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Similarity Solutions for MHD Visco-Elastic Flow over a Permeable and Non-Linearly Quadratic Stretching Sheet

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Abstract
A notable study on similarity solutions for visco-elastic (Walters liquid B' model) fluid flow over a permeable and nonlinearly stretching sheet is investigated in the presence uniform magnetic field. Recently Raptis and Perdikis [1] studied the similarity solutions for boundary layer flow over an impermeable quadratic non-linear stretching sheet using a stream function of the kind $\psi = \alpha x f(\eta) + \beta x^2 g(\eta)$ Also Neil and Kelson [2] have worked on the paper Raptis and Perdikis [1] and pointed out the fundamental error in their problem [1] and shown that similarity solutions do not exist for this choice of $\psi = \alpha x f(\eta) + \beta x^2 g(\eta)$. Thus in the present technical paper it is studied that similarity solutions do not exist for visco-elastic boundary layer flow also with the above choice of ψ

Introduction
The study of boundary layer behavior over a continuously moving flat wall finds wide applications in technological manufacturing process in industry. These includes aerodynamic extrusion of plastic sheets, rolling and extrusion in manufacturing process, the cooling of an infinite metallic plate in a cool bath, the boundary layer along a liquid film in condensation process and the controlled cooling system.

Raptis and Perdikis [1] have studied about the viscous flow over a non-linearly stretching sheet in the presence of chemical reaction and magnetic field. Neil and Kelson [2] made a note on similarity solutions for viscous flow over an impermeable and non-linearly (quadratic) sheet. Takhar et al. [3] have analyzed MHD asymmetric flow past a semi infinite moving plate. Crane [4] has studied the flow past a stretching plate. Rao et al. [5] have studied the momentum and heat transfer in a power-law fluid with arbitrary injection/suction at a moving wall. Prasad et al. [6] investigated about the momentum and heat transfer in visco-elastic fluid flow in a porous medium over a non-isothermal stretching sheet. Hopwell [7] analyzed the momentum and heat transfer over a continuous moving surface in power law fluid. Acrivose et al. [8] noted on momentum and heat transfer in laminar boundary-layer flows of non-newtonian fluids past external surfaces. Howell et al. [9] made a study on momentum and heat transfer on a continuous moving surface in power law fluid. Markovitz and Coleman [10] have investigated the visco-elastic boundary layer flow over a stretching plastic sheet in polymer processing. Sonth et al. [11] have studied the effect of heat and mass transfer flow in a visco-elastic fluid over an accelerating surface with added heat source. Gupta and Gupta [12] have extended the study of stretching sheet problem . Chaim [13] analyzed the magneto hydrodynamic heat transfer flow over a non- isothermal stretching sheet. Kumaran and Ramanaiah [14] analyzed the boundary layer flow over a stretching sheet. Yih [15] studied about the influence of non-Darcy forced convection flow over a flat plate with variable wall temperature in a porous medium. Vajravelu and Nayfeh [16] made an analysis of the convective heat and mass transfer flow in an electrically conducting fluid past a stretching sheet through porous medium. Vajravelu [17] investigated in the convection flow with heat and mass transfer past a stretching sheet with suction or blowing. Surmadevi et al. [18] have analyzed the boundary layer flow caused by a stretching/continuously moving sheet for different thermo-physical situations using variety of fluid models and boundary conditions. Amkadni et al. [19] have discussed the exact solutions of laminar magneto hydrodynamic viscous flow over a stretching sheet. Cortell [20] has studied flow and heat transfer of a fluid through porous media over stretching surface with internal heat absorption and blowing. Troy et al. [21] have analyzed the uniqueness of flow of a second order fluid past a stretching sheet in the presence of magnetic field. Idress et al. [22] have studied visco-elastic flow past a stretching sheet in a saturated porous medum. Naseem and Khan [23] have done the work on boundary layer flow past a stretching plate with suction. McCormack and Crane [24] had a comprehensive study on boundary layer flow caused by stretching of flat surface and between two surfaces under different physical situations.

Coleman and Noll [25] have studied an approximation theorem for functional with applications in continuoum mechanics. Dunn and Fosdick [26] have discussed thermodynamics stability and boundedness of fluids of complexity 2 and fluids of second grade. Fosdick and Rajagopal [27] have studied anomalous features in the model of second order fluids.

