Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.57, 2016
www.iiste.org

Modulating Mathematical Function to be used in Electron Objective Lens

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Abstract
A theoretical study has been carried out on the magnetic field resulting from a modulating mathematical function to produce a magnetic pole to be used for designing the objective magnetic lens in the electron optical devices. This function has only one effective optimization parameter (a) which is named half of the half-width of the magnetic field that can affect on the design and focal properties of the magnetic lens. At first we will be seen the peak of the magnetic field shifted out of a symmetry point (z=0) by a distance approximately equals to the half half-width of the magnetic field. Therefore, some axial transformations have been carried out to make this peak symmetric around the symmetry axis. The present work has shown the possibly of obtaining a magnetic pole of the magnetic field in which the peak shifted off symmetry point.
Keywords: Electron optics, Objective Magnetic Lenses, Aberration.

1. Introduction
As generally known, the spherical and chromatic aberration coefficients in the electron microscope are smaller, when the intensity of the lens field is stronger near the specimen and the focal length is smaller. Therefore, it is naturally desirable that the specimen be placed near the center of the lens. However, the position of the specimen has so far been chosen or placed on the side of the electron source with respect to the center of the lens which focuses the image of the specimen, in order to utilize the lens function more efficiently. However, when the inside of the lens is made a vacuum, the focal length is always shortened owing to the spherical aberration, and the cross-over of the off-axial electron beams is formed always nearer the electron source than that of the paraxial beams. The distance between them increases rapidly at a rate proportional to the cube of the distance from the beam to the axis. Therefore, if the specimen is placed near the center of the lens in order to reduce the spherical and chromatic aberration coefficients, the off-axial beams reach the center part of the specimen and mix with the paraxial beams. Thus the image becomes obscure owing to the large aberration, which consequently requires the very small aperture for the clear image. However, such a small aperture reduces the visual field and brings about difficulties in manufacturing the maintenance of the aperture, which is far from the practical use **[1]**.

2. Modulating Mathematical Function
A modulating mathematical field function has been suggested to represent objective electron lens given by the formula equation:

$$B_z(z) = \left[\frac{a^2}{\pi\left[(2z-a)^2 + a^2\right]}\right] \tag{1}$$

Where a, is an optimization parameter which represents half of the half-width of the field distribution. In the present work the mathematical field formula given in equation (1) has been used to represent the density of axial magnetic field distribution $B_z(z)$ of the electron objective lens. The gradient of the magnetic field distribution and its curvature can be evaluating analytically using the following equation;

$$B'_z(z) = \left[\frac{-4a^2(2z-a)}{\pi\left[(2z-a)^2 + a^2\right]^2}\right] \tag{2}$$

In the inverse design procedure, it is necessary to compute the axial magnetic scalar potential along the optical axis of the lens under considerations. However, by using Ampere's circuited law the B_z component of the fields along the axis of symmetry obey Ampere's law;

$$\int_{-\infty}^{\infty} B_z \, dz = \mu_o NI \tag{3}$$

Where μ_o is the magnetic permeability in vacuum ($4\pi \times 10^{-7}$ Hm^{-1}) and NI is the number of Ampere-turns of the magnetizing coil. For a permanent magnetic lens, the integral of equation (3) is always nil, since NI = 0 in this case. Whenever, these conditions are satisfied, the pole-piece surfaces can be regarded as equipotential surfaces. Hence, the magnetic field lines intersect the surfaces of the pole-pieces in an orthogonal way, this means that the magnetic scalar potential supplied by the excitation is concentrated at the air-gap between the pole-pieces, i.e., magnetic scalar potential drop must constitute most of at least 95%, the total ampere turns produced by the excitation coil. According to that, Ampere's law given by equation (3) can be written as follows;

$$\int_{z_s}^{z_f} B_z\, dz = \mu_o NI = \mu_o (V_{z_s} - V_{z_f}) \tag{4}$$

Where V_{z_s} and V_{z_f} are the magnetic scalar potentials at the terminal points z_s and z_f at the optical axis respectively **[2]**. With aid of the equation (4);

$$B_z(z) = -\mu_o (dV_z/dz) \tag{5}$$

The trigonometric integral method can be used to integrate equation (1) along the optical axis to obtain the axial magnetic scalar potential distribution at each axial coordinate z, the result will be as follows;

$$V(z) = -\left[\left(\frac{a}{2\mu_o \pi}\right) tan^{-1}\left(\frac{(2z-a)}{a}\right)\right]_{z_s}^{z_f} \tag{6}$$

