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The Mean and Variance of Photon Number for Twin Light Beams Generated by Parametric Oscillator

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Abstract
A detailed photon statistics of twin light beams with the same and different frequencies produced by the process of parametric oscillation has been presented. We have described the process of the parametric oscillation with first-order Hamiltonian regardless of whether the twin light beams have the same or different frequencies. According to our study we have observed that the mean photon number of the cavity mode decreases with the cavity damping constant increase, which implies that the more photons are escaped from the cavity for the large damping constant and the mean photon number of residue pump mode increases with the amplitude of pump mode.

Introduction

Light has played a special role in our attempt to understand nature both classically and quantum mechanically. In classical description light consists of waves with well defined amplitude and phase, but this is not the case when we treat light quantum mechanically. Quantum optics is a field of quantum physics that deals with the interaction of photons with matter and the quantum properties of the light generated by various quantum optical systems. The calculation of variance in a quantum state leads to the determination of the total noise of that state. The knowledge of the noise level of a state is essential to estimate the value of such a state in practice [3]. The variance in a two mode state are defined as mean-square uncertainties in the real and imaginary parts of the annihilation operators of the mode [4]

Master Equation

The process of parametric oscillation [1] leading to the creation of twin light modes with the same or different frequencies can be described by the Hamiltonian

$$\hat{H} = i\gamma\left(\hat{b}^{\dagger} - \hat{b}\right) + i\varepsilon\left(\hat{b}^{\dagger}\hat{a}_1\hat{a}_2 - \hat{b}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\right)$$

where $\hat{a}_1$ and $\hat{a}_2$ are the annihilation operators for the light modes emitted from the top and intermediate levels of the fundamental mode, $\hat{b}$ is the annihilation operator for the pump mode and part of the pump mode that emerges from non-linear crystal without being down-converted (residue mode), ε is the coupling constant, and γ is proportional to the amplitude of the coherent light deriving the pump mode. We assume that the operators $\hat{a}_1$ and $\hat{a}_2$ commute and satisfy the commutation relation

$$\left[\hat{a}_1, \hat{a}_1^{\dagger}\right] = \left[\hat{a}_2, \hat{a}_2^{\dagger}\right] = 1$$

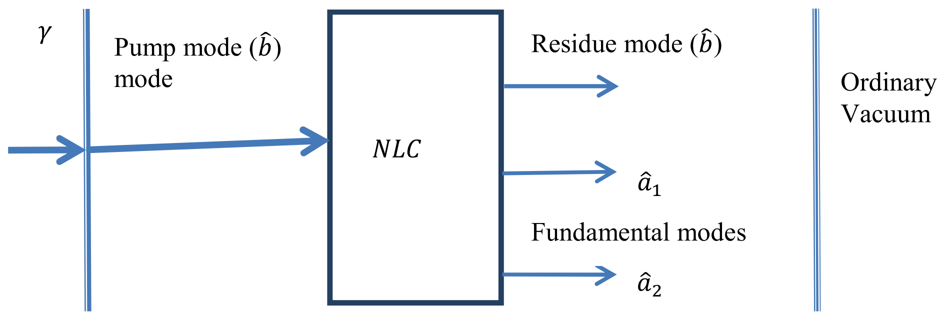

Fig.1: Plot of the external pumping radiation of frequency (ω) is down converted to the fundamental (signal-signal or signal-idler) mode of frequencies (ω_s, ω_i) by the nonlinear crystal.