Mathematical Formulation and Solution

We consider a laminar steady state incompressible MHD visco-elastic second order fluid flow over a porous infinite stretching sheet.

Following the postulates of gradually fading memory, Coleman and Noll [25] derived the constitutive equation of second order fluid in the form.

$$T = -Pl + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{3}$$

Where T is the Cauchy stress tensor, - P l is the spherical stress due to constraint of incompressibility, μ is the dynamics viscosity; α_1 and α_2 are the material moduli. A_1 and A_2 are the first two Rivlin-Ericksen tensors and they are defined as

$$A_1 = (grad\ q) + (grad\ q)^{\tau} \tag{4}$$

$$A_2 = {dA_1}/{dt} + A_1 (grad\ q) + (grad\ q)^{\tau} A_1 \tag{5}$$

The model equation (3) was derived by considering upto second-order approximation of retardation parameter. Dunn and Fosdick [26] have given the range of values of material moduli μ, α_1 and α_2 as

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0 \tag{6}$$

The fluid modeled by (3) with the relationship (6) is compatible with the thermodynamics. The third relation is the consequence of satisfying the Clausis-Duhem inequality by fluid motion and the second relation arises due to the assumption that specific Helmholtz free energy of the fluid takes its minimum values in equilibrium. Later on Fosdick and Rajagopal [27] have reported, by using the data reduction from experiments that in the case of a second order fluid the material moduli μ, α_1 and α_2 should satisfy the relation

$$\mu \geq 0, \alpha_1 \leq 0, \alpha_1 + \alpha_2 \neq 0 \tag{7}$$

It is reported that the fluids modeled by equation (3) with the relationship (7) exhibit some anomalous behavior. It must be mentioned that second order fluid, obeying model equation (3) with $\alpha_1 < \alpha_2, \alpha_1 < 0$ although exhibits some undesirable instability characteristics. The second order approximation is valid at low shear rate [2]. Now in literature the fluid satisfying the model equation (3) with $\alpha < 0$ is termed as second order fluid and with $\alpha > 0$ is termed as second grade fluid [2].

Thus the usual boundary layer equations for the present study are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}.\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} - \frac{\upsilon}{k^1}u \tag{9}$$

Here u and v are the velocity components in x- and y- directions, respectively. υ is the kinematic coefficient of viscosity, k_0 is the elastic parameter. Hence in the case of second order fluid flow k_0 takes positive value as α_1 takes negative value and other quantities have their usual meanings. In equation (9) it is assumed that the normal stress is the same order of magnitude as that of the shear stress in addition to usual boundary layer approximations.

Boundary Conditions on Velocity

For the present physical problem, where the stretching of the boundary surface is assumed to be such that the flow directional velocity is linear function of the flow directional co-ordinate, we employ the following boundary conditions (Raptis and Perdikis [1]).

$$u(y=0) = ax + bx^2, \quad v(y=0) = 0, \quad \lim_{y\to\infty} u = 0 \tag{10}$$

and the following similarity transformations

$$\eta = \sqrt{\frac{a}{\upsilon}}\, y, \quad \psi = \sqrt{a\upsilon}\, x\, f(\eta) + \frac{bx^2}{\sqrt{\frac{a}{\upsilon}}} g(\eta) \tag{11}$$

Where $u = {\partial \psi}/{\partial y}$ and $v = -{\partial \psi}/{\partial x}$ on substitution these into equations (8), (9) and (10) above the following coupled ordinary differential equations are obtained.

$$f'^2 - ff'' = f''' - k_1\left[2f'f''' - ff^{IV} - f''^2\right] - (k_2 + M_n)f' \quad (12)$$

$$3f'g' - 2gf'' - fg'' = -k_1[2f'g'' + 4g'f''' - fg^{IV} - 2gf^{IV} - 3f''g''] + (k_2 + M_n)g' \quad (13)$$

$$g'^2 - gg'' = g''' - k_1\left[2g'g''' - gg^{IV} - g''^2\right] \quad (14)$$

$$k_1 = {k_0 a}/{\upsilon} \qquad k_2 = {\upsilon}/{k'cx} \quad M_n = \frac{\sigma B_0^2}{\rho cx}$$

