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The Impact of Non-Uniform Heat Source/Sink on Chemically Reacting MHD Flow Over An Expanding Sheet

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Abstract
The impact of uneven heat source/sink on the magnetohydrodynamic liquid film flow of Casson fluid in the presence of chemical reaction is investigated theoretically. The transformed governing boundary layer equations are solved numerically by adopting the bvp4c Matlab package. The impact of pertinent parameters, namely, Casson parameter, non-uniform heat source/sink parameters, chemical reaction parameter, magnetic field parameter, unsteadiness parameter and film thickness parameter on the flow, thermal and concentration fields are discussed with the help of graphs. Numerical illustrations of local Nusselt and Sherwood numbers are presented and discussed with the help of tables.
Keywords: Chemical reaction, MHD, film flow, non-uniform heat source/sink.

1. Introduction

The flow and heat transfer in non-Newtonian fluids has a wide range of applications in both science and engineering. Some of their applications include the heat transfer fluids (cooling or heating), solar energy, nuclear reactors, etc. Heat transfer in the flow over a stretching surface plays important role in industries like polymer engineering, aerodynamics, wire drawing, stretching of plastic films, etc. Hayat et al. [1] investigated the heat and mass transfer of chemically reacting second grade flow. Rohini et al. [2] studied the convective heat transfer in the flow over a shrinking surface at a stagnation point region. Further, Najib et al. [3] extended this work by considering the flow over a stretching/shrinking cylinder in the presence of chemical reaction. The effect of heat source/sink on MHD radiative flow of a visco elastic fluid over a stretched surface was numerically studied by Cortell [4]. Yih [5] analyzed the free convective radiative heat transfer in the flow over an isothermal cone.

The mixed convection flow in porous media in the presence of chemical reaction was numerically studied by Rashad et al. [6]. Deductive group technique for MHD coupled heat and mass transfer natural convective flow of non- Newtonian power law fluid over a vertical cone through porous medium was studied by Parmar and Timol [7]. Variable temperature effects on heat and mass transfer in the flow over a truncated cone was illustrated by Chamkha et al. [8]. Garoosi et al. [9] reported the two phase flow of a nanofluid over a square cavity. The stagnation-point flow of a non-Newtonian fluid over a stretching sheet with induced magnetic field effect was studied by Sandeep et al. [10]. Ramana Reddy et al. [11] studied the effect of non-uniform heat source/sink on MHD nanofluid flow over a non-uniform thickness sheet with slip effects. Multiple solutions for the Falkner-Skan flow past a stretched surface was numerically repored by Riley and Weidman [12]. The forced convection flow over a wedge with variable viscosity was studied by Vol [13]. The effect of suction and injection on Falkner-Skan equation for flow past a moving wedge was studied by Ishak et al. [14]. Flow and heat transfer behavior of MHD dusty nanofluid past a porous stretching/shrinking cylinder at different temperatures was studied by Sandeep and Sulochana [15]. Three-dimensional flow of nanofluid with Cattaneo–Christov double diffusion was numerically studied by Hayat et al. [16].

The effect of variable thermal conductivity on Cattaneo-Christov flow of generalized Burgers fluid was studied by Waqas [17]. Aligned magnetic field effect on liquid film flow of nanofluid was numerically illustrated by Sandeep [18]. Numerical exploration of magnetohydrodynamic nanofluid flow suspended with magnetite nanoparticles was studied by Sandeep et al. [19]. UCM flow over a melting surface with cross diffusion was numerically studied by Jayachandra Babu and Sandeep [20]. Analysis of boundary layer formed on an upper horizontal surface of a paraboloid of revolution within nanofluid flow in the presence of thermophoresis and Brownian motion of 29 nm was illustrated by Koriko et al. [21]. Effect of cross diffusion on MHD flow of Oldroyd-B fluid with double stratification was studied by Sandeep and Gnaneswara Reddy [22]. Numerical analysis of plane and parabolic flow of MHD Carreau fluid with buoyancy effects was studied by Mohan Krishna et al. [23]. Ramana Reddy et al. [24] studied the frictional heating effect on radiative ferrofluid flow over a slendering stretching sheet with aligned magnetic field. Very recently, the researchers [26-30] studied the heat transfer in MHD flow over various flow geometries.