We may refer to a Hamiltonian of the form described by Eq. (1) as first order Hamiltonian. We next seek to calculate the master equation and operator dynamics for the twin light modes by applying the pertinent Hamiltonian described by Eq. (1). The master equation for the system under consideration turns out to be

$$\frac{d\hat{\rho}}{dt} = \gamma\left(\hat{b}^{\dagger}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger} + \hat{\rho}\hat{b} - \hat{b}\hat{\rho}\right) + \varepsilon\left(\hat{b}^{\dagger}\hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{b}^{\dagger}\hat{a}_1\hat{a}_2 + \hat{\rho}\hat{b}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{b}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\rho}\right)$$
$$+\frac{k_1}{2}\left(2\hat{a}_1\hat{\rho}\hat{a}_1^{\dagger} - \hat{a}_1^{\dagger}\hat{a}_1\hat{\rho} - \hat{\rho}\hat{a}_1^{\dagger}\hat{a}_1\right) + \frac{k_2}{2}\left(2\hat{a}_2\hat{\rho}\hat{a}_2^{\dagger} - \hat{a}_2^{\dagger}\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_2^{\dagger}\hat{a}_2\right),$$

where, k_1 and k_2 are the cavity damping constant of the signal-idler mode.

Operator Dynamics

At issue here is to find, the equation of evolution of the expectation values of the cavity mode operators, and their steady state solutions. Using the relation

$$\frac{d\langle\hat{A}\rangle}{dt} = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right) \tag{4}$$

On account of the properties of the trace operator, and Eq.(2) along with the fact that the operators $\hat{a}_1$ and $\hat{a}_2$ commute, we find

$$\frac{d\langle\hat{a}_1\rangle}{dt} = -\frac{k_1}{2}\langle\hat{a}_1\rangle - \varepsilon\langle\hat{b}\hat{a}_2^\dagger\rangle \tag{5}$$

$$\frac{d\langle\hat{a}_2\rangle}{dt} = -\frac{k_2}{2}\langle\hat{a}_2\rangle - \varepsilon\langle\hat{b}\hat{a}_1^\dagger\rangle \tag{6}$$

$$\frac{d\langle\hat{a}_1^\dagger\hat{a}_1\rangle}{dt} = -k_1\langle\hat{a}_1^\dagger\hat{a}_1\rangle - \varepsilon\langle\hat{b}\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle - \varepsilon\langle\hat{b}^\dagger\hat{a}_1\hat{a}_2\rangle \tag{7}$$

$$\frac{d\langle\hat{a}_2^\dagger\hat{a}_2\rangle}{dt} = -k_2\langle\hat{a}_2^\dagger\hat{a}_2\rangle - \varepsilon\langle\hat{b}\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle - \varepsilon\langle\hat{b}^\dagger\hat{a}_1\hat{a}_2\rangle \tag{8}$$

$$\frac{d\langle\hat{a}_1\hat{a}_2\rangle}{dt} = -\frac{1}{2}(k_1 + k_2)\langle\hat{a}_1\hat{a}_2\rangle - \varepsilon\langle\hat{b}\hat{a}_1^\dagger\hat{a}_1\rangle - \varepsilon\langle\hat{b}\hat{a}_2^\dagger\hat{a}_2\rangle - \varepsilon\langle\hat{b}\rangle. \tag{9}$$

On taking $k_1 = k_2 = k$, the steady-state solutions of the above equations become

$$\langle\hat{a}_1\rangle = -\frac{2\varepsilon}{k}\langle\hat{b}\hat{a}_2^\dagger\rangle, \tag{10}$$

$$\langle\hat{a}_2\rangle = -\frac{2\varepsilon}{k}\langle\hat{b}\hat{a}_1^\dagger\rangle, \tag{11}$$

$$\langle\hat{a}_1^\dagger\hat{a}_1\rangle = -\frac{\varepsilon}{k}\langle\hat{b}\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle - \frac{\varepsilon}{k}\langle\hat{b}^\dagger\hat{a}_1\hat{a}_2\rangle, \tag{12}$$

$$\langle\hat{a}_2^\dagger\hat{a}_2\rangle = -\frac{\varepsilon}{k}\langle\hat{b}\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle - \frac{\varepsilon}{k}\langle\hat{b}^\dagger\hat{a}_1\hat{a}_2\rangle, \tag{13}$$

$$\langle\hat{a}_1\hat{a}_2\rangle = -\frac{\varepsilon}{k}\langle\hat{b}\hat{a}_1^\dagger\hat{a}_1\rangle - \frac{\varepsilon}{k}\langle\hat{b}\hat{a}_2^\dagger\hat{a}_2\rangle - \frac{\varepsilon}{k}\langle\hat{b}\rangle. \tag{14}$$