Then we get the transformed boundary conditions as

$$f(0) = g(0) = 0, \qquad f'(0) = g'(0) = 1, \qquad f''(0) = g''(0) = 0$$
$$\lim_{\eta\to\infty} f' = \lim_{\eta\to\infty} g' = 0 \qquad \lim_{\eta\to\infty} f'' = \lim_{\eta\to\infty} g'' = 0 \quad (15)$$

On comparison of the transformed equations (12) to (14) with the corresponding boundary conditions (15), Raptis and Perdikis omitted equation (14) in their analysis and also this omission is evident in the work of Takhar et al. [3]. Actually equation (14) is obtained by equating the co-efficient terms of x^3 on both sides that arise after substituting in equation (9). Thus inclusion of equation (14) significantly changes the outcome of the attempt to find similarity solutions for viscous and visco-elastic boundary layer flow as the results are obtained from three differential equations rather than two ODEs describing the solutions to find f (η) and g (η). Further it is noticed that the complete solution to the set of equations (12) to (14) cannot be found due to the inability to meet the impermeability boundary conditions. Equation (14) is highly non-linear ordinary differential equation and has a general solution as g (η) =α $e^{\beta\eta}$ where α and β are constants. On comparing with the relevant boundary conditions given in equation (15) shows that it is not possible to find non-zero values of α and β such that all four of the required boundary conditions for g (η) are satisfied. It is also noted that the solutions to equation (14) was non-unique or a closed form solution for g (η) was not possible. Therefore assuming that $g^{11}(0)$ is finite, it is clear from equation (14) which is evaluated at η = 0 as $g(0)\,g''(0) - g'^2(0) = 0$ that a solution for g (η) that satisfies the required surface conditions given in equation (15) cannot be found.

Relationship with the Viscous Impermeable Stretching Sheet Problem

A note on similarity solutions for viscous flow due to a impermeable and non-linearly(quadratic) stretching sheet has been analyzed by Neil [2] and he made the comment on the study of Raptis and Perdikis [1] that at no stage they might have included equation (6) of Neil [2] and Raptis and Perdikis [1]. Besides analysis was made on similarity solutions for viscous flow due to a permeable and non-linearly (quadratic) stretching sheet by the authors cited in Kumaran [14] they used the stream function of the form given by equation (11) but with the choice of $g(\eta)=f^1(\eta)$. In the present analysis the stream function given by equation (11) is used and consequently with which similarity solutions can be successfully found and with this choice of $g(\eta)=f^{1}(\eta)$ equation (13) turns out to be redundant where as the solution of equation (12) is of the form $f(\eta) = \frac{1}{\alpha}\left[1 - e^{-\alpha\eta}\right]$ which also satisfies equation (14) with the value of $\alpha = \sqrt{K^1} \quad where \quad K^1 = (M_n + K_2)$

Exact solution to equations (12), (13) and (14) for $M_n = K_2 = 0$

As per the above discussion, the studies Raptis [1] and Takhar et al. [3] do not constitute the complete mathematical description of the solution for the impermeable and quadratically stretching sheet. Moreover as a test case they examined the suitability of numerical methods to solve coupled ODEs provided that accurate solutions are available in comparative studies. Hence closed form exact analytical solutions are highly desirable to this end where as Raptis et al. [1] and Takhar et al. [3] presented only numerical results of limited usefulness. Neil [2] presented and made a note on the work of Raptis and Perdikis [1] presented desirable analytical results for viscous fluid.

For the case $M_n = K_2 = 0$ an exact solution to equations (12), (13) and (14) in closed form can be obtained by initially that the exact solution to equation (12) is $f(\eta) = 1 - e^{-\eta}$.

Additionally Neil [2] noted that their equation of momentum which is third order homogeneous and linear in g (η) for viscous fluid albeit with variable co-efficients. On examining fourth order momentum equation (13) for visco-elastic fluid for $K_2 = M_n = 0$ has the exact analytical solution of Crane [4] i.e. $f(\eta) = 1 - e^{-\eta}$.

