santosh et al. (2015). Raju et al. (2015) discussed the effect of aligned magneticfield on the flow of a ferro fluid over a plate. He extended this work by considering the flow over a stretching surface.

The study of heat transfer of a viscous nanofluid over a nonlinearly stretching sheet in the presence of thermal radiation was stated by Hady et al. (2012). The Magnetohydrodynamic boundary surface of a Casson fluid in the presence of nano particles was investigated by Hussain et al. (2015). Mohan Krishna et al. (2013, 2014, 2015) discussed the effect of thermal radiation by considering different types of flows. Ramana Reddy et al. (2014) studied the aligned magneticfield, radiation and chemical reaction effects on unsteady dusty viscous flow. Sandeep et al. (2012-15) presented their valuable contributions to analyze the heat and mass transfer performance of the different types of fluids. Sugunamma and Sandeep (2011) discussed an unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate through porous media.

The main aim of this study is to investigate the influence of nonlinear thermal radiation and viscous dissipation on three-dimensional MHD Casson fluid flow over a stretching surface in the presence of chemical reaction. The transformed governing equations are solved numerically using Runge-Kutta based shooting technique. The influence of non-dimensional parameters on velocity, temperature and concentration profiles along with the friction factor, local Nusselt and Sherwood numbers are discussed with the help of graphs and tables.

2. Problem statement

Consider a three-dimensional, steady, incompressible flow of a Casson fluid over a stretching sheet. It is

considered that the sheet is stretched along the xy -plane while fluid is placed along the z -axis. Moreover, it is considered that constant magnetic field is applying normal to the fluid flow. The heat transfer process is taken in to account. It is assumed that induced magnetic field is negligible. Here, we assumed that sheet has stretched with the linear velocities (Fig. 1) $u = ax$ and $v = by$ along the xy -plane, respectively, with constants a and b . The rheological equation of state for an isotropic flow of a Casson fluid can be expressed as follows:

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{p_z}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left(\mu_B + \frac{p_z}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c \end{cases}$$

In the above equation $\pi = e_{ij}e_{ij}$ and e_{ij} denotes $(i, j)^{th}$ component of the deformation rate, π be the product of the component of deformation rate itself, π_c be critical value of this product based on the non-Newtonian model, μ_B be the plastic dynamic viscosity of the Casson fluid and p_z be the yield stress of the fluid.

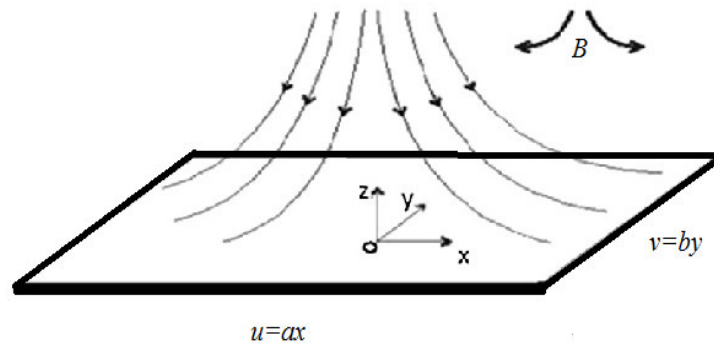


Fig. 1 Physical model and coordinate system

The conservation equation of continuity, momentum and energy equation is as follows: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$, (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B^2}{\rho} u - \frac{\nu}{K_p} u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B^2}{\rho} v - \frac{\nu}{K_p} v, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial z^2} + \frac{16\sigma^*}{3\rho c_p k^*} \frac{\partial}{\partial z} \left(T^3 \frac{\partial T}{\partial z} \right) + \frac{\mu}{\rho C_p} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial w}{\partial z} \right)^2, \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_m \frac{\partial^2 C}{\partial z^2} - k_l (C - C_\infty), \quad (5)$$

Where u, v and w are the velocity components in the x, y and z -direction, respectively, $\beta = \mu_B \sqrt{2\pi_c} / p_z$ is the Casson fluid parameter, B is the magnetic induction, T is the temperature, ν be the viscosity, K_p is the permeability, ρ is the density of the fluid, k is the thermal conductivity, D_m is the mass diffusivity, k_l is the chemical reaction and c_p is specific heat capacitance.

The associated boundary conditions of Eqn. (2)-(5) are as follows:

$$u = U_w(x) = ax, \quad v = V_w(x) = by, \quad w = 0$$

$$-k \frac{\partial T}{\partial z} = h_f(T_f - T), -D_m \frac{\partial C}{\partial z} = h_s(C_f - C) \text{ at } z = 0 \quad (6)$$

$$u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty \quad (7)$$

In the above expression, U_w and V_w are stretching velocities in the x and y directions, respectively. h_f is the convective heat transfer coefficient and T_f is the convective fluid temperature below the sheet, h_s is the mass transfer coefficient and C_f is the fluid concentration below the sheet.

Introducing the following similarity transformations

$$u = axf'(\eta), \quad v = byg'(\eta), \quad w = -\sqrt{a/v}(f(\eta) + g(\eta))$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = z\sqrt{a/v}, \quad \phi(\eta) = \left(\frac{c - c_\infty}{c_f - c_\infty} \right) \quad (8)$$

Equation (1) is automatically satisfied by (8) and Equations. (2) - (5) become

$$\left(1 + \frac{1}{\beta}\right) f''' - (f')^2 - (f + cg)f'' - (M + K)f' = 0, \quad (9)$$

$$\left(1 + \frac{1}{\beta}\right) g''' - (g')^2 - (f + cg)g'' - (M + K)g' = 0 \quad (10)$$

$$\left[1 + R(1 + (\theta_w - 1)\theta)^3\right] \theta'' + R\left[3(\theta_w - 1)\theta^2\right] \left[1 + (\theta_w - 1)\theta\right]^2 + \text{Pr} Ec (f' + cg')^2 \quad (11)$$

$$\frac{1}{Sc} \phi'' + (f + cg)\phi' - Kr\phi = 0 \quad (12)$$

With the boundary conditions

$$f = 0, \quad g = 0, \quad f' = 1, \quad g' = c, \quad (13)$$

$$\theta' = -Bi_1(1 - \theta(0)), \quad \phi' = -Bi_2(1 - \phi(0))$$

$$f' = 0, \quad g' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \text{ as } \eta \rightarrow \infty \quad (14)$$

