Advances in Physics Theories and Applications
ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 6, 2012
www.iiste.org

# The Anisotropies of The Universe-Variable Speed of Light – Matter –Antimatter System: A Disjecta Membra-Eventum Tantum Model For Attribution of Matter Abundance Contrast Antimatter

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**Abstract**.

We rest a case that variable speed of light is responsible for the anisotropies of the universe and draw a parallel between Bank "Deposits" and "Advances" for "matter" and "Antimatter" system. As Deposits are more than advances and form the plenipotentiary source for "advances", the "appearance" and "disappearance" of photons keep the energy levels constant. As Assets are equal to Liabilities, notwithstanding the "Profit", a suggestion is made to write The General Theory based on the Transfer Scroll of the General Ledger in a Bank.

**Introduction:**

Philosophy merges with ontology; ontology merges with univocity of being. Analogy has always a theological vision, not a philosophical vision. One becomes adapted to the forms of singular consciousness, self and world. The univocity of being does not mean that there is one and the same being. on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta, like gravity, Like electromagnetism; the constancy of gravity does not mean there does not exist total gravity, the universal theory depends upon certain parameters and it is disjoint. Conservations of energy and momentum is one, but they hold good for each and every disjoint system; so there can be classification of systems based on various parametric representationalities of the theory itself. This is very important. Like one consciousness, it is necessary to understand that the individual consciousness exists, so does the collective consciousness and so doth the evolution too. These are the aspects which are to be borne in my mind in unmistakable terms .The univocity of being signifies that being is a voice that is said and it is said in one and the same "consciousness". Everything about which consciousness is spoken about. Being is the same for everything for which it is said like gravity, it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum ; it is the ultimate form for all of the forms; and all these forms are disjointed. It brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity, highest affirmation of a Theory be it GTR or QFT. Like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. A single instance for all that exists , a single phantom for all the living, single voice for every hum of voices, or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring If consciousness is one unique event in which all the events communicate with each other; Univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible SENSES of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and

 

Consciously. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality; Despite philosophical overtones, the point we had to make is clear. **There doth exist different systems for which universal laws are applied and they can be classified. And there are situations and conditions under which the law itself breaks; this is the case for dissipations or detritions coefficient in the model**.

We incorporate the following :

**Anisotropies Of The Universe**
**Variable Speed Of Light**
**Matter**
**Antimatter**

**What is the matter with antimatter?**

Physicists at the Stanford Linear Accelerator Center (SLAC) in California and the High Energy Accelerator Research Organization (KEK) in Japan are colliding particles and anti-particles at high energies to study minute differences between the ways matter and antimatter **interact.** Their goal is to contribute to our understanding of the workings of the universe at its largest and smallest scales, from revealing the origin of matter shortly after the Big Bang, to uncovering the secrets of elementary particles and their interactions.

After decades of particle physics experiments, we now know that every type of particle has a corresponding antimatter particle, called an anti-particle. A particle and its anti-particle are identical in almost every way - they have the same mass, for example - but they have opposite charges. The existence of the positron, the positively charged anti-particle of the negative electron, was first hypothesized by Dirac in 1928. Its existence was experimentally proven in 1933 by Anderson, who received the 1936 Nobel Prize for this achievement. Since then, physicists have discovered the anti-particles of all the known elementary particles, and have even been able to combine positrons with antiprotons to make antihydrogen "antiatoms".

Matter and antimatter are created together.

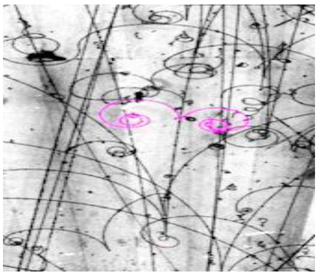

Fig 1. Particles and antiparticles (such as the pair highlighted in pink) are **created in** pairs from the energy released by the collision of fast-moving particles with atoms in a bubble chamber. Since particles and antiparticles have opposite electrical charges, they curl in opposite directions in the magnetic field applied to the chamber.

