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Transient Free Convection in a Vertical Channel with Variable Temperature and Mass Diffusion

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Abstract
Effects of thermal radiation and constant mass diffusion on the transient laminar free convective flow in a vertical channel with have been analyzed. The exact solutions of the governing equations have been obtained by using the Laplace transform technique. The influences of the parameters on the velocity field, temperature distribution, concentration in the fluid, shear stress, rate of heat and mass transfers have been presented either graphically . It is seen that the velocity and temperature fields decrease with an increase in radiation parameter. On the other hand, the rate of heat transfer at the plate $\eta = 1$ decreases with an increase in either radiation parameter or Prandtl number. Further, the shear stress at the plate $\eta = 0$ increases with an increase in either radiation parameter or thermal Grashof number or mass Grashof number.
Keywords: MHD flow, Transient Free convection, thermal Grashof number, mass Grashof number, thermal radiation, Heat transfer, and Mass transfer.

1 Introduction

Natural convection flows resulting from combined heat and mass transfer have been investigated extensively in past decades because of their relevance to many applications in engineering and environmental processes. For example, such flow situations may relate to the design of chemical processing equipment, solar energy collectors, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields, as well as manufacture of polymer films. The variation of concentrations from point to point generates a concentration gradient in the flow, which results in the transportation of one constituent from the higher concentration region to the lower concentration region to achieve the state of concentration equilibrium. This phenomenon is known as mass transfer. In recent times, the study of free convective mass transfer flow has become an object of extensive research, as the effects of heat transfer along with mass transfer are the dominant features in many engineering applications such as rocket nozzles, cooling of nuclear reactors, high sinks in turbine blades, high-speed aircraft and their atmospheric re-entry, chemical devices, and process equipment. Recently Rao et.al.[1] have studied about the phenomenon of heat and mass transfer in the object of extensive research due to its applications in Science and Technology. Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasisolid bodies such as earth and so on. If the temperature of surrounding fluid is rather high, radiation effects play an important role. In such cases one has to take into account the effect of thermal radiation and mass diffusion. The study of Magnetohydrodynamic (MHD) flows have stimulated considerable interest due to its important applications in cosmic fluid dynamics, meteorology, solar physics and in the motion of Earth's core (Cramer and Pai [2]). Das et.al.[3]-[4] also described free convection is a physical process of heat and mass transfer involving fluids which originates when the temperature as well as species concentration change causes density variations inducing buoyancy forces to act on the fluid. Natural convection flows between two long vertical plates have attracted substantial interest in the context of metallurgical fluid dynamics, nuclear reactors, heat exchangers, re-entry aerothermodynamics, astronautics, geophysics, cooling appliances in electronic instruments and applied mathematics observed by Das et.al.[5]-[6]. Investigation of natural convection transport processes due to the coupling of the fluid flow and heat transfer is a challenging as well as interesting phenomenon. It has been extensively studied between vertical plates because of its importance in many engineering applications. Ostrach [7] has first presented the steady laminar free-convective flow of a viscous incompressible fluid between two vertical plates. Ostrach [7] has also studied the combined effects of a free and forced convective laminar flow and heat transfer between two vertical plates. Mass transfer is the net movement of mass from one location of higher concentration, usually meaning a stream, phase, fraction or component, to that of a lower concentration. Mass transfer occurs in many processes, such as absorption, evaporation, drying, precipitation, membrane filtration, and distillation. Mass transfer is the basis for many biological and chemical processes. Biological processes include the oxygenation of blood and the transport of ions across membranes within the kidney[8]. Another process of heat transfer is radiation through electromagnetic waves. Radiation is a process in which energetic particles or energetic waves travel through a vacuum, or through matter-containing media that are not required for their propagation[9].

The effect of heat source on heat transfer and mass diffusion is of immense importance in several physical problems. In literature there are many others using the importance of temperature dependent heat source on the heat transfer of various fluids. However, they ignored space dependent heat source effect which is of immense important in the heat transfer analysis. Some authors have also studied and presented the significance of space dependent heat source in addition to the temperature dependent heat source such as Narahari et. al.[10], Pantokratoras et. al.[11], Pop et. al.[12], Al-Amri et. al.[13], Chauhan [14], Rajput et.al.[15]. Effect of radiation on transient natural convection flow between two vertical plates and effects of radiation on MHD free convective Couette flow in a rotating system have been analyzed by Das et.al.[17]-[18]. Narahari [19]-[20] also investigated transient free convection flow between two long vertical parallel plates with constant temperature and mass diffusion and effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary. Recently unsteady MHD free convective flow along a vertical porous plate embedded in a porous medium with heat generation, variable suction and chemical reaction effects have been studied by Ibrahim [21] and Kothiyal et.al.[22] . The study of radiation interaction with convection for heat and mass transfer in fluids is quite significant.

The aim of the present paper is to study the transient free convection in a vertical channel with variable temperature and mass diffusion. The governing equations are normalized and then solved using the Laplace Transform technique. The solutions are expressed in terms of exponential and complementary error function form. The results are discussed with the help of graphs.

2 Formulation of the problem and its solution

Consider an unsteady laminar transient free convection of a viscous incompressible electrically conducting and radiating fluid between two infinitely long vertical parallel plates with constant temperature and mass diffusion in the presence of transverse magnetic field. Let the channel plates be separated by a distance h. The x-axis is taken along one of the plates in the vertically upward direction and the y axis is taken normal to the plate. Initially, at time $t = 0$, the two plates and the fluid are assumed to be stationary, at the same temperature θ. At time $t > 0$, the plate starts move. It is assumed that the radiative heat flux in the x - direction is negligible as compared to that in the y - direction. As the plates are infinite in length, the velocity and temperature fields are functions of y and t only.

