Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol.28, 2014
www.iiste.org

Study of Potential Energy Surface for ^{154,160}Dy Isotopes by IBA-1

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Abstract :

Energy levels of ^{154,160}Dy isotopes have been studied in the frame- work of the interacting boson approximation model (IBA-1). The contour plot of the potential energy surfaces, V(β,γ), shows that the nucleus ^{154}Dy is deformed and has transitional characters between SU(5) and O(6) limits, and the nucleus ^{160}Dy is deformed and has rotational characters, SU(3). Levels energy spectra belonging to the g,β,γ bands and electric quadrupole moment (Q) are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

Keywords: energy levels; potential energy surface; ^{154,160}Dy; SU(3).

1. Introduction :

General algebraic group techniques, applied to the **I**nteracting **B**oson **A**pproximation (IBA), have rather successfully described the low-lying collective properties of a wide range of nuclei. In the relatively simple Hamiltonian of the model, the collective states are described by a system of interacting s- and d-bosons carrying angular momenta 0 and 2, respectively, which define an overall symmetry [1-3]. The IBM Hamiltonian has exact solutions in three dynamical symmetry limits [SU(5),SU(3) and O(6)], which are geometrically analogous to the an harmonic vibrator, axial rotor and γ-unstable rotor, respectively. More generally, the Hamiltonian can be expressed in terms of an invariant operator of that chain of symmetries, and a shape phase transition between the dynamical symmetry limits results [4-6]. The analytic description of the structural change at the critical point of the phase transition being still an open problem, the Hamiltonian must be diagonalized numerically. Pan *et al* [7], proposed a new solution based on the affine SU(1,1) algebraic technique, which determines the properties of nuclei in the SU(5)↔O(6) transitional region of IBM-1[7-8].

In the simplest version of the interacting boson approximation, its assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, its assumed that the particle configurations which are most important in shaping the properties of the low-lying states are these in which identical particles are coupled together forming pairs of angular momentum 0 and 2 [1,9,10].

2. Interacting Boson Approximation :

The interacting boson model of Arima and Iachello has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model it was assumed that low-lying collective states of
even-even nuclei could be described as states of a given (fixed) number *N* of bosons. Each boson could occupy two levels one with angular momentum *L* = 0 (*s*-boson) and another, usually with higher energy, with *L* = 2 (*d*-boson). In the original form of the model known as IBM-1, proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, associated with it [11-16].

The Hamiltonian employed for the present calculation is given as [17,18]:

$$\hat{H} = \varepsilon\hat{n}_d + a_o\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \qquad \textbf{(1)}$$

Where;

n_d is the number of boson; P.P, L.L, Q.Q, $T_3.T_3$ and $T_4.T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons respectively; ε is the boson energy; and a_o, a_1, a_2, a_3, a_4 is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions respectively.

In O(6)→SU(5) transition region, nuclei have transitional properties between (SU(5)) and (O(6)) and the Hamiltonian is give by [12] :

$$\hat{H}^{(I+III)} = \varepsilon\hat{n}_d + a_o\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \qquad \textbf{(2)}$$

The properties of the nuclei fall in this transitional region depends on the ratio (ε'/a_o), if this ratio is large means nuclei properties are near to SU(5) limit and when the ratio is small the properties will be near O(6) limit.

Hamiltonian function operator for rotational limit SU(3) in terms of creation and annihilation operators can be given according to the following equation [19-22] :

$$\hat{H} = a_1 \hat{I}^2 + a_2 \hat{Q}^2 \tag{3}$$

The electric quadrupole moment ($Q2_1^+$) is [23] :

$$Q_{2_1^+} = -\alpha_2 \sqrt{\frac{16\pi}{40}} \frac{2}{7} (4N + 3) \tag{4}$$

As to the potential energy surface operator is given by [24] :

$$V(N,\beta,\gamma) = \frac{N}{1+\beta^2}\left(\varepsilon_s + \varepsilon_d \beta^2\right) + \frac{N(N-1)}{\left(1+\beta^2\right)}\left(A_1\beta^2 + A_2\beta^3 \cos 3\gamma + A_3\beta^2 + A_4\right) \tag{5}$$

Where;
β is the magnitude of nuclear deformation taken the values (0-2.4);
γ asymmetry angle taken the values (0^o-60^o);A_1,A_2,A_3,A_4 parameters relationship with the function of the surface potential.