From the physicists' point of view, what is strange about antimatter is that we do not see more of it. When we **collide** high-energy particles in accelerators, their energy is **converted into** equal amounts of matter and antimatter particles (according to Einstein's famous formula, the energy (E) it takes to create matter and antimatter of total mass (m) is E=mc^2

For example, you can see matter and antimatter particles **created in** the bubble chamber photo on the left. The photo shows many bubble tracks **generated by** charged particles **passing through** superheated liquid. Due to the magnetic field applied to the chamber, positive particles **curl to the right** and **negative particles curl** to the left. The two curled tracks highlighted in pink show an electron-positron pair **created by** the collision of a gamma ray photon (a highly energetic particle of light) with an atom in the chamber, in a process called pair production.

**<u>Where did all the antimatter go?</u>**

**Since we see matter and antimatter created in equal amounts in particle experiments, we expect that shortly after the Big Bang, when the universe was extremely dense and hot, equal amounts of matter and antimatter were created from the available energy. The obvious question is, therefore, where did the antimatter go? To this seemingly intractable problem our explanation is that like Deposits are more in the Bank compared to Advances, Matter is more in the universe because antimatter is produced by the matter itself, like "Advances" are attributable to ""Deposits". This then brings in to question "Conservation of Energy" compliance. One can only say that "Energy" is produced and dissipated on par to the "Profits" in a Bank, and this is done by the creation and destruction of particles continuously. Towards the end of obtention of a "Transfer Scroll", it is necessary that we open a parallel "General Ledger", so that all the transactions are recorded, and no transaction or transformation of one type of energy to another is left out. So the "General Ledger" is nothing but mere a General Theory Of "All Happenings in the Universe", including the appearance and disappearance of the particles. Conservation of Energy is explained on the same term that one explains "Assets" and "Liabilities" in a Bank. Both are always equal. They are equal despite the fact that one Bank branch makes Profit and the other branch makes "Loss". Now this can be comparable to the creation and destruction of matter that is continuously taking place and "appearance" or "Disappearance" of Charges. Each "Department" say "Earth" or "Moon" or "space one" "Space n" in which such transactions take place is recorded. There is of course, experimentation impracticability, unviability and infeasibility in the sense that every "Transaction" cannot be measured. However, we give "General Theory Of All Transactions That Take Place In The Universe". Verily, there is a contrast nonpareil; When the Transformation of one type of Energy takes place in to another, this shall be recorded notwithstanding that it shall be immediately "annulled" by the "corresponding countervailing reaction" So General Ledger provides " "The General Theory of Debit and Credit of all the transactions of the Universe". Conservation of Energy is maintained for the "Profits (in terms of Energy" is balanced immediately by the Loss of Energy on the other side" To put up any justification argumentation would be allowing the subject object problem raise its head again; and such a revitalization, rejuvenation and resurrection would clearly make aggravated and exacerbated. Freud was clear in stating that mind divides and disintegrates: and the only time subject= object satisfaction would be at Mother's breast. Later on the division starts…THIS MODEL IS THE FIRST STEP IN THAT DIRECTION.**

**One survivor for every billion**

Based on numerous astronomical observations and the results of particle physics and nuclear physics experiments, **<u>we deduce</u>** that all the matter in the universe today is only about a billionth of the amount of matter that **<u>existed during</u>** the very early universe. As the universe expanded and cooled, almost every matter particle **<u>collided with</u>** an antimatter particle, and the two turned into two photons - gamma ray particles - in a process called annihilation, the opposite of pair production. But roughly a billionth of the matter particles survived, and it is those particles that now make the galaxies, stars, planets, and all living things on Earth, including our own bodies.

**The universe and the particles**

The survival of a small fraction of the matter particles indicates that, unlike what we wrote above, matter and antimatter are not exactly identical. There is a small difference between the ways they interact. This difference between matter and antimatter was first observed in particle accelerators in 1964 in an experiment for which Cronin and Fitch were awarded the 1980 Nobel Prize, and its connection to the existence of matter in the universe was realized in 1967 by Sakharov.

