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Fluid Mechanics Analysis of a Voice Coil Needle Free Jet Injector

Erastus O. Ogunti, Olaoluwa R Popoola [2*]

[1,2] Department of Electrical/Electronic Engineering, Federal University of Technology Akure.

PMB 704, Akure, Ondo state Nigeria

* E-mail of corresponding author: orpopoola@futa.edu.ng

Abstract

This paper considers the analytic modelling of the voice coil actuated needle free jet injection. It first investigates the currently existing model for the injection and making some alterations based on the system behavior and basic definitions in physics. These alterations yielded a new set of equations which were solved using the Runge-Kutta-Felberg formular. Second order, forth order and fifth order curve fitting of the numerical model were performed in order to develop a polynomial that gives the direct non-differential pressure-time relationship for the needle free jet injection (developed, to the best knowledge of the authors, for the first time). This fifth order polynomial presented the least error, measured by the least of the norm of residuals. The model for the voice coil activator was developed by writing force balance equations based on Newton's second law of motion. The transfer function of the motor was then used to investigate the behavior of the voice coil actuator. Finally, the two systems (voice coil motor and injection) were linked up by setting up an analogous electromechanical system to see the behavior of the voice coil motor in the injection. Where the mechanical parts represent the injection pressure profile.

Keywords: Needle Free Jet Injection, Voice coil motor

1. Introduction

Current designs for the jet injection system are based on a totally new actuator capable of delivering a well-controlled jet stream at extremely high speeds and also with a very high precision. To achieve this, the new actuator used is the linear voice coil actuator. It application to the Needle Free Jet Injection System (NFJIS) has been investigated since 2006 (Dawn et al, 2006). Due to the complexity involved in the design, work is still going on, in order to make the device applicable in safe drug delivery for humans. (Bhagyashri et-al, Sept 2013) In order to design a well optimized adaptable controller for the NFJIS, a detailed understanding of the plant, which in this case, is the needle free injection is necessary. Studies of the jet injection model were carried out and the model developed earlier by Baker and Sander (Bakers, 1999) was modified based on first principle physics definitions. The behavior of the model with respect to the voice coil was observed and it showed compensators will be required for the design of voice coil actuated jet injection system. The design, adaptation and optimization of the injection controller will be shown in future papers.

2. Needle Free Jet Injection

For centuries man has battled with various forms of sicknesses. The use of drugs has almost been always imperative whenever there is one sickness or the other. Health practitioners have hence been faced with the task of getting the pharmaceuticals into circulation in the body. Methods of getting drugs into the body circulatory system have been looked into through the years amongst these methods; two that have chiefly stood out are ingestion by oral route and transdermal delivery.

As a substitute for the ingestion by oral route, Transdermal drug delivery enables the avoidance of gastrointestinal absorption, with its associated pit falls of enzymatic and pH associated deactivation. Transdermal delivery has many advantages over conventional modes of drug administrations, it thus avoids hepatic first pass metabolism and improves patient compliance. This approach of drug delivery is more permanent in case chronic disorders like hypertension or diabetes which requires long term dosing to maintain therapeutic drug concentration. These systems are easy to apply when desired (Sampath, 2010). Transdermal drug delivery systems (TDDS) are defined as self-contained, discrete dosage forms which, when applied to intact skin, deliver the drug(s), through the skin, at a controlled rate to systemic circulation

Development of new therapeutic drugs offers promise for the treatment of many diseases, but effective and efficient delivery of drugs remains a significant challenge. Various methods of drug delivery have been tried over the years. Large molecule and protein-based drugs and vaccinations typically are administered via hypodermic needle since they cannot be absorbed through the skin or taken orally (Mitragotri, 2005).

Due to the increased benefit of Transdermal Drug Delivery (TDD) systems, various methods have been implemented. Methods of transdermal drug delivery include: TDD by Micro needles, TDD by Iontophoresis, TDD by Sonophoresis, and TDD by Needle/Jet Injection. This work focuses on improvement of Transdermal Drug Delivery by Jet Injection.