In the above expression $M = \frac{\sigma B_0^2}{\rho a}$ is a magnetic parameter, $K = \frac{v}{aK_p}$ is the porosity parameter,

$\text{Pr} = \frac{\rho c_p}{k}$ is Prandtl number, $R = \frac{16\sigma^* T_\infty^3}{3kk^*}$ is the radiation parameter,

$Bi_1 = \frac{h_f}{k} \left(\sqrt{v/a}\right)$, $Bi_2 = \frac{h_s}{D_m} \left(\sqrt{v/a}\right)$ are the Biot numbers and $c = b/a$ is the stretching parameter and

Schmidt number $Sc = \frac{D_m}{v}$ and Eckert number $Ec = \frac{a}{T_\infty Cp(\theta_w - 1)}$.

3. Skin friction and local Nusselt number

Expression for skin friction coefficient C_f on the surface along the x and y -directions, which are denoted by C_{fx} and C_{fy} , respectively are as follows:

$$C_{fx} = \frac{\tau_{wx}}{\rho u_w^2}, \quad C_{fy} = \frac{\tau_{wy}}{\rho u_w^2} \quad (15)$$

Where τ_{wx} , and τ_{wy} are the wall shear stress along x and y -directions, respectively.

$$Re_x^{1/2} C_{fx} = \left(1 + \frac{1}{\beta}\right) f''(0), Re_x^{1/2} C_{fy} = \left(1 + \frac{1}{\beta}\right) g''(0) \quad (16)$$

where $Re_x = u_x(x) x / \nu$ is the local Reynolds number which depends on the stretching velocity $u_w(x)$.

The local Nusselt and Sherwood numbers in non-dimensional form are given by

$$Re_x^{-1/2} Nu = -\theta'(0), Re_x^{-1/2} Sh = -\phi'(0), \quad (17)$$

4. Results and discussion

The system of equations (9) to (12) subjected to the boundary conditions (13) and (14) solved numerically. The study is conducted on Newtonian and Non-Newtonian fluids to illustrate the effects of different governing parameters like, the Magnetic field parameter M , Radiation parameter R , wall temperature θ_w , Stretching parameter c , Chemical reaction Kr , Biot number Bi_1, Bi_2 and Schmidt number Sc on velocity, temperature and concentration profiles of the flow. For numerical results we considered $Sc = 2, M = 1, Bi_1 = 0.8, K = c = R = Kr = Sr = Bi_2 = 0.5, Pr = 3, Ec = 0.2, \theta_w = 1.1$, these values are kept as common in entire study except the varied values as displayed in the respective graphs and tables.

Figs. 2-5 show the effect of magnetic field parameter on velocity, temperature and concentration profiles of the flow. It can be seen that the result is true since the velocity decreases with increase in magnetic parameter M . It is evident that an increase in the magnetic field parameter depreciates the velocity and enhances the temperature and concentration profiles of the flow. It is due to the fact that the increasing magnetic field parameter improves the opposite force to the flow of direction which is called Lorentz force. This shows that the Lorentz force slows down the motion of electrically conducting fluid.

Figs. 5 and 6 show that an increase in the radiation parameter and temperature parameter enhances the temperature profiles of the flow. This is due to the fact that a rise in the radiation parameter releases the heat energy to the flow. This agrees with the general physical behaviour of the radiation parameter. Figs. 8 -11 depict the effect of stretching ratio parameter on velocity, temperature and concentration profiles of the flow. It is clear that an increase in the stretching ratio parameter declines $f'(\eta)$, temperature and concentration profiles. But it showed reverse action in the velocity profiles $g'(\eta)$.

Fig. 12 illustrates the effects of chemical reaction parameter on the concentration profile of the flow. It is clear that an increase in the chemical reaction parameter declines the concentration profile of the flow. Figs. 13 and 14 show the effect of Biot numbers on temperature and concentration profiles. It indicates that the increase in Biot number Bi_1 increases the temperature profiles and an increase in the Biot number Bi_2 enhances the concentration profiles of the flow. Physically, the Biot number is expressed as the convection at the surface of the body to the conduction within the surface of the body. When thermal gradient is applied on the surface, the ratio governing the temperature inside the body varies significantly, while the body heats or cools over a time.

Table 1 shows the variation in friction factors, local Nusselt and Sherwood numbers for different values of non-dimensional parameters. It is evident that magnetic field parameter have tendency to reduce the friction factors along with heat and mass transfer rate. Chemical reaction parameter have tendency to enhance the mass transfer rate. Radiation parameter declines the heat transfer rate and Biot numbers are shown mixed response in heat and mass transfer performance.

