Study Projectile Motion With Different Initial Conditions Using Digital Image

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Abstract:
The aim of this research is building algorithms to study projectile motion in tow dimension and tracking the object in sequence frames of digital image .
Computer program has written in visual basic language (version 6) depend on mathematical models to detect a motion of object in two–dimensions (2-D)with different initial conditions like initial velocity, the height of object from the earth and the angle of motion, to calculate important variables in motion such as distance, displacement, velocity, speed and the energy (kinetic and potential).
Color digital images of type (bmp) and (RGB) color model were used in the study for easy handling them, after determining the center of the image on the x-axis, and y-axis and tracking movement on the basis of the center, and the results were expected to conform to the movement of the body.
Key words: Projectile, Motion, Digital Images

Introduction:

The human Interested in nature since time immemorial, the movement of celestial bodies was a matter of curiosity and admiration for him and tried, still, uncover the secrets of nature, from the largest celestial bodies and the end of the smallest components of the atom and the nucleus. Human attention to these phenomena not only for curiosity and wonder, but also to take advantage of them and put them to his service in various ways.
The study of the movement of objects and move the spine in the body of physics because it describes how and why objects move and how you can take advantage of this movement[1].
We recognize that motion represents a continuous change in the position of an object. In physics we are concerned with three types of motion: translational, rotational, and vibrational. A car moving down a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion[2]. In this research, we are concerned only with translational motion. In our study of translational motion, we describe the moving object as a particle regardless of its size.
It is important to recognize that various changes can occur when a particle accelerates. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Finally, both the magnitude and the direction of the velocity vector may change simultaneously[3].
The motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration.

Motion in Two Dimensions

The real world is three-dimensional, so why do we bother with two-dimensional motion? First, two-dimensional motion is easier to describe, easier to deal with mathematically, and easier to sketch on a piece of flat paper. This makes two-dimensional motion a good place for introducing concepts that are peculiar to motion in more than one dimension. Second, many objects actually do exhibit motion in a plane, motion that needs only two dimensions for its complete description. Any motion under constant acceleration can always be described in terms of just two dimensions. Even if the acceleration is not constant, many objects still move in a plane[4].

The Equations of the Projectiles:

Anyone who has observed a baseball in motion (or, for that matter, any other object thrown into the air) has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze as: (1) the free-fall acceleration g is constant over the range of motion and is directed downward and (2) the effect of air resistance is negligible. Let us choose our reference frame such that the y direction is vertical and positive is upward. The projectile leaves the origin with speed Vo, and the vector Vo makes an angle (θ) with the horizontal[2,5].

From the definitions of the cosine and sine functions we have:

$V_{ox} = Vo * cos\theta \quad V_{oy} = Vo * sin\theta$ (1)

V_{ox} and V_{oy} are the horizontal and the vertical component of initial velocity respectively.

First, two elementary formulae are called upon relating to projectile motion:

$$x = V_{ox}\, t \Rightarrow t = \frac{x}{Vox} \quad (2)$$

$$y = V_{oy}\, t - \frac{1}{2} g t^2 \quad (3)$$

$$V_{fy} = V_{oy} - gt \quad \text{and} \quad V_{fy} = V_{ox} = constant \quad (4)$$

$$V_{tot} = \sqrt{{V_{ox}}^2 + {V_{fy}}^2} \quad (5)$$

Where x and y are the position of projectile in x and y coordinate at any time t.

V_{tot} is total velocity of projectile at any position.

g the gravitational acceleration(9.81 m/s^2) near the Earth's surface and be negative for falling object.

Second the time of flight *(t)* is the time it takes for the projectile to finish its trajectory[6].

$$d = \frac{V_{ox}}{g}\left(V_{oy} + \sqrt{{V_{oy}}^2 + 2gyo}\right) \quad (6)$$

$$t = d/V_{ox} = \left(V_{oy} + \sqrt{{V_{oy}}^2 + 2gyo}\right)\Big/ g \quad (7)$$

d the total horizontal distance traveled by the projectile. y_0 the initial height of the projectile.

