Operational Readiness of a Complex System under Different Weather Conditions

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Abstract

The authors have considered a complex system composed of single repairable unit and operating in '*n*' multiple environmental conditions. The system may either go to complete breakdown shape due to common cause failure or it may go to any one of the abnormal weather operation states. The system is repairable from the degraded state. The failed system is assumed to be repaired back to its normal weather and abnormal weather operation states.

Keywords: Availability/Reliability Analysis, Repairable Parallel System, Laplace transform, Ergodic system , Abel Lemma, supplementary variable technique and Steady- state behavior.

1. Introduction

The authors have therefore considered a complex system composed of single repairable unit and operating in '*n*' multiple environmental conditions. The system may either go to complete breakdown shape due to common cause failure or it may go to any one of the abnormal weather operation states. The system is repairable from the degraded state. The failed system is assumed to be repaired back to its normal weather and abnormal weather operation states. The failure rates are exponential while repair rates follow general time distributions.

In general there are 3-types of weather condition i.e. cold, hot and warm standby. Cold standby means that the redundant components cannot fail while they are waiting. Earlier many researchers [1, 4, 14] have discussed when an operative unit fails, its repair starts immediately but in practical problem it may not possible always. In practice, system do not always fail with major breakdown. Igichart and Igichart and lemoline; Arti R.(1993); Singh, S.B.(1998) developed various mathematical models consists two types of failure namely major and minor. Using Supplementary variables technique Laplace transforms of various state probabilities have been evaluated. Numerical examples have also been added to highlight the important results.

2. Assumption

(*i*). Initially at $t = 0$, the system operate in its normal weather mode.

(*ii*). Failures are statistically independent.

(*iii*). Repair facility is available at every state of transition.

(*iv*). Repairs follow general time distribution while failures follow exponential time distribution.

(*v*). The system has three modes *viz*; good, degraded and failed.

(*vi*). The system cannot move from one abnormal weather operation state to another.

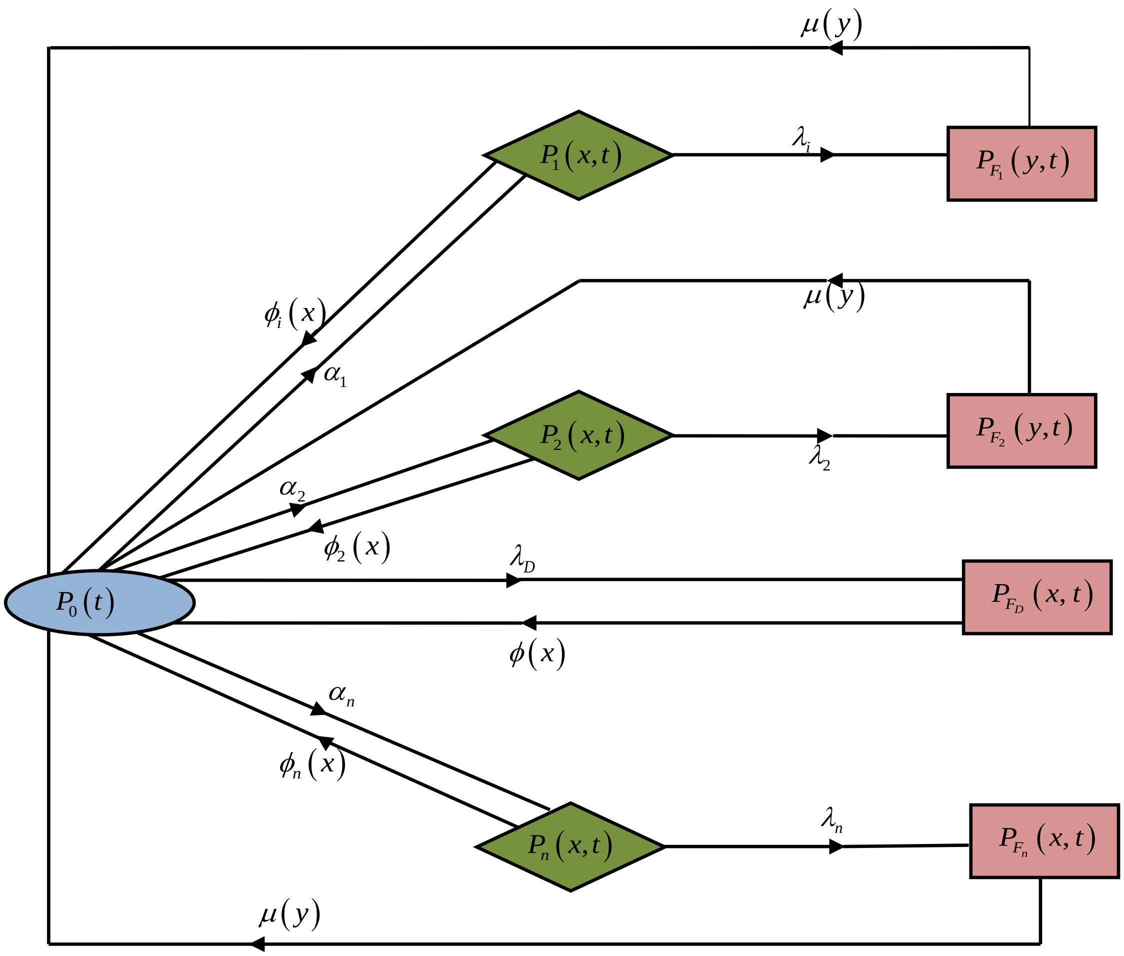
(*vii*). After repair, system works like a new one