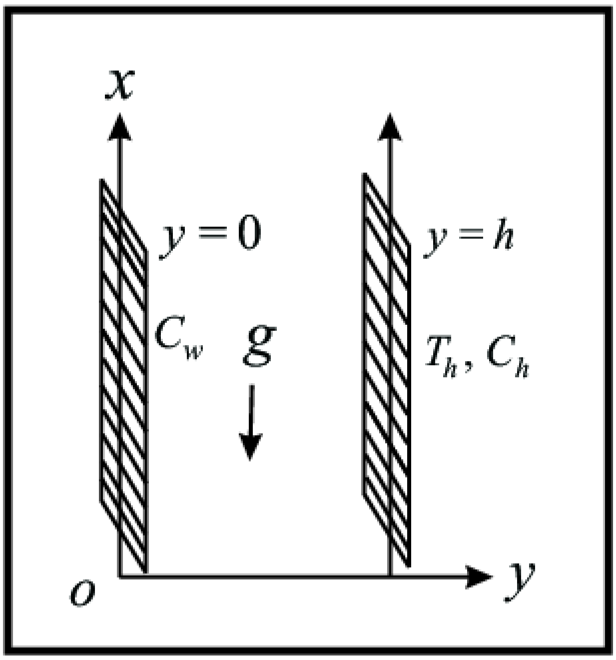

Fig.1: Geometry of the problem

Now the Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density is assumed to depend on the temperature according to the equation of state

$$\rho = \rho_0[1-\beta(T-T_h)], \tag{1}$$

where T is the fluid temperature, ρ the fluid density, β the coefficient of thermal expansion and ρ_0 is the temperature at the entrance of the channel. Then under the usual Boussinesq's approximation, the flow of a radiating fluid is shown to be governed by the following system of equations are:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_h) + g\beta^*(C - C_h), \tag{2}$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{3}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} - K_1^*(C - C_h), \tag{4}$$

where u is the velocity in the x-direction, C the species concentration of the fluid, D the mass diffusivity, t the time, g the acceleration due to gravity, ν the kinematic viscosity, β^* the concentration expansion coefficient, c_p the specific heat at constant pressure, q_r the radiative heat flux and K_1^* is the chemical reaction parameter. For small velocities the heating due to viscous dissipation is neglected in the energy equation (2).

The initial and boundary conditions are

$$t \le 0 : u = 0,\ T = T_h,\ C = C_h \ \ for\ all\ \ y,$$

$$t > 0 : \begin{cases} u = 0,\ T = T_h + (T_0 - T_h)t/t_0, C = C_h + (C_0 - C_h)t/t_0 \ \text{ at } \ y = 0, \\ u = 0,\ T = T_h,\ C = C_h \ \text{ at } \ y = h. \end{cases}$$

It has been shown by Cogley et al.[16] that in the optically thin limit for a non-gray gas near equilibrium, the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4(T - T_h)\int_0^\infty K_{\lambda_h}\left(\frac{\partial e_{\lambda_h}}{\partial T}\right)_h d\lambda, \tag{5}$$

where K_{λ_h} is the absorption coefficient, λ is the wave length, e_{λ_h} is the Planck's function and subscript $'h'$ indicates that all quantities have been evaluated at the temperature T_h which is the temperature of the plate at time $t \le 0$. Thus our study is limited to small difference of plate temperature to the fluid temperature.

On the use of the equation (5), equation (3) becomes:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - 4(T - T_h)I, \tag{6}$$

where

$$I = \int_0^\infty K_{\lambda_h}\left(\frac{\partial e_{\lambda_h}}{\partial T}\right)_h d\lambda, \tag{7}$$

Introducing non-dimensionless variables as given below:

$$\eta = \frac{y}{h}, \tau = \frac{t}{t_0}, t_0 = \frac{h^2}{\nu}, u_1 = \frac{hu}{\nu},\ \theta = \frac{T - T_h}{T_0 - T_h},\ \phi = \frac{C - C_h}{C_0 - C_h}, \tag{8}$$

equations (2), (6) and (4) become

$$\frac{\partial u_1}{\partial \tau} = \frac{\partial^2 u_1}{\partial \eta^2} + Gr\theta + Gc\phi, \tag{9}$$

$$Pr\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} - R\theta, \tag{10}$$

$$Sc\frac{\partial \phi}{\partial \tau} = \frac{\partial^2 \phi}{\partial \eta^2} - K_1\phi \tag{11}$$

where $Gr = \dfrac{g\beta(T_0 - T_h)h^2}{\nu^2}$ the thermal Grashof which approximates the ratio of the buoyancy force to the viscous force acting on a fluid, $Gc = \dfrac{g\beta^*(C_0 - C_h)h^2}{\nu^2}$ the mass Grashof number which is a measure of the relative importance of transport of chemical species, $Pr = \dfrac{\rho\nu c_p}{k}$ the Prandtl number which measures the ratio of momentum diffusivity to the thermal diffusivity, $R = \dfrac{4Ih^2}{K}$ is the radiation parameter, $Sc = \dfrac{\nu}{D}$ Schmidt number and $K_1 = \dfrac{K_1^* h^2}{D}$ is the chemical reaction rate.

The initial and boundary conditions for u_1, ϕ and θ are

$$\tau \le 0: u_1 = 0,\ \phi = 0,\ \theta = 0 \ \text{for all} \ \eta$$

$$\tau > 0: \begin{cases} u_1 = 0,\ \theta = \tau,\ \phi = \tau, \ \text{at} \ \eta = 0 \\ u_1 = 0,\ \phi = 0,\ \theta = 0 \ \text{at} \ \eta = 1 \end{cases} \tag{12}$$

On the use of the Laplace transformation, equations (7) - (9) become

$$s\bar{u}_1 = \frac{d^2\bar{u}_1}{d\eta^2} + Gr\bar{\theta} + Gc\bar{\phi} \tag{13}$$

$$sPr\bar{\theta} = \frac{d^2\bar{\theta}}{d\eta^2} - R\bar{\theta} \tag{14}$$

$$sSc\bar{\phi} = \frac{d^2\bar{\phi}}{d\eta^2} - K_1\bar{\phi} \tag{15}$$

where

$$\bar{u}_1(\eta, s) = \int_0^\infty u_1(\eta,\tau)e^{-s\tau}d\tau,\ \ \bar{\theta}(\eta, s) = \int_0^\infty \theta(\eta,\tau)e^{-s\tau}d\tau \ \text{and} \ \bar{\phi}(\eta, s) = \int_0^\infty \phi(\eta,\tau)e^{-s\tau}d\tau. \tag{16}$$

The corresponding boundary conditions for $\bar{u}_1$, $\bar{\phi}$ and $\bar{\theta}$ are

$$\bar{u}_1 = 0,\ \bar{\theta} = \frac{1}{s^2}, \bar{\phi} = \frac{1}{s^2}, \ \text{at} \ \eta = 0,$$
$$\bar{u}_1 = 0,\ \bar{\theta} = 0,\ \bar{\phi} = 0 \ \text{at} \ \eta = 1. \tag{17}$$

