Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.8, No.2, 2018
www.iiste.org

Modeling a Life Queue-Event Data for Selected ATM Service-Point(s)

Johnson Olanrewaju Victor[1*] Aladesote Olomi Isaiah[1] Aliu Hassan[2]

1. Department of Computer Science, The Federal Polytechnic, Ile-Oluji, Ondo State, Nigeria
2. Department of Mathematics/Statistics, Rufus Giwa Polytechnic, Owo, Ondo State, Nigeria

* E-mail of the corresponding author: easywaylan@gmail.com

Abstract

Automatic Teller Machine (ATM) is meant to solve a lot of banking services in today technological world. But the problems its meant to solve in most cases is impeded with a lot of bottlenecks such as network failure, in sufficient funds and long delay in loading the ATM with cash. Meanwhile banking system service is known with First-Come-First-Serve (FCFS) discipline known as queue. Effective queue utilization is based on how efficient is the system in terms of service rate to the number of arrivals per time. The current developments in Rufus Giwa Polytechnic as our case study has also witness deployment of some ATMs within the school. But the incessant and unending long queues being experienced in the ATM point is a matter of serious concern. Dataset of queue records were collected and analyzed. The results show that the available ATM points (2 points) were overloaded, service utilization from each service point was above the threshold, and cases of long waiting time were noticed. While experimenting with R language the mean arrival rate and mean service rate for more than two servers a better performance in the system was observed. To a large extent better service delivery is achievable with 4 to 5 ATM service points with a trade-off of likely idle time to be experience while the customers (most students are on holidays).

Keywords: Automatic Teller Machine (ATM), First-Come-First-Serve (FCFS), Queue, Service utilization, dataset.

1. Introduction

One of the indicators of development in Information Technology both in the Banking sector and business world is the Automatic Teller Machine (ATM). It is meant to offer a range of services to people whether at the on-site of such machine or any Point-of-Sale (POS). Two types of ATMs had been an issue of concern; one of which is the branch ATM, the other being the out-of-branch ATM (Vasumathi and Dhanavanthan, 2010). The effort, be it any type deployed, is to maximize profit, reduce cost and satisfy customers optimally in the most generally acceptable international standard and practices (Bakari, Chamalwa and Baba, 2014). Meeting these needs on the other hand requires that services are rendered to customer without much *delay.* But one thing remain pertinent in the developing countries like Nigeria where ATMs are restricted to the banking vicinity; *queue*.

Queue, known as a waiting line is a general and practicable occurrence of everyday life. They are formed when customers (human or things) demanding service have to wait in that their number exceeds the number of servers available; or the facility does not work efficiently or takes more than the time prescribed to service a customer. Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some service facilities stand idle when the total number of service facilities exceeds the number of customers requiring service (Sharma, 2007; Bakari et al, 2010). According to Taha, (2003) queue is “defined as simply a waiting line”, while Hiray, (2008) puts it in similar way as a waiting line by two important elements: the population source of customer from which they can draw and the service system. The population of customer could be finite or infinite.

Some estimates state that Americans spend 37 billion hours per year waiting in lines. Whether it is waiting in line at a grocery store to buy items or checking out at the cash registers, waiting in line at the bank for a teller, or waiting at an amusement park to go on the newest ride, we spend a lot of time waiting. We wait in lines at the movies, campus dining rooms, the registrar’s office for class registration, at the Division of Motor Vehicles, and even at the end of the school term to sell books. A waiting line system (or queueing system) is defined by two elements: the population source of its customers and the process or service system itself. Examples of objects that must wait in lines include a machine waiting for repair, a customer order waiting to be processed, subassemblies in a manufacturing plant (that is, work-in process inventory), electronic messages on the internet, and ships or railcars waiting for unloading. In a waiting line system, managers must decide what level of service to offer. A low level of service may be inexpensive, at least in the short run, but may incur high costs of customer dissatisfaction, such as lost future business and actual processing costs of complaints. A high level of service will cost more to provide and will result in lower dissatisfaction costs. Because of this trade-off, management must consider what the optimal level of service to provide is (Prenhall, 2014).

Waiting line management has the greatest dilemma for managers seeking to improve on the investment of their operation; as customers do not tolerate waiting intensely. Whenever customer feels that he/she has waited too long at a station for a service, they would either opt out prematurely or may not come back to the station next time when needed a service. This would of course reduce customer demand and in the long run reduce revenue and profit. Moreover, longer waiting time might increase cost because it equals to more space or facilities, which means additional cost on the management (Anderson, 2007).

