Bicriteria in n x 2 Flow Shop Scheduling Under Specified Rental Policy, Processing Time and Setup Time Each Associated with probabilities Including Job Block

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Abstract

This paper is an attempt to obtains an optimal solution for minimizing the bicriteria taken as minimizing the total rental cost of the machines subject to obtain the minimum makespan for n jobs 2 machines flowshop problem in which the processing times and independent set up times are associated with probabilities including the job block concept. A heuristic approach method to find optimal or near optimal sequence has been discussed. The proposed method is very simple and easy to understand and also provide an important tool for the decision makers. A computer programme followed by a numerical illustration is give to clarify the algorithm.

Keywords: Flowshop Scheduling, Heuristic, Processing Time, Set Up Time, Rental Cost and Job Block.

1. Introduction

In flowshop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machines will optimize some well defined criteria. Every job will go on these machines in a fixed order of machines. The research into flow shop problems has drawn a great attention in the last decades with the aim to increase the effectiveness of industrial production. Recently scheduling, so as to approximate more than one criterion received considerable attention. The bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. The bicriteria scheduling problems are generally divided into three classes. In the first class, the problem involves minimizing one criterion subject to the constraint that the other criterion to be optimized. In the second class, both criteria are considered equally important and the problem involves finding efficient schedules. In the third class, both criteria are weighted differently and an objective function as the sum of the weighted functions is defined. The problem considered in this paper belongs to the first class.

Smith (1956) whose work is one of the earliest considered minimization of mean flow time and maximum tardiness. Wassenhove and Gelders (1980) studied minimization of maximum tardiness and mean flow time explicitly as objective. Some of the noteworthy heuristic approaches are due to Sen et al. (1983), Dileepan et al.(1988), Chandersekharan (1992), Bagga(1969), Bhambani (1997), Narain (2006), Chakarvrthy(1999), Singh T.P. et al. (2005), and Gupta et al.(2011). Setup includes work to prepare the machine, process or bench for product parts or the cycle. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant. The basic concept of equivalent job for a job – block has been investigated by Maggu & Das (1977) and established an equivalent job-block theorem. The idea of job-block has practical significance to create a balance between a cost of providing priority in service to the customer and cost of giving service with non-priority. The two criteria of minimizing the maximum utilization of the machines or rental cost and minimizing the maximum makespan are one of the combinations of our objective function reflecting the performance measure.

2. Practical Situation

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology. Further the priority of one job over the other may be significant due to the relative importance of the jobs. It may be because of urgency or demand of that particular job. Hence, the job block criteria become important.

3. Notations

S: Sequence of jobs 1,2,3,….,n

S_k: Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -------

M_j: Machine j, j= 1,2

M: Minimum makespan

a_{ij}: Processing time of i^{th} job on machine M_j

p_{ij}: Probability associated to the processing time a_{ij}

s_{ij}: Set up time of i^{th} job on machine M_j

q_{ij}: Probability associated to the set up time s_{ij}

A_{ij}: Expected processing time of i^{th} job on machine M_j

S_{ij}: Expected set up time of i^{th} job on machine M_j

A'_{ij} : Expected flow time of i^{th} job on machine M_j

β: Equivalent job for job – block

C_i: Rental cost of i^{th} machine

$L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j

$t'_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $E_j(S_k)$

$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k

$U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $E_j(S_k)$

$R(S_k)$: Total rental cost for the sequence S_k of all machine

3.1 Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as :

$$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + a_{ij} \times p_{ij} + s_{(i-1)j} \times q_{(i-1)j} \quad \text{for} \quad j \geq 2.$$

$$= \max(t_{i-1,j}, t_{i,j-1}) + A_{i,j} + S_{(i-1),j},$$

where ,$A_{i,,j}$= Expected processing time of i^{th} job on j^{th} machine

$S_{i,j}$= Expected setup time of i^{th} job on j^{th} machine.

3.2 Definition

Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by $t'_{i,j}$ and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j},$$

Also $t'_{i,j} = \max(t_{i,j-1}, t_{i-1,j}) + A_{i,j} + S_{i-1,j}$.

4. Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on 1^{st} machine.

5. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let A_{ij} be the expected processing time and $S_{i,j}$ be the expected setup time of i^{th} job on j^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1				Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}

4	a_{41}	p_{41}	s_{41}	q_{41}	a_{42}	p_{42}	s_{42}	q_{42}
5	a_{51}	p_{51}	s_{51}	q_{51}	a_{52}	p_{52}	s_{52}	q_{52}

(Table 1)

Mathematically, the problem is stated as

Minimize $U_j\left(S_k\right)$ and

Minimize $R\left(S_k\right)=\sum_{i=1}^{n} A_{i1}\times C_1 + U_j\left(S_k\right)\times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

6. Theorem

The processing of jobs on M_2 at time $L_2=\sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof. Let $t'_{i,2}$ be the completion time of i^{th} job on machine M_2 when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let P(n) : $t'_{n,2}=t_{n,2}$

Basic step: For n = 1, j =2;

$$t'_{1,2}=L_2+\sum_{k=1}^{1} A_{k,2}+\sum_{k=1}^{1-1} S_{k,2}=\sum_{k=1}^{1} I_{k,2}+\sum_{k=1}^{1} A_{k,2}+\sum_{k=1}^{1-1} S_{k,2}$$
$$=\sum_{k=1}^{1} I_{k,2}+A_{1,2}=I_{1,2}+A_{1,2}=A_{1,1}+A_{1,2}=t_{1,2}$$

$\therefore$ P(1) is true.

Induction Step: Let P(m) be true, i.e., $t'_{m,2}=t_{m,2}$

Now we shall show that P(m+1) is also true, i.e., $t'_{m+1,2}=t_{m+1,2}$

Since $t'_{m+1,2}=\max(t_{m+1,1},t'_{m,2})+A_{m+1,2}+S_{m,2}$

$$=\max\left(t_{m+1,1},L_2+\sum_{i=1}^{m} A_{i,2}+\sum_{i=1}^{m-1} S_{i,2}\right)+A_{m+1,2}+S_{m,2}$$

$$=\max\left(t_{m+1,1},\left(\sum_{i=1}^{m} I_{i,2}+\sum_{i=1}^{m} A_{i,2}+\sum_{i=1}^{m-1} S_{i,2}\right)+I_{m+1}\right)+A_{m+1,2}+S_{m,2}$$

$$=\max\left(t_{m+1,1},t_{m,2}+I_{m+1}\right)+A_{m+1,2}+S_{m,2}$$

$$=\max\left(t_{m+1,1},t'_{m,2}+\max\left(\left(t_{m+1,1}-t_{m,2}\right),0\right)\right)+A_{m+1,2}+S_{m,2} \qquad \text{(By Assumption)}$$
$$=\max\left(t_{m+1,1},t_{m,2}\right)+A_{m+1,2}+S_{m,2}$$
$$=t_{m+1,2}$$

Therefore, *P(m+1)* is true whenever *P(m)* is true.

Hence by Principle of Mathematical Induction P(n) is true for all n i.e. $t'_{n,2}=t_{n,2}$ for all n.

*Remark***:** If M_2 starts processing the job at $L_2=t_{n,2}-\sum_{i=1}^{n} A_{i,2}-\sum_{i=1}^{n-1} S_{i,2}$, then total time elapsed $t_{n,2}$ is not altered and M_2 is engaged for minimum time. If M_2 starts processing the jobs at time L_2 then it can be easily shown that $t_{n,2}=L_2+\sum_{i=1}^{n} A_{i,2}+\sum_{i=1}^{n-1} S_{i,2}$.

7. Algorithm

Step 1: Calculate the expected processing times and expected set up times as follows

$$A_{ij} = a_{ij} \times p_{ij} \quad \text{and} \quad S_{ij} = s_{ij} \times q_{ij} \qquad \forall i, j$$

Step 2: Calculate the expected flow time for the two machines A and B as follows

$$A'_{i1} = A_{i1} - S_{i2} \quad \text{and} \quad A'_{i2} = A_{i2} - S_{i1} \quad \forall i.$$

Step 3: Take equivalent job $\beta(k,m)$ and calculate the processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ on the guide lines of Maggu and Das (1977) as follows

$$A'_{\beta 1} = A'_{k1} + A'_{m1} - \min\left(A'_{m1}, A'_{k2}\right), \; A'_{\beta 2} = A'_{k2} + A'_{m2} - \min\left(A'_{m1}, A'_{k2}\right).$$

Step 4: Define a new reduces problem with the processing times A'_{i1} and A'_{i2} as defined in step 2 and jobs (*k*, *m*) are replaced by single equivalent job β with processing time $A'_{\beta 1}$ and $A'_{\beta 2}$ as defined in step 3.