3. Notations

α_i	:	Constant failure rates from state P_0 to $P_i \sum_i \alpha_i = \alpha$
λ_i	:	Constant failure rates from state P_i to $P_{F_i} \sum_i \lambda_i = \lambda$
$\overline{f}(s)$	:	Laplace transform of $f(t)$
λ_D	:	Constant failure rate from state P_0 to P_{F_D}
$\phi_n(x) \backslash \phi(x) \backslash \mu(y)$	:	General repair rates from state P_i to P_0, P_{F_D}, P_{F_i} to P_0
$P_0(t)$	:	The probability that at time t, the system is in good state.
$P_i(x,t)\Delta$	:	The probability that at time t, the system is in degraded state due to the abnormal weather operation and elapsed repair time lies in interval $(x, x+\Delta).i = 1, 2, ..., n$
$P_{F_i}(y,t)\Delta$	:	The probability that at time t, the system is in failed state and elapsed repair time lies in interval $(y, y+\Delta)$

$P_{F_D}(x,t)\Delta$: The probability that at time t, the system is in failed state due to common cause failure and elapsed repair time lies in interval $(x, x+\Delta)$.

Figure 1 represents the state transition diagram of the system



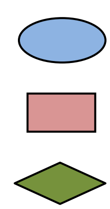


Fig. 1: Transition State Diagram

4. Formulation of the Mathematical Model

By using the probability consideration and continuity arguments, we get the following difference-differential equations governing the behaviour of the system:

$$\left[\frac{\partial}{\partial t}+\sum \alpha_i+\lambda_D\right]P_0(t)=\sum_i \int_0^\infty P_i(x,t)\phi_i(x)dx \quad + \quad +\sum_i \int_0^\infty P_{F_i}(y,t)\mu(y)dy+\int_0^\infty P_{F_D}(x,t)\phi(x)dx \tag{1}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\phi_i(x)+\lambda_i\right]P_i(x,t)=0,\ \forall i=1,2,3----n \tag{2}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu(y)\right]P_{F_i}(y,t)=0,\ \forall i=1,2,3----n \tag{3}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\phi(x)\right]P_{F_D}(x,t)=0 \tag{4}$$

4.1 Boundary Conditions

$$P_i(0,t)=\alpha_i P_0(t),\ \forall i=1,2,-----,n \tag{5}$$

$$P_{F_i}(0,t)=\lambda_i P_i(t),\ \forall i=1,2,-----,n \tag{6}$$

$$P_{F_D}(0,t)=\lambda_D P_0(t) \tag{7}$$

4.2 Initial Conditions

$P_0(0)=1$, Otherwise zero (8)

