

The Analysis of Queuing Theory Process for Customer Services Delivery: A Case Study of MUET Examination Department Jamshoro.

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Abstract

The application of Queuing models as technique of Queue solution in issuing the certificates like (pass certificates, marks certificates, transcript and Degree) in examination department MUET, Jamshoro. Specially this study attempts to look at the problem of long Queues in Examination department MUET Jamshoro. The variables measured include arrival rate (λ) and service rate. They were analyzed for the data of April 2017 to May 2018 and the efficiency in students satisfactory through the use of multichannel queuing models which were compared for a number of queue performance. It was discovered that using a 11- server channel system was better than a 9-server model, 7-server model, 5-server model, 3-server model and 2-server model.

Keywords: students issuing certificates, multichannel Queuing Model, server efficiency, Queue Length, Queue Time.

Introduction

Queuing theory is a tool of mathematics which is used to analyze the probabilistic systems of customers and the servers. Queuing theory is also called as the theory of waiting line. Queuing theory is the branch of operational research that finds the connection between demand on a service system and the delays tolerated by the consumers of that system.

This is a branch of operations research because the conclusions can be utilized to make business decisions about the resources required to give service. This theory consists of many important applications, most of them are well-documented in the literature of probability, operations research, management science, and industrial engineering. Some instances are traffic flow (vehicles, aircraft, people, communications), scheduling (programs on a computer, jobs on machines, patients in hospitals), and facility design (banks, post offices, amusement parks, fast-food restaurants).

The situation of waiting line which are commonly experienced are; (i) cars waiting for green signal; (ii) students depositing fee at the counters; (iii) passengers waiting for bus/train; (iv) machines waiting for repairs; (v) work man waiting for tools. A group of people at some place to waiting for the service is known as queue.

In order to control the waiting time in the queue, the queuing theory can be implemented because the group of people do not like the wait of her/his to very notion of time loss, when the customers arrive too frequently they will have to wait for the service if they arrive the infrequently then the additional people arrives in the system the century ago A.K Erlang introduced the Queuing theory; A.K Erlang studied on telephone exchange congestion problem in the 20th century. Nowadays the telephone exchange, traffic control, manufacture system,

communication system, supermarket, patrol station, etc. use the queuing system. When an analyst has to design the performance of queuing system, then the queuing models prove to be as powerful tools. (A.H Hamdya, 2003).

Queuing theory based on probability concepts possible changes in its performance with modifications to the system. the mean system response time (waiting time in the queue plus service times), mean use of the service facility, distribution of the number of customers in the queue and in the system. Customers who are waiting for service is called queue discipline.

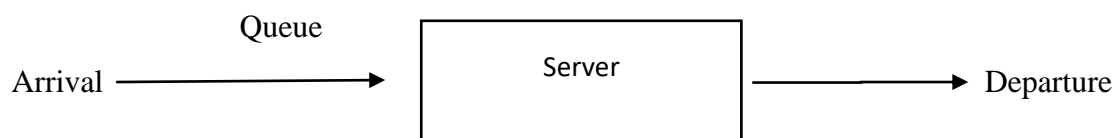
Queuing Discipline

- (i) First in first out (FIFO)
- (ii) Last in first out (LIFO)
- (iii) Service in random order (SIRO)
- (iv) Shortest processing time first (SPF)
- (v) Service according to priority (PR)

Queuing Models:

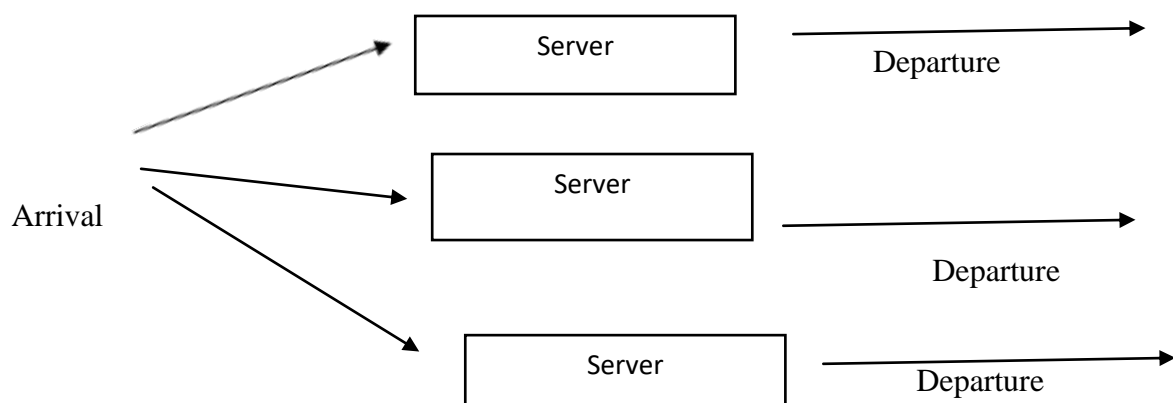
Some specific Waiting line models.

(1) Single-channel waiting line model In this model arrivals have single line and single server, Arrival are served on a first-in-first-out basis, every arrival waits to be served, service time vary from one customer to the next customer.



(2) Multiple-channels waiting line model

In this model arrival have single line and multiple channels, arrival is served on first come first served basis



(3) constant-service-time model

In this model service time is constant, when customers are processed according to a fixed cycle, as automatic car wash.

With the help of queuing models, we can change the behavior of queue and make a balanced between customers waiting time, and service capability and also their effect on the system.

Literature Review:

Cooray studied that the study of the sales checkout counter in supermarket checkout service and also the effectiveness of the models in term of utilization and queue length. in supermarket no anyone have to wait when servers in working (Cooray T.M.J.A, 2016).

I.M.Mankilik and H.N.Kana further studied that the mess system in Nigeria defense academy and analyzed the efficiency in mess system and also compare the six-channel is better than 4-channel,3-channel and 2-channel (I.M. Mankilik and H.N Kama, 2015).

Khalid Studied that the behavior and way of arrival of students in a university. They investigated student affairs department in many universities after investigating the result show that more than 70% students in university are unhappy. (M I Qureshi& Khalid Zaman, 2014)

Susila and Balambigai studied the analysis performance and staff planning in a telephone exchange and decide the various number of staffs achieve different management Queuing theory approach. (Susilamunisamy and Balambigai, 2007).

Rittu Mehandiratta studied on the heath care organization around the world and benefits required from the same. (Rittu Mehandiratta, 2011).

Ervin adi studied on the analysis of waiting time in fast food restaurants during the lunch time. (Ervin Adi,2012).

Tanzina studied on Bank ATM machine which is cannot provide the proper service to customers. than with help of queuing theory to solve this problem and maintain the customers' arrival and departure rate.(S.K. Dhar and Tanzina Rehman, 2013)

Methodology:

The present study is descriptive in nature. This study will be carried out on the process of student certificates similar to Mark sheet, Transcript and Degrees.

This data will be analyzed by queuing model $M/M/s$ which involves a single-line with multiple servers in the system.

The condition for probability that a service channel is busy given by $\rho = \frac{\lambda}{M\mu} < 1$ where λ mean arrival rate is, μ is mean service rate, M is the number of servers and ρ is called the service utilization factor that each server is busy. The totalservice rate must be greater than the arrival rate, that is $M\mu > \lambda$, and if $\lambda < M\mu$ the queue wouldeventually grow infinitely large.