Where z_s, and z_f, are the terminals of the magnetic lens field. The second derivative of the axial magnetic scalar potential distribution $V''(z)$ can be determined analytically from the equation;

$$V''(z) = \left[\frac{B_z'(z)}{\mu_o}\right] = \left[\frac{-4a^2(2z-a)}{\mu_o \pi\left[(2z-a)^2 + a^2\right]^2}\right] \tag{7}$$

3. Paraxial Ray Equation and Equipotential Surfaces

The trajectory of the charged particle (electron in the case of magnetic lens) inside a rotationally symmetric magnetic field can be controlled by the paraxial ray equation which is a second-order, linear, homogenous ordinary differential equation given by**[3]**;

$$r'' + \frac{\eta}{8V_r} B_z^2(z) r = 0 \tag{8}$$

Where η is the mass m to the charge e of the electron respectively, and V_r is defined as the corrected relativistically accelerating voltage which is given by **[4]**;

$$V_r = V_a (1 + e\, V_a / 2m\, c^2) = V_a (1 + 0.978x10^{-6}\, V_a) \tag{9}$$

Where V_a is the applied accelerating voltage. To obtain the electron beam trajectory inside the electron optical device (the magnetic lens in the present investigation) the paraxial ray equation can be solved analytically or numerically according to the constraints imposed in the study. By using the analytical solution of Laplace's equation, the shape of the pole-piece that would produce the desired magnetic field can be determined. Therefore, for an axially symmetric systems, the electrostatic or magnetic scalar potential distribution $V(R_P, z)$ in the plane can be calculated from the axial distribution of the same potential $V(z)$ by the following power series expansion **[5]**;

$$V(R_p, z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2}\left(\frac{R_p}{2}\right)^{2k} \frac{d^{2k} V(z)}{dz^{2k}} \tag{10}$$

Where R_p is the radial height of the pole-piece, V_P is the potential value at the pole-piece surface, which is equivalent to half of the lens excitation NI in the case of symmetrical charged particle lens, and $V''(z)$ is the second derivative of the magnetic scalar potential with respect to the z-coordinate. By taking the first two terms of equation (10) the equipotential surfaces (i.e. the magnetic pole-piece) can be determined from the following formula **[6]-[7]**;

$$R_P(z) = 2\left[\frac{V(z) - V_P}{V''(z)}\right]^{\frac{1}{2}} \tag{11}$$

4. Aberration Calculation

In electron-optical systems, some imperfections arise because of fundamental physical reasons. Spherical and chromatic aberrations are well- known examples of this: in round lenses of the type normally used in focusing of electrons and other charged particles, these aberrations are always positive and typically of the order of the focal length of the lens. Such aberrations are called fundamental aberrations [8]. It is found that if a beam of charged particles emerges from a point on the axis in the object plane z_o, the charged particles that emerge at very small angles to the optical axis intercept the latter farther away from the object plane than those that leave with large angles. That is mean; the outer regions of the lens have a stronger focusing effect than the regions close to the axis. A ray with initial slope α intersects the image plane z_i at a distance from the axis that is proportional to α^3. The result is that a point in the object plane will not be imaged as a point in the image plane but as a disc of radius Δr_i also sometimes symbolled d_{Cs} (see figure 1) which is equal to **[9]**;

$$\Delta r_i = M\, C_s\, \alpha^3 \tag{12}$$

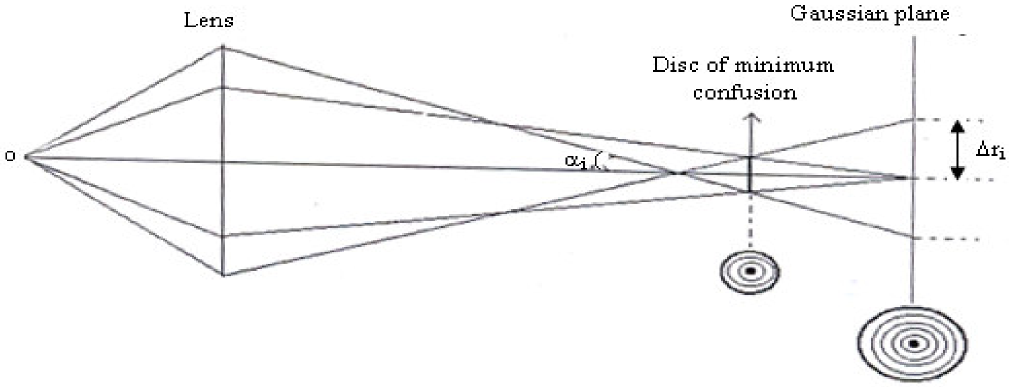

Figure 1: The origin of spherical aberration.