On the other hand, the evolution of the expectation value of the pump mode $\hat{b}$ is given by

$$\frac{d\langle\hat{b}\rangle}{dt} = -iTr([\hat{H},\hat{\rho}]\hat{b}) + \frac{k}{2}Tr\{(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b})\hat{b}\}, \tag{15}$$

in the absence of parametric oscillation ($\varepsilon = 0$), the $Eq.(1)$ becomes

$$\hat{H} = i\gamma(\hat{b}^\dagger - \hat{b}), \tag{16}$$

Upon dropping the noise operator, we can write the quantum langevin equation as

$$\frac{d\langle\hat{b}\rangle}{dt} = -\frac{1}{2}k\hat{b} + \gamma, \tag{17}$$

where k is the cavity damping constant. The steady-state solution of this equation is

$$\hat{b} = \frac{2\gamma}{k}. \tag{18}$$

Now substituting the value of $\hat{b}$ in Eqs. (10), (11), (12), (13) and (14), we get

$$\langle\hat{a}_1\rangle = -\frac{2\lambda}{k}\langle\hat{a}_2^\dagger\rangle, \tag{19}$$

$$\langle\hat{a}_2\rangle = -\frac{2\lambda}{k}\langle\hat{a}_1^\dagger\rangle, \tag{20}$$

$$\langle\hat{a}_1^\dagger\hat{a}_1\rangle = -\frac{\lambda}{k}\langle\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle - \frac{\lambda}{k}\langle\hat{a}_1\hat{a}_2\rangle, \tag{21}$$

$$\langle \hat{a}_2^\dagger \hat{a}_2 \rangle = -\frac{\lambda}{k} \langle \hat{a}_1^\dagger \hat{a}_2^\dagger \rangle - \frac{\lambda}{k} \langle \hat{a}_1 \hat{a}_2 \rangle, \tag{22}$$

$$\langle \hat{a}_1 \hat{a}_2 \rangle = -\frac{\lambda}{k} \langle \hat{a}_1^\dagger \hat{a}_1 \rangle - \frac{\lambda}{k} \langle \hat{a}_2^\dagger \hat{a}_2 \rangle - \frac{\lambda}{k}, \tag{23}$$

in which λ is determined by

$$\lambda = \frac{-2\gamma\varepsilon}{k}. \tag{24}$$

Applying Eq. (19) and (20), we easily find

$$\langle \hat{a}_1 \rangle = \langle \hat{a}_2 \rangle = 0. \tag{25}$$

In addition, by using Eqs. (21), (22) and (23) we get

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle = \frac{2\lambda^2}{k^2 - 4\lambda^2}, \tag{26}$$

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle = \langle \hat{a}_2^\dagger \hat{a}_2 \rangle, \tag{27}$$

$$\langle \hat{a}_1 \hat{a}_2 \rangle = -\frac{\lambda k}{k^2 - 4\lambda^2}. \tag{28}$$

It can also be readily asserted that

$$\langle \hat{a}_1^2 \rangle = \langle \hat{a}_2^2 \rangle = \langle \hat{a}_1^\dagger \hat{a}_2 \rangle = 0. \tag{29}$$

We take

$$\hat{a} = \hat{a}_1 + \hat{a}_2, \tag{30}$$

to be the annihilation operator for superposition of light modes $\hat{a}_1$ and $\hat{a}_2$, produced by the parametric oscillator. We can easily assure that

$$[\hat{a}, \hat{a}^\dagger] = 2. \tag{31}$$

We actualize that the superposition of the two light modes, with the same or different frequencies, constitutes a two-mode light. We wish to call superposed light modes with the same frequency and the superposed light modes with different frequencies. It also proves to be convenient to the parametric oscillator which produces the same frequencies as the degenerate parametric oscillator and the one which produces the different as the non-degenerate parametric oscillator. Finally, we would like to mention that the result described by $Eqs. (25) - (29)$ are valid for the signal-signal or signal-idler modes.