3. Results and Discussion

3.1. Energy Levels :

In this work we have studied the energy levels of even-even Dy (A=154) isotope which is classified to O(6)→SU(5) transition region and even-even Dy (A=160) isotope which is classified to rotational dynamical symmetry SU(3) by comparing the energy ratios $\frac{E0_2^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}, \frac{E4_1^+}{E2_1^+}$ with ideal values [25,26] for three dynamical symmetries SU(5), O(6) and SU(3) of IBM-1 (shown in figures (1) to (4)).

Table (1) presents the isotopes used in the present work according to its atomic mass number, total number of boson and the corresponding Hamiltonian parameters used in the IBM-Code according to O(6)→SU(5) transition region and rotational dynamical symmetry SU(3).
Figures (5) and (6) present values of the energy levels (present work), according to energy bands (g, β, and γ bands) in comparison with available experimental data.

This table list the new energy levels belong to, β_1, β_2, γ_1 and γ_2 bands with their spins and parties. The results show that, the β-band is a large extent emerge than the γ-band for the dynamical symmetry SU(3), while the emergence of γ-band is increasing for the isotopes having the transitional dynamical symmetry SU(5)-O(6).

The β-band is not difficult to see it in the dynamical symmetry SU(3), in the low spin states, while the γ-band is difficult to find it due to the high spin state.

3.2. Electric Quadrupole Moments ($Q2_1^+$) :

The quadrupole moment (Q) is an important property for nuclei. It is defined as the deviation from the spherical charge distribution inside the nucleus. From the quadrupole moment, we can determine if the nucleus is spherical, deformed oblate or prolate shapes.
Table shows theoretical and available experimental values of $Q2_1^+$ for ^{154}Dy and ^{160}Dy isotopes.

3.3. Potential Energy Surface (P.E.S.) :

One of methods to knowledge the deformation of nuclear structure, calculation the potential energy surface.
In the present work, we were used the IBM-1 analysis for the set of the plots potential energy surface function V(N,β,γ) calculate by using the parameters (A's) infer from (IBMP-Code) program, as shown in table (3).
Figures (7) and (8) show the potential energy surface as a function of deformed parameters (β,γ).

4. Conclusions :

1. The even-even ^{154,160}Dy isotopes have (66) protons and (88,91) neutrons respectively. The core is taken at major closed shell (82) for protons and neutrons. Therefore, the number of bosons were determined for ^{156}Dy and ^{158}Dy, is equal (11) and (14) bosons respectively.
2. The interacting boson model version one (IBM-1) gives us a very closing value with the experiment.
3. Since the energy levels depends on the total boson number so that only the ground state band will appear.

4. Hamiltonian parameters in table (1) are very small so that these parameters vary to any change may occurs in any one of these parameters, so that it is difficult to get the coincidence values between the energy levels in high energy states.
5. The electric quadruple moment increase as the mass number increase.
6. The nuclear deformation increases with the increasing of valance boson number.

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Table (1): The Hamiltonian Parameters Used in the IBM-Code for ^{154,160}Dy Isotopes.

Isotope	EPS MeV)(	P.P. (MeV)	L.L. (MeV)	Q.Q. (MeV)	$T_3.T_3$ (MeV)	$T_4.T_4$ (MeV)	CHI	SO6
^{154}Dy	0.8440	0.1660	0.0055	0.0000	0.0055	0.0000	-0.6322	1.0000
^{160}Dy	0.0000	0.0000	0.0122	-0.0044	0.0000	0.0000	-1.3220	1.0000