Where C_s is the spherical aberration coefficient of the lens which it has a magnification M. The spherical aberration will therefore cause blurring in the image of the lens. This aberration limits the resolution of the lens. The coefficient C_s depends on the geometry and excitation of the lens and also on the position of the object plane. The spherical aberration coefficient C_s of any axially symmetric magnetic lenses can be calculated using the following integral [**10**];

$$C_S = \frac{\eta}{128 V_r} \int_{z_o}^{z_i} \left[\left(\frac{3e}{mV_r} \right) + B_z^4(z) r_\alpha^4(z) + 8 B_z'^2(z) r_\alpha^4 - 8 B_z^2(z) r_\alpha^2 r_\alpha'(z) \right] dz \tag{13}$$

Where $r_\alpha(z)$ is the solution of the paraxial ray equation, and primes indicate differentiation with respect to the axial coordinate z. In light optics, chromatic aberration arises from the spread of wavelengths of light passing through a lens, coupled with the dependence of refractive index on the wavelength. Concerning with electron optics, it is known that the electron wavelength depends on the momentum or kinetic energy of each particle and therefore, if the electrons have a spread in kinetic energy, one may expect a variation in focusing power of a magnetic lens. However, spread in kinetic energy may be due to: (1) the variation in kinetic energy of the electrons emitted from the electron source; (2) fluctuations in the potential applied to accelerate the electrons; (3) not all electrons lose the same amount of energy due to inelastic scattering in the specimen, in the case of imaging lenses which follow the specimen in a transmission electron microscope column **[11]**. Chromatic aberration can be defined with the aid of the simplified diagram in figure 2. It is seen that rays of energy ($E+\Delta E$) will intersect the Gaussian image plane at a distance Δr_i from the axis (also sometimes symbolled d_{Cc}). However, when the angle of convergence of the rays is ${}_i$, then Δr_i is given by **[12]**;

$$\Delta r_i = C_{ci}\, \alpha_i \left(\frac{\Delta E}{E} \right) \tag{14}$$

Where C_{ci} is the chromatic aberration coefficient referred to image plane.

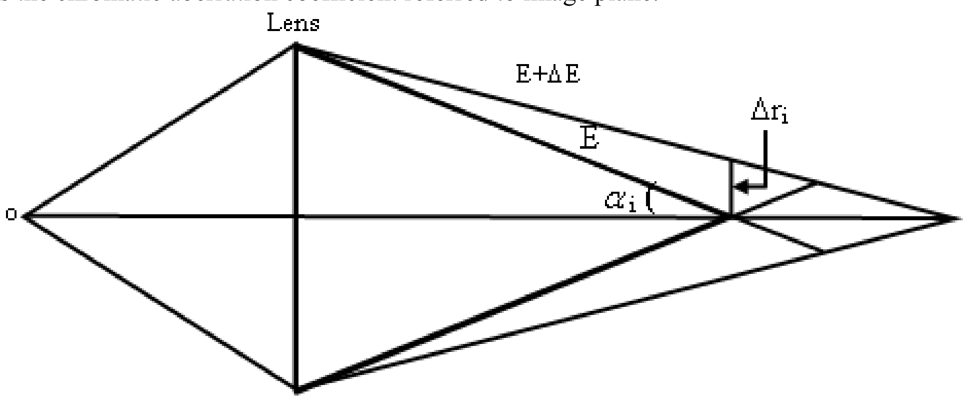

Figure 2: A simplified diagram defining the chromatic aberration.