Figure 1: Jet Injection with removable and variable drug chamber

Jet injection was first described by Hingson and Hughes as a drug delivery method that would make traditional needle injections obsolete. It is a method of drug delivery without needles that release a jet of high speed and pressure fluid into the body. They note "the fact that extremely fine high pressure jets are capable of piercing the human skin," and provide patient testing that showed jet injections were capable of delivering a skin analgesic and controlling diabetes in a patient. (Shergold and Fleck, 2006). But through the years its development has been dawdling due to various factors until the last decade when scientists renewed their interest in the work. Various methods have been used to implement drug delivery by the jet injection but the most of these methods are non-repeatable and uncontrollable. (Arora, 2007, Batisse et al, 2000, Chandan, 2011)

3. Methodology and Mathematical Analysis

This paper will develop a model for the needle free injection by first considering some basic laws in mechanics. We first start with law of mass conservation then the law of conservation of energy and finally, we write a force balance equation in order to develop a mathematical equation which will represent the needle-free system. Although prior work has been done in this regard but this work considered the model in two scenarios. First, when the injection is not delivering drugs to the skin, model will be developed and analyzed, then the model will also be investigated when the injection is delivering drugs to the skin. The second part will be reported in future work

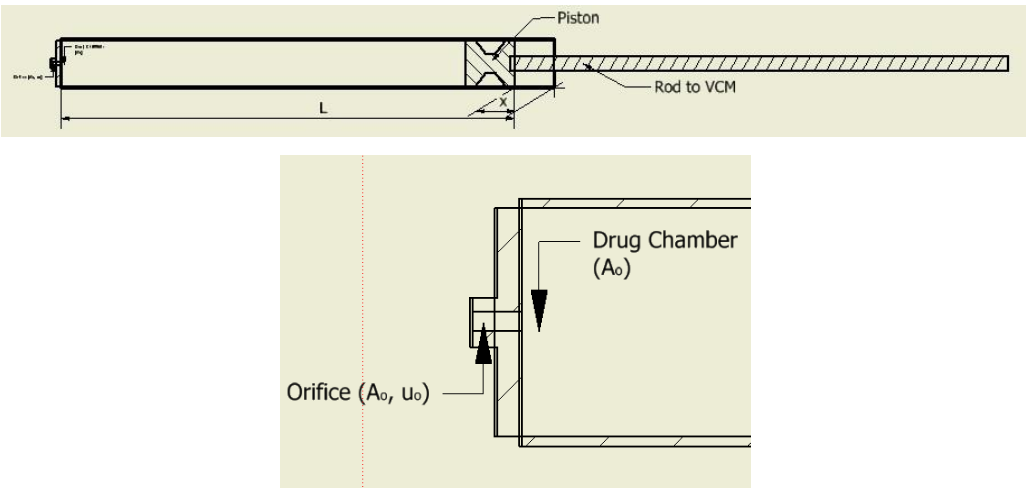


Figure 1: Injection system geometry

For the source of actuation in developing a needle free jet injection, only spring actuators was considered in the previous model (Baker and Sanders, 1999). But in this paper we will consider other common sources of actuation too. These sources includes the compressed gas actuation and the controllable linear voice coil actuation.

3.1 Modified Injection Model

By simplifying the flow of the fluid into a quasi-static process, Bakers and Sanders (1999) established mass conservation equations for the fluid and a force balance model for the driving piston. This relation was based on the compressibility of the fluid as expressed by the bulk modulus of the fluid. Although, the expression used is given as:

$$\frac{\Delta P}{B} = \frac{P}{B} = \frac{\Delta\rho}{\rho_0} \tag{1}$$

Where P represents pressure, B is the bulk modulus and ρ is the fluid density. This relation hardly stands to be true because bulk modulus is a measure of a fluids ability to resist compressibility. From the Dictionary of Physics and Electronics (p114), the bulk modulus of elasticity is defined as

$$B = \frac{\Delta P}{\frac{\Delta V}{V_0}} \tag{2}$$

Where:

ΔP is the increase in pressure

ΔV is the decrease in volume and

V_o is the original volume

From (2), we can write

$$\frac{\Delta P}{B} = \frac{\Delta V}{V_0} \tag{3}$$

From definition, the density of a fluid is given as the ratio of mass to volume of the fluid. Hence the volume varies in inverse proportion to the density of the fluid when mass is kept constant. Such that

$$\rho = \frac{m}{V} \text{ and } V \propto \frac{1}{\rho} \tag{4}$$

Hence we can conclusively write

$$\frac{\Delta V}{V_0} = \frac{\rho_0}{\Delta\rho} \tag{5}$$

Which implies equation (1) in a more appropriate form will be written as

$$\frac{\Delta P}{B} = \frac{P}{B} = \frac{\rho_0}{\Delta\rho} \tag{6}$$

Applying the continuum analysis assuming static incompressibility, a new needle injection mathematical model will be developed using the appropriate relation for the bulk modulus and density variations.

By the mass conservation assuming the needle-free injection only consists of the piston case diameter and the orifice diameter as shown in figure 1 above

$$m_t = m_c + m_o \tag{7}$$

$$\rho_o A_1 L = \rho A_1 (L - x) + \rho_o A_o \int u_o \, dt \tag{8}$$

But change in a quantity can be write as the difference between the final and initial value of the quantity. Hence we can write the chamber density value as

$$\rho = \rho_o + \Delta\rho \tag{9}$$

Dividing equation (8) by density at orifice and Substituting equation 9 into the resulting equation, we can write

$$A_1\left(1 + \frac{\Delta\rho}{\rho_o}\right)(L - x) - A_1 L + A_o \int u_o \, dt = 0 \tag{10}$$

Or

$$A_1\left(\frac{\Delta\rho}{\rho_o}\right)L - xA_1\left(1 + \frac{\Delta\rho}{\rho_o}\right) + A_o \int u_o \, dt = 0 \tag{11}$$

By the relation for the bulk modulus in equation (6),

$$\frac{\Delta\rho}{\rho_o} = \frac{B}{P} \tag{12}$$

Substituting (12) into (11)

$$\frac{A_1 BL}{P} - A_1 x - \frac{A_1 xB}{P} + A_o \int u_o \, dt = 0 \tag{13}$$

Differentiating (13) we have:

$$A_1 BL \frac{d}{dt}\left(\frac{1}{p}\right) - A_1 \frac{d}{dt}(x) - A_1 B \frac{d}{dt}\left(\frac{x}{p}\right) + A_o \frac{d}{dt}\left(\int u_o \, dt\right) = 0 \tag{14}$$

$$-\frac{A_1 BL}{p^2}\frac{dp}{dt} - A_1 \frac{dx}{dt} - \frac{A_1 B}{P}\frac{dx}{dt} + \frac{A_1 Bx}{P^2}\frac{dp}{dt} + A_o u_o = 0 \tag{15}$$

$$\frac{dp}{dt} = \frac{p^2 A_1 (1 + B/P)\frac{dx}{dt} - A_0 u_0 p^2}{A_1 B(x - L)} \tag{16}$$

$$\frac{dp}{dt} = \frac{(p + B)\frac{dx}{dt} - \frac{A_0}{A_p} u_0 p^2}{\frac{B}{p}(x - L)} \tag{17}$$

Bernoulli's principle applies in nozzles where flow accelerates and pressure drops as the channel diameter reduces. Hence we can write out equations of flow based on the law of conservation of energy which requires that the sum of kinetic energy and potential energy remains constant which implies that across a streamline,

$$\frac{\rho u_1{}^2}{2} + P_1 = \frac{\rho u_0{}^2}{2} + P_0 \tag{19}$$

$$\text{If, } P_1 - P_0 = P, \text{ and } u_0{}^2 \gg u_1{}^2, u_0 = \sqrt{\left(\frac{2P}{\rho}\right)} \tag{20}$$