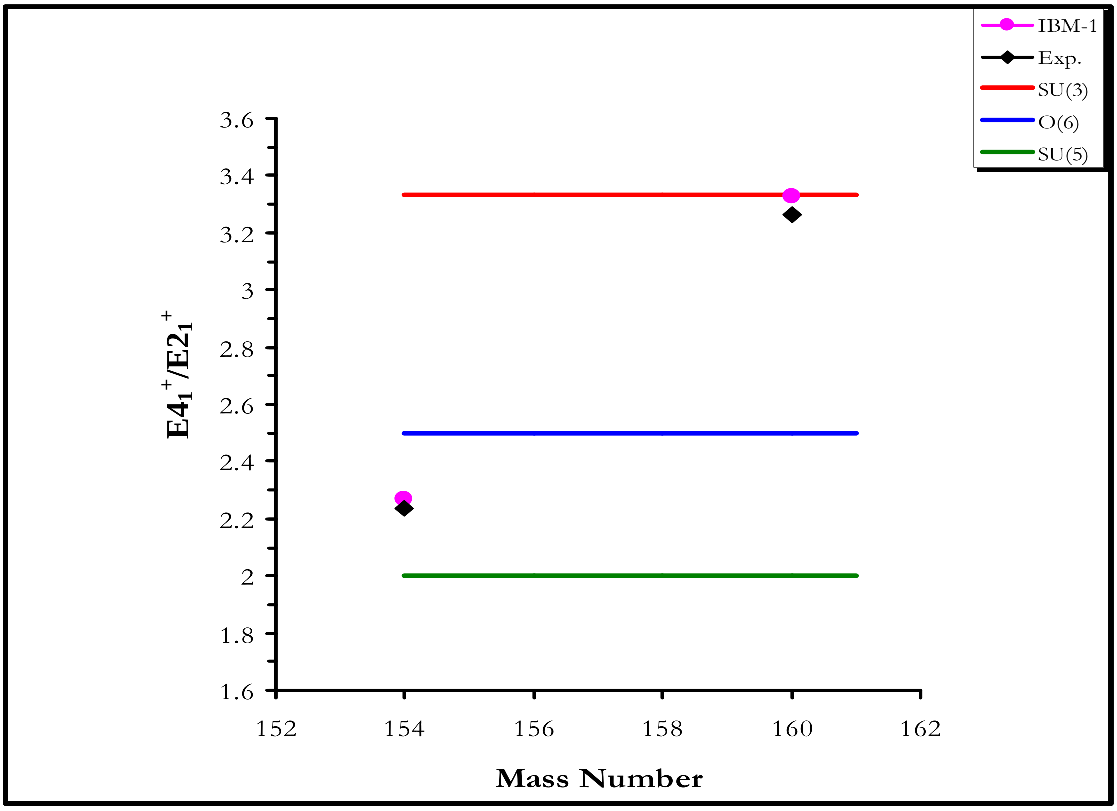

Figure (1): The Comparison of $E4_1^+/E2_1^+$ Theoretically, Experimentally [27-29] and with the Typical Values [25,26] for Each Limit.

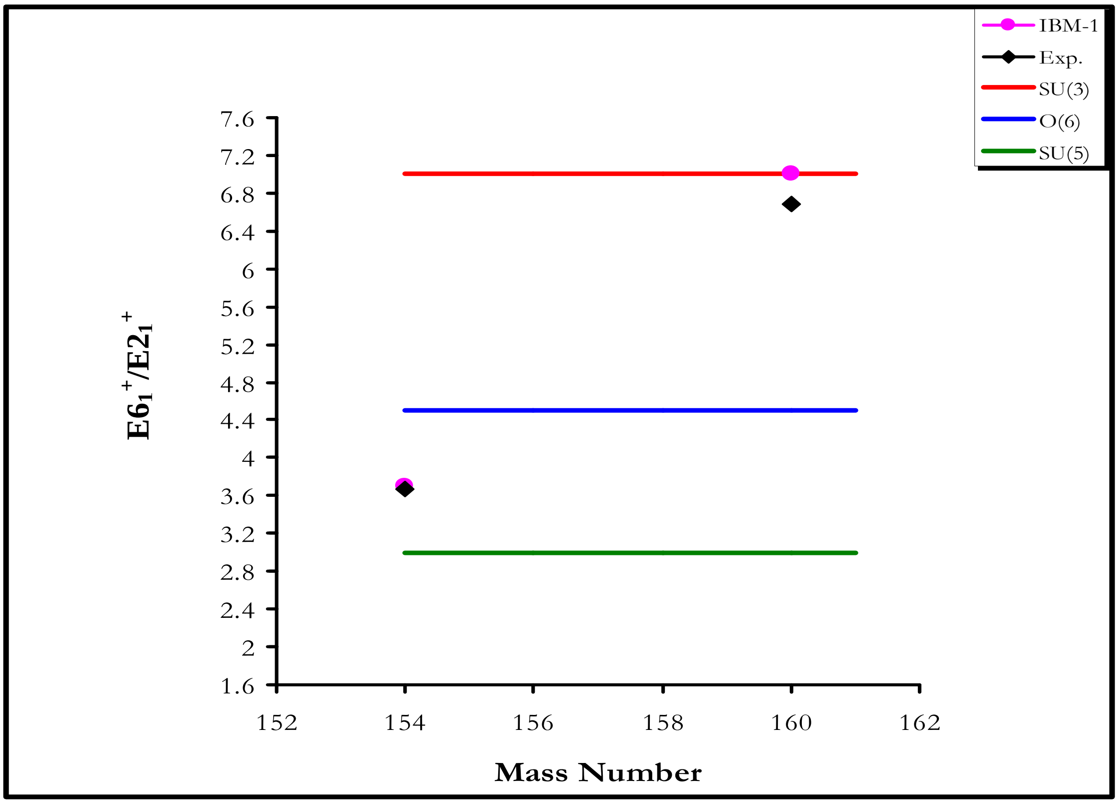

Figure (2): The Comparison of $E6_1^+/E2_1^+$ Theoretically, Experimentally [27-29] and with the Typical Values [25,26] for Each Limit.