In this work, the chromatic aberration coefficient C_c can be computed with the aid of the following integral formula

[13];

$$C_c = \frac{\eta}{8V_r} \int_{z_o}^{z_i} B_z^2(z)\, r_\alpha^2(z)\, dz \qquad (15)$$

5. The Magnetic Field and Reconstructed Pole-Piece Shape

To study the effect of the optimization parameter a, the following six values of a are selected (1, 2, 3, 4, 5, and 6 mm), the lens operates under the magnetic unsaturated mode, and the length of the lens is kept constant at 100 mm and estimate the effects which are resulted from the variations of parameter a on the objective lens properties. From the proposed model equation (1), it is noted that the distribution of the magnetic field will be shifted from the symmetry plane at (z=0) by a distance approximately equals to half of the half-width a, and consequently the distribution of the magnetic scalar potential as shown in figure 3. It should be mentioned that, this shift in the field and potential distributions makes these distributions are asymmetric about the symmetry plane. Therefore, some axial transformations have been carried out to make these distributions symmetric around the symmetry plane (at z=0) as shown in figure 4. However, figure 5, shows half of the half-width upper reconstructed magnetic pole-piece shape at a = 1mm**.**

6. The Objective Magnetic Lens

There is no variation in the maximum value of axial magnetic field density B_{max} (in tesla) with different values of the parameter a, as we shown in the figures 3 and 4. It is obvious that when the parameter a is increased the peak field value B_{max} of the corresponding fields is not affected. The lens excitation NI (Ampere-turn) is equal to the area under the curve of the magnetic field, and varying as a function of the different chosen values of the parameter a , shown in the figure 6. On other hand, the area under each field distribution NI is enlarged with increasing the parameter a, and naturalistic that's credible, because the increase in the half half-width means enlarge the bell shape of magnetic field, lead to increase in area under magnetic curve. This means that the field distribution would be more distributed along a large axial extension of the optical axis for large values of parameter a. The variation of scalar potential V with the values of half of the half-width (a-values) shown in figure 6, illustrated that the increasing in a-values leads to increasing the scalar potential and that's mathematically proofs in equation (4), because the rise in a-values leads to enlarge in the bell-shape of magnetic field (increasing in B_z-values) and finally increasing V-values.

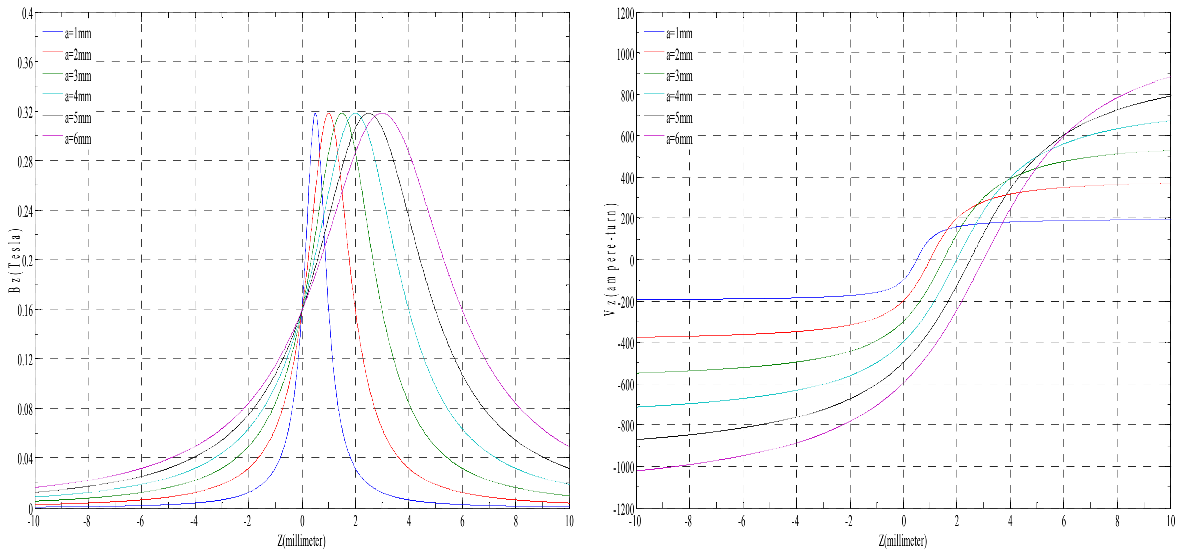

Figure 3: The axial magnetic field and magnetic scalar potential distributions, shifted from the symmetry plane for different values of the parameter a.