The queueing theory as a mathematical method of analyzing the congestions and delays of waiting in line is therefore adopted in solving waiting line problem (Investopedia, 2014). Queueing theory examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places and the number of "customers" (which might be people, data packets, cars, etc.).

Taking a critical look into recent developments in Rufus Giwa Poytechnic, Owo, one will agree that there have been a massive increase in infrastructural development ranging from opening of new sites for construction, lecture halls and offices, diverse renovations to establishment of new faculties to mention but a few. Arising from this is a growing population of 14,000+ estimate. Banking sectors are also not left out in the course of this development because of the essential services they render. Attention is drawn to the growing number of populace within the community both students and staff as a direct reflection of the developments. This teaming populace is expected to be rendered some services by the Institution. In order to remove bottlenecks known with manual processing systems technological innovations in Banking system are therefore deployed to alleviate these problems, in which ATM has been found practicable. Host community often take a solace by visiting the ATMs.

Drawing from the research carried out by Bakari et al (2005) on queueing process and its application to customer service delivery (a case study of Fidelity Bank Plc, Maiduguri). Observation method was adopted as their primarily method of data collection over a period of 10 working days. This we felt might not be provable enough to justify the correctness of the developed queue model for implementation. We, therefore rely on site data collection over a sample space at least 3000 datasets for a period of days in generating our queue data for modeling.

Hence, a conscious effort was taken in providing workable queue model for selected ATMs (Skye Bank and Zenith ATMs) on campus to alleviate the present situation. This will in turn ensure the delivery of maximum profit, cost reduction and specifically optimal customers' satisfaction putting into consideration the volatility of the operating environment (Augustine, 2013).

2. Aim and Objectives of the Study

The major focus of this research work was to provide the management of tertiary institution (Rufus Giwa Polytechnic as a pilot) and the banking sector at large with measures, standard and ideal practices for the deployment of ATMs into campuses. This in reflection will amount to effective and efficient delivery of services that will bring serenity to the operating environment, thereby ensuring compliance with the current developments.

Specific objectives include:

1) to study and identify the operating scenario of the selected ATM within a busy and tenable operating hours.
2) to model from the collected data using an appropriate Queue model;

3. Queueing Theory

Anyone who goes shopping or to a movie experiences the inconvenience of waiting in line. Not only do people spend time waiting in lines, but parts and products queue up prior to a manufacturing operation and wait to be worked on, machinery waits in line to be serviced or repaired, trucks line up to be loaded or unloaded at a shipping terminal. Waiting takes place in virtually every productive process or service. Since the time spent by people and things waiting in line is a valuable resource, the reduction of waiting time is an important aspect of operations management.

Waiting lines are analyzed with a set of mathematical formulae which comprise a field of study called *queueing theory* which was based on the origin work of A.K Erlang. Different queueing models and mathematical formulas exist to deal with different types of waiting line systems (Prenhall, 2014). *A queueing process therefore is a process in which customers arrive at some designated place where a service is being rendered. It assumed that the time between arrivals and the time spent in providing the service for a given customer follows a probabilistic laws* (Raluca, Devon, and Paul, 2005).

4. The Analysis of Waiting Line

Waiting lines form because people or things arrive at the servicing function, or server, faster than they can be served. This does not mean that the service operation is understaffed or does not have the capacity to handle the influx of customers. Most businesses and organizations have sufficient serving capacity available to handle its customers *in the long run.* Waiting lines result because customers do not arrive at a constant, evenly paced rate, nor are they all served in an equal amount of time. Customers arrive at random times, and the time required to serve each individually is not the same. A waiting line is continually increasing and decreasing in length (and is sometimes empty) and in the long run approaches an average rate of customer arrivals and an average time to serve the customer.