In this study, we investigated the effect of non-uniform heat source/sink on the magnetohydrodynamic liquid film flow of Casson fluid in the presence of chemical reaction. The transformed governing boundary layer equations are solved numerically by adopting the bvp4c Matlab package. The impact of pertinent parameters, namely, Casson parameter, non-uniform heat source/sink parameters, chemical reaction parameter, magnetic field parameter, unsteadiness parameter and film thickness parameter on the flow, thermal and concentration fields are discussed with the help of graphs.

2. Mathematical Formulation

Consider a 2D unsteady magnetohydrodynamic film flow of Casson fluid over a stretched sheet. The sheet is placed along the *x*-axis with velocity $u_w(x,t)=bx/(1-\alpha t)$, where b,α constants and the *y*-axis is perpendicular to it. It is assumed that the T_0 and T_r are the slit and reference temperatures and C_0 and C_r are the slit and reference concentration. A transverse magnetic field is applied along the flow direction as shown in Fig.1. The non-uniform heat source/sink and chemical reaction effects are taken into account.

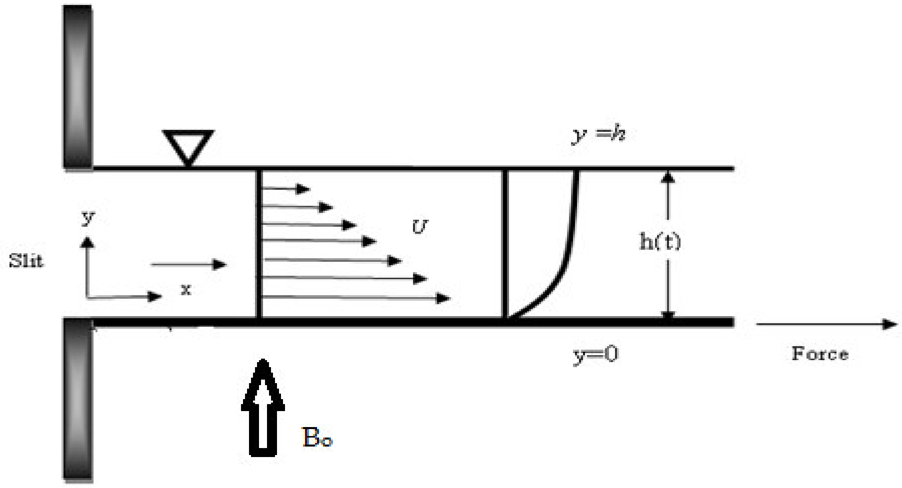

Fig. 1 Physical model of the problem

As per the assumptions above, the governing equations can be expressed as:

$$\frac{\partial^2\xi}{\partial x\partial y}-\frac{\partial^2\xi}{\partial y\partial x}=0, \tag{1}$$

$$\left(\frac{\partial^2\xi}{\partial t\partial y}-\frac{\partial^2\xi}{\partial y^2}\frac{\partial\xi}{\partial x}+\frac{\partial\xi}{\partial y}\frac{\partial^2\xi}{\partial x\partial y}\right)=\nu\left(1+\frac{1}{\delta}\right)\frac{\partial^3\xi}{\partial y^3}-\frac{\sigma}{\rho}\frac{\partial\xi}{\partial y}B^2(t), \tag{2}$$

$$\left(\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}\frac{\partial\xi}{\partial x}+\frac{\partial T}{\partial x}\frac{\partial\xi}{\partial y}\right)=\frac{k}{(\rho c_p)}\frac{\partial^2 T}{\partial y^2}+\frac{1}{(\rho c_p)}q''', \tag{3}$$