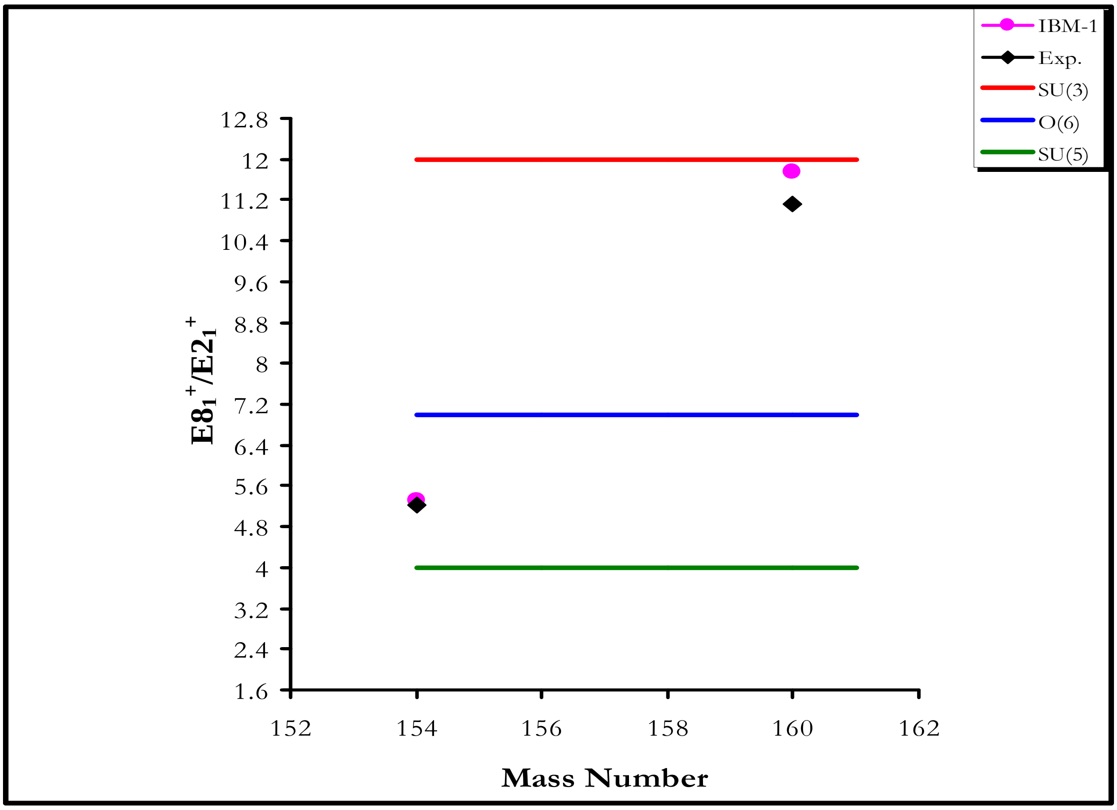

Figure (3): The Comparison of $E8_1^+/E2_1^+$ Theoretically, Experimentally [27-29] and with the Typical Values [25,26] for Each Limit.