Now we will allow (y_0) to be nonzero. Our equations of motion (3) are now[1]:

$$y = yo + V_{oy}\, t - \frac{1}{2} g t^2 \quad (8)$$

The Energy:

the kinetic energy (KE) of a particle of mass (m) moving with a velocity (V_{tot}) is defined as[7]:

$$KE = \frac{1}{2} m {V_{tot}}^2 \quad (9)$$

There is gravitational potential energy stored in the system. The product of the magnitude of the gravitational force (mg) acting on an object and the height (y) of the object. The symbol for gravitational potential energy is (PE), and so the defining equation is [7]:

$$PE = mgy \quad (10)$$

Then when the projectile has different velocities with time, that mean we can calculate the momentum (Mon) of it as[1]:

$$Mon = mV_{tot} \quad (11)$$

The Digital Images:

The human realizes what turned him from viewers from eye as receive images of optical power distributed in a particular order, and these images are called photographs of which is their information in the electrical signals transmitted to the brain, and when dealing with computer must be converted to digital form and this is the process of digitization in each of the coordinate space (x, y). The division of the image called image sampling while the digitization of wideband called quantization [8,9].

Finding the center of an object will help us to locate an object in the two-dimensional image plane. We can compute the center of an object by using the following equation[10,11]:

$$Cx = \frac{X_{\min} + X_{\max}}{2} \quad \text{and} \quad Cy = \frac{Y_{\min} + Y_{\max}}{2} \qquad (12)$$

Algorithm of the study:

Start algorithm

1. load the image and determine the center of it using equation(12).
2. Determine the initial conditions (Vo, h, m, θ).
3. Determine max number of motion steps and determine step length depending on d and t from equations 6 and 7.
4. Calculate x, y, V_{fy} , V_{tot} from equations 2, 3, 4, 5 respectively.
5. Determine 1st center of object point in image plane.
6. Remove all object points from the image plane.
7. Loop for k=0 To max number step some number, then determine the motion in x-direction, and y-direction using equations 8. These equations are at least varying with k-value to determine the new center of the object location using equation 12.
8. Save the result. Then remove the object again from the image .
9. calculate: KE, PE, and Mon from equations 9, 10 and 11 respectively.
10. End for.

End algorithm

The results and discussion:

We can use the previous algorithm in the study by changing the value of θ to get the results of projectile with different values of θ . Table 1 is the sample of projectile motion applying this algorithm with $\theta = 80$, and figure 1 represent the projectile motion using algorithm 1 for different values of θ .

The horizontal component of velocity Vx will be constant at any point of the path of projectile, therefore V_{tot}, KE, and Mon all of them depend on V_{fy} . From table 1, we can see that V_{fy} will be decrease until reaching to zero in maximum point (Hmax) because the projectile stopped. The kinetic energy and the momentum decrease with decreasing of V_{fy} until be zero approximately at Hmax, but the potential energy will be increase because depending on the vertical distance y.

Because of the gravity the projectile downward with negative value of velocity, so V_{fy} will be increase in value until the body hit earth and KE, Mon increase with it, but decreasing of PE because decreasing of y.

When we use another value of angle like $\theta = -45$ as shown in table 2, the projectile will behave as in table 1, but beginning with negative velocity because the body falling under the reference coordinate. When the angle increase the horizontal distance x will be decrease and inversely. Figure 1 represent the projectile motion with many angles (th=θ), we can see from the figure that when the angle increase the Hmax increase and vertical distance y, but increasing of horizontal distance x and table 3 is the summary of table 1 and 2. In table 3 we see that if the projectile project from big angle, the total distance d, maximum height Hmax and tatal time t will be increase, but when the angle is negative, these quantities are decreasing, there is notes that the component of velocity connected with θ too as equation 1.

Figure 2 represent the images of one motion of the projectile with some initial conditions as referred to in figure and the object chose arbitrary.

The previous algorithm can be used with different initial velocities and constant of another variables, the data in table 4, that mean we can control the projectile from changing with initial angle or speed, this is clear in figure 3 and the images of this case in figure 4. We can also change in height or the mass of the object.