$$\left(\frac{\partial C}{\partial t}+\frac{\partial\xi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\xi}{\partial x}\frac{\partial C}{\partial y}\right)=D_B\frac{\partial^2 C}{\partial y^2}-k_l(C-C_0), \tag{4}$$

with the conditions

$$\left.\begin{aligned}&u=u_w, v=0, T=T_s,\ C=C_s,\ \ at\ y=0\\&\frac{\partial u}{\partial y}=0,\frac{\partial T}{\partial y}=0,\frac{\partial C}{\partial y}=0\ \ at\ y=h,\ \ v=\frac{dh}{dt}\ \ as\ y=h(t)\end{aligned}\right\} \tag{5}$$

where ξ is the stream function, u and v are the velocity components, ρ is the density, μ is dynamic viscosity, σ is the electrical conductivity, $B(t)$ is the applied magnetic field, T and C are the temperature and concentration, (ρc_p) represent the specific heat capacitance, k is thermal conductivity, δ is the Casson parameter and k_l is the chemical reaction parameter.

In Eq. (3) $q'''=\frac{(T_s-T_0)ku_w}{x\nu}\left(A^*f'+B^*\frac{(T-T_0)}{(T_s-T_0)}\right)$ is the non-uniform heat source/sink parameter.

We now introduce the similarity transformations as

$$\zeta = \left(\frac{b}{v_f (1-\alpha t)} \right)^{0.5} y, \xi = \beta \left(\frac{v_f b}{(1-\alpha t)} \right)^{0.5} x f(\eta),$$

$$T = T_0 + T_r (bx^2 / 2v_f)(1-\alpha t)^{-1.5} \theta(\eta), B(t) = B_0 (1-\alpha t)^{-0.5} \quad (6)$$

$$C = C_0 + C_r (bx^2 / 2v_f)(1-\alpha t)^{-1.5} \phi(\eta),$$

where , ζ is the similarity variable, $\beta > 0$ is the dimensionless film thickness. f, θ and ϕ are dimensionless flow, thermal and concentration fields.
By making use of Eq. (5), the Eqs. (1) to (4) can be transformed as

$$\left(1+\delta^{-1}\right) f''' + \lambda \left(f'' f - (f')^2 - Sf' - Sf'' \frac{\zeta}{2} \right) - Mf' = 0, \quad (7)$$

$$\theta'' + \Pr \lambda \left(2f'\theta - f\theta' + \frac{3}{2} S\theta + \frac{1}{2} S\zeta\theta' \right) + A^* f' + B^* \theta = 0, \quad (8)$$

$$\phi'' + Sc\lambda \left(\frac{3}{2} S\phi + \phi' f + \frac{1}{2} S\zeta\phi' - 2f'\chi \right) - Kr\phi = 0, \quad (9)$$

with the transformed boundary conditions

$$f = 0, f' = 1, \theta = 1, \phi = 1, \text{ at } \zeta = 0, f = \frac{\lambda}{2}, \theta' = \phi' = 0, \text{ at } \zeta = 1, \quad (10)$$

where, $\Pr$ is the Prandtl number, S is unsteadiness parameter, λ is the dimensionless film thickness, M is magnetic field parameter, A^* and B^* are the space and temperature dependent heat source/sink, Sc is the Schmidt number and Kr is the chemical reaction parameter.

$$\Pr = \frac{(\mu c_p)}{k}, M = \frac{\sigma B_0^2}{b\rho}, S = \frac{\alpha}{b}, \lambda = \beta^2, Sc = \frac{\upsilon}{D_B}, Kr = \frac{k_l}{b}, \quad (11)$$

For engineering interest, the local Nusselt and Sherwood numbers are given by

$$\mathrm{Re}_x^{-0.5} Nu_x = -\frac{1}{\beta}\left(1+\frac{1}{\delta}\right)\theta'(0), \mathrm{Re}_x^{-0.5} Sh_x = -\phi'(0), \quad (12)$$

where $\mathrm{Re}_x = \frac{u_w x}{v_f}$ is the local Reynolds number.