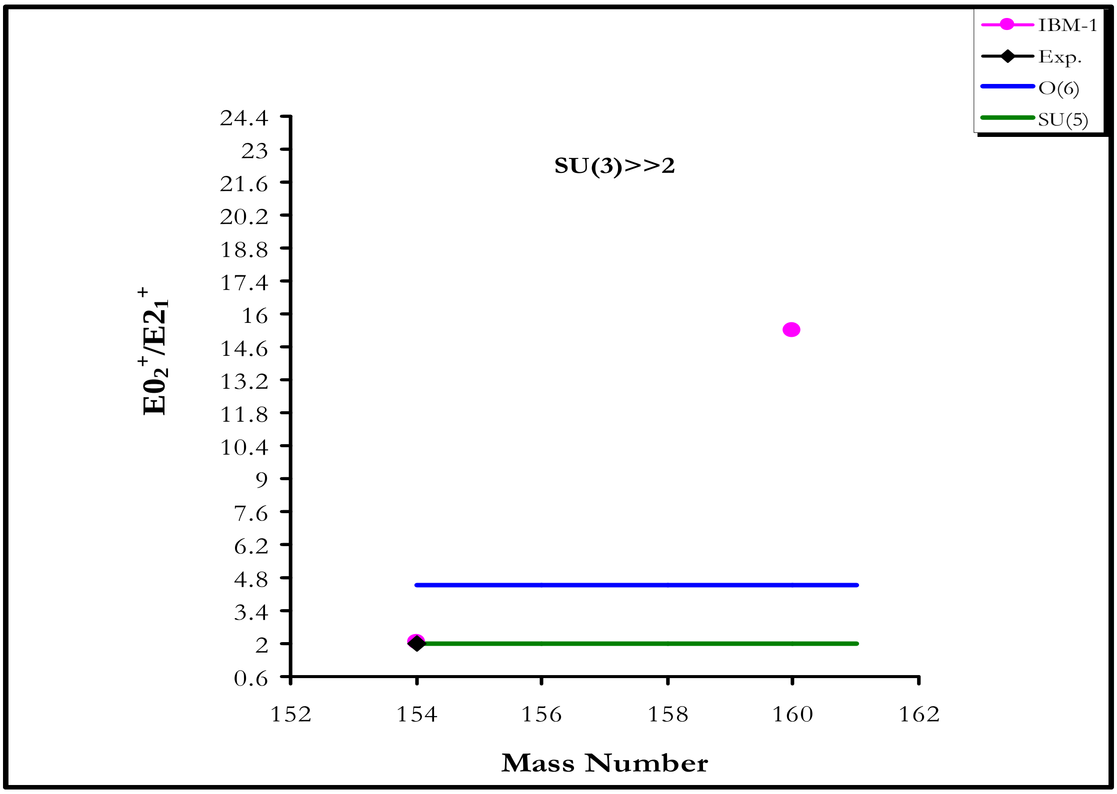

Figure (4): The Comparison of $E0_2^+/E2_1^+$ Theoretically, Experimentally [27-29] and with the Typical Values [25,26] for Each Limit.