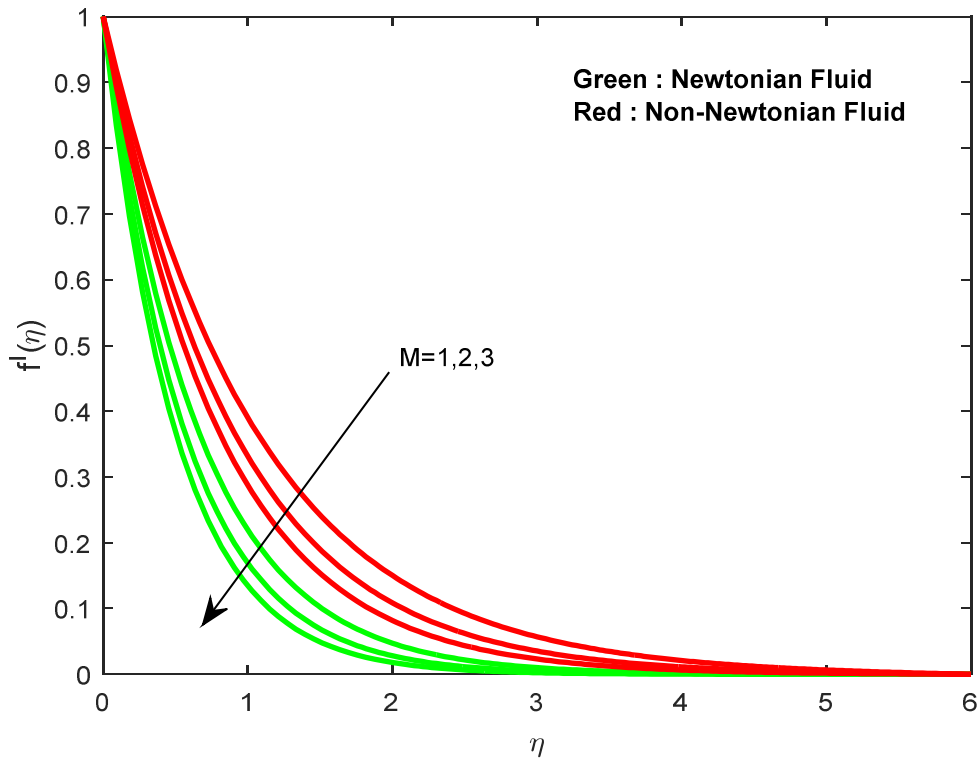


Fig.2 Velocity profiles for various values of Magnetic parameter M

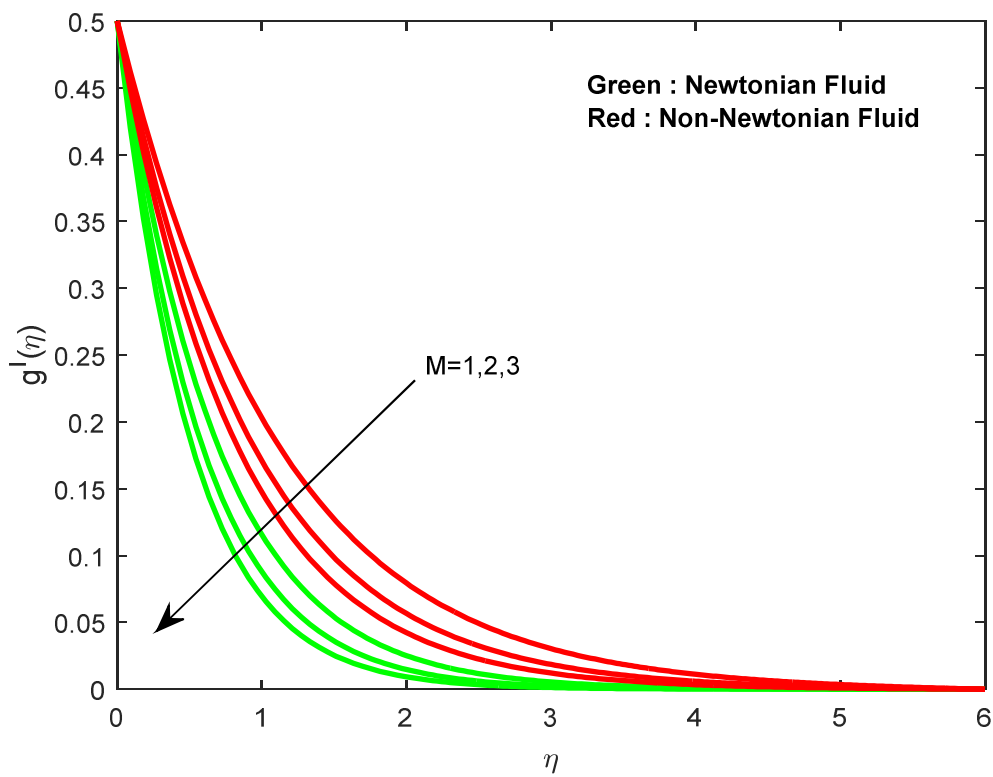


Fig.3 Velocity profiles for various values of Magnetic parameter M

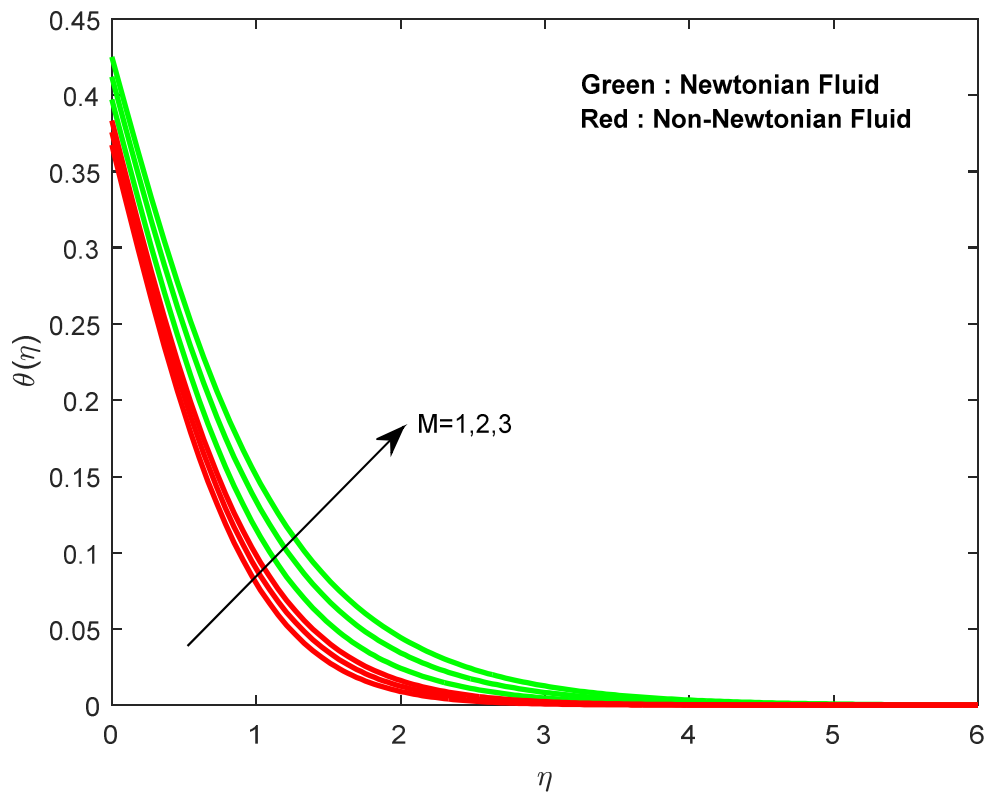


Fig.4 Temperature profiles for various values of Magnetic parameter M

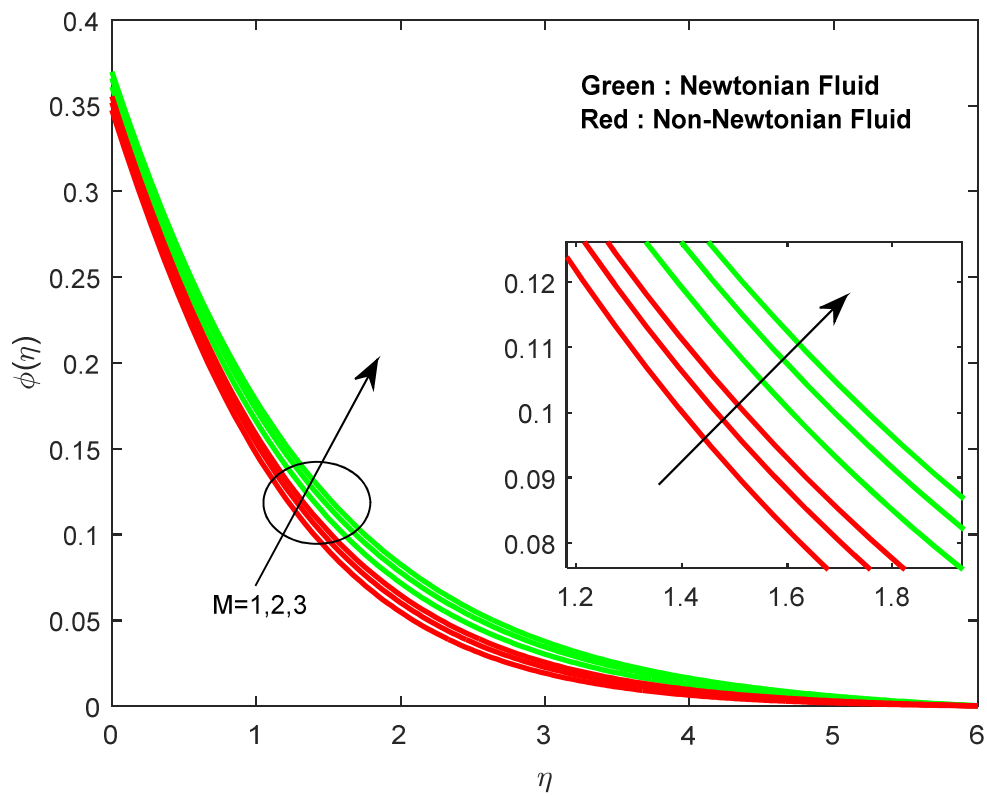


Fig.5 Concentration profiles for varies values of Magnetic parameter M

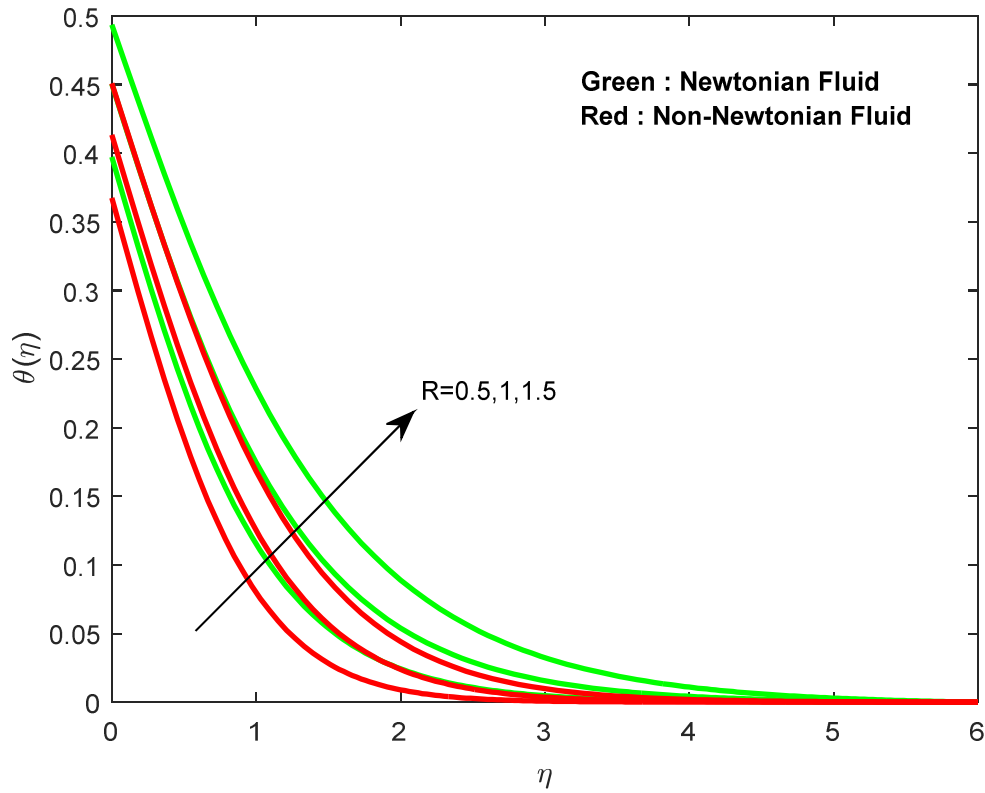


Fig.6 Temperature profiles for various values of Radiation parameter R

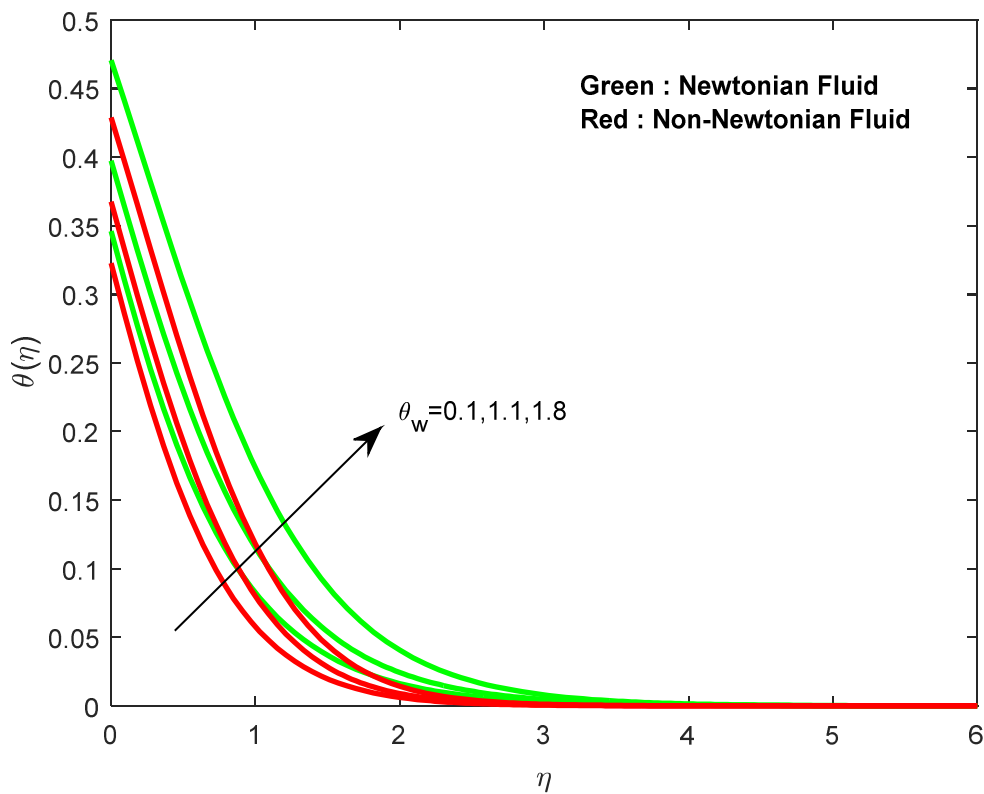


