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Theoretical Study for Neodymium-Doped Optical Fiber Laser

Mudhir Shihab Ahmed and Amjed Abdul hameed Salman
Department of Physics – College of Education (Ibn Al-Haitham) – University of Baghdad, Iraq

Abstract:
In this paper, as the first the rate equation for a four-level system in end-pumped fiber laser have been solved analytically with minimum approximation and the lasing output power versus pump power has been derived. The results were applied by a numerical solution for Nd^{3+} doped fiber laser.
Keywords: Neodymium, rate equations, silica glass, ZBLAN glass.

1. Introduction

The first fiber laser demonstrated in the early 1960's was doped with neodymium ions. Lasers based on Nd^{3+} have been the subject of intense research ever since, Devices such as pulsed and continuous-wave lasers, super fluorescence sources, and amplifiers, have all been demonstrated with Nd^{3+}-doped fibers [1,2]. Furthermore, the first cladding-pumped fiber laser was also doped with Nd^{3+}. Currently, Nd^{3+}-based lasers are still the focus of a great deal of research, because of attractions of the four-level nature of dominating (0.9-1.4) µm transition, and because of the widely available, efficient be used ad pump sources [3,4]. And a linear cavity consists of a Fabry-Perot cavity can be formed from the end Fresnal reflection at the fiber facets [5, 6]. So, we analytically solved a rate equations in a single-end pumped Nd^{3+}-doped fiber laser, and a numerical solution for the lasing output power equation when the core of fiber is silica glass for the designs Lycom(a), Lycom(b), York, and IVA at laser operating (1065 nm) and (1333nm), and ZBLAN glass for design Le Verre Fluor at laser operating (1048 nm) and (1317 nm) , because these transition wavelength are operation in a four-level system

2. Theoretical model:

2.1 Rate Equation:

Consider the four – level scheme as shown in Fig (1).

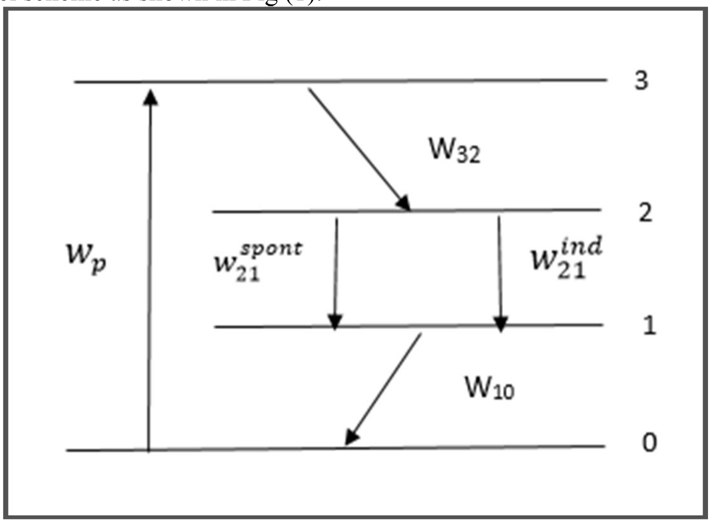

Fig (1) . The scheme of a four – level system .

In this scheme, a higher state (level 3) is excited directly by the pumping mechanism, and this state decays quickly and nonradiatively to the upper laser level (2). Stimulated emission then occurs from level (2) to level (1), which is the optical gain transition. In contrast, the four – level system can achieve population inversion with only a small number of atoms raised out of the ground – state [7]. Generally, it is easiest to obtain amplification and lasing with a four – level system because not as much pump energy must be wasted in removing atoms from the ground state.

Where ($N_1 = N_3 \cong 0$), population inversion is therefor, readily obtained, since $\Delta N \cong N_2$, which is positive for any rate of pumping. It is also clear level (2) is in effect being directly populated by the pump.

With the above approximations, only a single rate equation is required to describe the level populations in the four – level system. The rate of change of the upper laser state population can be written as [7, 8]:

$$\frac{dN_2}{dt} = N_o W_p - N_2 W_{21}^{ind} - \frac{N_2}{T_2} \;\ldots\ldots\ldots\; (1)$$

Where (T_2) is the fluorescence lifetime of level (2) been used . (W_P) is the pump rate , defined as the

probability per unit time that an atom is promoted by the pump from the ground state up to level (3) .($W_p = \frac{I_p \sigma P}{h\upsilon_p}$)
And (W_{21}^{ind}) is the probability per unit time for an induced transition ($W_{21}^{ind} = \frac{I\sigma L}{h\upsilon_L}$) [7,8].