Conclusions:

This study very important to known the shape that the projectile will be taken, and all the parameters we need:

1. Calculation the total velocity of the body in the study with the components, the kinetic, the momentum and the potential energy, which were very important in study moving object in nature.

2. When the angle of trajectory increase the total distance, vertical component of velocity, total time and maximum height reached will be increase and via.

3. We can change initial conditions according to the study, and what we need from the study.

4. The images in the study can be uniform shape or not, whoever we can determine the center and study them.

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Table 1: projectile motion with *yo*=100, *Vo*=10, *m*=0.2, $\theta = 80$

No. frames	Cx	Cy	X	Y	Vyf	Vtot	KE	PE	Mon
1	27	124	0.163	100.88	8.928	9.095	8.273	197.728	1.8191
2	33	125	0.489	102.385	7.088	7.298	5.325	200.675	1.46
3	39	127	0.815	103.543	5.248	5.528	3.056	202.944	1.1055
4	45	127	1.141	104.356	3.41	3.83	1.463	204.54	0.77
5	50	128	1.467	104.822	1.568	2.34	0.55	205.453	0.468
6	56	128	1.793	104.944	-0.272	1.758	0.31	205.69	0.352
7	62	128	2.1193	104.721	-2.1123	2.734	0.7477	205.252	0.55
8	68	127	2.445	104.15	-3.99	4.317	1.864	204.136	0.863
9	74	126	2.77	103.236	-5.793	6.047	3.657	202.343	1.209
10	80	125	3.097	101.976	-7.63	7.83	6.127	199.873	1.566
11	86	123	3.423	100.37	-9.473	9.630	9.275	196.725	1.926
12	92	121	3.75	98.419	-11.313	11.445	13.0991	192.9	2.289
13	97	119	4.076	96.122	-13.153	13.267	17.6	188.4	2.654
14	103	116	4.4	93.5	-14.993	15.093	22.78	183.22	3.0186
15	109	113	4.728	90.492	-16.833	16.922	28.636	177.364	3.385
16	115	110	5.054	87.158	-18.673	18.754	35.17	170.83	3.75
17	121	106	5.38	83.48	-20.513	20.587	42.38	163.62	4.117
18	127	102	5.706	79.455	-22.353	22.42	50.268	155.732	4.484

Table 2: projectile motion with *yo*=100, *Vo*=10, *m*=0.2, $\theta = -45$

No. frames	Cx	Cy	X	Y	Vyf	Vtot	KE	PE	Mon
1	27	123	0.454	99.53	-7.7	10.4545	11.939	195.07	2.1
2	35	122	0.91	99.011	-8.33	10.93	11.94	194.06	2.186
3	43	121	1.362	98.456	-8.96	11.413	13.027	192.97	2.283
4	52	121	1.816	97.86	-9.59	11.914	14.194	191.8	2.383
5	60	120	2.27	97.224	-10.218	12.43	15.44	190.6	2.4852
6	68	120	2.725	96.55	-10.85	12.95	16.77	189.234	2.589
7	76	119	3.1788	95.83	-11.48	13.48	18.17	187.83	2.697
8	84	118	3.63	95.074	-12.11	14.02	19.66	186.344	2.8
9	92	117	4.087	94.3	-12.74	14.567	21.219	184.78	2.91
10	101	116	4.54	93.44	-13.365	15.12	22.862	183.138	3.024
11	109	116	4.995	92.56	-13.994	15.679	24.6	181.417	3.136
12	117	115	5.449	91.641	-14.623	16.243	26.385	179.615	3.249
13	125	114	5.9	90.68	-15.253	16.812	28.265	177.735	3.362
14	133	113	6.358	89.68	-15.8822	17.385	30.225	175.776	3.477
15	141	112	6.812	88.64	-16.512	17.962	32.263	173.737	3.59
16	150	111	7.266	87.561	-17.141	18.45	34.38	171.62	3.71
17	158	109	7.72	86.44	-17.77	19.126	36.44	169.422	3.83
18	166	108	8.174	85.278	-18.4	19.7	38.855	167.145	3.94