Fig.7 Temperature profiles for various values of wall temperature θ_w

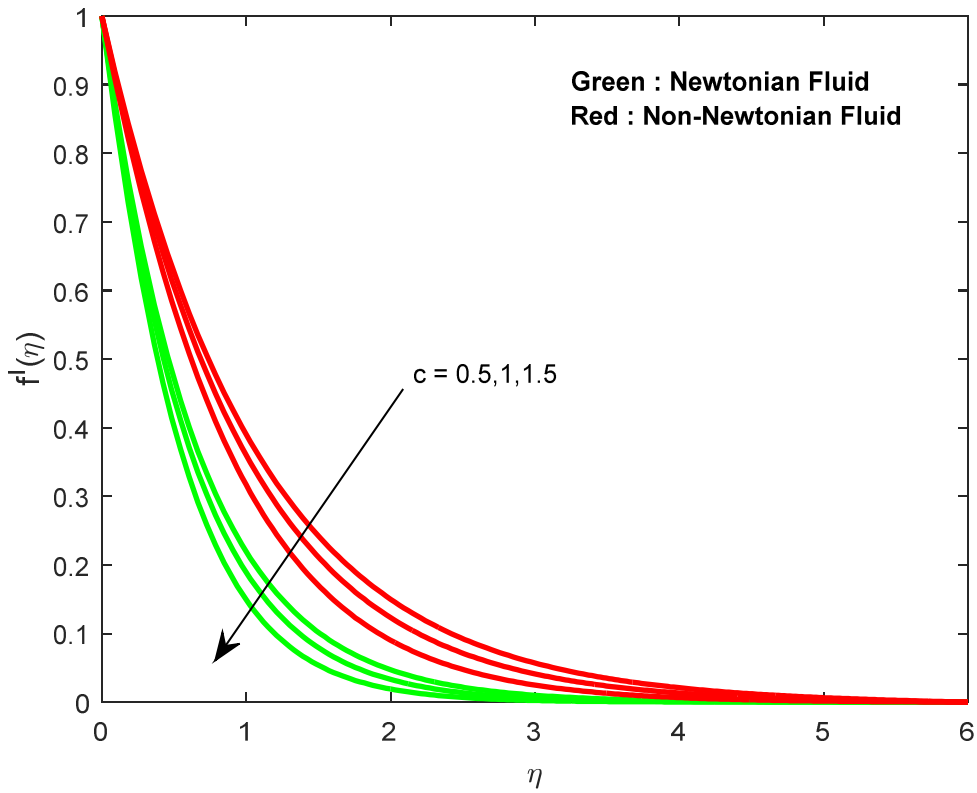


Fig.8 Velocity profile for various values of Stretching parameter c

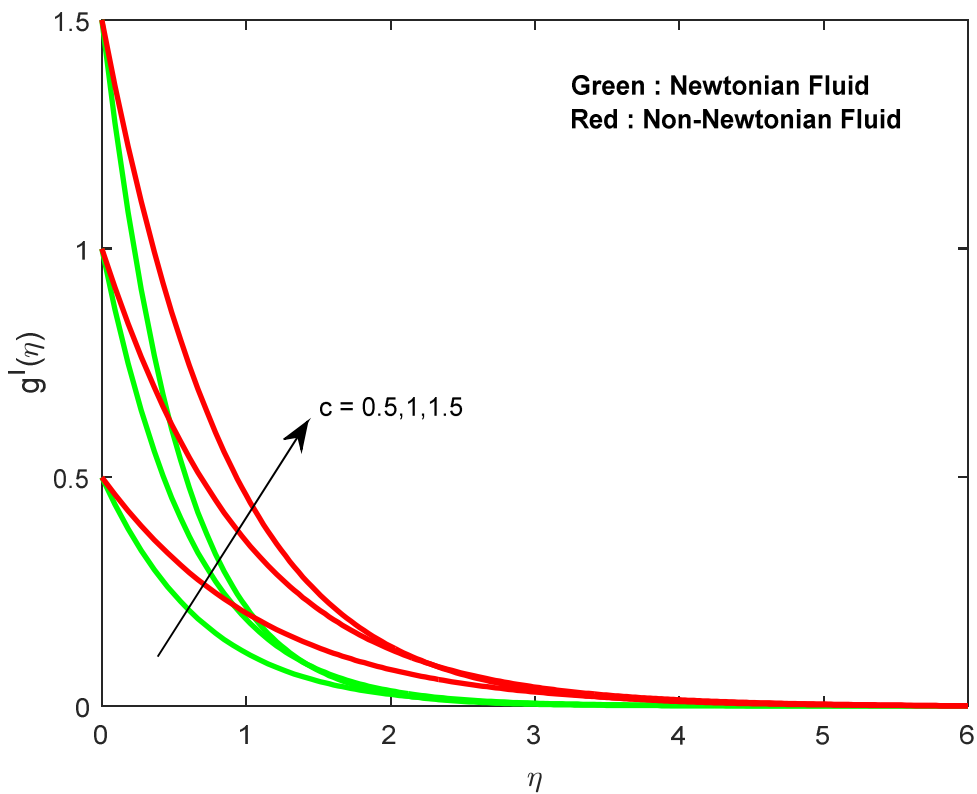


Fig.9 Velocity profiles for varies values of stretching parameter c

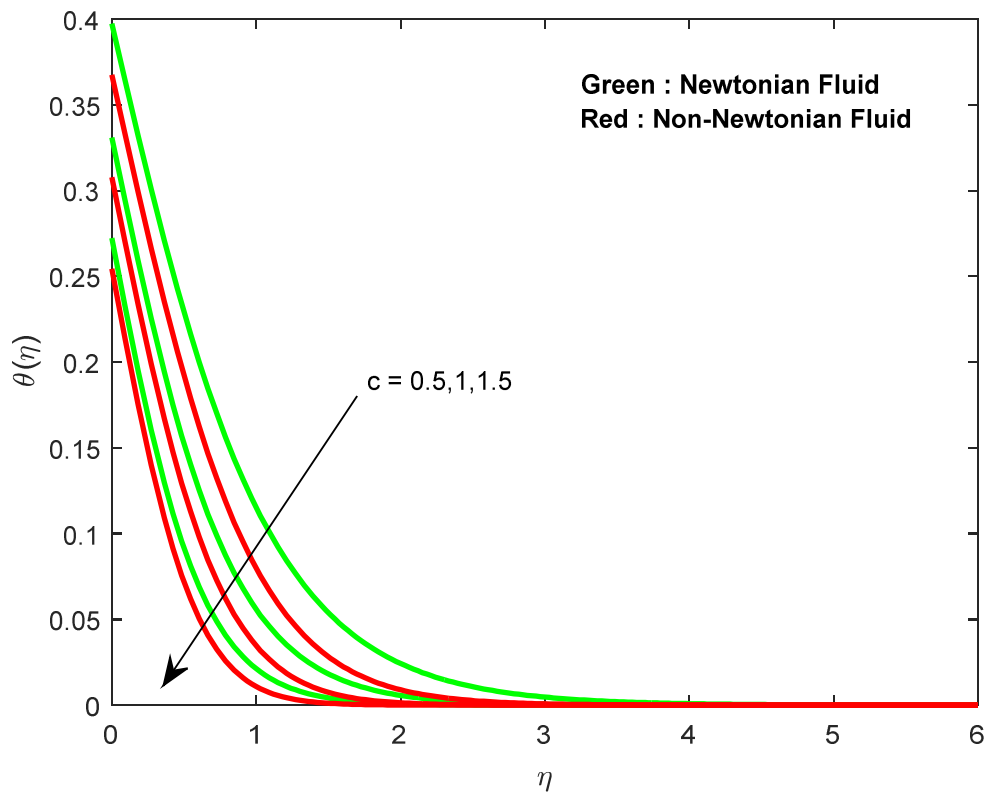


Fig.10 Temperature profiles for various values of Stretching parameter c

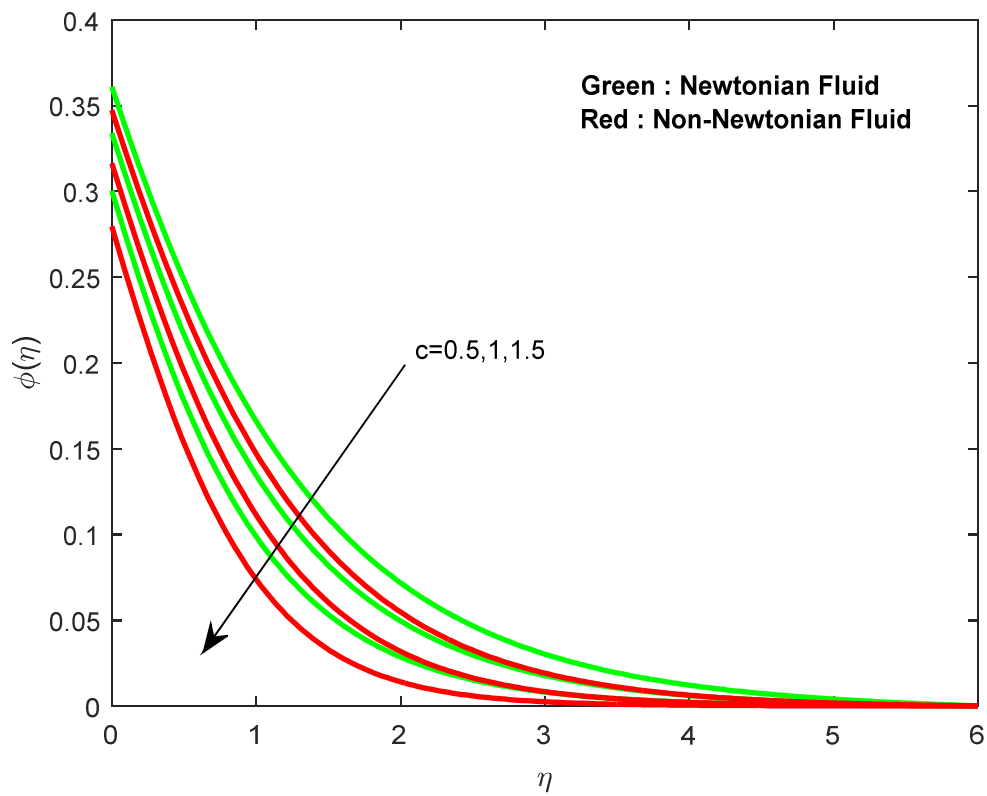


Fig.11 Concentration profiles for varies values of stretching parameter c

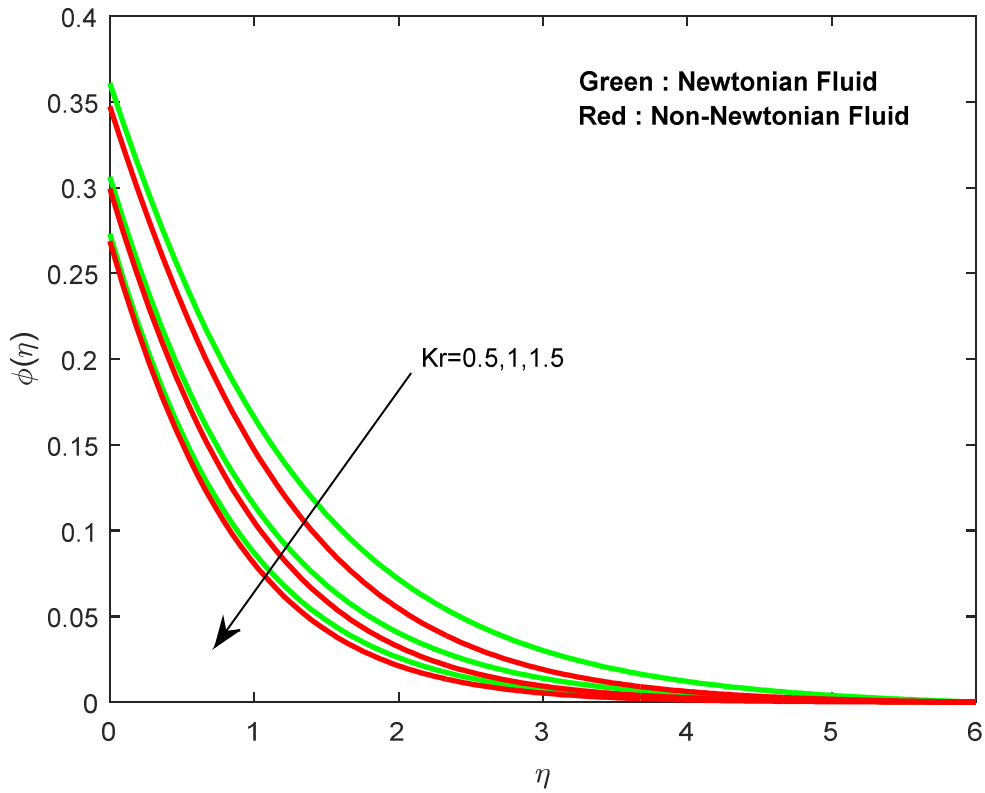