4.1 Elements of a Waiting Line

The basic elements of a waiting line, or **queue,** are arrivals, servers, and the waiting line. The relationship between these elements is shown in Figure 1, for the simplest type of *waiting line system,* a single server with a single queue. This is commonly referred to as a *single-channel* queueing system. Other waiting line components are briefly defined below.

a. *The Calling Population:* This is the source of the customers to the queueing system, and it can be either *infinite* or *finite.* An infinite calling population assumes such a large number of potential customers that it is always possible for one more customer to arrive to be served

b. *The Arrival Rate:* This means the rate at which customers arrive at the service facility during a specified period of time. This rate can be estimated from empirical data derived from studying the system or a similar system, or it can be an average of these empirical data. We further assume that arrivals at a service facility conform to some probability distribution. Arrivals could be described by many distributions, but it has been determined (through years of research and the practical experience of people in the field of queueing) that the number of arrivals per unit of time at a service facility can frequently be defined by a *Poisson distribution.*

c. *Service Times:* The queueing theory arrivals are described in terms of a *rate* and service in terms of *time.* Service times in a queueing process may also be any one of a large number of different probability distributions. The distribution most commonly assumed for service times is the *negative exponential distribution.* Although this probability distribution is for service *times,* service must be expressed as a *rate* to be compatible with the arrival rate.

d. *Queue Discipline and Length:* The queue discipline is the order in which waiting customers are served. The most common type of queue discipline is *first come, first served*-(FCFS or FIFO). Other possibilities are *last in, first out (LIFO), random (Service in Random Order (SIRO) or with Priority.* Often customers are scheduled for service according to a predetermined appointment, such as patients at a dentist's office or diners at a restaurant where reservations are required. These customers are taken according to a prearranged schedule regardless of when they arrive at the facility. Queues can be of an infinite or finite size or length. An infinite queue can be of any size with no upper limit and is the most common queue structure. For example, it is assumed that the waiting line at an ATM could be as long as possible, if necessary. A finite queue is limited in size.

e. *Service Mechanism:* Waiting line processes are generally categorized into four basic structures, according to the nature of the service facilities: single-channel-single-phase; single-channel-multiple-phase; multiple-channel-single-phase; and multiple-channel-multiple-phase processes. The number of channels in a queueing process is the number of parallel servers for servicing arriving customers. The number of phases**,** on the other hand, denotes the number of sequential servers each customer must go through to complete service. An example of a *single-channel-single-phase* queueing operation is a post office with only one postal clerk waiting on a single line of customers. A post office with several postal clerks waiting on a single line of customers is an example of a *multiple-channel-single-phase* operation.

f. *Operating Characteristics:* The mathematics used in queueing theory do not provide an optimal, or "best," solution. Instead they generate measures referred to as *operating characteristics* that describe the performance of the queueing system and that management uses to evaluate the system and make decisions. It is assumed these operating characteristics will approach constant, average values after the system has been in operation for a long time, which is referred to as a *steady state.*

g. Kendall-Lee Notation: Kendall in 1953 and lee in 1966 came-up with a much simpler notation that describes the characteristics of a queue termed Kendall-Lee notation. Let us denote the system by

$$A / B / m / K / n / D;$$

Where

A: distribution function of the interarrival times,

B: distribution function of the service times,

m: number of servers,

K: capacity of the system, the maximum number of customers in the system including the one being serviced,

n: population size, number of sources of customers,

D: service discipline. Exponentially distributed random variables are notated by M, meaning Markovain or memoryless.

h. *Little's Queueing Formula:* It is pertinent to determine the various waiting times and queue size for particular components of the system in order to make judgment about how to run the system. Suppose L denotes the average number of customers in the queue at any given time, assuming that the steady-state has reached. We can break that into *Lq*; average number of customers waiting in the queue and *Ls*; average number of customers in the service; and since the customers in the system can only either be in the queue or in service, this implies that: *L=Lq+Ls*. Moreover, we can say W denotes the average time a customer spends in the queueing system. Wq is the average time spends in the queue; while Ws is the average time spends in the service. Therefore, *W=Wq+Ws*. Let λ denotes the arrival rate into the system, meaning thereby, the number of customers arriving the system per unit time, thus,

$$L=W\lambda,\ Lq=Ws\ \lambda,\ Ls=Wq\lambda$$

$$L_q + L_s = W_q\lambda + W_s\lambda \tag{1}$$

5. Related Work

Bakari et al (2005) did a research work on queueing process and its application to customer service delivery (a case study of Fidelity Bank Plc, Maiduguri). Observation method was primarily their method of data collection over a period of 10 working days. The study reveals that the traffic intensity (ρ) is 0.96, otherwise known as the utilization factor is less the one (i.e. ρ<1). It was concluded that the system operates under steady-state condition. Thus, the value of the traffic intensity, which is the probability that the system is busy, implies that 96% of the time period considered during data collection the system was busy as against 4% idle time. This indicates high utilization of the system.