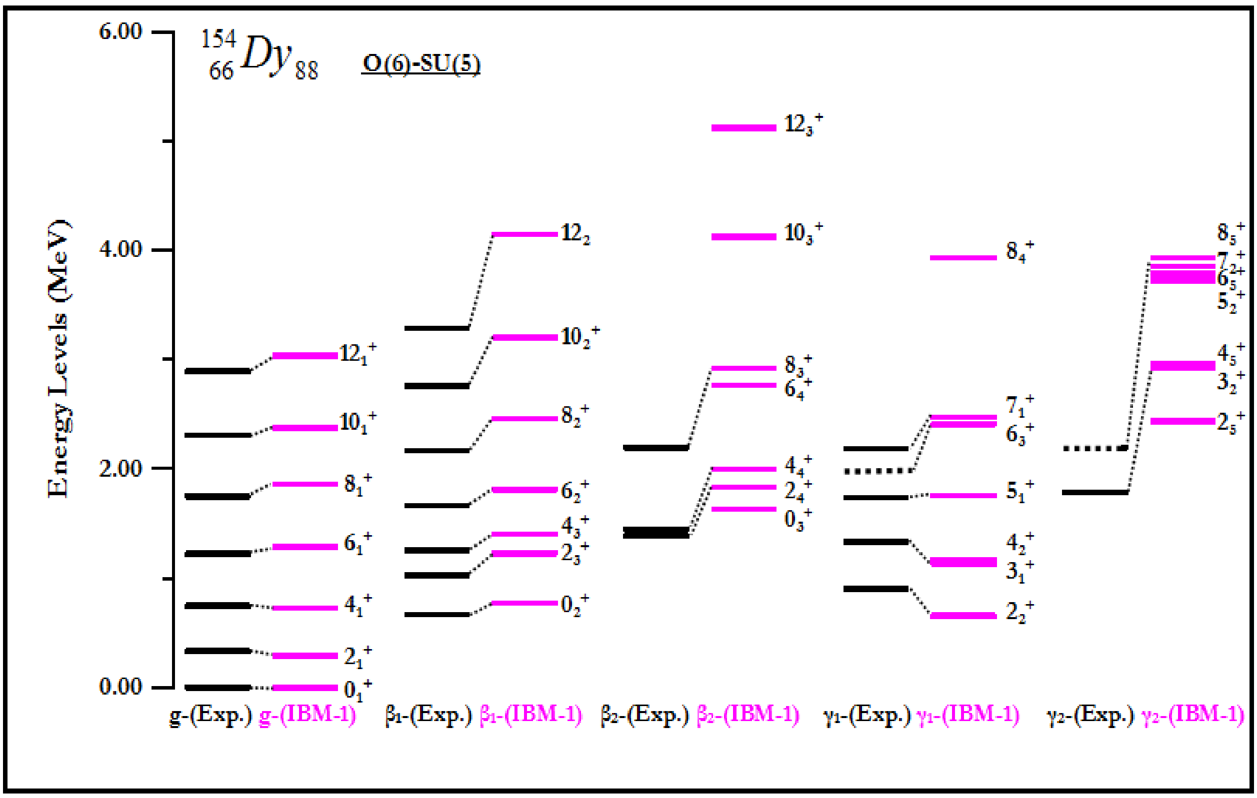

Figure (5): Comparison between Experiment [27,28] and Calculated Energy Levels for ^{154}Dy Isotope.

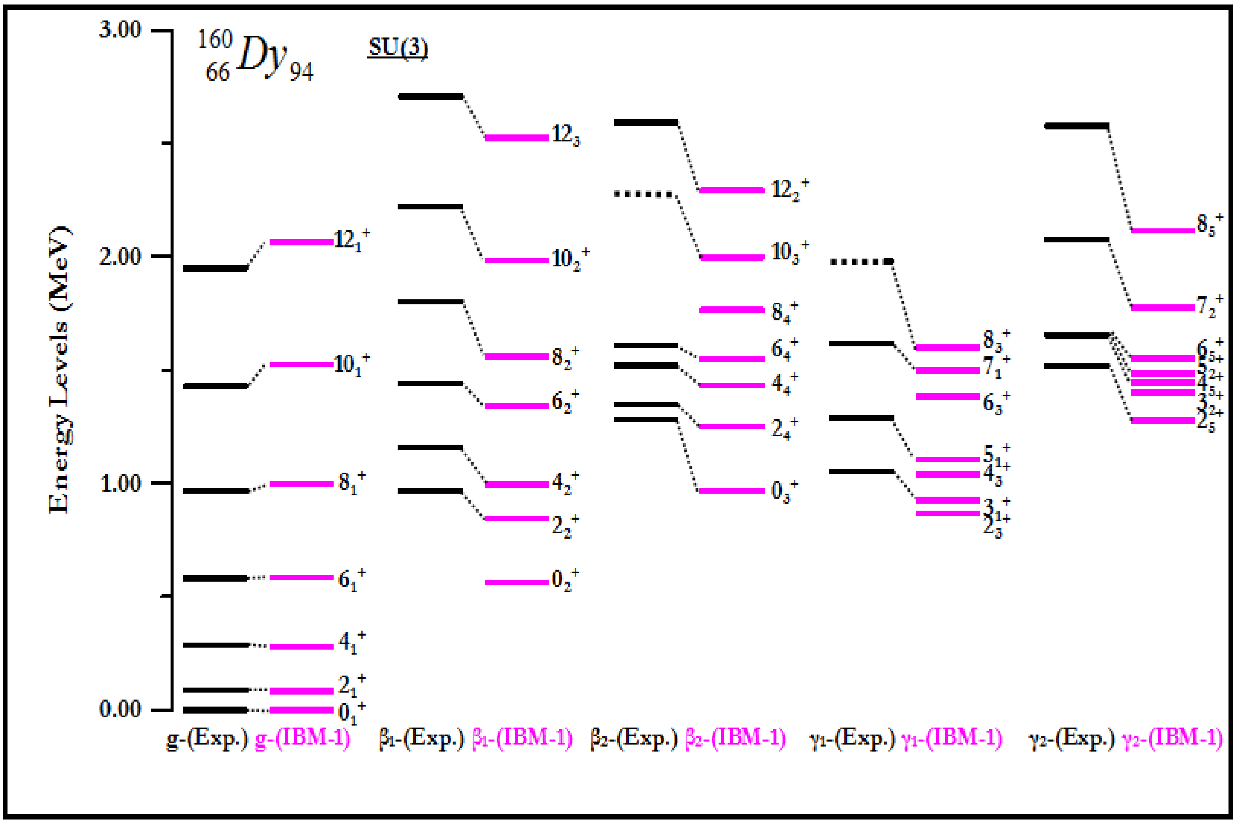

Figure (6): Comparison between Experiment [27,29] and Calculated Energy Levels for ^{160}Dy Isotope.