Fig.12 Concentration profiles for varies values of Chemical reaction Kr

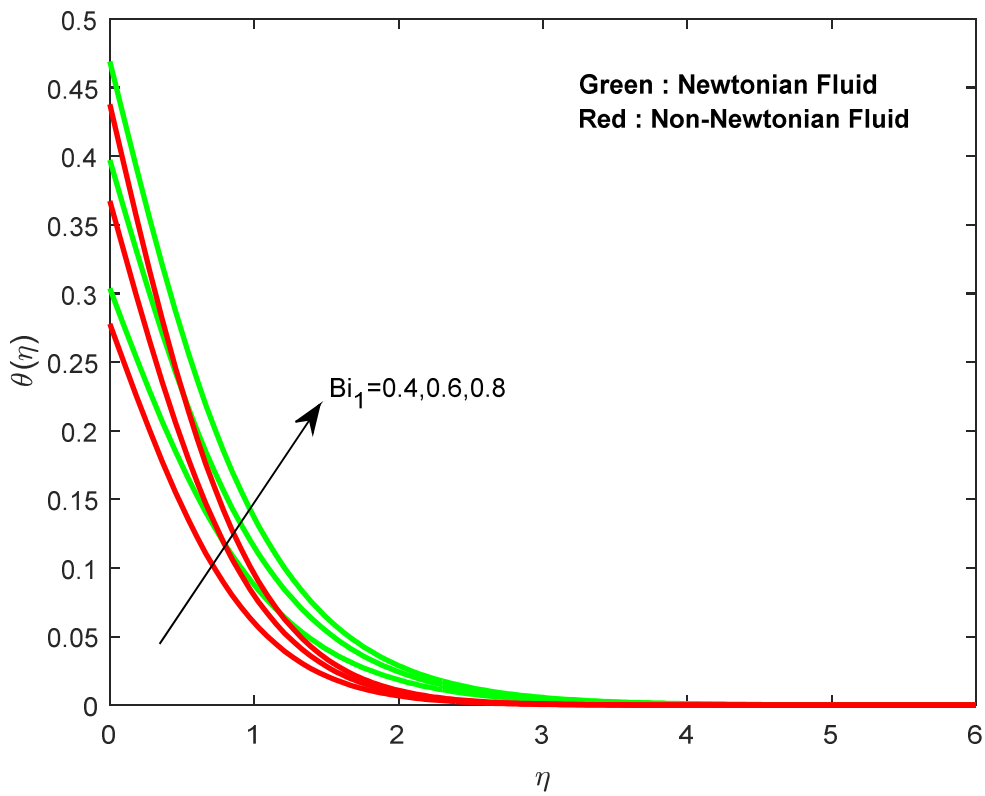


Fig.13 Temperature profiles for various values of Biot number Bi_1

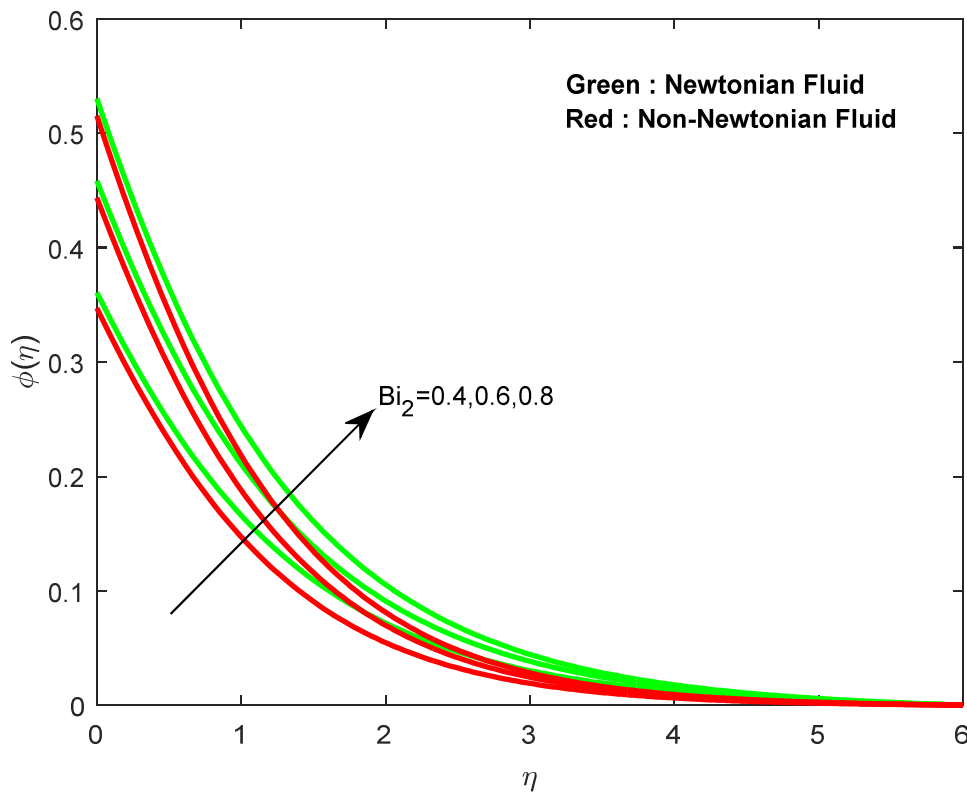


Fig.14 Concentration profiles for varies values of Biot number Bi_2

Table 1 Variation in $f''(0)$, $g''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Casson fluid at different non dimensional parameters

M	R	c	Kr	Bi_1	Bi_2	$f''(0)$	$g''(0)$	$-\theta'(0)$	$-\phi'(0)$
1						-0.928995	-0.433610	0.379430	0.261061
2						-1.093475	-0.520730	0.374424	0.259277