Step 5: Using Johnson's technique [1] obtain all the sequences S_k having minimum elapsed time. Let these be S_1, S_2, ----------.

Step 6 : Compute total elapsed time $t_{n2}(S_k)$, $k = 1,2,3,$----, by preparing in-out tables for S_k.

Step 7 : Compute $L_2(S_k)$ for each sequence S_k as $L_2(S_k) = t_{n,2}(S_k) - \sum_{i=1}^{n} A_{i,2}(S_k) - \sum_{i=1}^{n-1} S_{i,2}(S_k)$.

Step 8 : Find utilization time of 2nd machine for each sequence S_k as $U_2(S_k) = t_{n2}(S_k) - L_2(S_k)$.

Step 9 : Find minimum of $\{(U_2(S_k)\}$; $k = 1,2,3,\ldots.$

Let it for sequence S_p. Then S_p is the optimal sequence and minimum rental cost for the sequence S_p is

$$R(S_p) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_p) \times C_2.$$

8. Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
int n,j, f=1;
float a1[16],b1[16],g[16],h[16],sa1[16],sb1[16], macha[16],machb[16],cost_a,cost_b,cost;
int group[16];//variables to store two job blocks
float minval,minv,maxv, gbeta=0.0,hbeta=0.0;
void main()
{       clrscr();
        int a[16],b[16],sa[16],sb[16],j[16];
        float p[16],q[16],u[16],v[16], maxv;
        cout<<"How many Jobs (<=15) : "; cin>>n;
if(n<1 || n>15)
{       cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting"; getch(); exit(0); }
for(int i=1;i<=n;i++)
{       j[i]=i;
                cout<<"\nEnter the processing time and its probability, Setup time and its probability of "<<i<<" job for
        machine A : ";
        cin>>a[i]>>p[i]>>sa[i]>>u[i];
                cout<<"\nEnter the processing time and its probability, Setup time and its probability of "<<i<<" job for
        machine B : ";
```

```
        cin>>b[i]>>q[i]>>sb[i]>>v[i];
//Calculate the expected processing times and set up times of the jobs for the machines:
        a1[i] = a[i]*p[i];b1[i] = b[i]*q[i]; sa1[i] = sa[i]*u[i];sb1[i] = sb[i]*v[i];}
        cout<<"\nEnter the rental cost of Machine A:"; cin>>cost_a;
        cout<<"\nEnter the rental cost of Machine B:"; cin>>cost_b;
        cout<<endl<<"Expected processing time of machine A and B: \n";
for(i=1;i<=n;i++)
        {cout<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t"; cout<<sa1[i]<<"\t"<<sb1[i]; cout<<endl; }
//Calculate the final expected processing time for machines
        cout<<endl<<"Final expected processing time of machin A and B:\n";
        for(i=1;i<=n;i++)
        {       g[i]=a1[i]-sb1[i];h[i]=b1[i]-sa1[i]; }
        for(i=1;i<=n;i++)
        {cout<<"\n\n"<<j[i]<<"\t"<<g[i]<<"\t"<<h[i]; cout<<endl; }
        cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):"; cin>>group[0]>>group[1];
 //calculate G_Beta and H_Beta
        if(g[group[1]]<h[group[0]])
        {       minv=g[group[1]];}
else
        {       minv=h[group[0]];}
        gbeta=g[group[0]]+g[group[1]]-minv,hbeta=h[group[0]]+h[group[1]]-minv;
        cout<<endl<<endl<<"G_Beta="<<gbeta;
        cout<<endl<<"H_Beta="<<hbeta;
        int j1[16]; float g1[16],h1[16];
for(i=1;i<=n;i++)
        {if(j[i]==group[0]||j[i]==group[1])
        {       f--; }
else
        { j1[f]=j[i];}
        f++; }
j1[n-1]=17;
for(i=1;i<=n-2;i++)
        {g1[i]=g[j1[i]];h1[i]=h[j1[i]];}
g1[n-1]=gbeta;h1[n-1]=hbeta;
        cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
        {cout<<j1[i]<<"\t"<<g1[i]<<"\t"<<h1[i]<<endl;}
float mingh[16];
char ch[16];
```