Hence equation 16 can be re-written as

$$\frac{dp}{dt} = \frac{(p + B)\frac{dx}{dt} - \frac{A_0}{A_p}\left(\frac{2P^3}{\rho}\right)^{\frac{1}{2}}}{\frac{B}{p}(x - L)} \tag{21a}$$

This equation does not factor in the viscous loss which modifies the velocity at the orifice from $\sqrt{\frac{2p(t)}{\rho}}$ to $\sqrt{\frac{2p(t)}{\left(1 + \frac{YL}{D}\right)\rho}}$. (Chen and Zhou, 2011) Hence the new model of the injection will be

$$\frac{dp}{dt} = \frac{(p + B)\frac{dx}{dt} - \frac{A_0}{A_p}\sqrt{\frac{2p(t)^3}{\left(1 + \frac{YL}{D}\right)\rho}}}{\frac{B}{p}(x - L)} \tag{21b}$$

where γ is the friction factor, and D is the internal diameter of the barrel.

3.2 *Voice coil actuated Force balance equations:*

For the force balance equations, we will consider the forces that act on the piston when in motion. Forces acting on the piston essentially includes: force from the actuator (which in this case will be force from the voice coil actuator), force due to compression of fluid, and friction force due to resistance generated by the movement of the o-ring seal against the walls of the drug cylindrical chamber. The resultant of this three forces is the force that produces motion based on Newton's third law of motion. For a spring actuator by Hooke's Law we have

$$m\frac{d^2x}{dt^2} = kx - A.p - Fr \tag{22}$$

Where m is mass of piston, x is compressional spring displacement, and Fr is the force due to friction. The resultant force will be analyzed in detail in the next section. For the voice coil powered injector, assuming no impact gap the driving force is Lorentz-force that is governed by:

$$F = \int i(dl \; x \; B) = Bl.\,i(t) = Bl.\frac{V(t)}{R} = Bl.\frac{V_r \exp\left(-\frac{t}{RC}\right)}{R} \tag{23}$$

Where F is force vector

i is electric current
dl is a differential element of length(l) of the conductor
B = magnetic flux density vector (magnetic induction)

The R.H.S. of Equation (23) assumes the current passing through the coil is supplied by an energy-storage capacitor as described in Taberner (2006). V is the voltage of the coil, R is the resistance of the coil, Vr is discharging voltage at the beginning and C is the capacity of the capacitor.

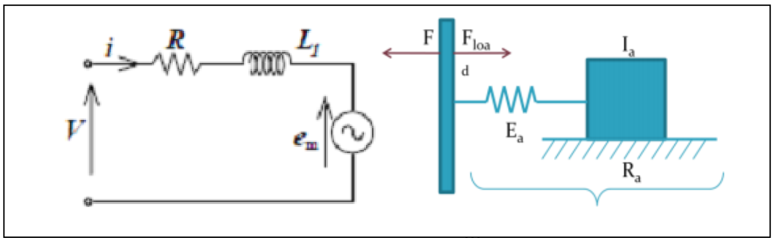
3.3 *Mathematics of Voice Coil Actuator Injection*

The above figures show the electrical model and the mechanical model of the voice coil actuators. The model gives an idea of what happens to the actuator during operation. By Lorentz force law, the force that acts on a charged particle through a magnetic field is given by:

$F = qv \times B$, Where F is the Lorentz force, q is the amount of charge, B is the magnetic flux density. Since the wire is in a straight line and at right angles to the magnetic field, this transforms to the well-known formular for the force on a wire

$$F = iL \times B = iLB\sin\theta = iLB = k_f\, i(t) \tag{24}$$

Equation 24 above shows the linear relationship between the force and current by Lorentz theorem which will be used in the design of the mathematical model. From the electrical circuit diagram, we can write the electric circuit model as:



$$V(t) = U(t) = Ri(t) + L\frac{di}{dt} + e_m(t) \text{ but } e_m(t) = k_b v(t) \tag{25}$$