The Mean and Variance of the Photon Number

We define the photon number of the two-mode sub-harmonic light, $\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle$. Then using Eq.(30) and taking into account Eq.(29), we easily find

$$\bar{n} = \langle \hat{a}_1^\dagger \hat{a}_1 \rangle + \langle \hat{a}_2^\dagger \hat{a}_2 \rangle, \qquad 32$$

so considering Eqs.(26) and (27) there follows

$$\bar{n} = \frac{4\lambda^2}{k^2 - 4\lambda^2}. \qquad 33$$

This represents the mean photon number of the signal-signal or signal-idler modes. We observe that the mean photon number for the conventional Hamiltonian is half of the result given by Eq.(33). This unexpected result must be due to representation of the twin light beams with the same frequency by second- order annihilation and creation operators in the conventional Hamiltonian.

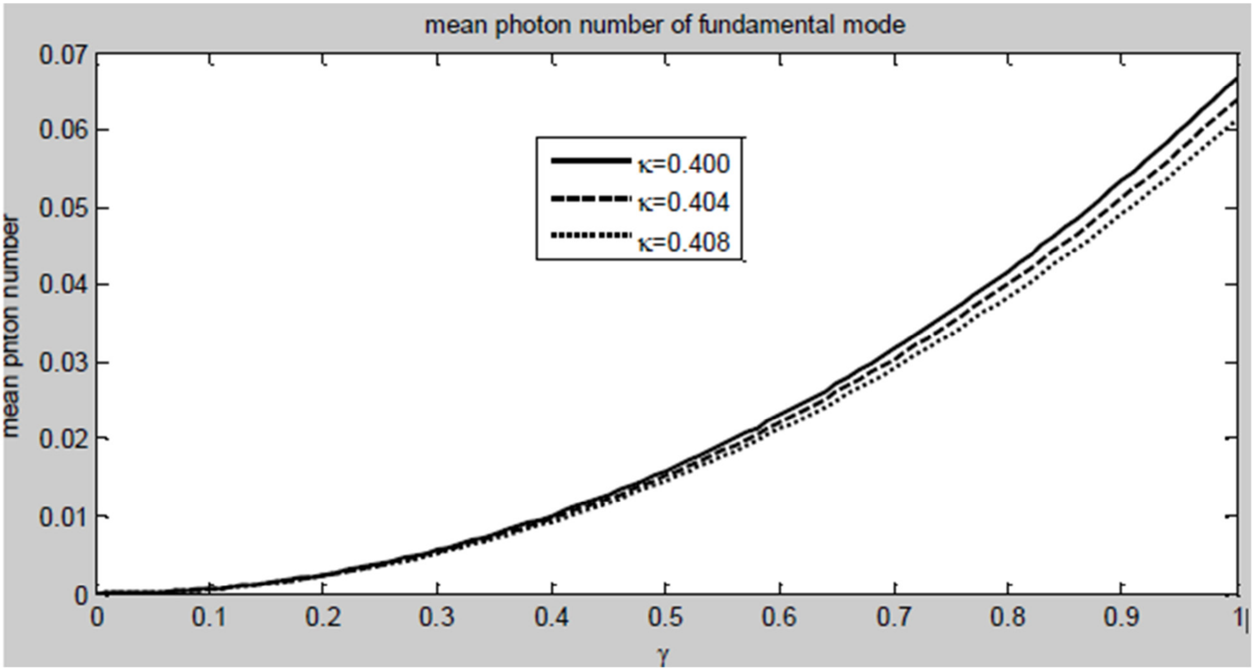

Fig. 2: Plots of the the mean photon number [Eq.(33)] versus γ for ε = 0.01, and different values of κ ,for κ = 0.400(solid curve), κ = 0.404 (dashed curve) and κ = 0.408 (doted curve).