Solutions of equations (13)-(15) subject to the boundary conditions (17) are easily obtained and are given by

$$\bar{\phi}(\eta, s) = \frac{1}{s^2}\frac{\sinh\sqrt{sSc + K_1}(1-\eta)}{\sinh\sqrt{sSc + K_1}}, \tag{18}$$

$$\bar{\theta}(\eta, s) = \frac{1}{s^2}\frac{\sinh\sqrt{sPr + R}(1-\eta)}{\sinh\sqrt{sPr + R}}, \tag{19}$$

$$\bar{u}_1(\eta, s) = c_1\cosh\sqrt{s}\eta + c_2\sinh\sqrt{s}\eta + \frac{1}{s^2}\left[\frac{Gr}{(1-Pr)(s-\alpha)}\frac{\sinh\sqrt{sPr+R}(1-\eta)}{\sinh\sqrt{sPr+R}}\right.$$

$$+\frac{Gc}{(1-Sc)(s-\delta)}\frac{\sinh\sqrt{sSc+K_1}(1-\eta)}{\sinh\sqrt{sSc+K_1}}\Bigg], \tag{20}$$

where $\alpha=\dfrac{R}{1-Pr}$ and $\delta=\dfrac{K_1}{1-Sc}$.

The inverse Laplace transforms of (18) - (19) give the solutions for the mass concentration, temperature and velocity distributions respectively as follows:

$$\begin{aligned}\phi(\tau,\eta)&=\tau\frac{\sinh\sqrt{K_1}(1-\eta)}{\sinh\sqrt{K_1}}\\&+\frac{Sc}{\sqrt{K_1}(\cosh 2\sqrt{K_1}-1)}\left[(1-\eta)\sinh\sqrt{K_1}\cosh\sqrt{K_1}(1-\eta)\right.\\&\left.-\cosh\sqrt{K_1}\sinh\sqrt{K_1}(1-\eta)\right]+\sum_{m=1}^{\infty}\frac{2m\pi e^{s_1\tau}}{Scs_1^2}\sin m\pi\eta,\end{aligned} \tag{21}$$

$$\begin{aligned}\theta(\tau,\eta)&=\tau\frac{\sinh\sqrt{R}(1-\eta)}{\sinh\sqrt{R}}\\&+\frac{Pr}{\sqrt{R}(\cosh 2\sqrt{R}-1)}\left[(1-\eta)\sinh\sqrt{R}\cosh\sqrt{R}(1-\eta)\right.\\&\left.-\cosh\sqrt{R}\sinh\sqrt{R}(1-\eta)\right]+\sum_{m=1}^{\infty}\frac{2m\pi e^{s_2\tau}}{Prs_2^2}\sin m\pi\eta,\end{aligned} \tag{22}$$

$$\begin{aligned}u_1(\tau,\eta)&=\frac{Gr}{1-Pr}\left[-\frac{1}{6\alpha}(1-\eta)\eta(2-\eta)+\frac{1}{\alpha^2}(\tau\alpha+1)\left\{1-\eta-\frac{\sinh\sqrt{R}(1-\eta)}{\sinh\sqrt{R}}\right\}\right.\\&-\frac{Pr}{\alpha\sqrt{R}(\cosh 2\sqrt{R}-1)}\\&\times\left\{(1-\eta)\sinh\sqrt{R}\cosh\sqrt{R}(1-\eta)-\cosh\sqrt{R}\sinh\sqrt{R}(1-\eta)\right\}\\&\left.+\sum_{m=1}^{\infty}2m\pi\sin m\pi\eta\left\{\frac{e^{s_2\tau}}{Prs_2^2(s_2-\alpha)}-\frac{e^{s_3\tau}}{s_3^2(s_3-\alpha)}\right\}\right]\\&+\frac{Gc}{1-Sc}\left[-\frac{1}{6\delta}(1-\eta)\eta(2-\eta)+\frac{1}{\delta^2}(\tau\delta+1)\left\{1-\eta-\frac{\sinh\sqrt{K_1}(1-\eta)}{\sinh\sqrt{K_1}}\right\}\right.\\&-\frac{Sc}{\delta\sqrt{K_1}(\cosh 2\sqrt{K_1}-1)}\\&\times\left\{(1-\eta)\sinh\sqrt{K_1}\cosh\sqrt{K_1}(1-\eta)-\cosh\sqrt{K_1}\sinh\sqrt{K_1}(1-\eta)\right\}\\&\left.+\sum_{m=1}^{\infty}2m\pi\sin m\pi\eta\left\{\frac{e^{s_1\tau}}{Scs_1^2(s_1-\delta)}-\frac{e^{s_3\tau}}{s_3^2(s_3-\delta)}\right\}\right]\end{aligned} \tag{23}$$

where $s_1=-\dfrac{1}{Sc}(m^2\pi^2+K_1)$, $s_2=-\dfrac{1}{Pr}(m^2\pi^2+R)$ and $s_3=-m^2\pi^2$.

Also for large time ($\tau \gg 1$), equation (21)-(23) becomes:

$$\phi(\tau,\eta)=\tau\frac{\sinh\sqrt{K_1}(1-\eta)}{\sinh\sqrt{K_1}}$$
$$+\frac{Sc}{\sqrt{K_1}(\cosh 2\sqrt{K_1}-1)}\left[(1-\eta)\sinh\sqrt{K_1}\cosh\sqrt{K_1}(1-\eta)\right.$$
$$\left.-\cosh\sqrt{K_1}\sinh\sqrt{K_1}(1-\eta)\right] \tag{24}$$
$$\theta(\tau,\eta)=\tau\frac{\sinh\sqrt{R}(1-\eta)}{\sinh\sqrt{R}}$$
$$+\frac{Pr}{\sqrt{R}(\cosh 2\sqrt{R}-1)}\left[(1-\eta)\sinh\sqrt{R}\cosh\sqrt{R}(1-\eta)\right.$$
$$\left.-\cosh\sqrt{R}\sinh\sqrt{R}(1-\eta)\right] \tag{25}$$
$$u_1(\tau,\eta)=\frac{Gr}{1-Pr}\left[-\frac{1}{6\alpha}(1-\eta)\eta(2-\eta)+\frac{1}{\alpha^2}(\tau\alpha+1)\left\{1-\eta-\frac{\sinh\sqrt{R}(1-\eta)}{\sinh\sqrt{R}}\right\}\right.$$
$$-\frac{Pr}{\alpha\sqrt{R}(\cosh 2\sqrt{R}-1)}$$
$$\left.\times\left\{(1-\eta)\sinh\sqrt{R}\cosh\sqrt{R}(1-\eta)-\cosh\sqrt{R}\sinh\sqrt{R}(1-\eta)\right\}\right]$$
$$+\frac{Gc}{1-Sc}\left[-\frac{1}{6\delta}(1-\eta)\eta(2-\eta)+\frac{1}{\delta^2}(\tau\delta+1)\left\{1-\eta-\frac{\sinh\sqrt{K_1}(1-\eta)}{\sinh\sqrt{K_1}}\right\}\right.$$
$$-\frac{Sc}{\delta\sqrt{K_1}(\cosh 2\sqrt{K_1}-1)}$$
$$\left.\times\left\{(1-\eta)\sinh\sqrt{K_1}\cosh\sqrt{K_1}(1-\eta)-\cosh\sqrt{K_1}\sinh\sqrt{K_1}(1-\eta)\right\}\right]$$