Physicists call this <u>difference CP violation</u>. Jargon, but it just means that if you are conducting a particular experiment on particles, from which **<u>you deduce</u>** a certain theory of the laws of physics, then conducting the same experiment on anti-particles would lead you **<u>to deduce</u>** different laws. <u>The only way to end up with a consistent set of physical laws is to incorporate the matter-antimatter difference into your theory</u>. Because this difference is small, conducting any old experiment would not reveal it. For example, if your experiment involves gravity, you would find that apples are attracted by massive bodies like the earth, and that anti-apples are also attracted by massive bodies. So gravity **<u>affects</u>** matter and antimatter identically, and this experiment would **not reveal CP violation**. A much more sophisticated experiment is required.

**Sophisticated experiment**

The new generation of experiments at SLAC and KEK, called BaBar and Belle, offer new tools with which to probe the nature of CP violation, hopefully shedding light on what happened a tiny fraction of a second after the Big Bang, and expanding our understanding of elementary particles and their interactions. These experiments work as follows: an accelerator accelerates electrons and positrons to high energies. They are **then "stored**" in bunches of about a hundred billion particles each, running around in a circular accelerator called a storage ring at about 0.99997 of the speed of light. Electrons are made to go one way, and positrons go the other way, so that the bunches cross through each other every time they go around the ring.

**Making quarks**

On some bunch crossings, a positron and an electron come close enough to collide, and the high energy that they have been given by the accelerator **turns into a** new particle & anti-particle pair: a B meson and its anti-particle, called a B-bar meson (mesons are particles composed of a quarkand an anti-quark). These mesons undergo radioactive decay within about a picosecond (a trillionth of a second). Because they are quite heavy - their mass is about five times that of the proton - they can **decay i**n numerous ways into different combinations of lighter particles.

Physicists have built a living-room size detector around the collision point in order to detect the lighter particles which **are produced** in the decay of the two mesons. These detectors allow them to identify the types of particles produced, measure their momenta and energies, and trace them to their points of origin to within less than a 10th of a millimeter.

The huge amounts of data collected by the detectors is stored in large databases and analyzed by computer "farms" with many hundreds of computers. Together, BaBar and Belle have produced almost 300 million B mason & B-bar meson pairs, and physicists around the world are hard at work analyzing the mountains of data and publishing their results. 300 million is a large number, but when it comes to some CP violation measurements, it can be barely enough.

**Measuring CP violation**

Physicists detect differences between matter and antimatter and **determine the** strength of CP violation by measuring the ways the B and the B-bar **decay**. For example, decays into particular sets of particles exhibit a peculiar time structure which is different for B and B-**bar decays**.

To expose this behavior, the physicists conduct the following analysis:

First, they select "events" in which they see one of the heavy mesons undergoing the desired decay. This is done by looking at all particle signatures in the detector and determining which combinations of particles may have been produced in the decay of interest, given the constraints imposed by Einstein's theory of relativity.

Next, they analyze the decay products of the other meson **to determine** whether it was a B or a B-bar. This process is called "tagging", and it **makes use of** the fact that B-bar meson decay tends **to produce a** certain particle, such as an electron, whereas the decay of a B **usually produces** the corresponding anti-particle, such as a positron.

Third, by measuring the points of origin of the decay products of the two mesons, they can find the distance between them, which is typically about a quarter of a millimeter. They divide this distance by the velocity with which the mesons move, to obtain the difference between their decay times, known as dt, which is typically about a picosecond.

Finally, they plot the number of events observed in different ranges of dt.

**Observation template**:

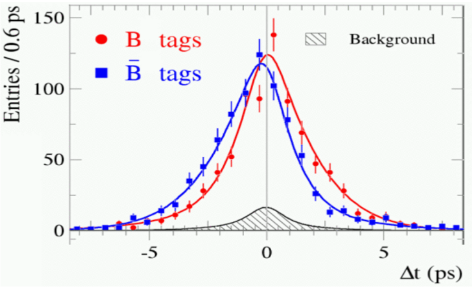


Fig 2. The difference between the red and the blue lines **shows the** difference in how a particle and its antiparticle behave. This is CP violation, and **indicates** that matter and anti-matter are not exactly opposites.