Another research work carried out by Bhavin and Pravin (2012) shows the arrival rate at a banking time is $\mathbf{1}$ customer per minute (cpm) while the service rate is $\mathbf{1.66}$ cpm. The average number of customers in the ATM is $\mathbf{1.5}$ and the utilization period is $\mathbf{0.60}$. Data were sourced from a bank ATM in a city with the adoption of Little's Theorem and M/M/1 queueing model. This means utilization is fairly above average. A simulation model was later proposed to confirm their result.

A research work of Vasumathi. et al (2010) considered a simulation techniques on 3 ATM services of 3 different Banks at Vellore Institute of Technology, Chennai. Data were sourced from these banks using observation method in a period of 2 months. Their result shows a low service delivery in some area of the campus and suggested a new installation despite its cost.

6. Materials and Methods

This research work painstakingly looked at the performance index of the Skye Bank ATMs deployed at Rufus Giwa Polytechnic, Owo. The system's characteristics of interest that will be examined in this research work include; number of arrivals (number of customers arriving to the service point at a given time), service time (the time it takes for one server to complete customer's service), the average number of customers in the system, and the average time a customer spends in the system. The results of the operating characteristics is used to evaluate the performance of the service mechanism and to ascertain whether customers are satisfied with the banks' services. The importance is to deliver quality service from the customers' perception of bank services.

Contrary to the methods adopted from research literatures reviewed in sourcing data, a field-work approach was adopted. A life data were collected at the ATM point(s) using tally and timing method for a period of days amounting to 3000 data set.

A generalized M/M/c/∞/FIFO model was adopted, where the first M denotes the inter-arrival time, the second M denotes the service time with c channels and First-In-First-Out discipline to analyze the data collected to derive the queue models and its parameters. Queue modeling program was written in ***R-language***.

7. Assumptions of Queueing Model

Model as an idealized representation of the real life situation; in order to keep the model as simple as possible however, some assumptions need to be made (Hira and Gupta, 2004). The following assumptions is made on the System

1) Poisson arrival (Random arrivals).
2) Inter-arrival time & Service time follow exponential distribution.
3) Multiple channel queue.
4) There is an infinite population from which customers originate.
5) The queue discipline is First-In- First-Out (FIFO).
6) The waiting area for customers is adequate.

7.1 M/M/c/∞/FIFO model

Establishing a foundation for this work, M/M/c/∞/FIFO model is analyzed with exponential interarrival times with mean 1/λ, exponential service times with mean 1/μ and a single serve. Customers are served in order of arrival. We require the utilization rate from

1) Customers in the sojourn time
2) Customers in the system
3) Service time in the sojourn time
4) Service time in the system
5) Waiting time in the sojourn time
6) Waiting time in the system

Thus **Error! Reference source not found.Error! Reference source not found.**

as

$$\rho = \frac{\lambda}{c\mu} < 1 \qquad \rho = \begin{cases} not\ busy, & \rho < 1 \\ busy, & \rho > 1 \end{cases} \tag{2}$$

8. Results and Discussion

In queue phenomenon, it is interesting to know that queue metrics for queue performances can be modeled from probabilistic (Markovian model) and Simulation (Discrete Event Simulation-DES) frameworks. This research employed a probabilistic approach and results were presented.

The performance metrics as of the queue model were determined using equations (2) given above. Table 1.0 therefore, shows the summary of the result using data collected for the period. The R script was used to generate graphical results presented in Appendix I.

From the Table 1.0, there are two notable outburst of utilization rate $\boldsymbol{\rho}.$ > 1 (it exceed 1), which implies that sever is over used and may be practicably down for service. It thus also means that the population (customers) in the particular day are extremely above the capacity of the server. In Appendix I, we detailed graphical presentation for our analysis putting into consideration a 5-server hypothetical model (i.e *when the channel* ***nS*** *is 1, 2, 3, 4 or 5*).

Figure 2 depicts the utilization rate considering server variation on the dataset for the same period. Using a one service channel shows an extremely high trends of utilization rate with notable point of the utilization rate exceeding normalcy. This same trend is also exhibited by 2 Servers deployment. But meanwhile the sharp high trends of utilization rate started reducing with 3 servers to 4 servers and finally to 5 servers. The implication is that with 1 or 2 server(s) considering the same infinite population, the tendency for the system to break down in terms of service is very high.