The I_P, I_L are the pumping and lasing intensity, σP, σL, are absorption and emission cross-section and pumping and lasing wavelength and υ_p, υ_L are a frequency of pumping and lasing respectively

When; (R) is the total number of atoms pumped up to level (2) per unit volume per unit time ($R = N_o W_P$), the Equation can be written as [8] :

$$\frac{dN_2}{dt} = R - N_2 \left(\frac{I\sigma L}{h\upsilon_L} + \frac{1}{T_2}\right) \ldots\ldots\ldots\ldots (2)$$

2.2 Threshold Condition:

The conditions under which lasing will occur can be determined by considering the simple laser cavity shown in Fig (2).

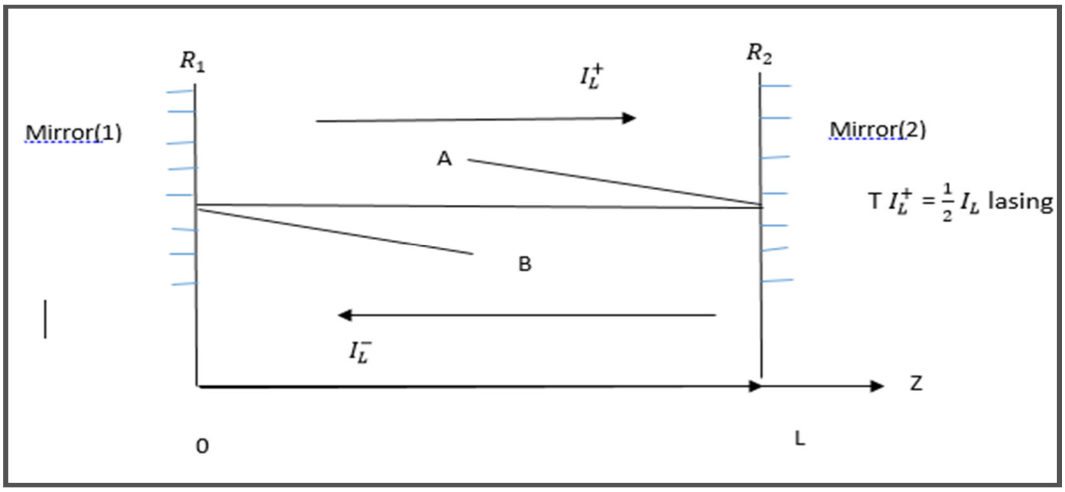

Fig (2) : fabry-perot cavity .

A uniform gain medium fills the region between two cavity mirrors. Which are separated by distance (1) and have reflectivities (R_1) and (R_2). Let us that there is a small amount of lasing intensity at point (A) that happens to be moving in a direction to ward mirror (2) , as shown . As the lasing makes a round – trip through the cavity from point (A) to point (B) it is amplified with again coefficient (G). Where (I_B) is the intensity of lasing arriving at point (B), the (I_B) can be written as [7, 8]:

$$I_B = R_1 R_2 e^{2(G - \alpha_L)L} I_A \ldots\ldots\ldots\ldots (3)$$

Where (I_A) is the intensity of lasing originating at point (A). For lasing output to occur, it is necessary that ($I_B > I_A$).

The threshold condition for laser oscillation then becomes:

$$R_1 R_2 e^{2(G - \alpha_L)L} \geq 1 \ldots\ldots\ldots\ldots (4)$$

At threshold ($G = G_{th}$), then the Eq. (4) become.

$$G_{th} = \alpha_L + \frac{1}{L} \ln \frac{1}{\sqrt{R_1 R_2}} \ldots\ldots\ldots\ldots (5)$$

Where (α_L) is the scattering losses at lasing wavelength (λ_L), ($\alpha_L = N_o \Gamma_L \sigma_L$);($N_o$) is the Nd^{3+}-concentration in the fiber core, and(Γ_L) is the power filling factor at lasing wave length (λ_L)

($\Gamma_L = 1 - \exp\left[-2\left(\frac{a}{w_L}\right)^2\right]$) [9,10],

If (w_L) is the mode field radius, for a fundamental mode (w_L) is defined by [9, 10] :

$$W_L = a\left[6.616 + \frac{1.660}{\sqcup 1.5} + \frac{0.987}{\sqcup 6}\right] \ldots\ldots\ldots (6)$$

Where ($\sqcup$) is the normalized frequency at lasing wavelength

($\sqcup = \frac{2\pi a N_A}{\lambda_L}$)

Where (a) is the radius of core fiber [9, 10].