5. Solution of the Model

On taking Laplace transform of equations (1) through (7) by using initial conditions one may obtain:

$$\left[s+\sum_i \alpha_i+\lambda_D\right]\bar{P}_0(s)=1+\sum_i \int_0^\infty \bar{P}_i(x,s)\phi_i(x)dx+\sum_i \int_0^\infty P_{F_i}(y,s)\mu(y)dy+\int_0^\infty P_{F_D}(x,s)\phi(x)dx \tag{9}$$

$$\left[\frac{\partial}{\partial x}+s+\phi_i(x)+\lambda\right]\bar{P}_i(x,s)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial y}+s+\mu(y)\right]\bar{P}_{F_i}(y,s)=0 \tag{11}$$

$$\left[\frac{\partial}{\partial x}+s+\phi(x)\right]\bar{P}_{F_D}(x,s)=0 \tag{12}$$

$$\bar{P}_i(0,s) = \alpha_i \bar{P}_0(s) \tag{13}$$

$$\bar{P}_{F_i}(0,s) = \lambda_i \bar{P}_0(s) \tag{14}$$

$$\bar{P}_{F_D}(0,s) = \lambda_D \bar{P}_0(s) \tag{15}$$

Integrating the equation (10) and using (13), one may get

$$\bar{P}_i(x,s) = \alpha \bar{P}_0(s) \exp\left[-(s+\lambda_i)x - \int_0^x \phi(x)\,dx\right]$$

$$\Rightarrow \qquad \bar{P}_i(s) = \alpha \bar{P}_0(s) D_{\phi_i}(s+\lambda_i) \tag{16}$$

Where, $D_{\phi_i}(s+\lambda_i) = \dfrac{1-\bar{S}_{\phi_i}(s+\lambda_i)}{(s+\lambda_i)}$

Integrating (11) and using (14) and (10), one may obtain

$$\bar{P}_{F_i}(y,s) = \lambda \bar{P}_i(s) \exp\left[-sy - \int_0^y \mu(y)\,dy\right]$$

$$\Rightarrow \qquad \bar{P}_{F_i}(s) = \lambda \bar{P}_0(s) D_{\phi_i}(s+\lambda_i) D_\mu(s) \tag{17}$$

Integrating (12) and using (15), one may obtain

$$\bar{P}_{F_D}(x,s) = \lambda_D \bar{P}_0(s) \exp\left[-sx - \int_0^y \phi(x)\,dx\right]$$

$$\Rightarrow \qquad \bar{P}_{F_D}(s) = \lambda_D \bar{P}_0(s) D_\phi(s) \tag{18}$$

By using relevant relations, (9) becomes

$$\bar{P}_0(s) = \frac{1}{A(s)} \tag{19}$$

Where, $A(s) = s + \alpha + \lambda_D - \sum_i \alpha_i \left[\bar{S}_{\phi_i}(s+\lambda_i) + \lambda D_{\phi_i}(s+\lambda_i)\bar{S}_\mu(s)\right] - \lambda_D \bar{S}_\phi(s)$ (20)

Thus, finally we have

$$\bar{P}_0(s) = \frac{1}{A(s)} \tag{21}$$

$$\bar{P}_i(s) = \frac{\alpha}{A(s)} D_{\phi_i}(s+\lambda), \ \forall i = 1, 2 --- n \tag{22}$$

$$\bar{P}_{F_i}(s) = \frac{\lambda\alpha}{A(s)} D_{\phi_i}(s+\lambda) D_\mu(s), \forall i = 1, 2 --- n \tag{23}$$

$$\bar{P}_{F_D}(s) = \frac{\lambda_D}{A(s)} D_\phi(s) \tag{24}$$

It is interesting to note that sum of equations (21) through (24) $= \dfrac{1}{s}$