Hence, $U(t) = Ri(t) + L\frac{di}{dt} + k_b v(t)$ (26)

From the mechanical diagram, we can write the mechanical model as:

$$m\frac{dv}{dt} = F - F_{load} - frictional\ damping = F - F_{load} - bv(t) \tag{27}$$

$$m\frac{dv}{dt} = k_f i(t) - PA - bv(t) \tag{28}$$

Hence we can write in s-domain as:

$$si(s) = \frac{1}{L}U(s) - \frac{R}{L}i(s) - \frac{k_b}{L}v(s) \tag{29}$$

$$sv(s) = \frac{k_f}{m}i(s) - \frac{A}{m}p(s) - \frac{b}{m}v(s) \tag{30}$$

Solving the above equations by superposition, we first set p(s) to 0 then later set u(s) to 0 in order to derive the transfer function of the injection model. It should be noted that the model of the voice coil actuator is the transfer function for p(s) =0.

When p(s) = 0, we have:

$$(Ls + R)i(s) = U(s) - k_b v(s) \tag{31}$$

$$i(s) = \left(\left(\frac{m}{k_f}s^2 - \frac{b}{k_f}s\right)x(s)\right) \tag{32}$$

$$\frac{x(s)}{U(s)} = \frac{k_f}{mLs^3 + (mR + Lb)s^2 + (bR + k_f k_b)s} \tag{33}$$

This is the model of the linear voice coil actuator motor we shall be working with.

In the same way, when U(s) =0, we have:

$$(sL + R)i(s) = -k_b v(s) \tag{34}$$

$$i(s) = \frac{1}{k_f}((ms^2 + bs)x(s) + p(s)A) \tag{35}$$

$$\frac{x(s)}{p(s)} = -A\frac{Ls + R}{mLs^3 + (mR + bL)s^2 + (bR + k_g k_b)s} \tag{36}$$

Finally by superposition, we have

$$x(s) = \frac{k_f}{mLs^3 + (mR + Lb)s^2 + (bR + k_f k_b)s}U(s) - A\frac{Ls + R}{mLs^3 + (mR + bL)s^2 + (bR + k_g k_b)s}P(s) \tag{37}$$

$$x(s) = \frac{k_f U(s) - A(Ls + R)p(s)}{mLs^3 + (mR + Lb)s^2 + (bR + k_f k_b)s} \tag{38}$$

Combining equations 38 and 21b we have a voice coil voltage vs displacement governing equation as:

$$x^2\emptyset(.) - x\emptyset(:) = \emptyset.U + k \tag{39}$$

Where x is the displacement, U is voice coil regulation voltage, $\emptyset, \emptyset(.), \emptyset(:)$ are the dynamic equations of the system as given in the following equations below.

$$\emptyset(.) = (mls^3 + (mR + lb)s^2 + (bR + k_f k_b)s)(A_1 s + A_1 Bs) \tag{40}$$

$$\emptyset(:) = (mls^3 + (mR + lb)s^2 + (bR + k_f k_b)s)A_0 u_0 \tag{41}$$

$$\emptyset = -k_f A_0 u_0 \tag{42}$$

$$k = A(ls + R)(A_1 BLs) \tag{43}$$

These equations use previously defined parameters with l representing inductance and L displacement length, B representing Bulk modulus.

This is the relationship between, input voltage to the voice coil U(s), displacement x(s) and stagnation pressure of the needleless injection p(s)