2.3 lasing intensity

The net fractional increase in intensity after propagating a distance (ΔZ) is ($G - G_{th}$) ΔZ, this fractional increase occurs in a time ($\Delta t = \frac{\Delta Z}{c}$), since the beam is moving with speed (c).

The fractional increase in intensity can then be written [7, 11]:

$\frac{\Delta I_L}{I_L}$ = (G – G_{th}) c Δt ………. (7)

At (ΔN = N_2) , then (G=δ_{LN_2}) and the cavity lifetime ($T_c = \frac{1}{cG_{th}}$) , the equation (7) become :

$\frac{\Delta I_L}{I_L}$ = c σ_L N_2 I - $\frac{I}{T_c}$ ………. (8)

Taking the limit Δt → 0 in eq. (8) gives an equation for the rate of change of lasing intensity, then yields of differential equation relation (I_L).

$\frac{dI_L}{dt}$ = c σ_L N_2 I_L - $\frac{I_L}{T_c}$ ………. (9)

In the steady state, Eqs. (2) And (9) can be written as [7, 11]:

0 = R – $N_2\left(\frac{I_L\sigma_L}{h\upsilon_L} + \frac{1}{T_2}\right)$ ………. (10)

0 = c σ_L N_2 I_L - $\frac{I_L}{T_c}$ ………. (11)

The solutions for (N_2) and (I_L) both above and below threshold can be found in the following way. Below threshold, (I_L) is very small, so $\left(\frac{I_L\sigma_L}{h\upsilon_L}\right)$ can be neglected compared to ($\frac{1}{T_2}$) in Eq. (10). The excited state population is then [9, 11, 12]:

N_2 = R T_2 ………. (12)

When (R) reaches the threshold value:

$R_{th} = \frac{N_{2th}}{T_2} = \frac{1}{c\sigma_L T_c T_2}$ ………. (13)

A further increase in (R) does not results in any additional increase in (N_2), because, the steady-state condition of Eq. (10) would then be violated. Instead, (N_2) become pinned at the threshold value ($N_2 = N_{2th} = R_{th}$ T_2).

Although (N_2) does not increase above threshold, the lasing intensity (I_L) does increase with increase with increasing (R).

Then Eq. (10) can be written as:

R – $R_{th}\left(\frac{I\sigma_L T_2}{h\upsilon_L} + 1\right)$ ………. (14)

And solving of Eq. (11) to give (I_L):

$I_L = \frac{h\upsilon_L}{\sigma_L T_2}\left(\frac{R}{Rth} - 1\right)$ ………. (15)

The lasing intensity (I_L) is increase linearly with excitation rate (R) , this linear increase in lasing intensity above threshold , along with the pinning of the population inversion , are key identifying features of laser action.

2.4 Threshold pump power:

The pump power is absorbed as it propagates down the fiber core, and this causes the pump intensity to vary with position along the fiber.

Consider pump power of intensity (I_p (o)) and wave length (λ_p) that is coupled into the core of fiber, the fiber length is (L). And the core has cross–sectional are is (A_{eff}), In this case, the pump intensity decays exponentially with (Z) according to Beer's a law [11,12] :

$I_p(Z) = I_{p(0)}\, e^{-\propto_p Z}$ ……….(16)

Where ($\propto_p$) is the Scohering losses ($\propto_p = \Gamma_p N_o \sigma_p$), Γ_p is the power filling ($\Gamma_p = \frac{A\ effc}{A\ clad}$) at pumping wavelength, Where (Aclad =πb^2), (b) is the radius of cladding, for a multi-mode fiber ($A_{effc} = \pi a^2$). The (w_p) is the mode field radius at pumping wavelength, for a fundamental mode is defined by [9] :

$W_p = a\left[0.761 + \frac{1.237}{V1.5} + \frac{1.429}{V6}\right]$ ………. (17)

Where (V) is normalized frequency at the pumping wavelength (V =$2\pi a N_A/\lambda_p$)[9].