6. Ergodic behaviour of the system

By using Abel's Lemma; $\lim_{s \to 0} s\bar{F}(s) = \lim_{t \to \infty} F(t) = F(say)$, provided the limit on R.H.S. exists, we obtain the following time independent probabilities from equations (21) through (24) as

$$P_0 = \frac{1}{A'(0)} \tag{25}$$

$$P_i = \frac{\alpha}{A'(0)} D_{\phi_i}(\lambda_i) \tag{26}$$

$$P_{F_i} = \frac{\lambda\alpha}{A'(0)} D_{\phi_i}(\lambda_i) M_\mu \tag{27}$$

$$P_{F_D} = \frac{\lambda}{A'(0)} M_\phi \tag{28}$$

Where, $$A'(0) = \left[\frac{d}{ds} A(s)\right]_{s=0} \tag{29}$$

M_k = Mean time to repair k^{th} unit

7. Evaluation of up and down state probabilities

We have,

$$\bar{P}_{up}(s) = \frac{1}{s+\alpha+\lambda_D}\left[1 + \frac{\alpha}{s+\lambda}\right] \tag{30}$$

Also, $$\bar{P}_{down}(s) = \frac{1}{s} - \bar{P}_{up}(s) \tag{31}$$

8. Reliability Analysis

We have for the system,

$$R(s) = \frac{1}{s+\alpha+\lambda_D} \tag{32}$$

Inverting this, we have

$$R(t) = \exp\left[-(\alpha+\lambda_D)t\right] \tag{33}$$

9. M.T.T.F. Evaluation

We know that

$$M.T.T.F. = \lim_{s \to 0} \bar{R}(s) = \frac{1}{\alpha+\lambda_D} \tag{34}$$

10. Numerical Computation

For a numerical example, let us consider_

$\alpha = 0.001, \lambda_D = 0.002$ and putting in equation (33) one obtains

$$R(t) = \exp(-0.003t)$$

and $$M.T.T.F. = \frac{1}{\alpha + 0.02}$$

11. Interpretation

11.1 Table 1 and Figure 2; forecast the reliability of the model w.r.t. time and their corresponding curve.

S.No.	*t*	R(t) = exp(-0.003 *t*)
1	0	1
2	1	0.997004496
3	2	0.994017964
4	3	0.991040379
5	4	0.988071713
6	5	0.98511194
7	6	0.982161032
8	7	0.979218965
9	8	0.97628571
10	9	0.973361242
11	10	0.970445534
12	11	0.96753856
13	12	0.964640293
14	13	0.961750709
15	14	0.958869781
16	15	0.955997482

Table 1: Reliability function as time

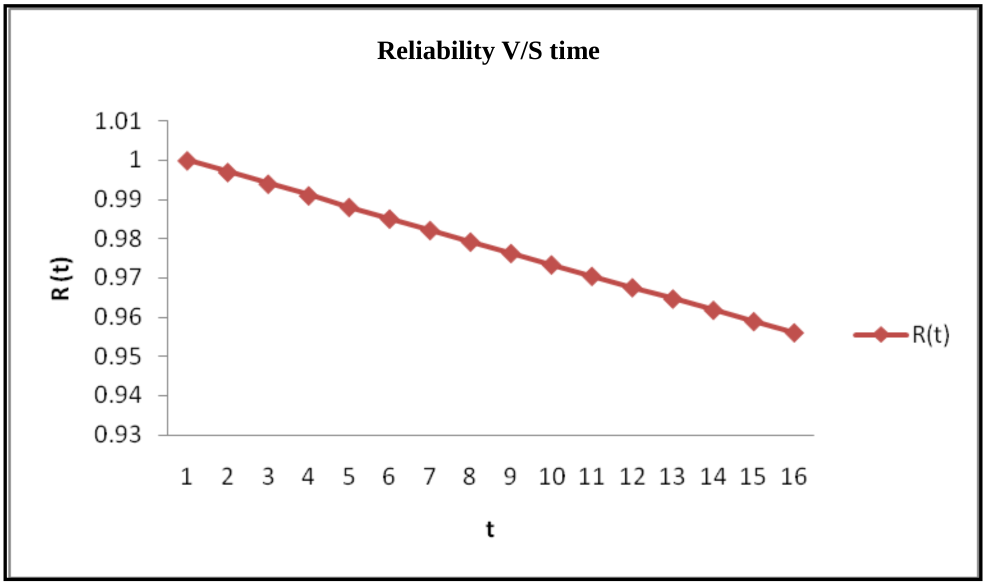


Figure 2: Reliability function as time

11.2 Table 2 and figure 3 reveal that as α increases, MTTF goes on decreases and ultimately the variation becomes negligible, and their corresponding curve