Further on inspection it can be seen that $g(\eta)=e^{-\eta}$ is the complementary function of equation (13) and the general solution for $g(\eta)$ can be obtained by using standard methods such as **reduction of order**.

After the lengthy procedure with $M_n = K_2 = 0$ we found an exact solution in the closed form that satisfies the momentum equations (12) and (13) with the respective boundary conditions (15) are given by

$$f(\eta)=1-e^{-H\eta} \tag{16}$$

and $$g(\eta)=2-e^{-H\eta}+4H\eta e^{-H\eta}-e^{-2H\eta}/7 \tag{17}$$

Results and Discussion

Fig. 1 & 2 shows that $f(\eta)$ decreases with increasing values of visco-elastic parameter k_1 (fig. 1) and decreases with increasing values of permeability parameter k_2 for fixed value of magnetic parameter at any given point above the sheet. It may also be seen from the figures 1 & 2 that the transverse velocity $f(\eta)$ decreases with increase in k_2 due to the inhibiting influence of the permeability.

Fig. 3 & 4 show the effect of visco-elastic parameter k_1 (fig. 3) and magnetic parameter M_n (fig. 4) on the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of longitudinal velocity distribution along the boundary layer. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of increase in the strength of magnetic field is observed. The effect of magnetic field is to provide rigidity to the electrically conducting fluid.

From fig. 5 & 6 which are drawn for transverse velocity $g(\eta)$ vs .η for different values of k_2 and M_n =0.5 (fig. 5) and M_n =0.75 (fig. 6) and for fixed value of k_1 respectively. It is evident from the figs. 5 & 6 that k_2 and M_n are the increasing functions of velocity profiles and M_n gives the steeper gradients in the transverse velocity profile. The effect of magnetic field is to provide rigidity to the electrically conducting fluid.

From fig. 7 & 8 which are drawn for longitudinal velocity g^1 vs η for different values of k_2 and for Mn = 0.25 and 0.5 respectively. It is evident from the figure 7 & 8 that k_2 is the decreasing function of longitudinal velocity and k_2 gives the steeper gradient in the velocity profiles. The effect of porous medium is to provide rigidity to the magneto hydrodynamic visco-elastic fluid flow.