- M= number of servers
- λ = Average arrival rate
- μ =Average of departure rate at each server

(A) The possibility that there is no any customers in the system is

$$p_0 = \frac{1}{\left(\sum_{n=0}^{M-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right) + \frac{1}{M!} \left(\frac{\lambda}{\mu} \right)^M \frac{M\mu}{M\mu - \lambda}} \text{ For } M\mu > \lambda$$

(B) The average number of customers in the system

$$L_s = \frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^M}{(M-1)!(M\mu-\lambda)^2} p_0 + \frac{\lambda}{\mu}$$

(C) The average time a customer spends in the waiting line and being serviced is

$$W_s = \frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^M}{(M-1)!(M\mu-\lambda)^2} p_0 + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

(D) The average number of customers or units in line waiting for service is

$$L_q = L_s - \frac{\lambda}{\mu}$$

(E) The average time a customer spends in the queue waiting for service is

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Results and Discussion

This paper reviews a queuing model for multiple servers. The average queue length calculates by tora software at each hour. We have compared this average for six different model's **M/M/2, M/M3, M/M/5, M/M/7, M/M/9 and M/M/11**, used to estimate a queue length: a single-queue multi-server. The data was collected from the examination department MUET Jamshoro, it comprises the student's arrival date for issuing the certificates like Pass Certificates, Marks Certificates, Transcript and Degree.

Table 1 to Table 6 representing the Arrival rate(λ), Departure rate (μ), Service utilization factor (ρ), average no of students in system (L_s), average no of students in Queue (L_q), average time a student waiting in line (W_s) and average time a student spends in queue (W_q).

The examination department of MUET Jamshoro consist on M/M/11 model. (Table#6) indicating that the system is stable as compared to the other models. (Table#7) indicates that the maximum time spend by a customer in the queue is 16.559 for the model M/M/2. The result of our observation is that the queuing process of examination department MUET Jamshoro is 99.99% stable because no any student in queue.

Conclusion:

This paper has introduced the basic concepts of queuing models, and their application to student's arrival for getting certificates. Students facing problems especially during the time of getting the Degree. So both, students and staff of Examination department, face problems but in MUET the examination department is stable. For the smooth running of the system it is suggested during the months of April and May when the arrival rate increases which causes the work load and stress making the communication between staff and students so examination department can apply M/M/11 model in this situation to ease out the queue while during the other months when the work load is decrease or arrival rate is too short the M/M/7 model can be used to shift the surplus staff in any other departments.

Table#01 **M/M/2 Model**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|----------------|---------------------------|-------------------------|----------------------------|--|---------------------------------------|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>50.390%</u> | <u>1.35079</u> | <u>0.34299</u> | <u>0.17430</u> | <u>0.04426</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>50.957%</u> | <u>1.37660</u> | <u>0.35745</u> | <u>0.15211</u> | <u>0.03950</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>52.531%</u> | <u>1.45107</u> | <u>0.40043</u> | <u>0.21853</u> | <u>0.06031</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>53.615%</u> | <u>1.50490</u> | <u>0.43260</u> | <u>0.17199</u> | <u>0.04944</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>57.658%</u> | <u>1.72748</u> | <u>0.57430</u> | <u>0.32779</u> | <u>0.10898</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>58.942%</u> | <u>1.80644</u> | <u>0.62759</u> | <u>0.29469</u> | <u>0.10238</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>56.206%</u> | <u>1.64326</u> | <u>0.51913</u> | <u>0.20388</u> | <u>0.06441</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>71.724%</u> | <u>2.95426</u> | <u>1.51977</u> | <u>0.28406</u> | <u>0.14613</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>64.627%</u> | <u>2.21960</u> | <u>0.92705</u> | <u>0.07532</u> | <u>0.03146</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>72.296%</u> | <u>3.02929</u> | <u>1.58335</u> | <u>0.17561</u> | <u>0.09179</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>89.343%</u> | <u>8.85558</u> | <u>7.06871</u> | <u>0.61412</u> | <u>0.49020</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>92.686%</u> | <u>13.15410</u> | <u>11.30037</u> | <u>0.99803</u> | <u>0.85739</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>98.423%</u> | <u>62.91269</u> | <u>60.94423</u> | <u>4.19978</u> | <u>4.06837</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>99.569%</u> | <u>231.83225</u> | <u>229.84086</u> | <u>16.70261</u> | <u>16.55914</u> |