The gain coefficient varies with (Z) , we first express it in terms of the level populations as [11,12]:

$G_{(Z)}$ = [N_2 (Z) – N_1(Z)] σ_L ………. (18)

Where N≪N_2 has been assumed for the four – level transition.

$G_{(Z)}$ = N_2 (Z) σ_L ………. (19)

By using $W_p=\frac{I_P\sigma_P}{h\upsilon_P}$, R=$N_o W_P$, $\alpha_p = \Gamma_p N_o \sigma_P$ and Eq.(12), the eq.(19) can be written as:

$G_{(Z)} = \frac{\propto_p\sigma_L T_2}{\Gamma_p h\upsilon_p} I_{P(Z)}$ ………. (20)

For the small section (d_Z) of fiber, the increment in lasing intensity is [7,12] :

dI = I $G_{(Z)}$ dz ………. (21)

By using Eq.(16) and (20) the Eq.(21) can be integrating over the fiber length (L), we obtain:

$\mathrm{Ln}\frac{I_2}{I_1} = \frac{\sigma_L T_2 I_{p(0)}}{\Gamma_p h\upsilon_p}(1-e^{-\propto_p L})$ ………. (22)

Were (I_1) and (I_2) is the lasing intensity at mirror (1) and mirror (2) respectively. The single-pass gain was denoted previously ($\gamma = \frac{I_2}{I_1}$) then

$\ln \gamma = \frac{\sigma_L T_2 I_{p(0)}}{\Gamma_p h\upsilon_p}$ [1-exp ($-\propto_p$ L)]………. (23)

At threshold ($\ln \gamma = \ln \gamma_{th} = G_{th}L$) and $I_{P(0)} = \frac{p_{th}}{A\ eff}$, the Eq.(23) can be written as:

$P_{th} = \frac{A\ eff\ h\upsilon_p \Gamma_p G_{th} L}{\sigma_L T_2[1-\exp(-\propto_p L)]}$ ………. (24)

2-5-Lasing output power:

The output power from the laser can be determined by referring to Fig (2). The wave inside the resonator has the form of a standing wave, which is equivalent to the superposition of two counter prorating beams of intensities (I_-) and (I_+) as showed.

Each of these has half the intensity (I) of the wave in the cavity, so ($I_- = I_+ = I/2$). Assume for simplicity that the left mirror (R_1), and that the other mirror has a transmission (T), so ($T = 1 – R_2$). Wave will then have the cavity only through the right mirror (R_2), with an intensity ($TI_T = \frac{1}{2}$ TI), the power exiting the laser becomes [7, 12]:

$P_{out} = \frac{1}{2}$ IT A eff ………. (25)

BY using Eq. (18), the Eq . (25) Become:

$P_{out} = \frac{1}{2}\frac{h\upsilon_L}{\sigma_L T_2}$ A eff ($\frac{R}{R_{th}}$-1) ………. (26)

Where the saturation lasing intensity is ($I_L^s = \frac{h\upsilon_L}{\sigma_L T_2}$), The input power to laser can be taken as the absorbed pump power is given by [7] :

$P_{in} = P_{abs} = Rh\nu_p$ A eff L = $P_{(o)}$[1-exp($-\propto_p$L)] ………. (27)

Were $P_{(o)}$ is power of the sources pumping. And threshold pumping can be written as [7]:

$P_{th} = R_{th}h\nu_p$ A eff L = $\frac{Aeff\ h\nu_p \Gamma_P G_{th} L}{\sigma_L T_2[1-\exp(-\alpha_P L]}$………. (28)

By using Eqs.(27) and (28), the Eq.(26) can be written as:

$P_{out} = \frac{1}{2}$ A eff $\frac{TI_s}{P_{th}}$ [P_{in} - P_{th}] ………. (29)

2-6 Efficiency:

Eq. (29) can be further manipulated the simple form [7, 12]:

$P_{out} = \eta_s$ [P_{in} - P_{th}] ………. (30)

Combining Eq. (29) with Eq. (30), the slope efficiency giving:

$\eta_s = \frac{1}{2}$ Aeff T $\frac{I_s}{P_{th}}$ ………. (31)

By using Eqs. (13) and (28) the Eq. (31) can be written as:

η_s = T $\frac{h\nu_L}{h\nu_p}\frac{1}{2\ln\gamma + h}$ ………. (32)

3. Results and Discussion

All the coefficients related the designs that is used in case of host is Silica and ZBLAN for Nd^{+3} – doped optical fiber which is used in numerical emulation through Mathlab program to calculate the lasing output power (P_{out}) as shown in table [14], while table (2) shows the wavelength of pumping source that is used in this research, and the wavelengths related lasing emission (λ_L).