4. Results and Discussion

The mechanical system of the injection was modelled by solving equations 21 and 22 and the result shown in figure 3 below. Initial conditions were set to 0 meter for displacement and initial speed but initial pressure was set as Fs/A which is the initial pressure once the spring plunger mechanism is set free. Fs is the due to energy stored in the spring and A is the area of the piston. The plot of the variation of pressure with time and although the results does not agree with the results of continuum analysis (Baker and Sandars,1999), it agrees with his shock wave analysis and also with results of other researchers (Chen, 2010, Kai and Hua, 2011, Dawn et al, 2006). It shows there will be a plunging force on the injection which creates an increasing pressure over the course of injection. This result also shows the initial pressure for the system is not zero. The justification of this is that at time (t<0), the compressed spring has internal energy which will be transferred to the injection once released hence at time (t=0), the initial pressure cannot be zero. The pressure rises due to a corresponding increase in force from spring since $F = kx$ will increase, at constant k, with an increase in x as the displacement of the plunger moves from $x = 0$ to $x = L$. By performing the basic curve fitting of the graph above with second forth and fifth order polynomials for increased accuracy, we develop the relational equation between pressure p and time t given by equations shown in the figures 4 and 5 below:

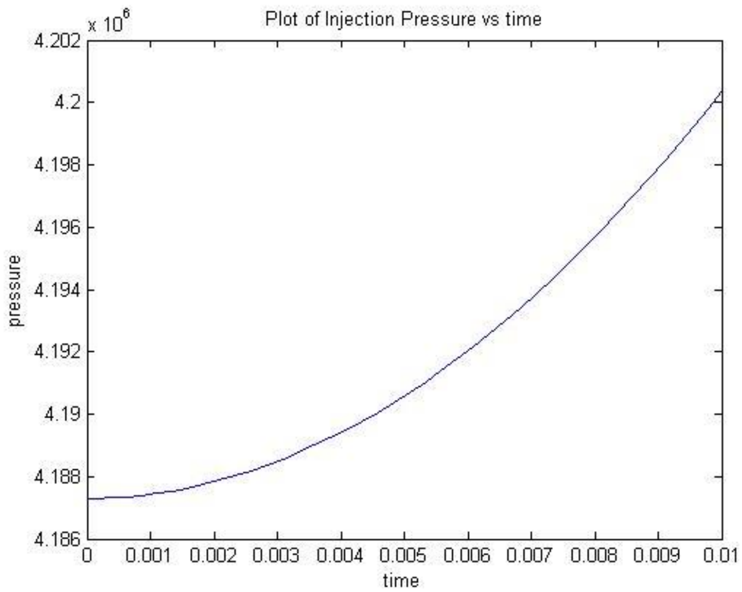


Figure 3: Pressure time variation of the new model

By performing the basic curve fitting of the graph above with second forth and fifth order polynomials for increased accuracy, we develop the relational equation between pressure p and time t given by equations shown in the figures 4 and 5 below where y represents the injection stagnation pressure p(t) and x represents time t.