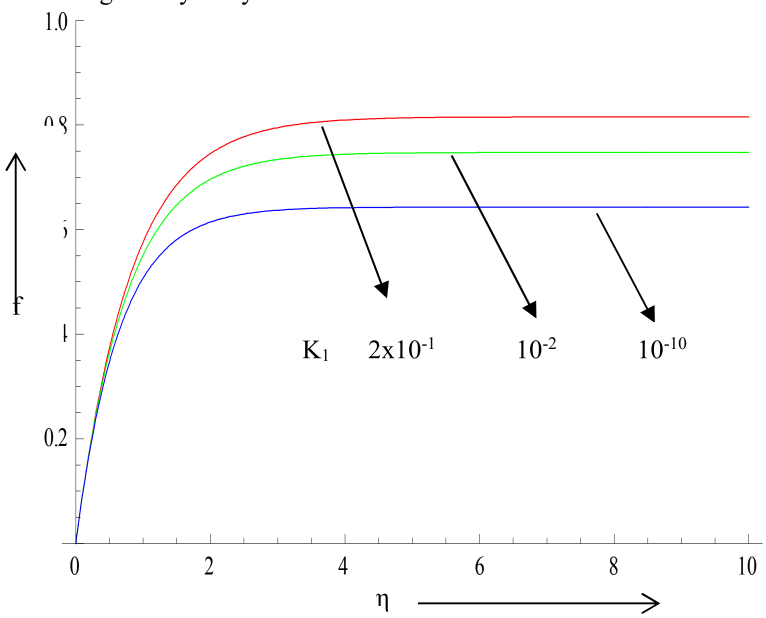

Fig. 1: Plot of Velocity profile for various values of H , k_2 =2 and M_n =0.25

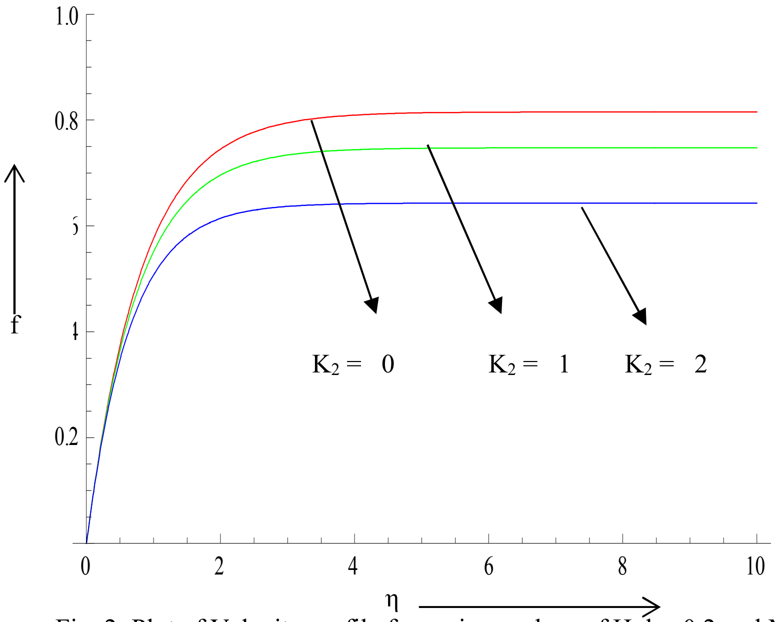

Fig. 2: Plot of Velocity profile for various values of H, k_1=0.2 and M_n =0.25

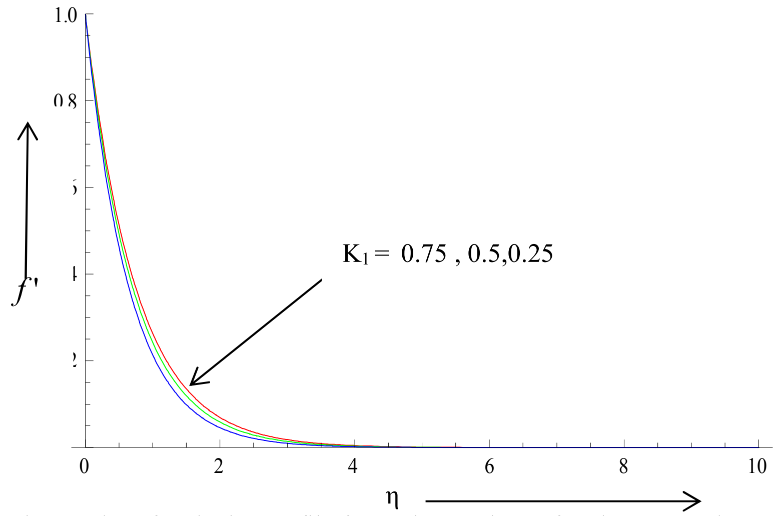

Fig. 3: Plot of Velocity profile for various values of H, k_2 =0.2 and M_n =0.5

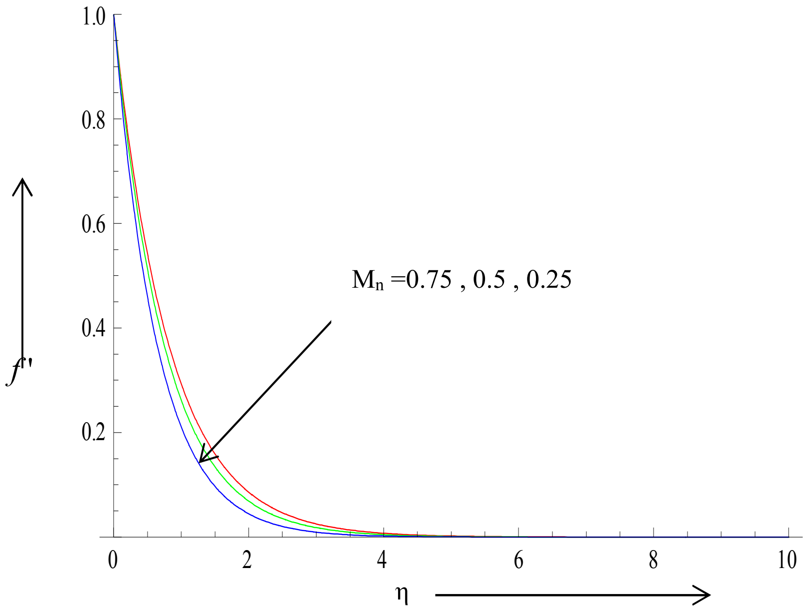

Fig. 4: Plot of Velocity profile for various values of H, k_1=1 and k_2 =0.2

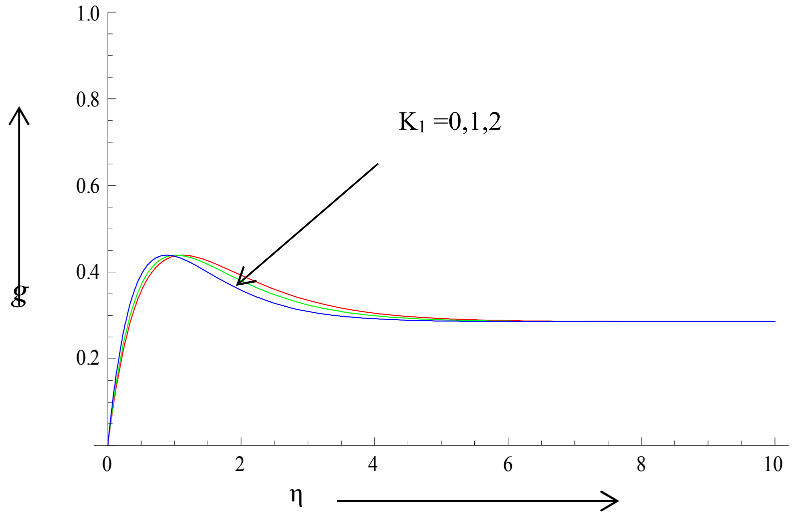

Fig. 5: Plot of Velocity profile for various values of H, k_2 =0.2 and M_n =0.75

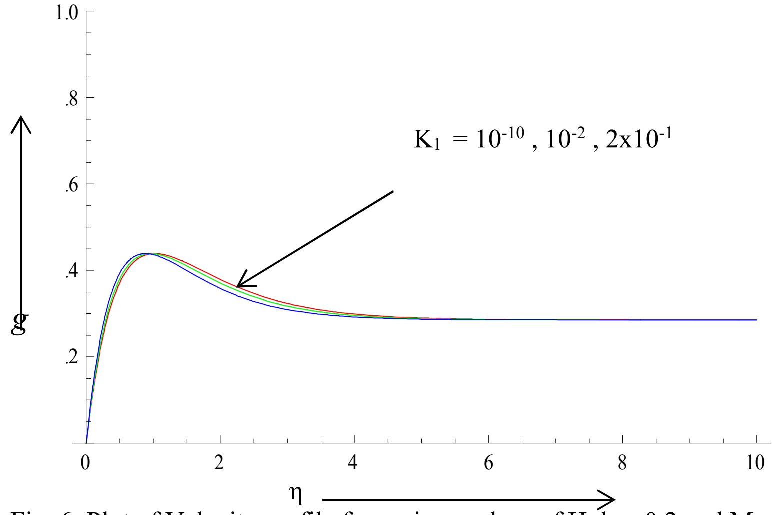