Table#02 **M/M/3 Model**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|----------------|---------------------------|-------------------------|----------------------------|--|---------------------------------------|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>33.59%</u> | <u>1.05468</u> | <u>0.04687</u> | <u>0.13609</u> | <u>0.00605</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>33.97%</u> | <u>1.06814</u> | <u>0.04900</u> | <u>0.11803</u> | <u>0.00541</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>35.02%</u> | <u>1.10592</u> | <u>0.05528</u> | <u>0.16655</u> | <u>0.00833</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>35.743%</u> | <u>1.13226</u> | <u>0.05996</u> | <u>0.12940</u> | <u>0.00685</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>38.44%</u> | <u>1.23335</u> | <u>0.08018</u> | <u>0.23403</u> | <u>0.01521</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>39.29%</u> | <u>1.26645</u> | <u>0.08760</u> | <u>0.20660</u> | <u>0.01429</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>37.47%</u> | <u>1.19651</u> | <u>0.07238</u> | <u>0.14845</u> | <u>0.00898</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>47.81%</u> | <u>1.63054</u> | <u>0.19606</u> | <u>0.15678</u> | <u>0.01885</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>43.08%</u> | <u>1.41985</u> | <u>0.12730</u> | <u>0.04818</u> | <u>0.00432</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>48.19%</u> | <u>1.64866</u> | <u>0.20273</u> | <u>0.09557</u> | <u>0.01175</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>59.56%</u> | <u>2.30114</u> | <u>0.51428</u> | <u>0.15958</u> | <u>0.03566</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>61.79%</u> | <u>2.46504</u> | <u>0.61131</u> | <u>0.18703</u> | <u>0.04638</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>65.61%</u> | <u>2.78854</u> | <u>0.82008</u> | <u>0.18615</u> | <u>0.05475</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>66.37%</u> | <u>2.86094</u> | <u>0.86955</u> | <u>0.20612</u> | <u>0.06265</u> |

Table#03 **M/M/5 Model**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|-----------------------|---------------------------|-------------------------|----------------------------|--|---------------------------------------|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>20.15%</u> | <u>1.00880</u> | <u>0.00100</u> | <u>0.13017</u> | <u>0.00013</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>20.38%</u> | <u>1.02021</u> | <u>0.00106</u> | <u>0.11273</u> | <u>0.00022</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>21.01%</u> | <u>1.05189</u> | <u>0.00126</u> | <u>0.15842</u> | <u>0.00019</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>21.44%</u> | <u>1.07371</u> | <u>0.00140</u> | <u>0.12271</u> | <u>0.00016</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>23.06%</u> | <u>1.15526</u> | <u>0.00209</u> | <u>0.21921</u> | <u>0.00040</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>23.57%</u> | <u>1.18120</u> | <u>0.00235</u> | <u>0.19269</u> | <u>0.00038</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>22.48%</u> | <u>1.12595</u> | <u>0.00182</u> | <u>0.13970</u> | <u>0.00023</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>28.68%</u> | <u>1.44128</u> | <u>0.00679</u> | <u>0.13858</u> | <u>0.00065</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>25.85%</u> | <u>1.29642</u> | <u>0.00388</u> | <u>0.04399</u> | <u>0.00013</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>28.91%</u> | <u>1.45303</u> | <u>0.00709</u> | <u>0.08423</u> | <u>0.00041</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>35.73%</u> | <u>1.80878</u> | <u>0.02191</u> | <u>0.12544</u> | <u>0.00152</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>37.07%</u> | <u>1.88035</u> | <u>0.02662</u> | <u>0.14267</u> | <u>0.00202</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>39.36%</u> | <u>2.00505</u> | <u>0.03659</u> | <u>0.13385</u> | <u>0.00244</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>39.82%</u> | <u>2.03029</u> | <u>0.03890</u> | <u>0.14627</u> | <u>0.00280</u> |