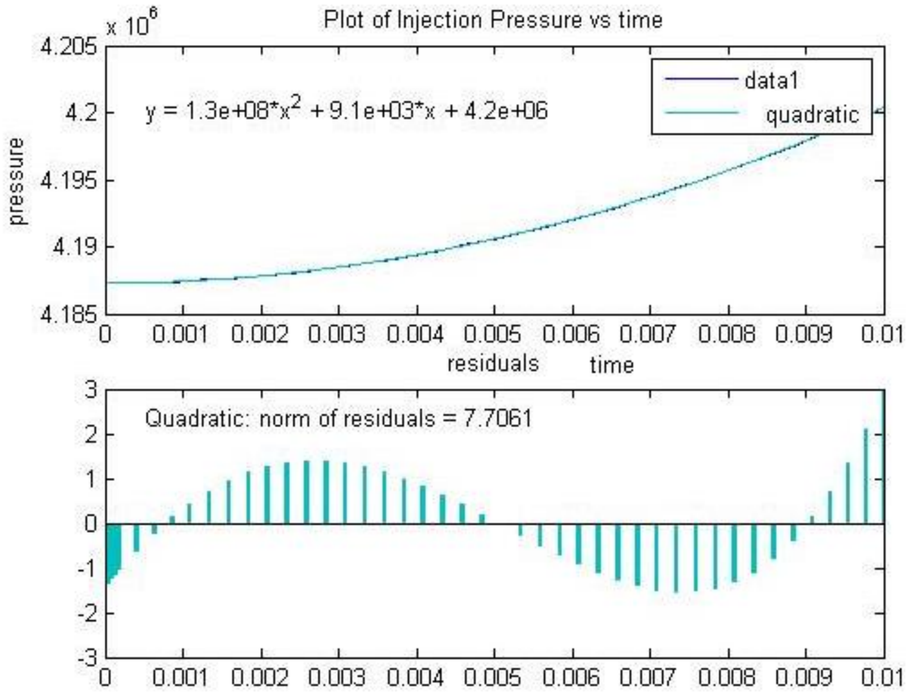


Figure 4: Second order curve fitting for model

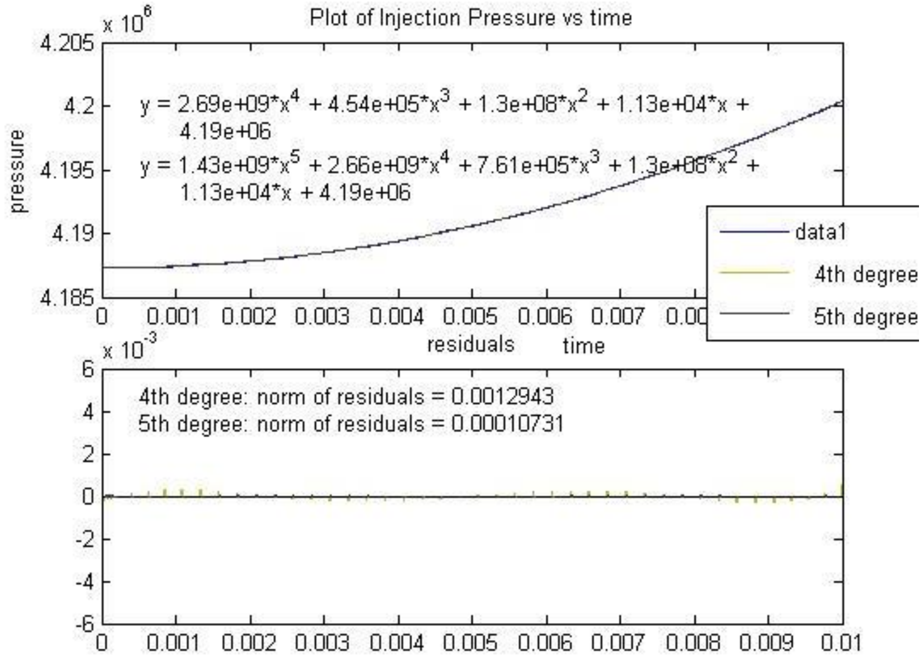


Figure 5: Forth and Fifth order curve fitting for model

It can be observed that using the fourth or fifth degree polynomials for pressure versus time relationship, error in the fitted curve was reduced to approximately 1×10^{-3}. This fitted model was then used to observe the behavior of voice coil on the injection.

The behavior of the voice coil actuator with respect to the injection system is shown in figure 6. To actualize the model data from a voice coil actuator NCC06-08-005-2X. Its specifications are available on the manufacturer's website. By using the data we arrived at the model for the voice coil motor. The response to a step response was not bound. This is not a desired response for the NFJIS. It is observed that this system is not stable (BIBO-stability) and is not controlled. Hence the next part of this work will be to develop a controller that will stabilize and control the linear motor. Also, based on report done by researchers, it has been discovered that for a very high current density, the operation of the voice coil actuator is non-linear and that the NFJIS operates at these high current density (Ian and Serge, 2011).