S.No.	α	M.T.T.F	M.T.T.F	M.T.T.F	M.T.T.F
		λD = 0.003	λD = 0.005	λD = 0.007	λD = 0.009
1	0.001	250	166.666667	125	100
2	0.002	200	142.857143	111.11111	90.909091
3	0.003	166.6666667	125	100	83.333333
4	0.004	142.8571429	111.111111	90.909091	76.923077
5	0.005	125	100	83.333333	71.428571
6	0.006	111.1111111	90.9090909	76.923077	66.666667
7	0.007	100	83.3333333	71.428571	62.5
8	0.008	90.90909091	76.9230769	66.666667	58.823529
9	0.009	83.33333333	71.4285714	62.5	55.555556
10	0.01	76.92307692	66.6666667	58.823529	52.631579
11	0.011	71.42857143	62.5	55.555556	50
12	0.012	66.66666667	58.8235294	52.631579	47.619048
13	0.013	62.5	55.5555556	50	45.454545
14	0.014	58.82352941	52.6315789	47.619048	43.478261
15	0.015	55.55555556	50	45.454545	41.666667
16	0.016	52.63157895	47.6190476	43.478261	40

Table 2: MTTF and different values of failure rate

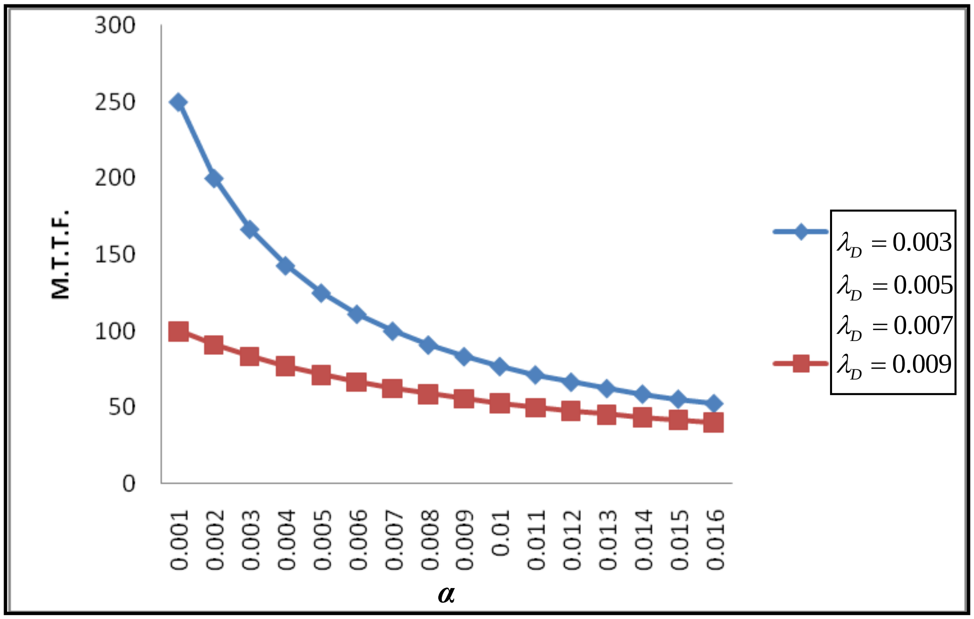


Figure 3: MTTF and different values of failure rate

12. Conclusion

Table 1 and Figure 2 provide information how reliability of the complex engineering repairable system changes with respect to the time when failure rate increases reliability of the system decreases.

Table 2 and Figure 3 reveal that as α increases, MTTF goes on decreases and ultimately the variation becomes negligible.

The further research area is widely open, where one may think of the application of MTTF and sensitivity analysis.

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