A plot appears to the left, with events in which the other Meson was "tagged" as a B shown in red, and those in which it was "tagged" as a B-bar shown in blue. You can see that the plots are not the same: events with a B tag tend to have a larger dt than events with a B-bar tag. This subtle difference is exactly what we are looking for: this is CP violation, observed for the first time in almost four decades!

Using their results, BaBar and Belle have measured with high precision a parameter called sin(2 beta), which describes part of the mechanism thought to be responsible for CP violation. According to our understanding of particle physics, if sin (2 beta) had been equal to zero, there would have been no CP violation, and matter and antimatter would have been identical. Recalling that the difference between matter and antimatter is necessary for the existence of matter in the universe today, a zero value for sin (2 beta) would have meant that the universe would have been a totally different place, with no stars or planets, not even people to ponder the mysteries of the universe and the underlying laws of physics.

Particle physicists are motivated to study CP violation both because it's an interesting phenomenon in its own right and because it is intimately **related to the** universe as a whole and to our very existence within it.

**What next?**

Having measured sin(2 beta), BaBar and Belle are now collecting more data about B and B-bar decays and measuring more CP violation parameters, to improve our understanding of the difference between matter and antimatter.

More data is coming from other experiments as well. Physicists at the CDFand D0 experiments in Fermilab are also studying the decays of B mesons **produced in** collisions of protons with anti-protons. Additional experiments using the Large Hadron Collider at CERN, which **will produce** B mesons copiously by colliding protons with protons at even higher energies, are scheduled to begin operation in a few years.

There are many open questions that these experiments seek to address. Some of the most intriguing questions are prompted by the fact that the matter-antimatter difference we see in the laboratory appears too small to be solely responsible for all the matter in the universe today. This suggests that there may be additional differences between matter and antimatter, additional sources of CP violation that we have not been able to detect yet, but which could have played an important role during the very early universe, when most matter and antimatter annihilated and a small fraction of the matter survived.

Physicists are searching for these unknown CP violation effects. We never know what exactly this quest will yield, but as has always been the case in the history of particle physics, we expect to learn a great deal about nature in the process. ( For more details Please see Abner Soffer of Colorado State University)

**Notation :**

**Anisotropies Of The Universe And Variable Speed Of Light:**

$G_{13}$ : Category One Of Anisotropies Of The Universe

$G_{14}$ : Category Two Of Anisotropies Of The Universe

$G_{15}$ : Category Three Of Anisotropies Of The Universe

$T_{13}$ : Category One Of Variable Speed Of Light

$T_{14}$ : Category Two Of Electromagnetism

$T_{15}$ :Category Three Of Electromagnetism

**Matter- Antimatter System**

$G_{16}$ : Category One Of Anti Matter

$G_{17}$ : Category Two Of Anti Matter

$G_{18}$ : Category Three Of Anti Matter

$T_{16}$ : Category One Of Matter

$T_{17}$ : Category Two Of Matter

$T_{18}$ : Category Three Of Matter

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$
$(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$ are Dissipation coefficients

**Governing Equations: Of The System Anisotropies Of The World And Variable Speed Of Light:**

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15}$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Governing Equations: System: Strong Nuclear Force And Weak Nuclear Force:**

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18}$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16}$$

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Electro Magnetic Force-Gravity-Strong Nuclear Force-Weak Nuclear Force-**

**The Final Governing Equations**

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}\right]G_{13}$

$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}\right]G_{14}$

$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}\right]G_{15}$

*Where* $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}\right]T_{13}$

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}\right]T_{14}$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}\right]T_{15}$

*Where* $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}$, $\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1)}(T_{14},t)}\right]G_{16}$

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1)}(T_{14},t)}\right]G_{17}$

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1)}(T_{14},t)}\right]G_{18}$

*Where* $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1)}(T_{14},t)}$ are second detrition coefficients for category 1, 2 and 3

