Numerically Modeling Solve by Homotopy Perturbation Method

Anmole Razzaq (Author)

National College of Business Administration & Economics

Abstract

In this paper, we used the homotopy perturbation method (HPM) to obtain a different approximate solutions of the Cauchy problem using in one dimensional nonlinear thermoelasticity. The comparison of the numerical solutions obtained by HPM with the exact solution shows the efficiency of the method of HPM.

Keywords: Homotopy perturbation method, Cauchy problem, nonlinear thermoelasticity, analytical approximate solution.

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1. Introduction

HPM does not require a small parameter in some equation and takes the full useful of the traditional perturbation methods and the homotopy problems. This problem is related a real life problem. Let consider the following nonlinear equation

$$v_t - a(v_x,\mu)v_{xx} + b(v_x,\mu)\mu_x = f(x,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1.1)$$

$$c(v_x,\mu)\mu_t + b(v_x,\mu)v_{xt} - d(\mu)\mu_{xx} = g(x,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .. (1.2)$$

And given initial condition of

$$v(x,0) = v^0(x), v_t(x,0) = v^1(x), \mu(x,0) = \mu^0(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (1.3)$$

Where,

V(x,t) is the anybody displacement who is the condition of equilibrium

μ(x,t) is the difference of body temperature who is include $T_0 = 0$

a, b, c and subscripts denote partial derivative are given smooth function.

***1.1* Implementation of HPM to problem:**

Now we define a, b, c, d, $v^0, v^1 and\ \mu^0$

$$a(v_x,\mu) = 2 - v_x\mu, b(v_x,\mu) = 2 + v_x\mu, c(v_x,\mu) = 1, d(v_x,\mu) = \mu \quad \ldots\ldots\ldots\ldots (1.4)$$

$$v^0(x) = \frac{1}{1+x^2}, v^1(x) = 0, \mu^0(x) = \frac{1}{1+x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.5)$$

And interchangin the R-H-S of equation (1) and (2) gives

$$f(x,t) = \frac{2}{1+x^2} - \frac{2(1+t^2)(3x^2-1)}{(1+x^2)^3}a(w,u) - \frac{2x(1+t)}{(1+x^2)^2}b(w,u) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.6)$$

$$g(x,t) = \frac{2}{1+x^2}c(w,u) - \frac{4xt}{(1+x^2)^2}b(w,u) - \frac{2(1+t^2)(3x^2-1)}{(1+x^2)^3}d(w,u) \quad \ldots\ldots\ldots\ldots (1.7)$$

Where a, b, c and d are defined in equation (4) so

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$w = w(x,t) = -\frac{2x(1+t^2)}{(1+x^2)^2}, u = u(x,t) = \frac{(1+t)}{(1+x^2)}$ …………………………………. (1.8)

The exact solution of equation (1) & (2)

$v(x,t) = \frac{(1+t^2)}{(1+x^2)}, \mu(x,t) = \frac{(1+t)}{(1+x^2)}$ ………………………………………………. (1.9)

If we put equation (4) into Equation (1) & (2), so we get

$v_t - (2 - v_x\mu)v_{xx} + (2 + v_x\mu)\mu_x - f(x,t) = 0$ ………………………………… (1.10)

$\mu_t + (2 + v_x\mu)v_{xt} - \mu\mu_{xx} - g(x,t) = 0$ …………………………………..... (1.11)

And now

$v_t - 2v_{xx} + v_x v_{xx}\mu + 2\mu_x + v_x\mu_x\mu = f(x,t) = 0$ …………………………… (1.12)

$\mu_t + 2v_{xt} + v_x v_{xt}\mu - \mu\mu_{xx} - g(x,t) = 0$ …………………………… (1.13)

And now we define the following different homotopies

$v_t + r\{-2v_{xx} + v_x v_{xx}\mu + 2\mu_x + v_x\mu_x\mu = f(x,t)\} = 0$ …………………..... (1.14)