3 Results and discussion

In order to get the physical insight into the problem, the numerical values of the velocity, temperature, concentration distribution, shear stress, are computed for different values of the system parameters such as radiation parameter R, Grashof number Gr, Prandtl number Pr and time tau. Fig.2 shows the fluid velocity decreases with increase in radiation parameter R. The influence of thermal Grashof number Gr on the fluid velocity u_1 is elucidated from Fig. 3. It can be observed that the fluid velocity u_1 increases for the increasing values of Gr. Grashof number Gr physically describes the ratio of bouyancy force to viscouss forces. Therefore, an increase in the values of Gr leads to increase in buoyancy forces, consequently the fluid velocity increases. Here the thermal Grashof number represents the effect of free convection currents. Physically, $Gr > 0$ means heating of the fluid of cooling of the boundary surface, $Gr < 0$ means cooling of the fluid of heating of the boundary surface and $Gr = 0$ corresponds the absence of free convection current.Fig.4 displays that an increase in Gc leads to increase in fluid velocity. Fig.5 shows that the fluid velocity decreases with an increase in Prandtl number. Physically, in heat transfer problems, the Prandtl number controls the the relative thickness of the momentum and thermal boundary layers. When Pr is small, it means that the heat diffuses very quickly compared to the velocity(momentum). This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer, i.e. for $Pr \ll 1$ the thermal diffusivity dominates and for $Pr \gg 1$ means that momentum diffusivity dominates. when $Pr = 1$, the boundary layer coincides. In the process of our calculation, the values of the Prandtl number Pr has been taken as 0.71 and 7.0 corresponding to the realistic fluids, i.e., air and water respectively. The mass transfer analog of the Prandtl number Pr is the Schmidt number Sc. We also obtained similar results from Fig.6 for Schmidt number Sc, i.e. an increase in Sc leads to decrease in fluid velocity. Fig.7 demonstrates that the fluid velocity decreases with an increase in chemical reaction parameter K_1. The values of Schmidt number Sc is chosen to be $Sc = 0.22, 0.45, 0.63$ and 0.77 representing diffusing chemical species of most common interest. The physics behind this observation is that the increased Schmidt number decreases chemical species molecular diffusivity, which ultimately reduces fluid velocity. Also, an increase of Sc (a predominance of diffusive transport of momentum over the mass) represents increase in the momentum boundary layer thickness with a fixed species diffusivity and this causes the decrease in velocity. Fig.8 illustrates that the the fluid velocity u_1 increases with an increase in the time parameter τ. This is due to increasing buoyancy effects in vertical channel as time progresses.

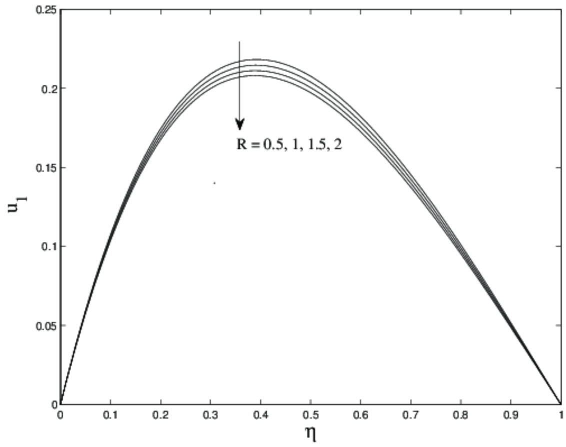

Fig 2. Velocity profiles for R when $Pr = 0.71$, $Gr = 5$, $Gc = 5$, $Sc = 0.6$ and $\tau = 0.5$

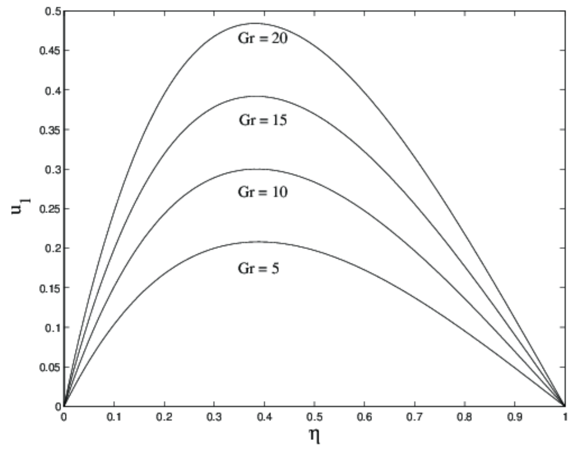

Fig 3. Velocity profiles for Gr when $Pr = 0.71$, $R = 2$, $Gc = 5$, $Sc = 0.6$ and $\tau = 0.5$

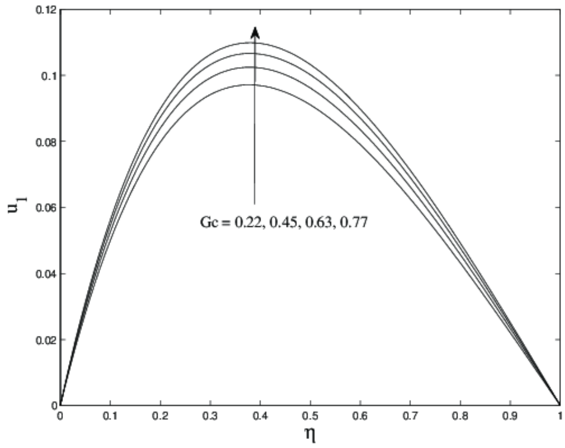

Fig 4. Velocity profiles for Gc when $Pr = 0.71$, $R = 2$, $Gc = 5$, $Sc = 0.6$ and $\tau = 0.5$.