Fig. 6: Plot of Velocity profile for various values of H, k_2 =0.2 and M_n =0.5

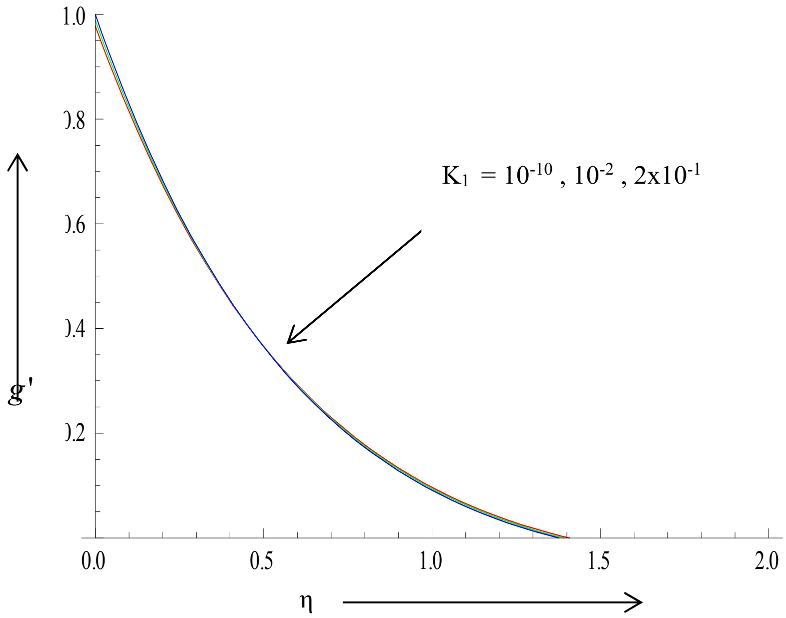

Fig. 7: Plot of Velocity profile for various values of H, k_2 =0.2 and M_n =0.25

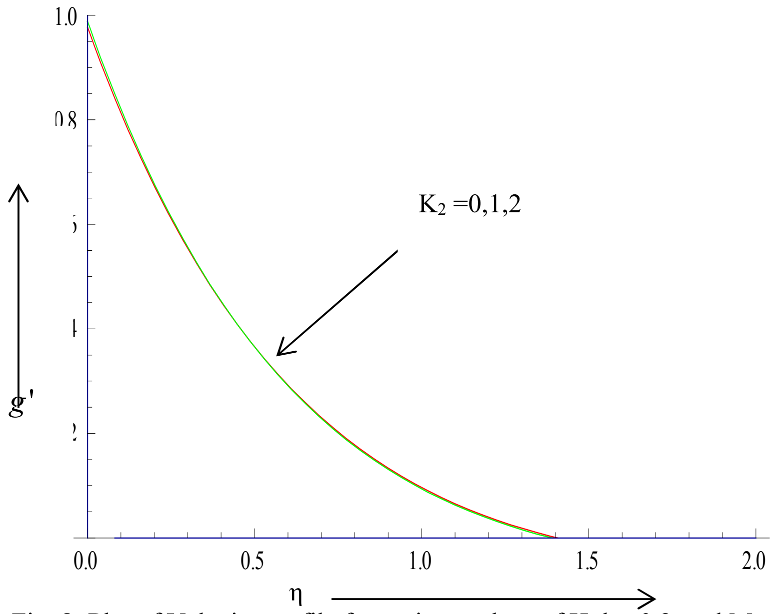

Fig. 8: Plot of Velocity profile for various values of H, k_1=0.2 and M_n =0.5

Conclusions

The investigative study of Raptis et al. [1] concerning similarity solutions for viscous flow over an impermeable non-linearly quadratic stretching sheet has been extended to visco-elastic boundary layer flow over a permeable stretching (non-linear and quadratic) sheet. It is found from the study of Neil and Kelson [2] that the theoretical development has omitted in the study of Raptis and Perdikis [1]. It is shown that using the choice of stream function of Neil and Kelson [2] similarity solutions cannot be found that are able to satisfy the required boundary conditions at the surface of the impermeable stretching sheet. The validity and theoretical findings of present study for $M_n = K_2 = 0$ for visco-elastic flow are well in agreement with the findings of Neil and Kelson [2] for viscous flow for $M_n = 0$.

References

1. C. Raptis and Perdikis, Viscous flow over a non-linearly stretching sheet in the presence of chemical reaction and magnetic field.