Table#04 **M/M/7 Model**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|----------------|---------------------------|-------------------------|----------------------------|--|---------------------------------------|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>14.39%</u> | <u>1.00782</u> | <u>0.00002</u> | <u>0.13004</u> | <u>0.00000</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>14.55%</u> | <u>1.01916</u> | <u>0.00002</u> | <u>0.11261</u> | <u>0.00000</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>15%</u> | <u>1.05065</u> | <u>0.00002</u> | <u>0.15823</u> | <u>0.00000</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>15.31%</u> | <u>1.07233</u> | <u>0.00002</u> | <u>0.12255</u> | <u>0.00000</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>16.47%</u> | <u>1.15321</u> | <u>0.00004</u> | <u>0.21883</u> | <u>0.00001</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>16.84%</u> | <u>1.17889</u> | <u>0.00005</u> | <u>0.19232</u> | <u>0.00001</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>16.05%</u> | <u>1.12416</u> | <u>0.00003</u> | <u>0.13947</u> | <u>0.00000</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>20.49%</u> | <u>1.43467</u> | <u>0.00019</u> | <u>0.13795</u> | <u>0.00002</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>18.46%</u> | <u>1.29264</u> | <u>0.00009</u> | <u>0.04386</u> | <u>0.00000</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>20.65%</u> | <u>1.44614</u> | <u>0.00020</u> | <u>0.08383</u> | <u>0.00001</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>25.52%</u> | <u>1.78775</u> | <u>0.00089</u> | <u>0.12398</u> | <u>0.00006</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>20.48%</u> | <u>1.85487</u> | <u>0.00115</u> | <u>0.14073</u> | <u>0.00009</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>28.12%</u> | <u>1.97019</u> | <u>0.00173</u> | <u>0.13152</u> | <u>0.00012</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>28.44%</u> | <u>1.99326</u> | <u>0.00187</u> | <u>0.14361</u> | <u>0.00013</u> |

Table#5 **M/M/9 Model**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|-----------------------|---|---|-----------------------------------|--|---|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>11.19%</u> | <u>1.00780</u> | <u>0.00000</u> | <u>0.13004</u> | <u>0.00000</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>11.32%</u> | <u>1.01914</u> | <u>0.00000</u> | <u>0.11261</u> | <u>0.00000</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>11.67%</u> | <u>1.05063</u> | <u>0.00000</u> | <u>0.15823</u> | <u>0.00000</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>11.91%</u> | <u>1.07230</u> | <u>0.00000</u> | <u>0.12255</u> | <u>0.00000</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>12.81%</u> | <u>1.15317</u> | <u>0.00000</u> | <u>0.21882</u> | <u>0.00000</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>13.09%</u> | <u>1.17885</u> | <u>0.00000</u> | <u>0.19231</u> | <u>0.00000</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>12.49%</u> | <u>1.12413</u> | <u>0.00000</u> | <u>0.13947</u> | <u>0.00000</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>15.93%</u> | <u>1.43449</u> | <u>0.00000</u> | <u>0.13793</u> | <u>0.00000</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>14.36%</u> | <u>1.29254</u> | <u>0.00000</u> | <u>0.04386</u> | <u>0.00000</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>16.06%</u> | <u>1.44594</u> | <u>0.00000</u> | <u>0.08382</u> | <u>0.00000</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>19.85%</u> | <u>1.78689</u> | <u>0.00003</u> | <u>0.12392</u> | <u>0.00000</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>20.59%</u> | <u>1.85376</u> | <u>0.00004</u> | <u>0.14065</u> | <u>0.00000</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>21.87%</u> | <u>1.96852</u> | <u>0.00006</u> | <u>0.13141</u> | <u>0.00000</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>22.12%</u> | <u>1.99146</u> | <u>0.00007</u> | <u>0.14348</u> | <u>0.00000</u> |