Table (2): Comparison between Theoretical and Available Experimental Values of $Q2_1^+$ for ^{154}Dy and ^{160}Dy Isotopes.

Isotope	$Q2_1^+$ (eb)	
	Theo.	Exp.[30]
^{154}Dy	-1.3080	-----
^{160}Dy	-2.1000	-2.0000 [30]

Table (3): Parameter Used for Potential Energy Surface Calculations in (IBMP-Code) Program for ^{154,160}Dy Isotopes.

Isotope	N	EPS	EPD	A_1	A_2	A_3	A_4
^{154}Dy	11	0.000	0.885	0.011	0.000	-0.083	0.000
^{160}Dy	14	-0.022	0.661	-0.002	-0.130	-0.180	0.000

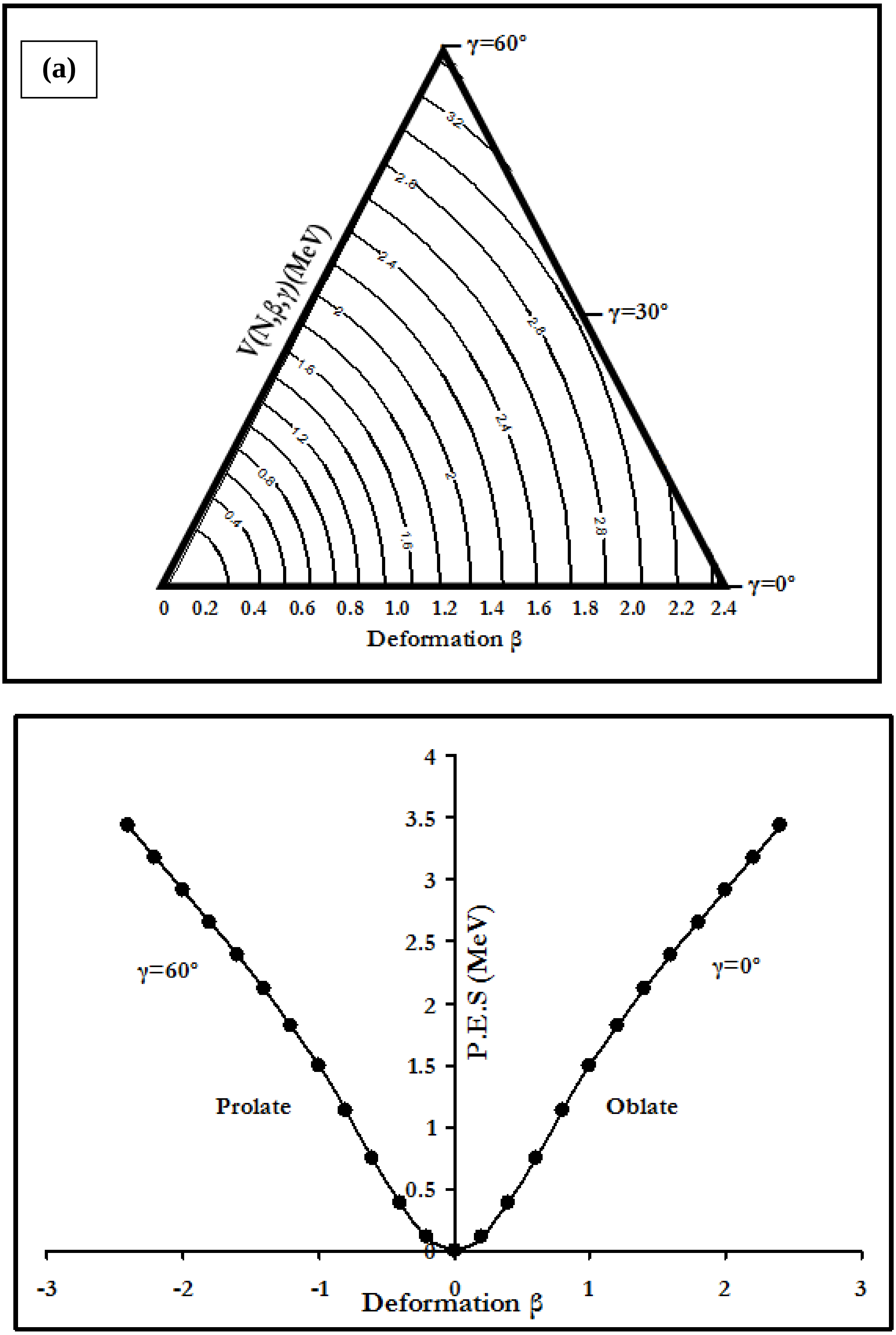

Figure: (7)
(a): The Contour Plot for the ^{154}Dy Isotope at γ=60°.
(b): The Axial Symmetric for the ^{154}Dy Isotope.