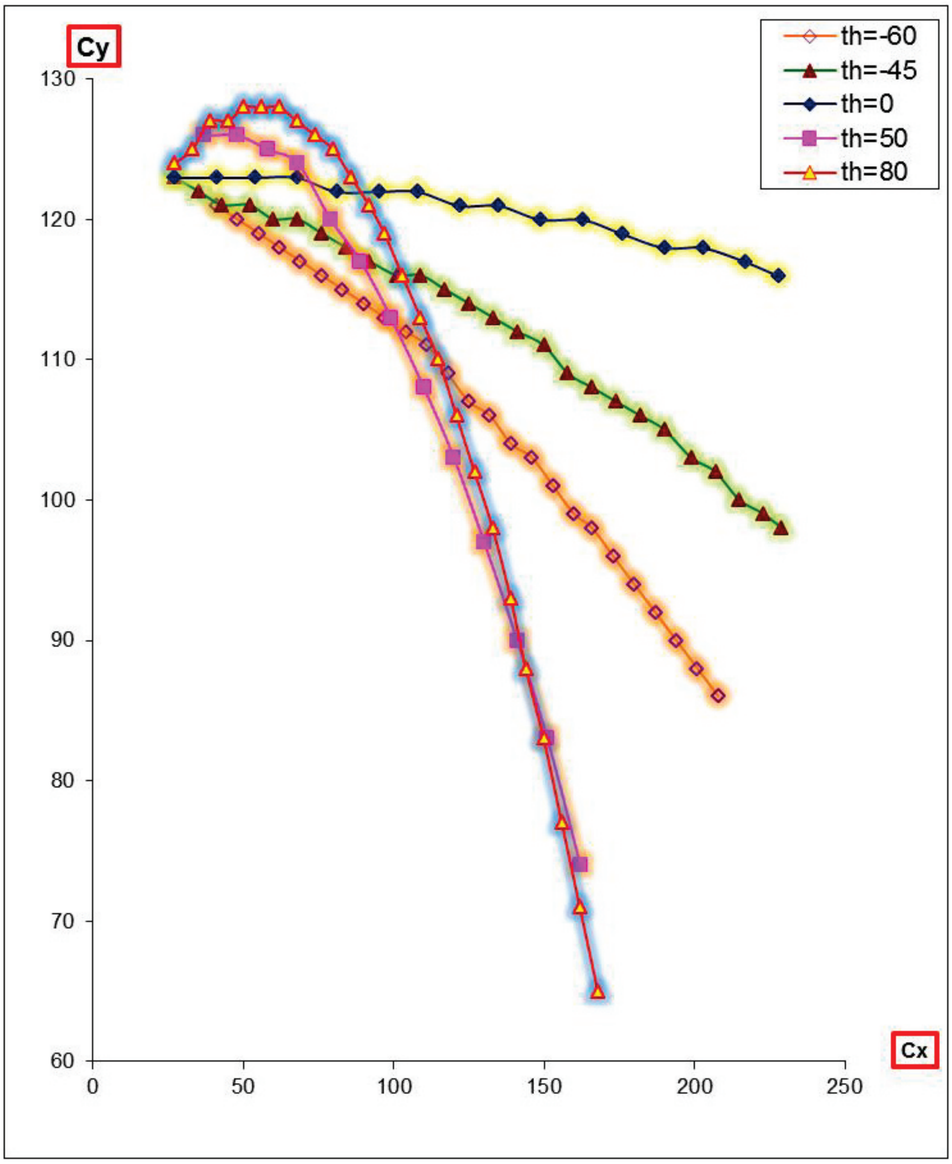

Figure 1: Projectile motion with yo= 100, Vo= 10, m= 0.2 and different angles (th=θ).