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1)}(G,t)}\right]T_{16}$

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1)}(G,t)}\right]T_{17}$

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1)}(G,t)}\right]T_{18}$

*Where* $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1)}(G,t)}$ are second detrition coefficients for category 1, 2 and 3

**Where we suppose**

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$

$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G, t) \leq \ (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14}, t) = (p_i)^{(1)}$

$\lim_{G \to \infty} (b''_i)^{(1)} (G, t) = \ (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T'_{14}|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$ . $(T'_{14}, t)$ and $(T_{14}, t)$ are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$ . It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$ , the first augmentation coefficient would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ :

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)},$ are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ :

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and \ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

**Where we suppose**

(F) $(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16,17,18$

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$

$(b_i'')^{(2)}(G, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)}$

(H) $\lim_{T_2 \to \infty} (a_i'')^{(2)} (T_{17}, t) = (p_i)^{(2)}$

$\lim_{G \to \infty} (b_i'')^{(2)} ((G_{19}), t) = (r_i)^{(2)}$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ :

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t}$

$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the SECOND augmentation coefficient would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [ (a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [ (b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)} ] < 1$$

**Theorem 1:** if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** $G_i(0)\,, T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ , $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ , $T_i(0) = T_i^0 > 0$

**PROOF:**

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i\,,\, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0\,,\, T_i(0) = T_i^0\,,\, G_i^0 \leq (\hat{P}_{13})^{(1)}\,, T_i^0 \leq (\hat{Q}_{13})^{(1)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i\,,\, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0\,,\, T_i(0) = T_i^0\,,\, G_i^0 \leq (\hat{P}_{16})^{(2)}\,, T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying CONCATENATED EQUATIONS into itself .Indeed it is obvious that

 

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} =$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

<u>**Definition of**</u> $\tilde{G}, \tilde{T}$ : $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

 

$(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)})|G_{13}^{(1)} - G_{13}^{(2)}|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$

$G_{13}^{(2)}|(a''_{13})^{(1)}(T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)}(T_{14}^{(2)}, s_{(13)})|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$|G^{(1)} - G^{(2)}|e^{-(\widehat{M}_{13})^{(1)}t} \leq$

$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left((G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)})\right)$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis (34,35,36) the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ and $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From the system it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\}ds_{(13)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(1)}t)} > 0$ for $t > 0$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ and $\left((\widehat{M}_{13})^{(1)}\right)_3$ :

**Remark 3:** if $G_{13}$ is bounded, the same property have also $G_{14}$ and $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)}G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}$ or $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ and $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below.

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$.

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

 

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.

The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions OF THE GLOBAL SYSTEM

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose

$(\widehat{P}_{16})^{(2)}$ $and$ $(\widehat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0)e^{-\left(\frac{(\widehat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{16})^{(2)}$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[((\widehat{Q}_{16})^{(2)} + T_j^0)e^{-\left(\frac{(\widehat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{16})^{(2)}\right] \leq (\widehat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying GLOBAL EQUATIONS into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\quad (\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

$$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$$

$$\int_0^t\{(a'_{16})^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)})|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)}|(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)}(T_{17}^{(2)}, s_{(16)})| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}|e^{-(\widehat{M}_{16})^{(2)}t} \leq$$

$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on

 

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$ :

**Remark 3:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 4**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below.

**Remark 5:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$.

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded.

The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

**Behavior of the solutions OF THE GLOBAL SYSTEM:**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

(a) $\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14}, t) + (a_{14}'')^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G, t) - (b_{14}'')^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ :

(b) By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ :

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :-

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)},\ if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)},\ if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)},\ if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined respectively

Then the solution of GLOBAL CONCATENATED EQUATIONS satisfies the inequalities

$$G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq \right.$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of GLOBAL EQUATIONS**

If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ :

(d) $\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big) - (b''_{17})^{(2)}\big((G_{19}),t\big) \leq -(\tau_1)^{(2)}$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ :

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ :

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

**Definition of** $(m_1)^{(2)}$ , $(m_2)^{(2)}$ , $(\mu_1)^{(2)}, (\mu_2)^{(2)}$ :-

(f) If we define $(m_1)^{(2)}$ , $(m_2)^{(2)}$ , $(\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)},$ ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$,