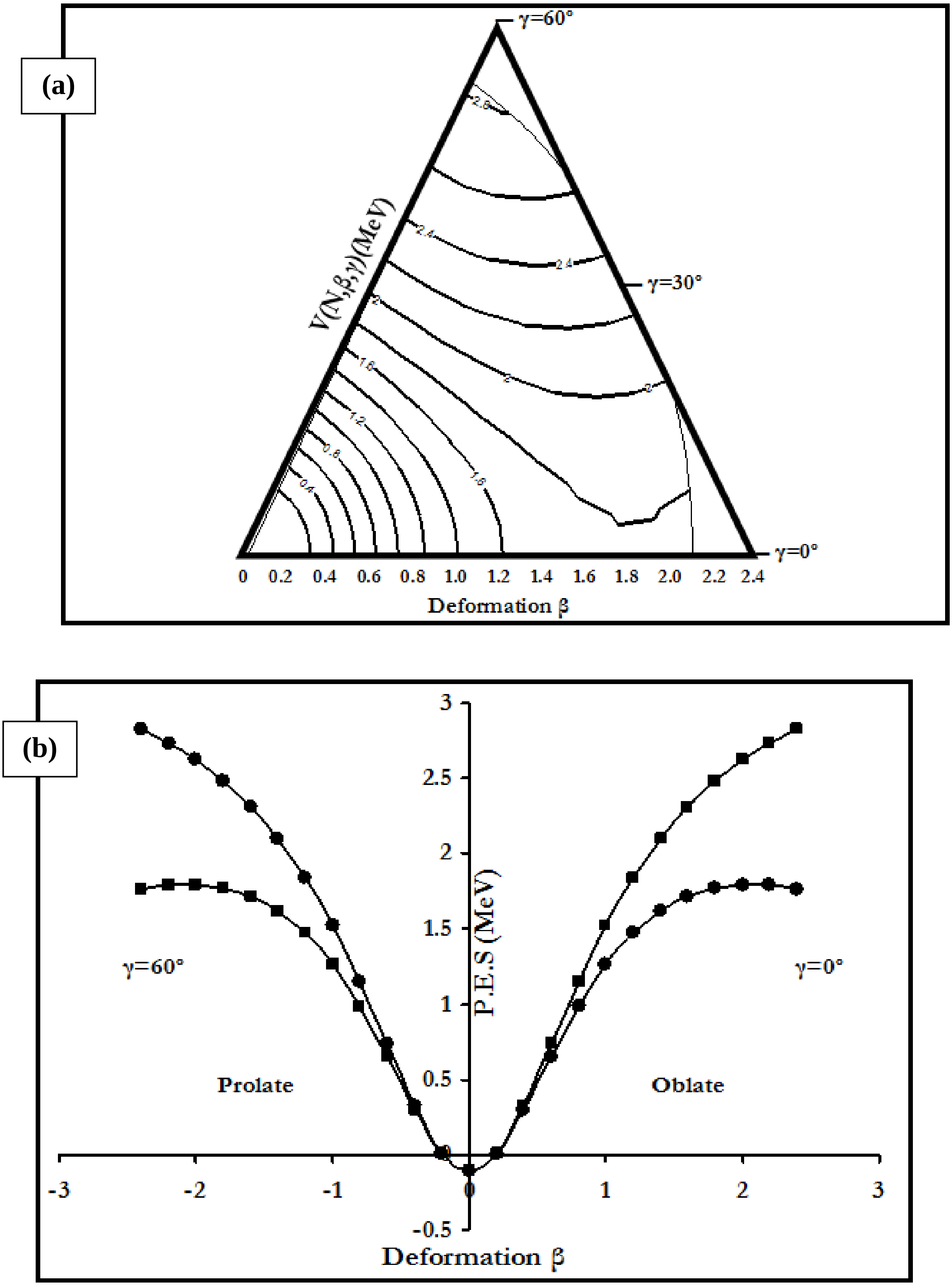

Figure: (8)
(a): The Contour Plot for the ^{160}Dy Isotope at γ=60°.
(b): The Axial Symmetric for the ^{160}Dy Isotope.