It can readily be observed that the mean photon number of the cavity mode decrease with the cavity damping constant increase. Since more photons escape from the cavity for large damping constant. Now applying Eq.(31), the photon-number variance of the two-mode sub-harmonic light is defined by

$$(\Delta n)^2 = \langle(\hat{a}^\dagger \hat{a}^\dagger)^2\rangle - \bar{n}^2, \qquad 34$$

Can be put in the form

$$(\Delta n)^2 = \langle \hat{a}^{\dagger 2}\hat{a}^{\dagger 2}\rangle + 2\bar{n} - \bar{n}^2. \qquad 35$$

Now applying the fact that $\hat{a}$ is a Gaussian variable with zero mean, we get

$$(\Delta n)^2 = 2\bar{n} + \bar{n}^2 + \langle \hat{a}^{\dagger 2}\rangle\langle \hat{a}^2\rangle \qquad 36$$

and on taking into account Eq.(31) along with Eq.(30), we arrive at

$$(\Delta n)^2 = 2\bar{n} + \bar{n}^2 + 4\langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle\langle \hat{a}_1 \hat{a}_2\rangle. \qquad 37$$

Hence in view of Eq.(33) along with Eq.(28), the photon-number variance of the two-mode parametric light takes the form

$$(\Delta n)^2 = \frac{8\lambda^2}{k^2-4\lambda^2} + \frac{16\lambda^2}{(k^2-4\lambda^2)^2} + \frac{4k^2\lambda^2}{(k^2-4\lambda^2)^2} \qquad 38$$

In addition, we note that the equation of evolution of the mean photon number for the pump mode can be written as

$$\frac{d}{dt}\langle \hat{b}^\dagger \hat{b}\rangle = -i\langle[\hat{b}^\dagger \hat{b}, \hat{H}]\rangle + \frac{1}{2}kTr\left[\left(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{\rho}\hat{b}^\dagger \hat{b}\right)\hat{b}^\dagger \hat{b}\right]. \qquad 39$$

Then using Eq.(2.36) and the fact that

$$\frac{1}{2}kTr\left[\left(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger \hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger \hat{b}\right)\hat{b}^\dagger \hat{b}\right] = -k\langle \hat{b}^\dagger \hat{b}\rangle, \qquad 40$$

we readily get

$$\frac{d}{dt}\langle \hat{b}^\dagger \hat{b}\rangle = -k\langle \hat{b}^\dagger \hat{b}\rangle + \gamma\langle \hat{b}\rangle + \gamma\langle \hat{b}^\dagger\rangle + \varepsilon\langle \hat{b}^\dagger \hat{a}_1 \hat{a}_2\rangle + \varepsilon\langle \hat{b}\hat{a}_1{}^\dagger \hat{a}_2{}^\dagger\rangle. \qquad 41$$

The steady-state solution of this equation is

$$\langle \hat{b}^\dagger \hat{b}\rangle = \frac{\gamma}{k}\langle(\hat{b} + \hat{b}^\dagger)\rangle + \frac{\varepsilon}{k}\left(\langle \hat{b}^\dagger \hat{a}_1 \hat{a}_2\rangle + \langle \hat{b}\hat{a}_1{}^\dagger \hat{a}_2{}^\dagger\rangle\right), \qquad 42$$

so that in view of Eq.(18) and Eq.(27), the mean photon number of the pump mode takes the form

$$\langle \hat{b}^\dagger \hat{b}\rangle = \frac{4\gamma^2}{k^2} - \frac{2\lambda^2}{k^2-4\lambda^2}. \qquad 43$$

The first term of Eq.(43) represents the mean photon number of the pump mode in the absence of the parametric oscillator and the second one represents the mean photon number of light mode a_1 or light mode a_2 or the mean photon number of the down converted pump mode. The difference of the two terms is the mean photon number of the pump mode that emerges from the nonlinear crystal without down-converted (residue mode). This is exactly what we would expect the mean photon number of the pump mode to be.