Table#06 **M/M/11**

| S:No | Arrival rate(λ) | Departure rate(μ) | Service utilization factor | Avg:no of students in system (L_s) | Avg:no of students in Queue (L_q) | Avg: time a student waiting in line (W_s) | Avg: time a student spends in queue (W_q) |
|-----------------------|---|---|-----------------------------------|--|---|---|---|
| April 2017 | <u>7.75000</u> | <u>7.69000</u> | <u>9.16%</u> | <u>1.00780</u> | <u>0.00000</u> | <u>0.13004</u> | <u>0.00000</u> |
| May 2017 | <u>9.05000</u> | <u>8.88000</u> | <u>9.26%</u> | <u>1.01914</u> | <u>0.00000</u> | <u>0.11261</u> | <u>0.00000</u> |
| June 2017 | <u>6.64000</u> | <u>6.32000</u> | <u>9.55%</u> | <u>1.05063</u> | <u>0.00000</u> | <u>0.15823</u> | <u>0.00000</u> |
| July 2017 | <u>8.75000</u> | <u>8.16000</u> | <u>9.74%</u> | <u>1.07230</u> | <u>0.00000</u> | <u>0.12255</u> | <u>0.00000</u> |
| August 2017 | <u>5.27000</u> | <u>4.57000</u> | <u>10.48%</u> | <u>1.15317</u> | <u>0.00000</u> | <u>0.21882</u> | <u>0.00000</u> |
| September 2017 | <u>6.13000</u> | <u>5.20000</u> | <u>10.71%</u> | <u>1.17885</u> | <u>0.00000</u> | <u>0.19231</u> | <u>0.00000</u> |
| October 2017 | <u>8.06000</u> | <u>7.17000</u> | <u>10.21%</u> | <u>1.12413</u> | <u>0.00000</u> | <u>0.13947</u> | <u>0.00000</u> |
| November 2017 | <u>10.40000</u> | <u>7.25000</u> | <u>13.04%</u> | <u>1.43448</u> | <u>0.00000</u> | <u>0.13793</u> | <u>0.00000</u> |
| December 2017 | <u>29.47000</u> | <u>22.80000</u> | <u>11.75%</u> | <u>1.29254</u> | <u>0.00000</u> | <u>0.04386</u> | <u>0.00000</u> |
| January 2018 | <u>17.25000</u> | <u>11.93000</u> | <u>13.14%</u> | <u>1.44593</u> | <u>0.00000</u> | <u>0.08382</u> | <u>0.00000</u> |
| February 2018 | <u>14.42000</u> | <u>8.07000</u> | <u>16.24%</u> | <u>1.78686</u> | <u>0.00000</u> | <u>0.12392</u> | <u>0.00000</u> |
| March 2018 | <u>13.18000</u> | <u>7.11000</u> | <u>16.85%</u> | <u>1.85373</u> | <u>0.00000</u> | <u>0.14065</u> | <u>0.00000</u> |
| April 2018 | <u>14.98000</u> | <u>7.61000</u> | <u>17.89%</u> | <u>1.96846</u> | <u>0.00000</u> | <u>0.13141</u> | <u>0.00000</u> |
| May 2018 | <u>13.88000</u> | <u>6.97000</u> | <u>18.10%</u> | <u>1.99139</u> | <u>0.00000</u> | <u>0.14347</u> | <u>0.00000</u> |

Table #07 Maximum and Minimum average time spends in queue for the six models.

| MODEL | MAXIMUM TIME IN QUEUE | | MINIMUM TIME IN QUEUE | |
|--------|-----------------------|-------|-----------------------|----------|
| | Wq | Month | Wq | Month |
| M/M/2 | 16.55914 | May | 0.03146 | December |
| M/M/3 | 0.06265 | May | 0.00432 | December |
| M/M/5 | 0.0028 | May | 0.00013 | December |
| M/M/7 | 0.00013 | May | 0.0000 | December |
| M/M/9 | 0.0000 | May | 0.0000 | December |
| M/M/11 | 0.0000 | May | 0.0000 | December |

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