According to pumping the four – level system for Silica and ZBLAN, that is used in this numerical emulation.

In this numerical semulation [8,13] , firstly determine the value of numerical aperture (N_A) for five designs, table (3) shows N_A values which is obtained the highest value of lasing output power (P_{out}) and efficiency (η) for the all designs.

For the four designs, figure (1) shows P_{out}values corresponding to pump power (P_o) of the Silica host at (λ_L=1064nm), while figure (2) shows the same result at (λ_L=1331nm), from the two figures, it can be noted that the highest values of P_{out} and η when used the design York, and its highest value at λ_L=1064nm, while figure (3) illustrated the P_{out} values corresponding to P_o values in case of using the host ZBLAN for λ_L=1048nmand λ_L=1317 nm, it can be noted that the highest value of P_{out}andηat λ_L=1048nm.

Table (1)

Parameters	Value (Silica)				Value (ZBLAN)	unit
	Lycom(a)	Lycom(b)	York	IVA	Le Verre	
N°	11.5×1025	5.6×1025	5×1025	5×1025	20×1025	Ion/m3
T2	430×10-6	485×10-6	460×10-6	480×10-6	445×10-6	Sec
T3	15×10-9	9.5×10-9	12×10-9	10.5×10-9	11×10-9	Sec
L	75×10-2	135×10-2	54×10-2	46×10-2	116×10-2	m
a	1.35×10-6	1.85×10-6	1.85×10-6	2.75×10-6	1.9×10-6	m

Table (2)

Parameters	Value (Silica)		Value (ZBLAN)		unit
	Nd1	Nd2	Nd4	Nd3	
λ_p	806×10-9	806×10-9	806×10-9	806×10-19	m
σ_p	2.5×10-24	2.5×10-9	2.5×10-24	2.5×10-24	m2
λ_L	1064×10-9	1331×10-9	1317×10-9	1048×10-19	m
σ_L	2×10-24	0.256×10-24	8.28×10-25	34.515×10-25	m2

Table (3)
The values of numerical aperture (NA)

Design	NA (Silica)		NA (ZBLAN)	
	λ_l=1064nm	λ_l=1331nm	λ_l=1048nm	λ_l=1317nm
Lycom(a)	0.045	0.060	-	-
Lycom(b)	0.035	0.045	-	-
York	0.035	0.054	-	-
IVA	0.025	0.035	-	-
Nd3	-	-	0.030	-
Nd4	-	-	-	0.040

Table (4)

parameters	value	unit
NA	0.045	-
a	1.35×10-6	m
b	3.375×10-6	m
L	46×10-2	m
R1	1	-
R2	0.9	-
N	5×1025	Ion/m2
T	485×10-6	sec

According to these results, the best one to obtained the highest value of Pout or η in case of using the host silica at λL=1064nm, through the coefficients related the four designs for this host, the optimal values have been determinate of wavelength, that is at this value can be obtained the highest values of Poutand η as show in in table (4), while figure (4) shows the values of Pout corresponding to Po that is calculated by using the optimal values.