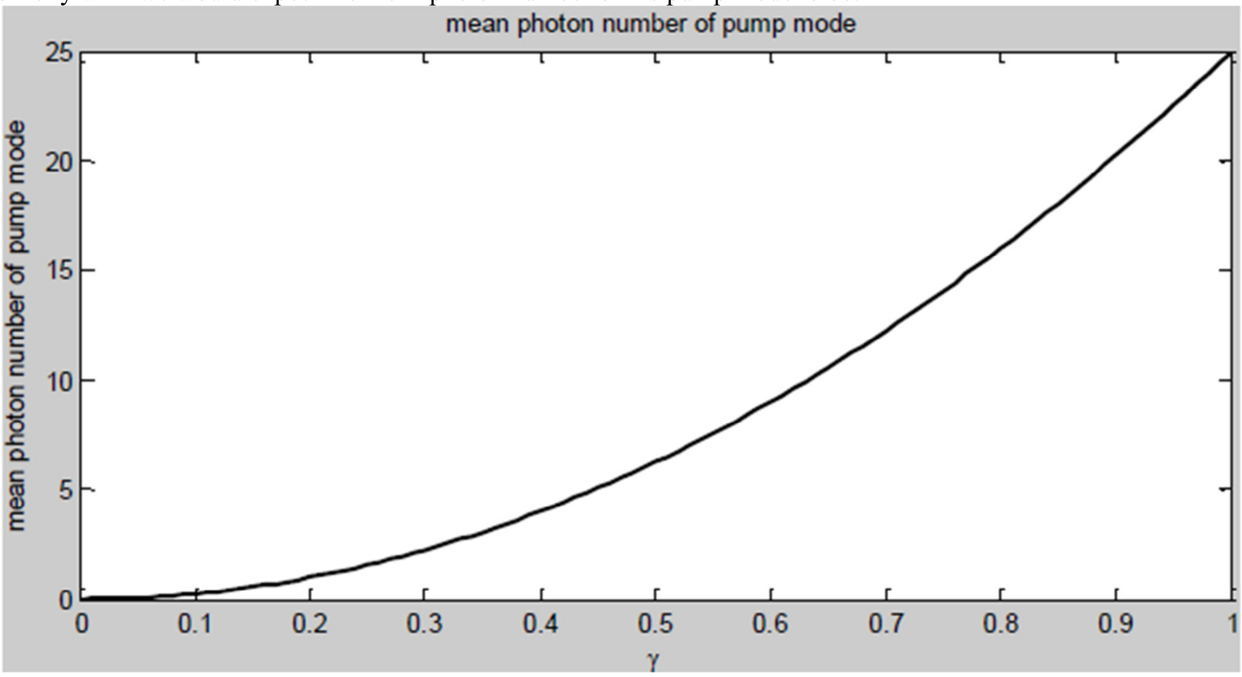

Fig. 3: Plots of the mean photon number of residue pump mode [Eq.(3.24)] versus γ for $\varepsilon = 0.01$, and $k = 0.4$.

Fig.(3.2) shows the mean photon number of residue pump mode increase with the amplitude of pump mode. Setting Eq.(43) to zero, one can find $\gamma = \frac{k}{4\varepsilon}\sqrt{(k^2 - 2\varepsilon^2)}$, at this point the incident photons were fully down-converted to fundamental (signal-signal or signal-idler) mode.

Conclusion

In this paper we seen that the mean and variance of photon number for twin light beams generated by parametric oscillator whose coupled to vacuum reservoir by the aid of steady state solution. Applying the steady state solution we have calculated variance and mean photon number for twin light beam. Our result shows that, we have the mean photon number of the cavity mode decreases with the cavity damping constant increase, which implies that the more photons are escaped from the cavity for the large damping constant and the mean photon number of residue pump mode increases with the amplitude of pump mode.

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