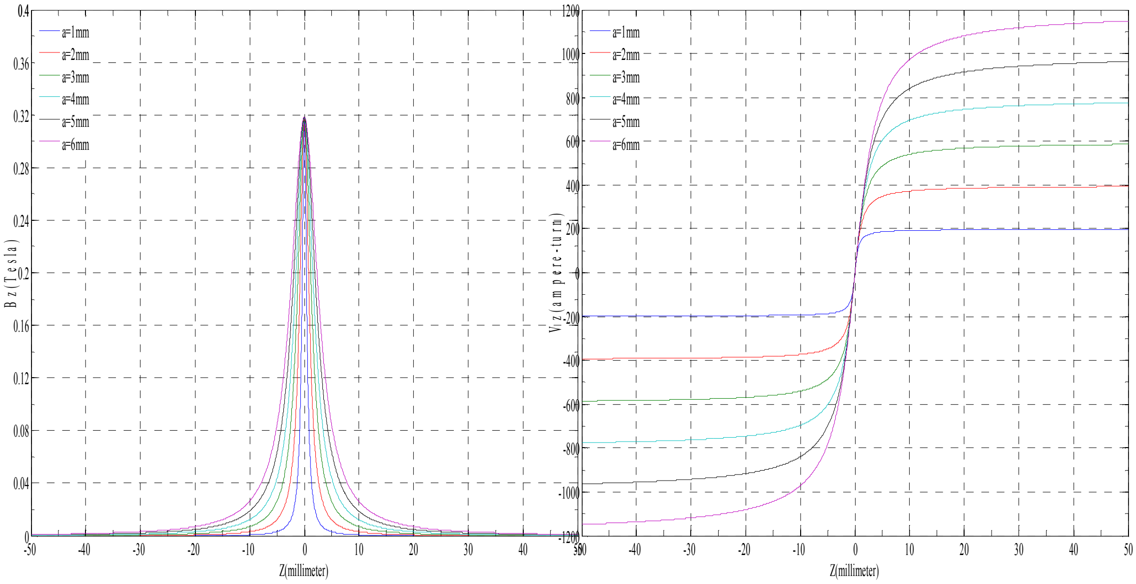

Figure 4: The symmetric of axial magnetic field and magnetic scalar potential distributions around the symmetry plane for different values of the parameter *a*.

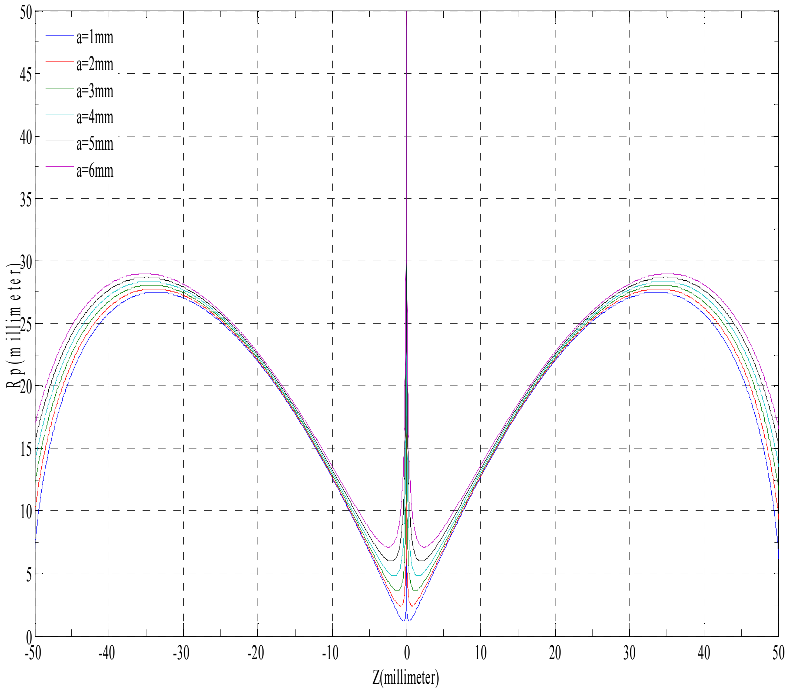

Figure 5: The half upper reconstructed magnetic pole-piece shape for different values of the parameter *a*.