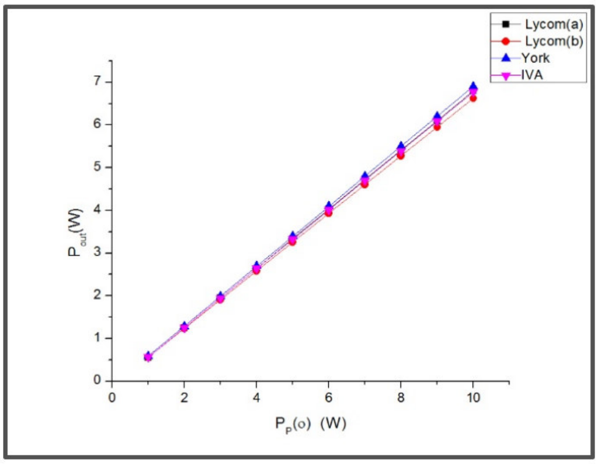

Fig (1): The Laser output power Vs pump power at the hot silica at 1964nm

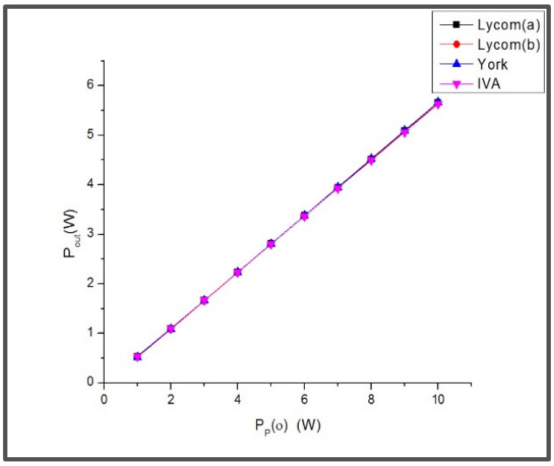

Fig (2): The Laser output power Vs pump power at the hot silica at 1331nm

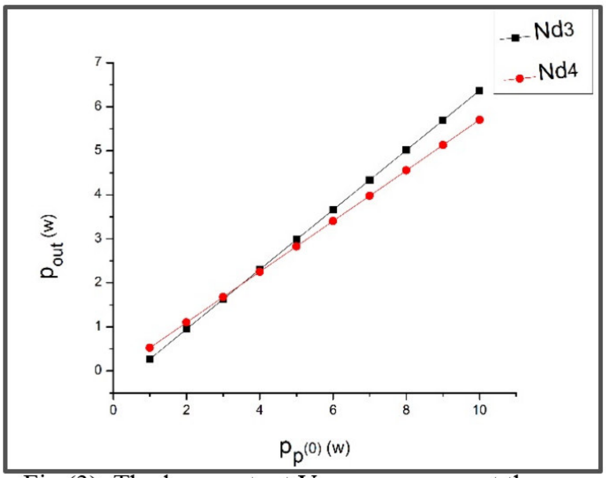

Fig (3): The laser output Vs pump power at the hot ZBLAN at 1048nm

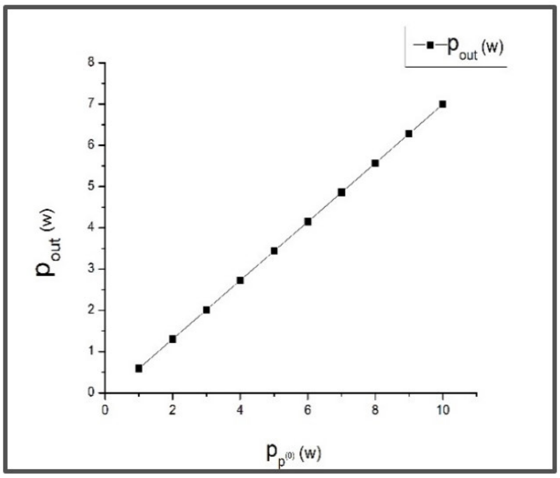

Fig (4): The laser output power Vs pump power at the hot ZBLAN at 1317nm

4. Conclusion

In this search, the equation of lasing output power (Pout) had been derived for optical fiber laser which process according to four-level system pump, the Pout values had been calculated for the two types of hosts represented by Silica and ZBLAN through the numerical emulation Nd+3-doped optical fiber.

In case of using the host Silica, the Pout values had been calculated for four different designs at (λL=1064nm) and (λL=1331nm), it is found that the Poutincreased linearly when Po increased. For both wavelengths, the highest values of λ and Pout when use the design York.

While in case of using the host (ZBLAN), the Pout has been calculated at (λL=1048nm) and (λL=1317 nm), and it is found that the highest values of λ and Pout at (λL=1048nm).

According to these results, to obtained the highest values of Pout and λ, it is must be using the host Silica at (λL=1064nm), then through the four- design coefficients for this host, the optimal values had been determinate.

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