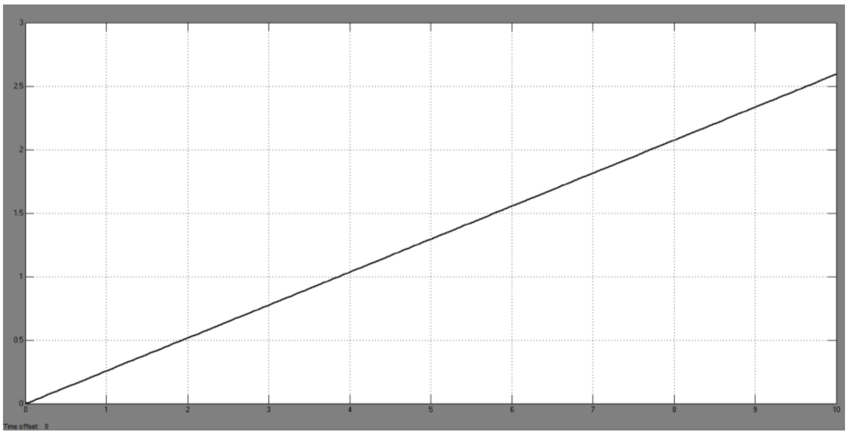


Figure 6: Step response of model voice coil actuator

Uncompensated feedback system was designed to test the response of the motor and the behavior was shown in figure 7, it reveals nonlinearities as observed by Ian and Serge, 2011, in the operations of the voice coil motor for the purposes of the needle free injection system. A non-linear controller will be developed in a further paper to reduce this effect and compensate the system for high accurate drug delivery.

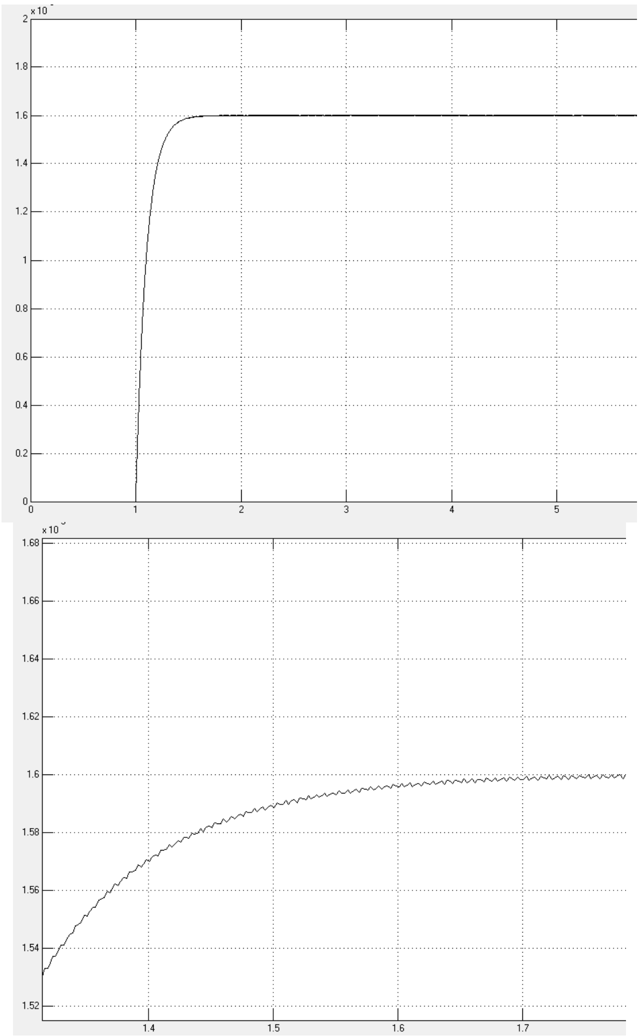


Figure 7: Step response of closed loop model actuator

5. Conclusion

This paper verified the mathematical model of the Needle Free Jet Injection with some alterations and then went further to observe the behavior of the system using a linear voice coil motor. The mathematical simulation of the motor was seen to have some effects of nonlinearities and a much slower response characteristics than that desired for rapid and highly repeatable drug delivery. The results of the work were compared with some experimental results and the model fits their results.

6. Further research

Most researchers for the needle free jet injection develop their investigations experimentally (Bhagyashri, 2013). But due to the high level of accuracy needed for drug deposition in the body via the skin, thorough mathematical analysis should be carried out before the development of the device by even modelling the skin and developing adaptable control system for therapeutic drug delivery and a wider range of drug administration.

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