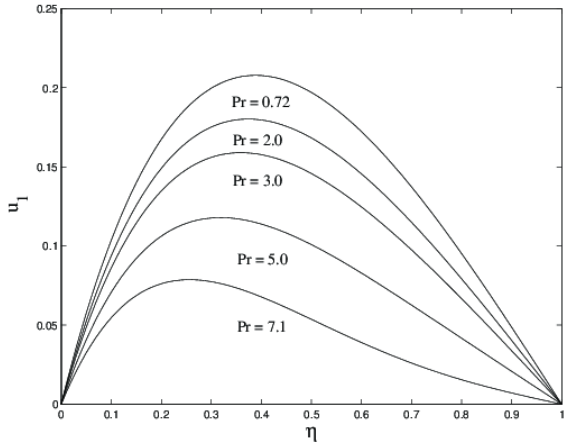

Fig 5. Velocity profiles for Pr when $R = 2$, $Gr = 5$, $Gc = 5$, $Sc = 0.6$ and $\tau = 0.5$

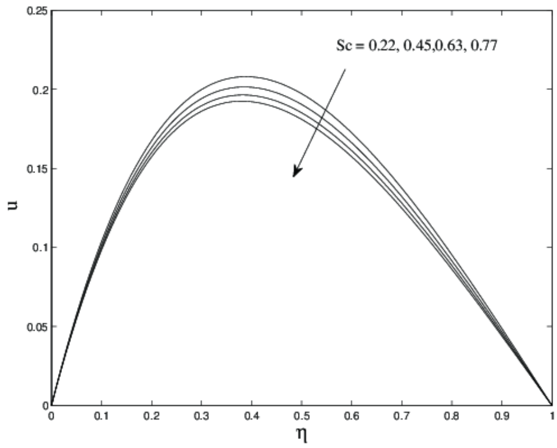

Fig 6. Velocity profiles for Sc when $Pr = 0.71$, $Gr = 5$, $Gc = 5$, $R = 2$ and $\tau = 0.5$

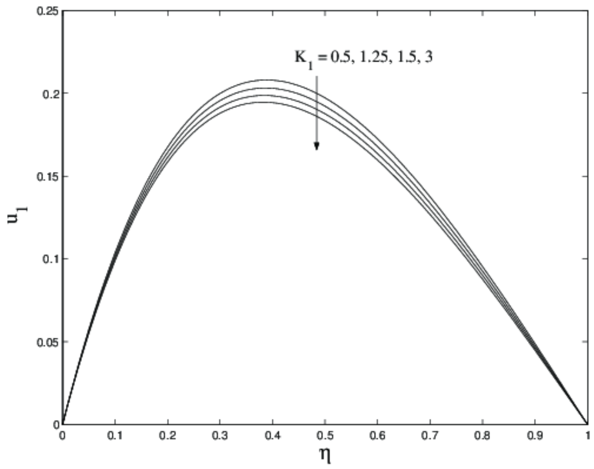

Fig 7. Velocity profiles for K_1 when $Pr = 0.71$, $Gr = 5$, $Gc = 5$, $Sc = 0.6$ and $R = 2$

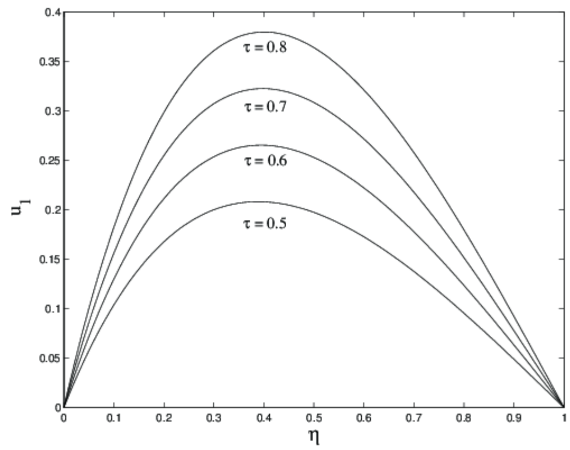

Fig 8. Velocity profiles for τ when $Pr = 0.71$, $Gr = 5$, $Gc = 5$, $Sc = 0.6$ and $R = 2$