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},$ ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$

and analogously

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},$ ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$ , ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T^0_{16}}{T^0_{17}}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},$ ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$

Then the solution of GLOBAL EQUATIONS satisfies the inequalities

$$G^0_{16}e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{16}(t) \leq G^0_{16}e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined

$$\frac{1}{(m_1)^{(2)}}G^0_{16}e^{((S_1)^{(2)}-(p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}}G^0_{16}e^{(S_1)^{(2)}t}$$

$$\Bigg(\frac{(a_{18})^{(2)}G^0_{16}}{(m_1)^{(2)}((S_1)^{(2)}-(p_{16})^{(2)}-(S_2)^{(2)})}\Big[e^{((S_1)^{(2)}-(p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\Big] + G^0_{18}e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)}G^0_{16}}{(m_2)^{(2)}((S_1)^{(2)}-(a'_{18})^{(2)})}\big[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\big] + G^0_{18}e^{-(a'_{18})^{(2)}t}\Bigg)$$

$$\boxed{T^0_{16}e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T^0_{16}e^{((R_1)^{(2)}+(r_{16})^{(2)})t}}$$

$$\frac{1}{(\mu_1)^{(2)}}T^0_{16}e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}}T^0_{16}e^{((R_1)^{(2)}+(r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)}T^0_{16}}{(\mu_1)^{(2)}((R_1)^{(2)}-(b'_{18})^{(2)})}\Big[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\Big] + T^0_{18}e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)}T^0_{16}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\Big[e^{((R_1)^{(2)}+(r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\Big] + T^0_{18}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$ :-

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**PROOF :** From GLOBAL EQUATSIONS we obtain

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

**Definition of** $\nu^{(1)}$ :- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}$, $\boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in CONCATENATED SYSTEM OF EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**PROOF** : From GLOBAL EQUATIONS we obtain (PLEASE REFER PART ONE OF THE PAPER)

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case,

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

 

**Definition of** $\nu^{(2)}(t)$ :-

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in GLOBAL SOLUTIONS we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

We can prove the following

**Theorem 3:** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied , then the system

If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ ,

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

$(a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right] G_{13} = 0$

$(a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right] G_{14} = 0$

$(a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right] G_{15} = 0$

$(b_{13})^{(1)} T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ] T_{13} = 0$

$(b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ] T_{14} = 0$

$(b_{15})^{(1)} T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ] T_{15} = 0$

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)} G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right] G_{16} = 0$

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0$

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0$

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0$

has a unique positive solution , which is an equilibrium solution for the GLOBAL SYSTEM

**Proof:**

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

(a) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Definition and uniqueness of** $\mathrm{T}^*_{14}$ :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]}$ , $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$

**Definition and uniqueness of** $\mathrm{T}^*_{17}$ :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $\mathrm{T}^*_{17}$ for which $f(\mathrm{T}^*_{17}) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}\mathrm{G}_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(\mathrm{T}^*_{17})]}$ , $G_{18} = \frac{(a_{18})^{(2)}\mathrm{G}_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(\mathrm{T}^*_{17})]}$

(c) By the same argument, the equations(CONCATENATED SET OF THE GLOBAL SYSTEM) admit solutions $G_{13}, G_{14}$ if

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - \left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

(d) By the same argument, the equations (SOLUTIONAL EQUATIONS OF THE GLOBAL EQUATIONS) admit solutions $G_{16}, G_{17}$ if

$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - \left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$

 

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^*$ such that $\varphi((G_{19})^*) = 0$

Finally we obtain the unique solution

$G_{14}^*$ given by $\varphi(G^*) = 0$ , $T_{14}^*$ given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution THE GLOBAL SYSTEM

$G_{17}^*$ given by $\varphi((G_{19})^*) = 0$ , $T_{17}^*$ given by $f(T_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]}$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL SYSTEM

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(1)}$ *and* $(b''_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:**_Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14}$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{17})^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \quad \frac{\partial (b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij}$$