$\mu_t + r\{2v_{xt} + v_x v_{xt}\mu - \mu\mu_{xx} - g(x,t)\} = 0$ …………………………… (1.15)

Assume the solution of equation (14) & (15), we get

$v = v_0 + rv_1 + r^2v_2 + r^3v_3 \ldots\ldots\ldots\ldots$ ………………………… (1.16)

$\mu = \mu_0 + r\mu_1 + r^2\mu_2 + r^3\mu_3 \ldots\ldots\ldots\ldots$ ………………………… (1.17)

Substituting equation 16 & 17 into equation 14 & 15 and equating the coefficients of like power r, we get the following set of differential equations so we get

$r^0: (v_0)_t = 0$

$(\mu_0)_t = 0$

$r^1: (v_1)_t - 2(v_0)_{xx} + (v_0)_x(v_0)_{xx}\mu_0 + 2(\mu_0)_x + (v_0)_x(\mu_0)_x\mu_0 - f(x,t) = 0$

$(\mu_1)_t - 2(v_0)_{xt} + (v_0)_x(v_0)_{xt}\mu_0 - \mu_0(\mu_0)_{xx} - g(x,t) = 0$

$r^2: (v_2)_t - 2(v_1)_{xx} + (v_0)_x(v_0)_{xx}\mu_1 + (v_0)_x(v_1)_{xx}\mu_0 + (v_1)_x(v_0)_{xx}\mu_0 + 2(\mu_1)_x + (v_0)_x(\mu_1)_x\mu_0 + (v_0)_x(\mu_0)_x\mu_1 + (v_1)_x(\mu_0)_x\mu_0 - f(x,t) = 0$

$(\mu_2)_t + 2(\mu_1)_{xt} + (v_0)_x(v_0)_{xt}\mu_1 + (v_0)_x(v_1)_{xt}\mu_0 + (v_1)_x(v_0)_{xt}\mu_0 - \mu_0(\mu_1)_{xx} - (\mu_0)_{xx}\mu_1 - g(x,t) = 0$

$r^3: (v_3)_t - 2(v_2)_{xx} + (v_1)_x(v_1)_{xx}\mu_1 + (v_0)_x(v_1)_{xx}\mu_2 + (v_0)_x(v_1)_{xx}\mu_1 + (v_0)_x(v_2)_{xx}\mu_0 + (v_1)_x(v_0)_{xx}\mu_1 + (v_2)_x(v_0)_{xx}\mu_0 + 2(\mu_2)_x + (v_0)_x(\mu_2)_x\mu_0 + (v_0)_x(\mu_1)_x\mu_1 + (v_0)_x(\mu_0)_x\mu_2 + (v_1)_x(\mu_1)_x\mu_0 + (v_1)_x(\mu_0)_x\mu_1 + (v_2)_x(\mu_0)_x\mu_0 - f(x,t) = 0$

$(\mu_3)_t + 2(v_2)_{xt} + (v_0)_x(v_0)_{xt}\mu_2 + (v_0)_x(v_1)_{xt}\mu_1 + (v_0)_x(v_2)_{xt}\mu_0 + (v_1)_x(v_0)_{xt}\mu_1 + (v_1)_x(v_1)_{xt}\mu_0 + (v_2)_x(v_0)_{xt}\mu_0 - (\mu_2)_{xx}\mu_0 - (\mu_0)_{xx}\mu_2 - (\mu_1)_{xx}\mu_1 - g(x,t) = 0$

…………..