3						-1.236412	-0.595323	0.369931	0.257860
	0.5					-0.928995	-0.433610	0.379430	0.261061
	1					-0.928995	-0.433610	0.351900	0.261061
	1.5					-0.928995	-0.433610	0.329343	0.261061
		0.5				-0.928995	-0.433610	0.379430	0.261061
		1				-0.975414	-0.975414	0.415254	0.273461
		1.5				-1.049260	-1.650940	0.447380	0.288164
			0.5			-0.928995	-0.433610	0.379430	0.261061
			1			-0.928995	-0.433610	0.379430	0.280264
			1.5			-0.928995	-0.433610	0.379430	0.292504
				0.4		-0.928995	-0.433610	0.288819	0.261061
				0.6		-0.928995	-0.433610	0.379430	0.261061
				0.8		-0.928995	-0.433610	0.449634	0.261061
					0.4	-0.928995	-0.433610	0.379430	0.261061
					0.6	-0.928995	-0.433610	0.379430	0.333645
					0.8	-0.928995	-0.433610	0.379430	0.387518

5. Conclusions

This study presents a numerical solution to investigate the influence of nonlinear thermal radiation and viscous dissipation on three-dimensional MHD Casson fluid flow over a stretching surface in the presence of chemical reaction. The transformed governing equations are solved numerically using Runge-Kutta based shooting technique. The influence of non-dimensional parameters on velocity, temperature and concentration profiles

along with the friction factor, local Nusselt and Sherwood numbers are discussed with the help of graphs and tables. The conclusions of the present study are as follows:

- A raise in magnetic field parameter decreases the friction factor.
- A raise in the stretching ratio parameter enhances the heat and mass transfer rate.
- An increase in the chemical reaction parameter enhances the mass transfer rate.
- An enhancement in radiation parameter enhances the thermal boundary layer thickness, so the temperature profile increases.

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