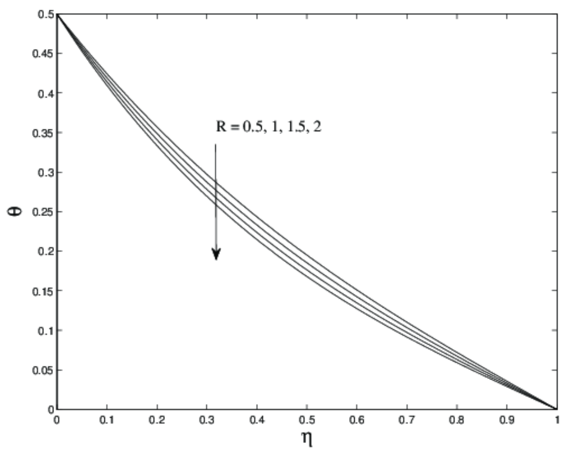

Fig 9. Temperature profiles for R when $Pr = 0.71$ and $\tau = 0.5$

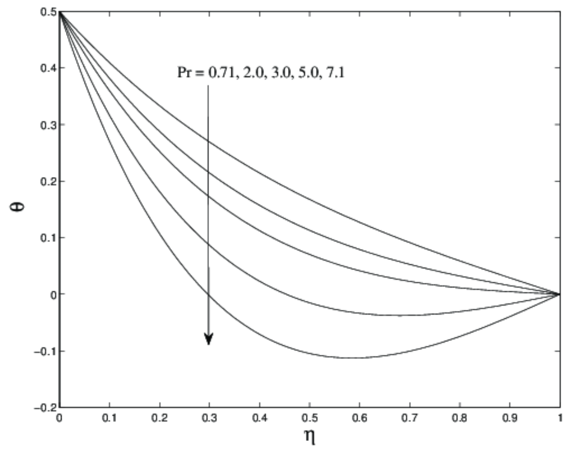

Fig 10. Temperature profiles for Pr when $R = 2$ and $\tau = 0.5$

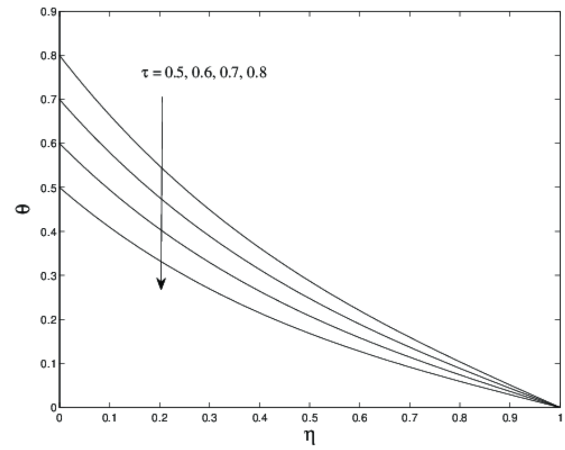

Fig 11. Temperature profiles for τ when $Pr = 0.71$ and $R = 2$

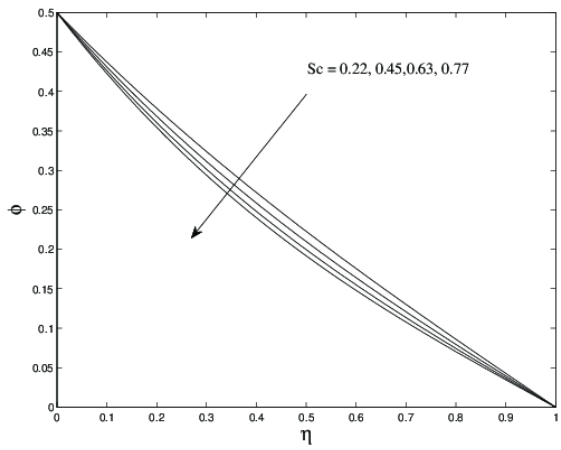

Fig 12. Concentration profiles ϕ for Sc when $\tau = 0.5$

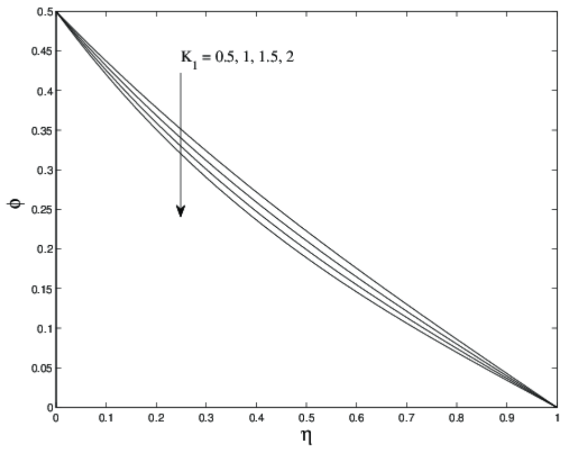

Fig 13. Concentration profiles ϕ for K_1 when $Sc = 0.6$

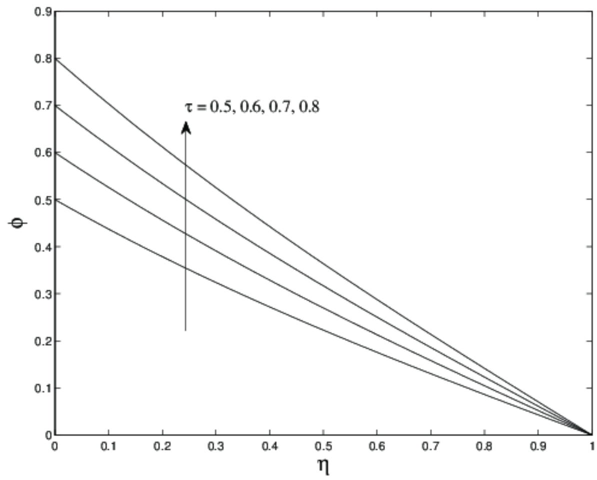

Fig 14. Concentration profiles ϕ for τ when $Sc = 0.6$

Fig.9 explains that the fluid temperature decreases with an increase in the radiation parameter R. Also Figs.10 and 11 reveal that an increase in Prandtl number Pr and τ result to decrease in the temperature distribution, because thermal boundary layer thickness decreases with an increase in Prandtl number Pr. An increase in the Prandtl number Pr means slow rate of thermal diffusion. The underlying physics behind this is when fluid attains a higher Prandtl number, its thermal conductivity is lowered down, so its heat conduction capacity diminishes. Thereby the the thermal boundary layer thickness gets reduced. Consequently, the heat transfer rate at the surface is increased. This is also very interesting results in Figs. 12 and 13 that an increase in Schmidt number Sc and chemical reaction rate K_1, concentration profile decreases. Fig.14 also reveals that concentration rate increases with increase in τ.

The rate of mass transfer at the plate $\eta = 0$ is given by

$$\phi_\eta(0,\tau) = \left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0}$$
$$= \sum_{m=1}^{\infty} \frac{2m^2\pi^2 e^{s_1\tau}}{Sc s_1^2} - \frac{\tau\sqrt{K_1}\cosh\sqrt{K_1}}{\sinh\sqrt{K_1}} \tag{26}$$
$$+ \frac{Sc}{\sqrt{K_1}(\cosh 2\sqrt{K_1} - 1)}\left[\sinh\sqrt{K_1}\left\{\sqrt{K_1}\sinh\sqrt{K_1} - \cosh\sqrt{K_1}\right\} + \sqrt{K_1}\cosh^2\sqrt{K_1}\right]$$
,

The rate of mass transfer $-\phi_\eta(0,\tau)$ at the plate $\eta = 0$ are presented for several values of Schmidt number Sc, K_1 and time τ. Fig.16 confirms us that concentration profile increases with an increase in K_1 whereas Fig.17 shows that the rate of mass transfer $-\phi_\eta(0,\tau)$ decreases with an increase in Schmidt number Sc. The variation of Schmidt number Sc shows that lesser the molecular diffusivity enhance the rate of mass transfer at the plate $\eta = 0$. With respect to time τ it is noticed that the rate of mass transfer decreases in progressing of time. The negative sign indicates that the mass transfers from the fluid to the plate.

The rate of heat transfer at the plate $\eta = 1$ is given by

$$\theta_\eta(1,\tau) = \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=1}$$
$$= \sum_{m=1}^{\infty}(-1)^m \frac{2m^2\pi^2}{Prs_2^2} - \frac{\tau\sqrt{R}}{\sinh\sqrt{R}}$$
$$+ \frac{Pr}{\sqrt{R}(\cosh 2\sqrt{R}-1)}\left[\sqrt{R}\cosh\sqrt{R} - \sinh\sqrt{R}\right] \quad (27)$$

The results of the rate of heat transfer $-\theta_\eta(1,\tau)$ at the plate $\eta = 1$ are presented for several values of radiation parameter R, Prandtl number Pr and time τ. It is seen from figures that the rate of heat transfer $-\theta_\eta(1,\tau)$ decreases with an increase in either Prandtl number Pr or radiation parameter R whereas it increases with an increase in time τ with light of the Fig.15. It is consistent with the fact that smaller values of Pr are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Pr and hence the rate of heat transfer is reduced. Since the radiation R causes a faster dissipation of heat and consequently heat is able to diffuse away from the plate and hence the rate of heat transfer is reduced.