```
for(i=1;i<=n-1;i++)
	{if(g1[i]<h1[i])
	{	mingh[i]=g1[i];   ch[i]='g';}
else
	{	mingh[i]=h1[i];   ch[i]='h'; }}
for(i=1;i<=n-1;i++)
{for(int j=1;j<=n-1;j++)
if(mingh[i]<mingh[j])
	{float temp=mingh[i]; int temp1=j1[i]; char d=ch[i];
	mingh[i]=mingh[j]; j1[i]=j1[j]; ch[i]=ch[j];
	mingh[j]=temp; j1[j]=temp1; ch[j]=d;}}
// calculate beta scheduling
float sbeta[16]; int t=1,s=0;
for(i=1;i<=n-1;i++)
	{if(ch[i]=='h')
	{	sbeta[(n-s-1)]=j1[i];    s++;}
else
if(ch[i]=='g')
	{	sbeta[t]=j1[i]; t++;}}
int arr1[16], m=1; cout<<endl<<endl<<"Job Scheduling:"<<"\t";
for(i=1;i<=n-1;i++)
	{	if(sbeta[i]==17)
	{ arr1[m]=group[0];    arr1[m+1]=group[1]; cout<<group[0]<<" " <<group[1]<<" "; m=m+2; continue;}
else
	{cout<<sbeta[i]<<" "; arr1[m]=sbeta[i]; m++;}}
//calculating total computation sequence
	float time=0.0,macha1[15],machb1[15]; macha[1]=time+a1[arr1[1]];
	for(i=2;i<=n;i++)
	{macha1[i]=macha[i-1]+sa1[arr1[i-1]]; macha[i]=macha[i-1]+sa1[arr1[i-1]]+a1[arr1[i]];}
	machb[1]=macha[1]+b1[arr1[1]];
//displaying solution
cout<<"\n\n\n\n\n\t\t\t	#####THE SOLUTION##### ";
cout<<"\n\n\t***************************************************************";
cout<<"\n\n\n\t	Optimal Sequence is : ";
for(i=1;i<=n;i++)
cout<<" "<<arr1[i]; cout<<endl<<endl<<"In-Out Table is:"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"\t"<<"Machine M2"<<endl;
cout<<arr1[1]<<"\t"<<time<<"--"<<macha[1]<<" \t"<<"\t"<<macha[1]<<"--"<<machb[1]<<" \t"<<"\t"<<endl;
for(i=2;i<=n;i++)
```