3. Results and Discussion

The set of differential equations (8) to (10) with the conditions Eq. (11) are solved by adopting bvp4c Matlab package. For the computation work we considered the pertinent parameters as $M = 2, Sc = 1, A^* = B^* = S = Kr = 0.5, \lambda = 0.3, \delta = 0.5$. Apart from the changed parameters as shown in the plots, these are conserved as invariable.

Figs. 2-4 depicts the impact of magnetic field parameter on the velocity, temperature and concentration profiles of Casson fluid. It is noticed that the increasing value of the magnetic field parameter suppresses the velocity profile, decline the temperature and concentration profiles of the flow. This may be due to the fact that the rising values of the magnetic field parameter develop the force called the Lorentz force, which works opposite to the flow field. The similar results has been observed for rising values of the Casson parameter, which is shown in Figs. 5-7. Physically, increasing the Casson parameter causes to reduce the viscous nature of the flow.

Figs. 8 and 9 display the influence of space and temperature dependent heat source/sink parameters on thermal field. It is evident that the boosting value of the non-uniform heat source/sink parameters enhances the thermal field of the flow. Generally, positive values of the space and temperature dependent heat source/sink parameters act like heat source parameters. Fig.10 illustrate the effect of chemical reaction parameter on concentration profiles of the flow. T is clear that the rising values of the chemical reaction parameter decline the concentration field.

The influence of unsteadiness parameter on temperature and concentration fields of Casson flow is depicted in Figs. 11 and 12. It is observed that the boosting value of the unsteadiness parameter enhances the concentration profiles and decline the thermal field of the flow. The effect of film thickness parameter on the flow, thermal and concentration profiles are depicted in Figs. 13-15. It is clear that the increasing value of the film thickness parameter decreases the flow, temperature and concentration fields.

The variations in the local Nusselt and Sherwood numbers at different pertinent parameters is displayed in Table.1. It is evident that the rising values of the magnetic field and Casson parameters declines the heat and mass transfer rate. The similar result has been observed in heat transfer rate for rising values of the non-uniform heat source/sink parameter. Chemical reaction parameter has a tendency to enhance the mass transfer rate. The increasing values of the unsteadiness and film thickness parameters boosts the heat and mass transfer rate of the Casson fluid.

Table 1 Variations in the local Nusselt and Sherwood numbers

M	δ	A^*	Kr	S	λ	$\mathrm{Re}_x^{-0.5} Nu_x$	$\mathrm{Re}_x^{-0.5} Sh_x$
1						0.433853	0.681873
2						0.405712	0.654255
3						0.381379	0.630876
	0.5					0.405712	0.654255
	1					0.377200	0.626908
	1.5					0.362321	0.612881
		0.1				0.660068	0.654255
		0.2				0.596479	0.654255
		0.3				0.532890	0.654255
			0.1			0.405712	0.343519
			0.2			0.405712	0.427949
			0.3			0.405712	0.507565
				0.1		0.140641	0.758896
				0.2		0.214328	0.734079
				0.3		0.282693	0.708401
					0.3	0.405712	0.654255
					0.4	0.701669	0.724268
					0.5	0.948128	0.792332

4. Conclusion

The impact of uneven heat source/sink on the magnetohydrodynamic liquid film flow of Casson fluid in the presence of chemical reaction is investigated theoretically. The transformed governing boundary layer equations are solved numerically by adopting the bvp4c Matlab package. Numerical observations of the present study are as follows:

- Non-uniform heat source/sink parameters enhances the temperature profiles of the flow.
- Unsteadiness and film thickness parameters have tendency to enhance the heat and mass transfer rate.
- Rising values of the magnetic field and Casson parameters decline the local Nusselt and Sherwood numbers.
- Increasing values of the chemical reaction parameter enhances the mass transfer rate.

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