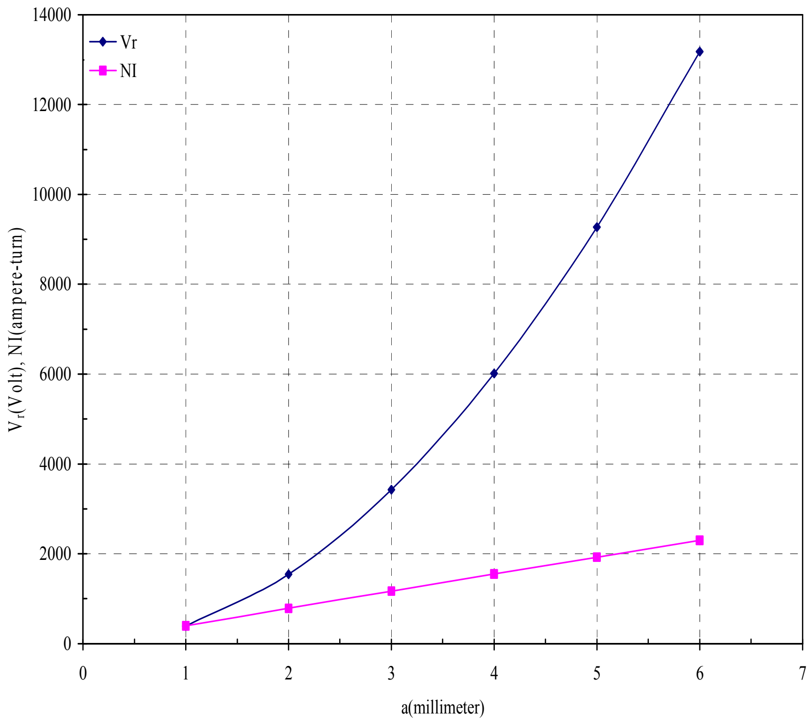

Figure 6: Magnetic field parameter *NI* and V_r as a function of *a***-**values.

7. Results and Discussion

In the present work the effect of the optimization parameter a on the design of symmetrical single magnetic electron lens and consequently the first order properties as well as third-order aberration has been taken under consideration. However, it should be mentioned that the optical properties and the lens configuration are not affected by changing the length of the lens. The electron beam trajectory along optical axis (axial distance *z*) represented at different *a*-values, shown in figure 7, according to paraxial ray equation (8) at zero magnification mode. This representation illustrate the refracting of electron beam trajectory toward the optical axis starting from z = -5 mm, approaching to the symmetry plane at (z=0). From figure 7, one can conclude that the stronger refracting happen in a smaller half of the half-width (a=1mm). This result lead to fact that the lens which has magnetic field with a small the half half-width can be a strong lens and smaller focal length as we shown in figure 8, where represent the focal length F_o as a function of *a*-values. From figure 8, we will be noticed that, the values of focal length increasing when *a*-values increased **[1]**.

Figure 8, illustrate the objective focal properties F_o , C_s , and C_c as a function of *a*-values, where increasing the spherical and chromatic aberration coefficients C_s , and C_c respectively, when *a*-values increased for the same reason above that state, the stronger lens which has smaller focal length and better properties with smaller aberration coefficients C_s , and C_c **[1]**. In addition, we can notice that, for this proposed mathematical model as a magnetic lens will has C_c-values higher than C_s-values, i.e. the formation image in this lens will be at small spherical aberrations than that with chromatic aberration, and that's proofs in figure 9 which is illustrate that, the radius of disc confusion for the spherical aberration d_{Cs} less than the chromatic aberration d_{Cc} .

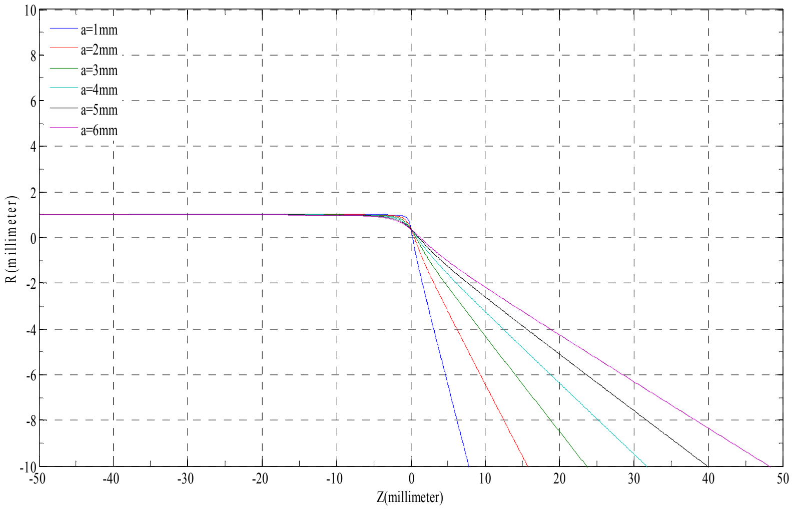

Figure 7: Electron beam trajectory along optical axis at different values of *a*.