And so on, the rest of the polynomials can be constructed in a similar manner. With the initial conditions equation (3) gives

$v_0(x) = \frac{1}{1+x^2}$ .... (1.18)

$v_1(x,t) = \frac{1}{105(1+x^2)^6}(10t^7(x - 3x^3) + 14t^6(x + x^2 - 3x^3 + x^4) + 42t^5(x + x^2 - 3x^3 + x^4) + 35t^4(1 + x + 2x^2 - 6x^3 - 2x^4 - 8x^6 - 3x^8) + 70t^3(2x^2 - 7x^3 + 2x^4 - 6x^5 - 4x^7 - x^9) + 105t^2(1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10})$ .... (1.19)

$\mu_0(x,t) = \frac{1}{1+x^2}$ .... (1.20)

$\mu_1(x,t) = \frac{1}{15(1+x^2)^5}(24t^5x^2 + 30t^4x^2 + 10t^3(1+2x^2-3x^4) + 30t^2(1-2x-6x^3-3x^4-6x^5-2x^7) + 15t(1+4x^2+6x^4+4x^6+x^8)$ .... (1.21)

solving in the same way, we can obtain $v_2(x,t)$, $\mu_2(x,t)$ and higher order approximations. Here, the numerical results are evaluated using $n = 2$ terms approximation of the recursive relations.

1.1.1 Question

$$\frac{d^2y}{dx^2} + y(x) = x$$

$$y(o) = \frac{dy}{dx} = 1$$

Now we solve

$L(y) = \frac{d^2y}{dx^2}$

$R(y) = Y(x)$

$N(y) = 0$

$f(r) = x$

According to HPM, we construct homotropy so,

$v(r,p) = \Omega \times [0,1]$

$H(v,p) = (1-P)[L(v) - L(y_0)] + P[L(v) + R(v) + N(v) - f(r)] =$

$$\Rightarrow (1-P)\left[\frac{d^2v}{dx^2} - \frac{d^2y_0}{dx^2}\right] + P\left[\frac{d^2v}{dx^2} + v(x) - x\right] = 0$$

$$\Rightarrow \frac{d^2v}{dx^2} - \frac{d^2y_0}{dx^2} - P\frac{d^2v}{dx^2} - P\frac{d^2y_0}{dx^2} + P\frac{d^2v}{dx^2} + Pv(x) - P(x) = 0$$

$$\Rightarrow \frac{d^2v}{dx^2} - \frac{d^2y_0}{dx^2} - P\frac{d^2y_0}{dx^2} + Pv(x) - P(x) = 0$$

$$\frac{d^2v}{dx^2} = \frac{d^2y_0}{dx^2} + P\frac{d^2y_0}{dx^2} - Pv(x) + P(x)$$

$$\frac{d^2v}{dx^2} = \frac{d^2y_0}{dx^2} + P(-v(x) + x - \frac{d^2y_0}{dx^2})$$

Now, we know that

$v(x) = v_0(x) + Pv_1(x) + P^2v_2(x) + \cdots$

Use

$v_0''(x) + Pv_1''(x) + P^2v_2''^{(x)} + \cdots = y_0'' - P(v_0(x) + Pv_1(x) + P^2v_2(x) + \cdots - y_0'' - x)$

And initial condition

$L(v) - L(y_0) = 0$

$L(v) = L(y_0)$

$v(x) = y_0$

$v_0(0) + pv_1(0) + Pv_2(0) + \cdots = y_0$

Comparing co-efficient,

$v_0(x) = y_0''$

$v_1''(x) = x = v_0(x) - y_0''$

$v_2(x) = v(x)$

Let

$y_0 = y_0(0) + xy'(0)$

$y_0 = v_0 = 1 + x.1 = 1 + x$

$v_1 = 0 + 0 + \int_0^x \int_0^x (1+x)dxdx = \frac{x^2}{2} + \frac{x^3}{6}$

$v_2 = 0 + \int_0^x \int_0^x \left(\frac{x^2}{2} + \frac{x^3}{6}\right) dxdx = \frac{x^4}{24} + \frac{x^5}{120}$

$y(x) = \lim_{p=1}(v_0 + v_1 + v_2 + \cdots)$

$y(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots = e^x$

1.1.2 Conclusion

In this research paper, we have applied HPM to obtain an approximation of the analytic solution of mathematically problem who arising in one dimensional nonlinear equation. We use this method who tell this equation is convergent equation. This result of HPM compare the other result or we say that the exact soltion, this result is very good result. This result is good and agreement result as compare the other result or recursive relation.

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