```
    {if((machb[i-1]+sb1[arr1[i-1]])>macha[i])
    maxv=(machb[i-1]+sb1[arr1[i-1]]);
else
    maxv=macha[i]; machb[i]=maxv+b1[arr1[i]];
cout<<arr1[i]<<"\t"<<macha1[i]<<"--"<<macha[i]<<" "<<"\t"<<maxv<<"--"<<machb[i]<<endl;}
cout<<"\n\n\nTotal Elapsed Time (T) = "<<machb[n]; cout<<endl<<endl<<"Machine A:";
for(i=1;i<=n;i++)
    {cout<<endl<<"Job "<<i<<" Computation Time"<<macha[i];}
cout<<endl<<endl<<"Machine B:";
for(i=1;i<=n;i++)
    {cout<<endl<<"Job"<<i<<" Computation Time"<<machb[i];}
float L2,L_2,min,u2,sum1=0.0,sum2=0.0;
for(i=1;i<=n;i++)
    {sum1=sum1+a1[i];sum2=sum2+b1[i];}
cout<<"\nsum1="<<sum1; L2=machb[n]; float sum_2,sum_3;arr1[0]=0,sb1[0]=0;
for(i=1;i<=n;i++)
    {sum_2=0.0,sum_3=0.0;
for(int j=1;j<=i;j++)
    {sum_3=sum_3+sb1[arr1[j-1]];}
for(int k=1;k<=i;k++)
    {sum_2=sum_2+b1[arr1[k]];}}
cout<<"\nsum_2="<<sum_2; cout<<"\nsum_3="<<sum_3; L_2=L2-sum_2-sum_3;
cout<<"\nLatest time for which B is taken on Rent="<<"\t"<<L_2; u2=machb[n]-L_2;
cout<<"\n\nUtilization Time of Machine M2="<<u2; cost=(sum1*cost_a)+(u2*cost_b);
cout<<"\n\nThe Minimum Possible Rental Cost is="<<cost;
cout<<"\n\n\t*****************************************************************";
getch();
}
```

9. Numerical Illustration

Consider 5 jobs, 2 machine flow shop problem with processing time and setup time associated with their respective probabilities as given in the following table and jobs 2, 4 are to be processed as a group job (2,4). The rental cost per unit time for machines M_1 and M_2 are 4 units and 6 units respectively. Our objective is to obtain optimal schedule to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines, under the rental policy P.

Job	Machine M_1				Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	18	0.1	6	0.1	13	0.1	3	0.2
2	12	0.3	7	0.2	8	0.3	4	0.3
3	14	0.3	4	0.3	16	0.1	6	0.2
4	13	0.2	7	0.3	14	0.2	5	0.1
5	15	0.1	4	0.1	6	0.3	4	0.2

(Table 2)

Solution:

As per step 1: Expected processing and setup times for machines M_1 and M_2 are as shown in table 3.

As per step 2: The expected flow times for the two machines M_1 and M_2 are as shown in table 4.

As per step 3: Here β= (2, 4)

$A'_{\beta 1}$ = 2.4 + 2.1 – 1.0 = 3.5, $A'_{\beta 2}$ = 1.0 + 0.7 - 1.0 = 0.7.

As per step 4: The new reduced problem is as shown in table 5.

As per step 5: Using Johnson's method optimal sequence is

S = 5 – 1 – β – 3 i.e. 5 – 1 – 2 – 4 – 3.

As per step 6: The In-Out table for the sequence S is as shown in table 6.

As per step 7: Total elapsed time $t_{n2}(S_1)$ = 19.8 units

As per Step 8: The latest time at which Machine M_2 is taken on rent

$$L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$$
$$= 19.8 - 9.9 - 3.1 = 6.8 \text{ units}$$

As per step 9: The utilization time of Machine M_2 is

$$U_2(S) = t_{n2}(S) - L_2(S) = 19.8 - 6.8 = 13.0 \text{ units}$$

The Biobjective In – Out table is as shown in table 7.

Total Minimum Rental Cost = $R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 = 13.7 \times 4 + 13.0 \times 6$ = 132.8 units.

10. Conclusion

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$ on M_2 will, reduce the idle time of all jobs on it. Therefore total rental cost of M_2 will be minimum. Also rental cost of M_1 will always be minimum as idle time of M_1 is always zero. The study may further be extending by introducing the concept of transportation time, Weightage of jobs, Breakdown Interval etc.

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Tables

Table 3: The expected processing and setup times for machines M_1 and M_2 are as follows:

Job	Machine M_1		Machine M_2	
I	A_{i1}	S_{i1}	A_{i2}	S_{i2}
1	1.8	0.6	1.3	0.6
2	3.6	1.4	2.4	1.2
3	4.2	1.2	1.6	1.2
4	2.6	2.1	2.8	0.5
5	1.5	0.4	1.8	0.8

Table 4: The expected flow times for the two machines M_1 and M_2 are

Job	Machine M_1	Machine M_2
I	A'_{i1}	A'_{i2}
1	1.2	0.7
2	2.4	1.0
3	3.0	0.4
4	2.1	0.7
5	0.7	1.4

Table 5: The new reduced problem is

Job	Machine M_1	Machine M_2
i	A'_{i1}	A'_{i2}
1	1.2	0.7
β	3.5	0.7
3	3.0	0.4
5	0.7	1.4

Table 6: The In-Out table for the sequence S is

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
5	0.0 – 1.5	1.5 – 3.3
1	1.9 – 3.7	4.1 – 5.4
2	4.3 – 7.9	7.9 – 10.3
4	9.3- 11.9	11.9 – 14.7
3	14.0– 18.2	18.2 – 19.8

Table 7: The Biobjective In – Out table is as follows

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
5	0.0 – 1.5	6.8 – 8.2
1	1.9 – 3.7	9.0 – 10.3
2	4.3 – 7.9	10.9 – 13.3
4	9.3- 11.9	14.5 – 17.3
3	14.0 – 18.2	18.2 – 19.8