 

taking into account equations (SOLUTIONAL EQUATIONS TO THE GLOBAL EQUATIONS) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right)$$

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$
$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$
$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$
$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$
$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$
$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$
$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G^*_{16}\right)\right]$$
$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T^*_{17} + (b_{17})^{(2)}s_{(16),(17)}T^*_{17}\right)$$
$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G^*_{17}\right)$$
$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T^*_{17} + (b_{17})^{(2)}s_{(16),(16)}T^*_{16}\right)$$
$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$
$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$
$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}G_{18}$$
$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}G^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G^*_{16}\right)$$
$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}T^*_{17} + (b_{17})^{(2)}s_{(16),(18)}T^*_{16}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

**Acknowledgments:**

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Article Abstracts, NASA Pages For Figures, Stanford Encyclopedia, Nature review articles, Face Book appraisals for the common man, ask a physicist column, Journal Reviews, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, We regret with great deal of compunction, contrition, and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

**References**

1. A Haimovici: “On the growth of a two species ecological system divided on age groups”. Tensor, Vol 37 (1982),Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday

2. Fritjof Capra: “The web of life” Flamingo, Harper Collins See "Dissipative structures” pages 172-188

3. Heylighen F. (2001): "The Science of Self-organization and Adaptivity", in L. D. Kiel, (ed), Knowledge Management, Organizational Intelligence and Learning, and Complexity, in: The Encyclopedia of Life Support Systems ((EOLSS), (Eolss Publishers, Oxford)

4. Matsui, T, H. Masunaga, S. M. Kreidenweis, R. A. Pielke Sr., W.-K, Tao, M. Chin, and Y. J Kaufman (2006),: “Satellite-based assessment of marine low cloud variability associated with aerosol, atmospheric stability, and the diurnal cycle”, J. Geophys. Res., 111, D17204,

5. Stevens, B, G. Feingold, W.R. Cotton and R.L. Walko, “Elements of the microphysical structure of numerically simulated nonprecipitating stratocumulus” J. Atmos. Sci., 53, 980-1006

6. Feingold, G, Koren, I; Wang, HL; Xue, HW; Brewer, WA (2010), “Precipitation-generated oscillations in open cellular cloud fields” *Nature, 466* (7308) 849-852, doi: 10.1038/nature09314, Published 12-Aug 2010

7. R Wood : “The rate of loss of cloud droplets by coalescence in warm clouds” J.Geophys. Res., 111, doi: 10.1029/2006JD007553, 2006

8. H. Rund : “The Differential Geometry of Finsler Spaces”, Springer-Verlag, Berlin, 1959

9. A. Dold, “Lectures on Algebraic Topology”, 1972, Springer-Verlag

10. S Levin “Some Mathematical questions in Biology vii ,Lectures on Mathematics in life sciences, vol 8” The American Mathematical society, Providence , Rhode island 1976

11. Davies, Paul (1986), The Forces of Nature, Cambridge Univ. Press 2nd ed.

12. Feynman, Richard (1967), The Character of Physical Law, MIT Press, ISBN 0-262-56003-8

13. Schumm, Bruce A. (2004), Deep Down Things, Johns Hopkins University Press

14. Weinberg, Steven (1993), The First Three Minutes*: A Modern View of the Origin of the Universe*, Basic Books, ISBN 0-465-02437-8

15. Weinberg, Steven (1994), Dreams of a Final Theory, Basic Books, ISBN 0-679-74408-8

16. Padmanabhan, T. (1998), After The First Three Minutes: The Story of Our Universe, Cambridge Univ. Press, ISBN 0-521-62972-1

17. Perkins, Donald H. (2000), Introduction to High Energy Physics, Cambridge Univ. Press, ISBN 0-521-62196-8

ISSN 2224-719X (Paper) ISSN 2225-0638 (Online)
Vol 6, 2012

18. Riazuddin (December 29, 2009). "Non-standard interactions". (Islamabad: Riazuddin, Head of High-Energy Theory Group at National Center for Physics) (1): 1–25. Retrieved Saturday, March 19, 2011.

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