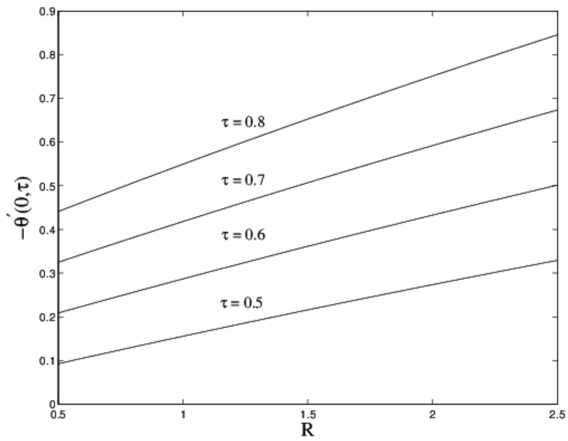

Fig 15. Temperature profiles θ for different time τ when $Sc = 0.6$

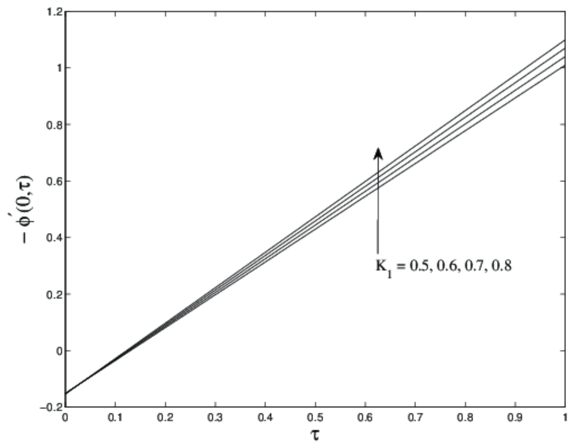

Fig 16. Concentration profiles ϕ for different K_1 when $Sc = 0.6$

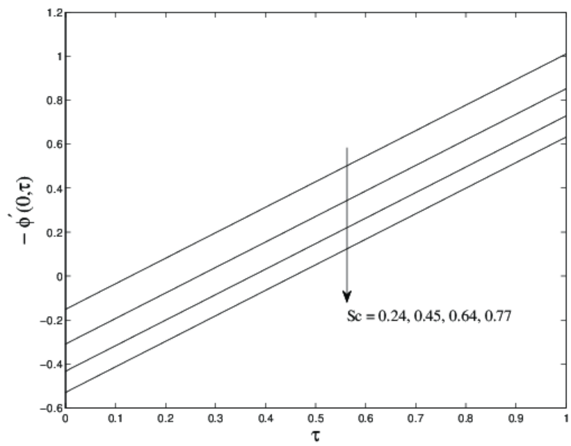

Fig 17. Concentration profiles ϕ for different Sc when $K_1 = 0.5$

The non-dimensional shear stress at the plate $\eta = 0$ is given by

$$\tau_x = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0}$$

$$= \frac{Gr}{1-Pr}\left[-\frac{1}{3\alpha}+\frac{1}{\alpha^2}(\tau\alpha+1)\left\{\frac{\sqrt{R}\cosh\sqrt{R}}{\sinh\sqrt{R}}-1\right\}\right.$$

$$+\frac{Pr}{\alpha\sqrt{R}(\cosh 2\sqrt{R}-1)}\left\{\sqrt{R}\sinh^2\sqrt{R}+\sinh\sqrt{R}\cosh\sqrt{R}-\sqrt{R}\cosh^2\sqrt{R}\right\}$$

$$\left.+\sum_{m=1}^{\infty}2m^2\pi^2\left\{\frac{e^{s_2\tau}}{Prs_2^2(s_2-\alpha)}-\frac{e^{s_3\tau}}{s_3^2(s_3-\alpha)}\right\}\right]$$

$$+\frac{Gc}{1-Sc}\left[-\frac{1}{3\delta}+\frac{1}{\delta^2}(\tau\delta+1)\left\{\frac{\sqrt{K_1}\cosh\sqrt{K_1}}{\sinh\sqrt{K_1}}-1\right\}\right.$$

$$+\frac{Sc}{\delta\sqrt{K_1}(\cosh 2\sqrt{K_1}-1)}\left\{\sqrt{K_1}\sinh^2\sqrt{K_1}+\sinh\sqrt{K_1}\cosh\sqrt{K_1}-\sqrt{K_1}\cosh^2\sqrt{K_1}\right\}$$

$$+\sum_{m=1}^{\infty} 2m^2\pi^2 \left\{ \frac{e^{s_1\tau}}{Scs_1^2(s_1-\delta)} - \frac{e^{s_3\tau}}{s_3^2(s_3-\delta)} \right\} \Bigg] \quad (28)$$

For the sake of engineering purposes, one is usually interested in the values of shear stress (skin friction) and the heat transfer at the surface of the channel. The shear stress is an important parameter in the heat transfer studies, since it is directly related to the heat transfer coefficients. The increased shear stress is generally a disadvantage in the technical applications, while the increased heat transfer can be exploited in some applications such as heat exchangers, but should be avoided in other such a gas turbine applications, for instances. Several values of the non-dimensional shear stress at the plate $\eta = 0$ are presented in Figs.18-22 for several values of thermal Grashof number Gr, mass Grashof number Gc, radiation parameter R, Schmidt number Sc and time τ. Figs.18 and 19 show that the shear stress τ_x at the plate $\eta = 0$ increases with an increase in either radiation parameter R or thermal Grashof number Gr or mass Grashof number Gc. Figs.20-22 reveal that the shear stress τ_x at the plate $\eta = 0$ decreases with an increase in Schmidt number Sc. Physically, it is true since an increase in Sc serves to increase momentum boundary layer thickness. Further, it is observed from Fig.16 that the shear stress τ_x at the plate $\eta = 0$ increases with an increase in time τ.