Table 3: Projectile motion for different angles

h=100 m=0.2 Vo=10					
θ	Vx	Vy	d	y	t
-60	5	-8.66	18.5973	100	3.7195
-45	7.071	-7.071	27.26	100	3.853
0	10	0	45.1754	100	4.51754
50	6.428	7.6604	34.49421	102.994	5.36635
80	7.365	9.85	9.78	104.95	5.633

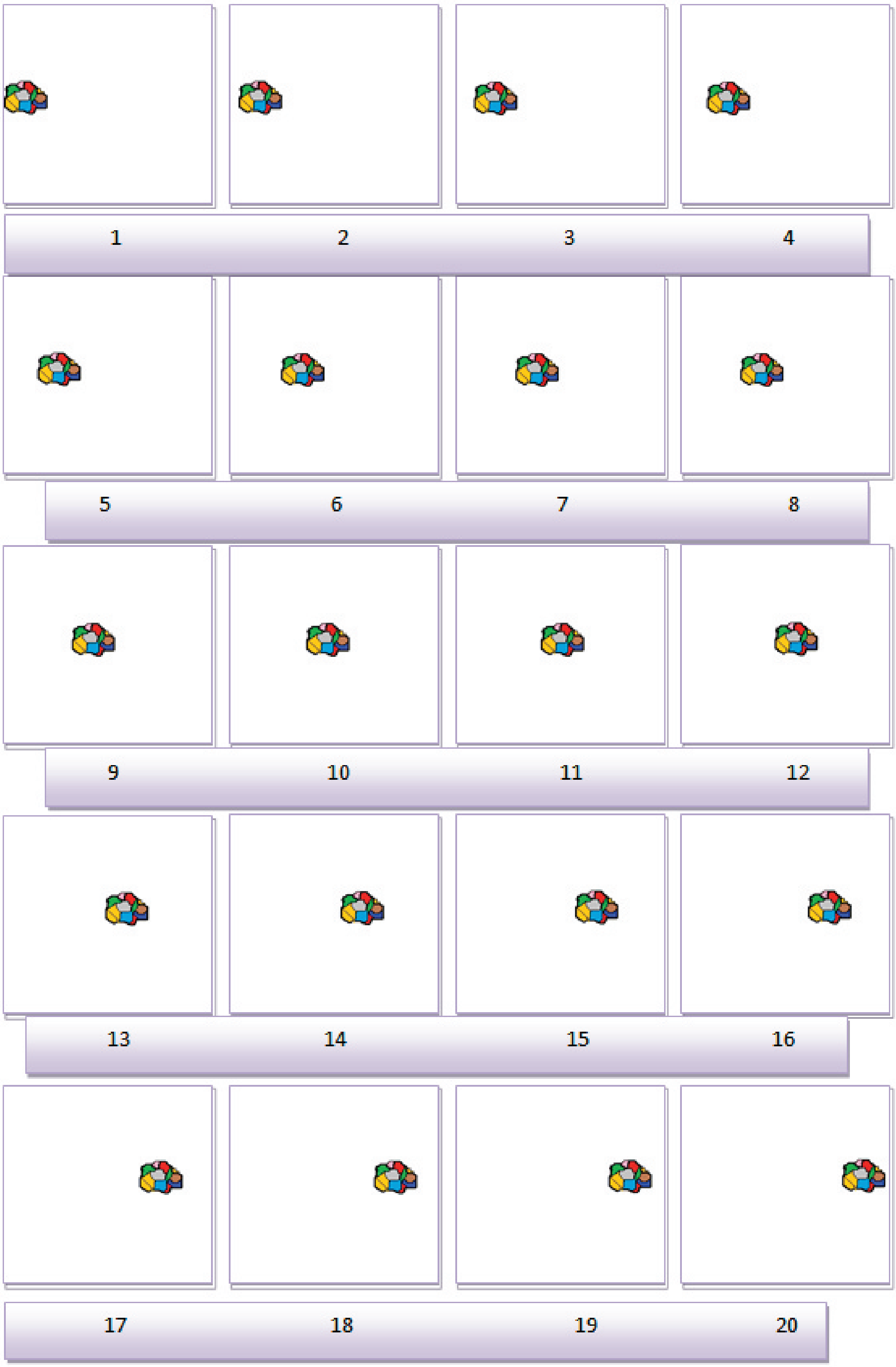

Figure 2: the images of projectile motion with yo= 100, Vo= 10, m= 0.2 and (th=80).