Int. J. Non-linear Mech. 41 (2006)527-529

2. A Neil and Kelson, Note on similarity solutions for viscous flow over a impermeable and non-linearly (quadratic) sheet.
Int. J. Non-linear Mech. 2011.04.025
3. H.S. Takhar, A.A. Raptis, C.P. Perdikis, MHD asymmetric flow past a semi infinite moving plate. *Acta Mech.* 65(1987) 287-290
4. L. Crane, Flow past a stretching plate.
Zangew. Math Phys. 21 (1970) 645-647
5. J.H. Rao, D.R Jeng,, and K.J. De Witt, Momentum and heat transfer in a power-law fluid with arbitrary injection/suction at a moving wall.
Int. J. Heat Mass Transfer, 42(1999) 2837-2847.
6. K.V. Prasad, M.S. Abel and S.K. Khan, Momentum and heat transfer in visco-elastic fluid flow in porous medium over non isothermal stretching sheet.
Int. J. of Num. Method for Heat and Fluid Flow 10, 8 786-801, 2000.
7. T.G. Hopwel l,, Momentum and heat transfer on a continuous moving surface in power law fluid.
Int. J. Heat Mass Transfer, 1997, 40(9): 1853-1861
8. A. Acrivos, M.J. Shah, and E.E .Petersen, Momentum and heat transfer in laminar boundary-layer flows of non-newtonian fluids past external surfaces.
A.I.Ch.E. Journal, 1960, 6:.312-317.
9. T.G. Howell, D.R. Jeng, and K.J. De Witt, Momentum and heat transfer on a continuous moving surface in power law fluid.
Int. J. Heat Mass Transfer, 40(1997): 1853-1861.
10. H. Markovitz and B.D. Coleman, Advances in Applied Mechanics, 8,
Academic Press, 196
11. R.M. Sonth, S.K. Khan, M.S. Abel and K.V. Prasad, Heat and mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation.
Heat and Mass Transfer., 38,213-220,2002
12. P.S. Gupta and A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing. *Canad. J. of Chem. Engg.,* 55, 744-746, 1977
13. T.C. Chiam Magneto hydrodynamic heat transfer over a non isothermal stretching sheet.
Acta Mechanica, 122, 169-179,1997.
14. V. Kumaran and G. Ramanaiah, A note on the flow over a stretching sheet.
Acta Mech., 116,229-233, 1996
15. K.A. Yih, Note: Blowing suction/ effect on non-darcy forced convection flow about a flat plate with variable wall temperature in porous media.
Acta Mechanica, 131 255-265,1998.
16. K .Vajravelu and J. Nayfeh, Convective heat transfer at a stretching sheet.
Acta Mech., 96, 47-54, 1993
17. K .Vajravelu, Convection heat transfer at a stretching sheet with suction or blowing.
J. of Math. Anal. Appl., 188,1002-1011, 1994.
18. C.D. Surmadevi, H.S. Takhar, U.N. Das, R.K. Deka, and A. Sarmah, 2004. Effect of variable viscosity on boundary layer flow along a continuously moving plate with variable surface temperature.
Heat and Mass Transfer, 40, 421-424
19. M. Amkadni, A. Azzaouzia and Z. Hammouch, on the exact solution of laminar MHD flow over a stretching flat plate.
Communications in Nonlinear and Numerical Simulation, 13, 359-364, 2008.
20. R. Cortell, 2005. Flow and heat transfer of a fluid through a porous medium over stretching surface with internal heat generation/absorption and suction/blowing.
Fluid Dynamics Research, 37,231-245.
21. W.C. Troy, E.A Overman, H.G.B. Ermentrout and J.P. Keener, Uniqueness of flow of a second order fluid past a stretching sheet.
Quart. Appl. Math., 44,753-755, 1987.
22. Idress, M.K., and Abel, M.S., 1996. Viscoelastic flow past a stretching sheet in porous media and heat transfer with internal heat source.
Ind. J. Theo. Phys., 44,233-244
23. A. Naseem and Khan, N, 2000. Boundary layer flow past a stretching plate with suction and heat transfer with variable conductivity.
Int. J. Eng. And Material Science, 7, 51-53.
24. P.D. McCormack, and L.J. Crane, 1973.

Physical Fluid Dynamics, Academic Press, New York

25. B.D. Coleman, W. Noll, An approximation theorem for functional with applications in continuous mechanics.
Arch. Ration. Mech. Anal. 6 (1960) 355-370
26. J.E. Dun, R.I. Fosdick, Thermodynamics stability and boundedness of fluids of complexity 2 and fluids of second grade.
Arch. Ration. Mech. Anal. 56 (1974) 191-252
27. R.I. Fosdick, K.R. Rajagopal, Anomalous features in the model of second order fluids.
Arch. Ration. Mech. Anal. 70 (1979) 145-152