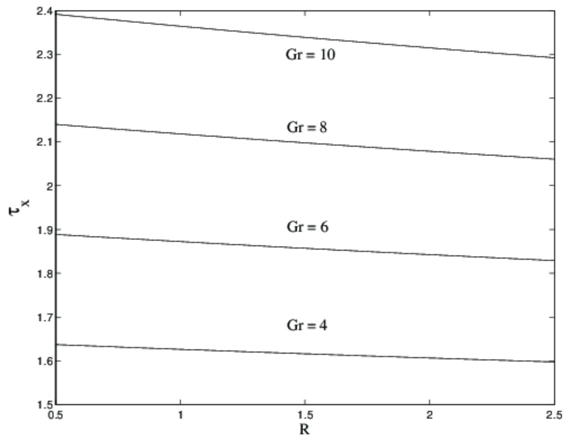

Fig 18. Shear stress τ_x for Gr when $Gc = 5$, $Sc = 0.6$, $Pr = 0.71$ and $\tau = 0.5$

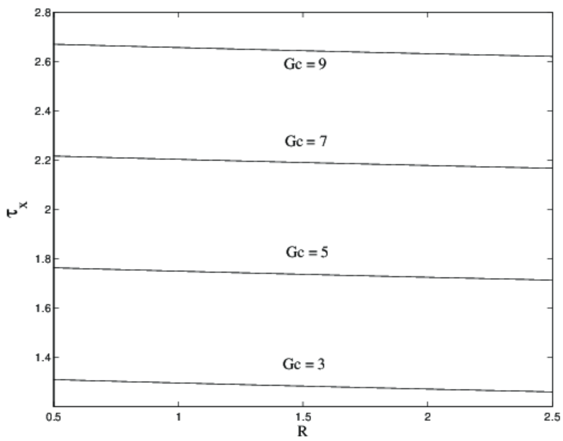

Fig 19. Shear stress τ_x for Gc when $Gr = 5$, $Sc = 0.6$, $Pr = 0.71$ and $\tau = 0.5$

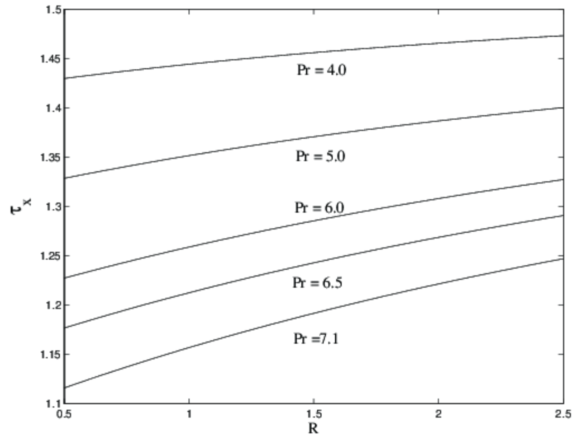

Fig 20. Shear stress τ_x for Pr when $Gc = 5$, $Gr = 5$, $Sc = 0.6$ and $\tau = 0.5$

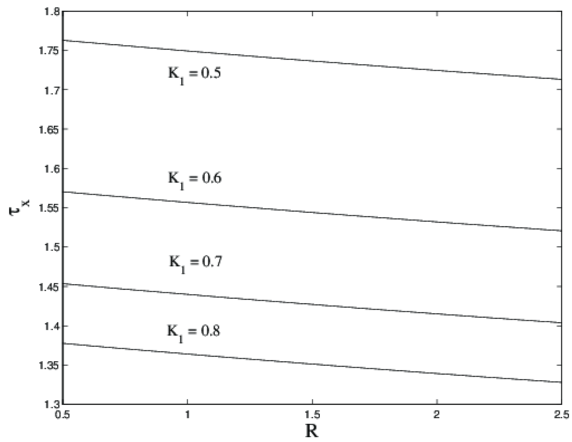

Fig 21. Shear stress τ_x for K_1 when $Gc = 5$, $Sc = 0.6$, $Pr = 0.71$ and $Gr = 5$

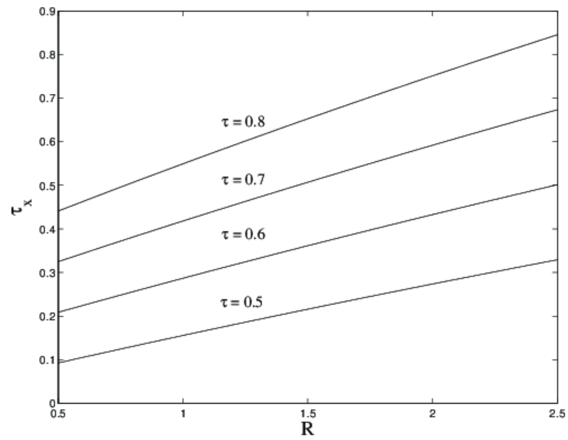

Fig 22. Shear stress τ_x for time τ when $Gc = 5$, $Sc = 0.6$, $Pr = 0.71$ and $Gr = 5$

4 Conclusion

An exact solution of the Transient free convection in a vertical channel with variable temperature and mass diffusion in the presence of thermal radiation and constant mass diffusion is presented. The dimensionless governing partial differential equations are solved by the usual Laplace transform technique. The effect of different parameters such as radiation parameter R, Grashof number Gr, Prandtl number Pr and time τ are studied. Our investigation leads us to the following conclusions:

• Velocity profile increases in the increase of Grashof number Gr, mass Grashof number Gc and τ; whereas it decreases in increase of radiation parameter R, Prandtl number Pr, Schmidt number Sc and chemical reaction rate K_1.

• Temperature profile decreases with radiation parameter R, Prandtl number Pr and time τ.

• Concentration profile increases with time τ and decreases with Schmidt number Sc and chemical reaction rate K_1.

• Shear stress for temperature profile increases with time τ.

• Shear stress for concentration profile increases with chemical reaction rate K_1 and decreases with Schmidt number Sc.

• Shear stress τ_x for velocity profile increases with Grashof number Gr, mass Grashof number Gc, time τ and decreases for Prandtl number Pr, chemical reaction rate K_1.

An increase in either thermal Grashof number or mass Grashof number leads to rise in the fluid velocity. Either radiation parameter or Schmidt number or Prandtl number has a retarding influence on the fluid velocity. Radiation has a retarding influence on the fluid temperature distribution. The temperature and concentration distributions in the fluid increase with an increase in time. Further, it is observed that the shear stress at the plate $\eta = 0$ increases whereas the rate of heat transfer at the plate $\eta = 1$ decreases with an increase in radiation parameter.

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