Table 4: Projectile motion for different initial velocities

=70 h=100 m=0.2θ					
Vo	Vox	Voy	d	h	t
5	1.71	4.698	8.5887	101.126	5.02234
7	2.394	6.578	12.54132	102.208	5.238
9	3.0781	8.457	16.814	103.649	5.4622

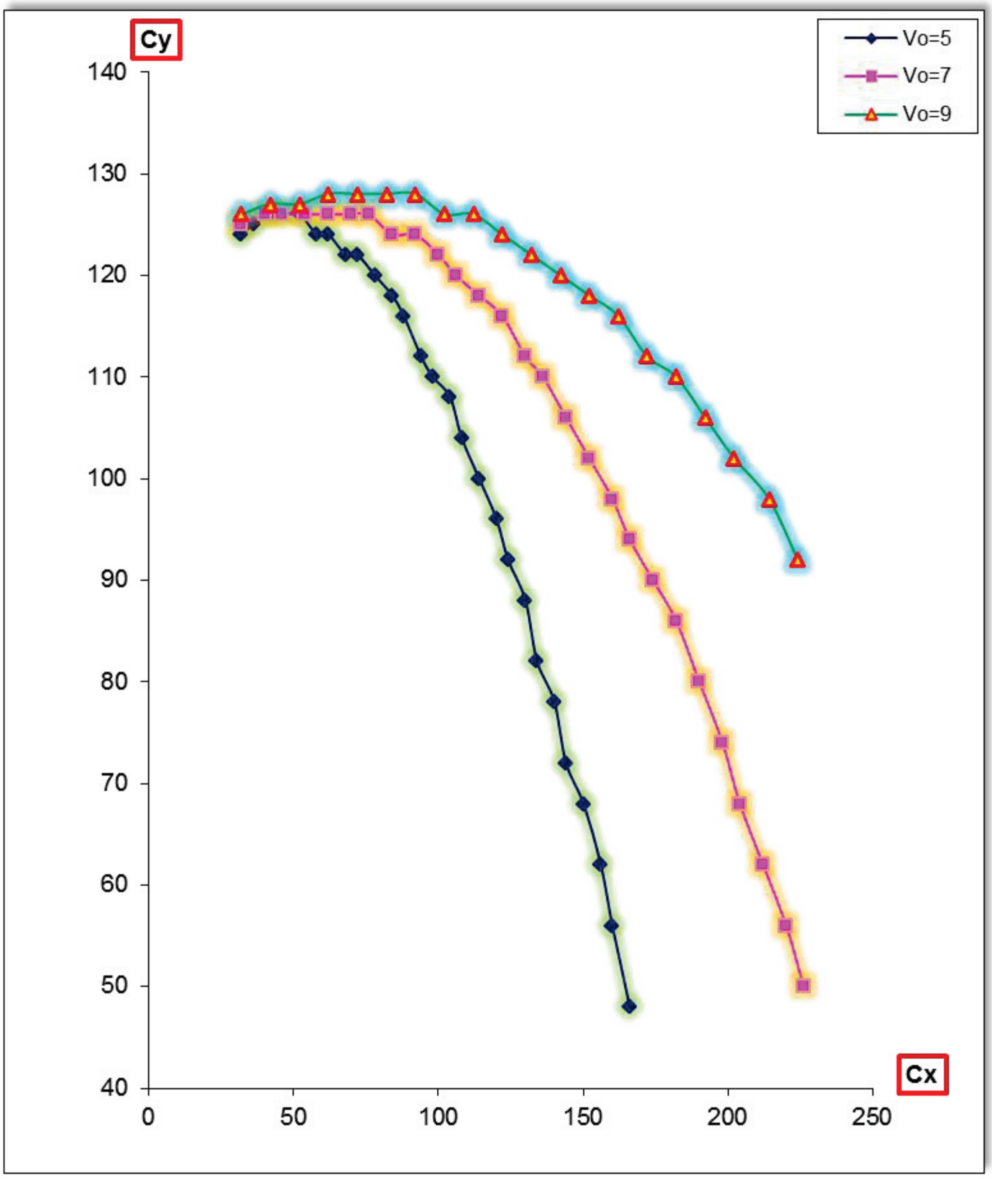

Figure 3: Projectile motion with yo= 100, θ = 70, m= 0.2 and different *Vo*.