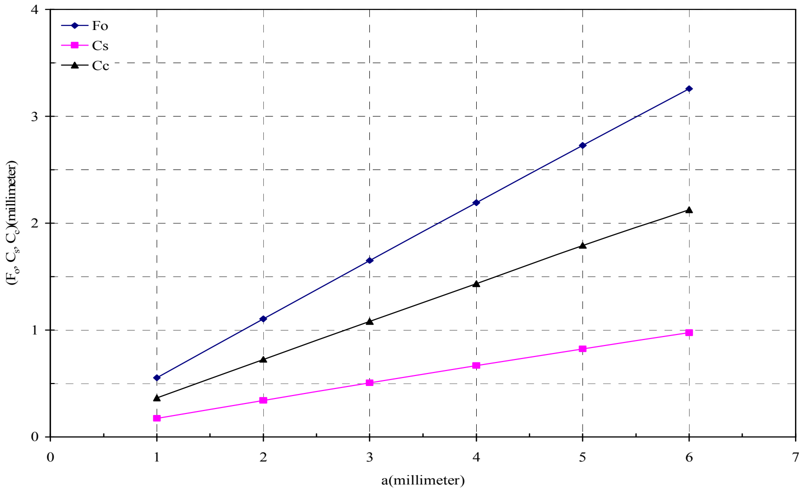

Figure 8: The objective focal properties F_o , C_s , and C_c as a function of *a*-values.

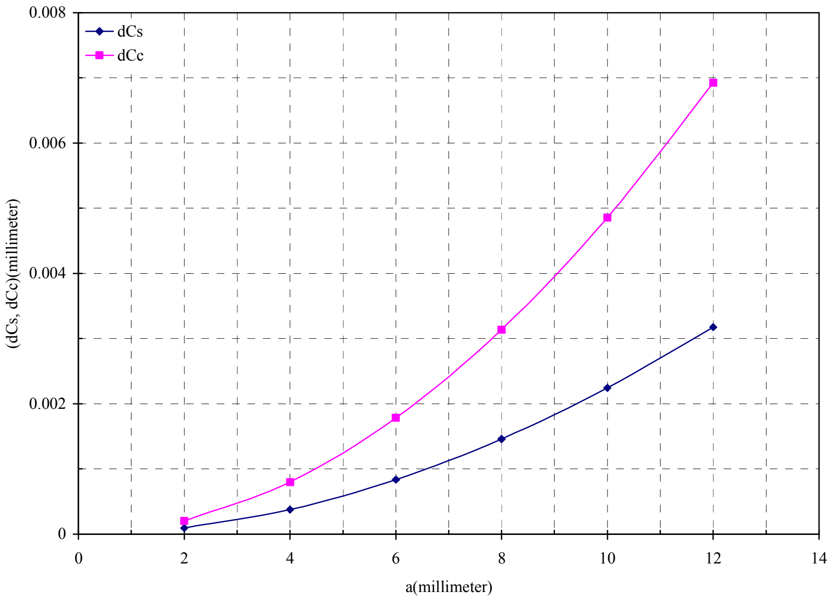

Figure 9: The radius of confusions disc at spherical and chromatic aberrations d_{Cs}, and d_{Cc} respectively, as a function of a-values.

8. Conclusions

In the current paper it can be concluded some important information can be summarized in the following points:

1) There is no variation in the maximum value of axial magnetic field density with different values of the half-width.

2) The field distribution would be more distributed (lens excitation is increased) along a large axial extension of the optical axis for large values of the half-width.

3) According to Ampere's law**,** the rise in the half-width values leads to enlarge the bell-shape of magnetic field and so on increasing the scalar potential.

4) The stronger refracting of electron beam trajectory toward the optical axis happens in a smaller half-width. Thus, the strong lens means smaller focal length in a smaller half-width.

5) For this proposed mathematical model as a magnetic lens will has C_c-values higher than C_s-values, and the radius of disc confusion for the chromatic aberration d_{Cc} higher than the spherical aberration d_{Cs} , i.e. the formation image in this lens will be at small spherical aberrations than that with chromatic aberration.

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