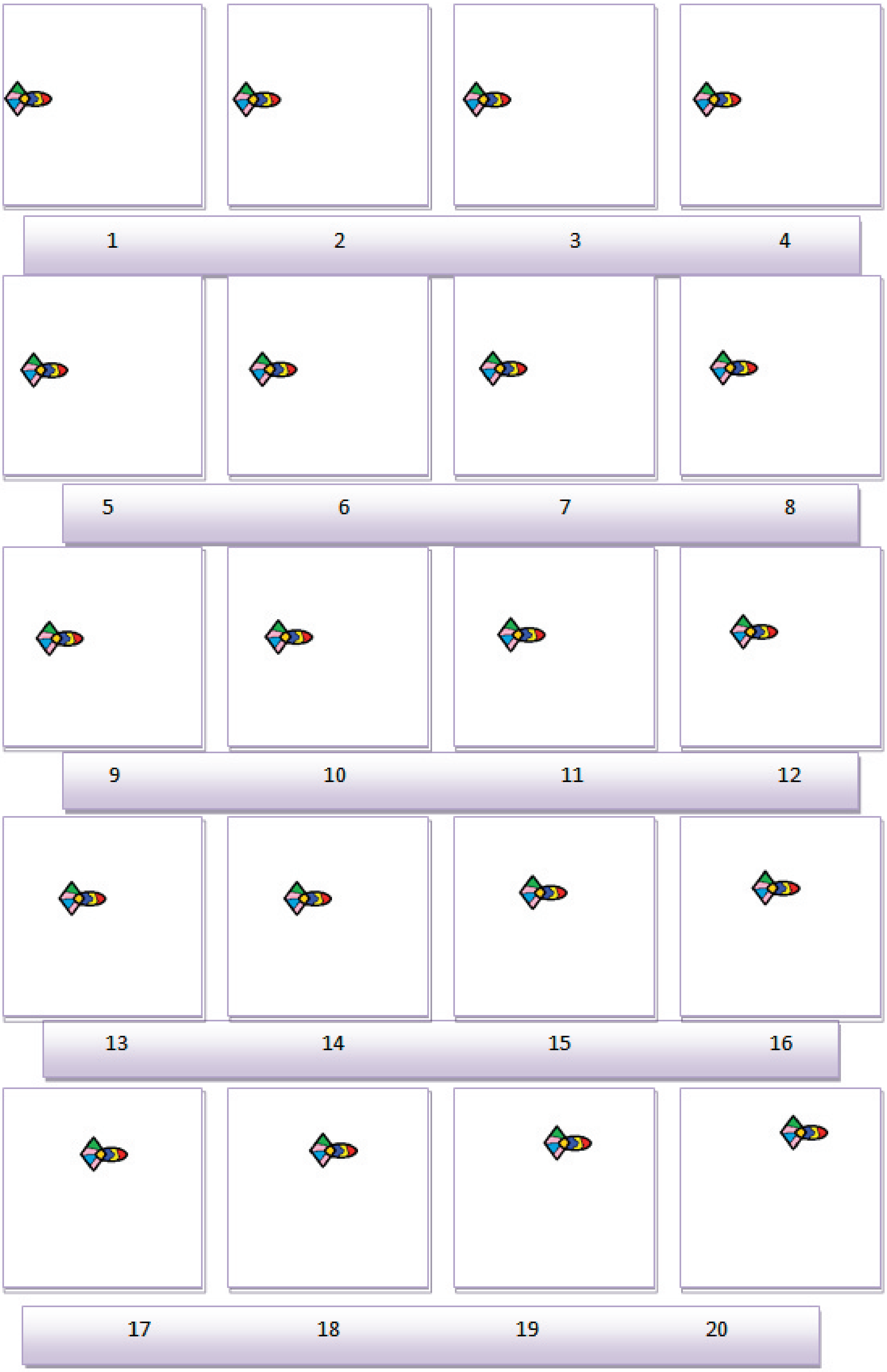

Figure 4: the images of projectile motion with yo= 100, θ = 70, m= 0.2 and Vo=7.