The experiment was also investigated with density plot. Figure 3 shows that with One (1) server, density was low with wider spread of utilization rate over 1. Density was seen to becoming high with low spread of utilization rate. This implies that at the point of 3 or more Servers in the system we expect performance to boost with less queue and overloading of ATM.

There was need to also inspect whether over-utilization rate experience within the experiment was detected in a particular service day or not. Figure 4 therefore shows the pattern of utilization rate and how it fairs for the days of the experiment. Our discovery was that there were notable period where utilization rate was**Error! Reference source not found.**. We further observed that if this is compared with Figure 5, it therefore suggest that with 1 server fitted utilization rate is over 1. Whereas as the server increases (specifically at 3 or more) utilization rate becomes normal. Further justification for the experiment was based on the number of customers in the queue or in the system, which is also a way of verifying the performance of the system. The longer the queue the longer it takes for a customer to get a service. Figure 6 and 7, depict the average number of customers in the system and in the queue respectively. The results show without reproof that with 1 server over-population were observed to be in the system or in the queue. This ultimately is bound to lead to over-burden of the system. This is also a case with 2 servers. Poor service performance is likely experience here and customers may tend to leave the queue (renege). Numbers of customers in the system or in the queue started reducing with a functional 3-server service points.

The third aspect of queue system performance analysis is the waiting time. We investigated this with a box plot where mean value can be visualized for better analysis. Figure 8 depicts a box plot for each of the servers' average waiting time against utilization. We observed that customers can be waiting on a queue with average of 252.90 hours when the queue is really built up in a 1 server scenario. This is a worst case ever as it is practically impossible for customers to wait that longer hours (amounting to days). The interpretation behind this could mean that the single server is not functioning well, service therefore delayed while many customers were trying to push their way to get served. Ultimately customers renege. Longer queue in most cases does not mean poor performance of the system in as much the waiting time is minimal (or bearable). But when queue are built up and waiting time is also on a high side definitely there is bound to be commotion, reneging and all sorts of inordinate attitude. With 4 or 5 servers specifically, we observed that customers are served optimally at less than 12 minutes. This may proofed preferable to impatient customers and is still cost effective for the banking institution.

9. Conclusion

Providing a workable queue model for real life application of this type requires a real life data collection of a very good and large sample space. This was the focus of this paper. From the results described in the paper, it was highly recommended that additional two or more ATM points be provided to the existing two. Thus, for 4 ATMs, the customer-server ratio will be 3,500 customers/ATM and for 5 ATMs, the customer-server ratio will be 2,800 customers/ATM on average. By submission it means to every additional 2,000 average growth in population one additional ATM be installed. With this, is expected that better service delivery will be achieved, less customers reneging and better profit for the banking institution. Meanwhile, this work has not considered discrete event simulation (DES) as basis of validating results obtained in the paper. Moreover is it opined that performance of ATM service system could be improved upon if results here are well and appropriately considered.

References

Anderson E. (2007), "A Note on Managing Waiting Lines", UT McCombs School of Business

Augustine A. N (2013), "Queueing Model as a Technique of Queue Solution in Nigeria Banking Industry", International Institute for Science, Technology and Education, Vol. 3, No 8. pp 188-195.

Bakari H.R, Chamalwa H, A and Baba A.M (2014), "Queueing Process and Its Application to Customer Service Delivery: A case Study of Fidelity Bank Plc, Maiduguri", International Journal of Mathematics and Statistics Invention, Volume 2 Issue 1, pp 14-21.

Bhavin P. and Pravin B. (2012), "Case Study for Bank ATM Queueing Model", International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622, Vol. 2, Issue 5, pp.1278-1284.

Hira D.S and Gupta P.K. (2004), "Simulation and Queueing Theory Operation Research S.C Chand and Company Ltd, New Delhi, India

Hiray J. (2008), "Waiting Lines and Queueing System", Article of Business Management

Inestopedia (2014), "Queueing Definition", retrieved 26th Oct, 2014 from http://www.investopedia.com/terms/q/queueing-theory.asp

Prenhall, (2014) "Queueing Theory and Waiting Lines Analysis, retrieved 30th Oct, 2014 from http://www.prenhall.com/divisions/bp/app/russellcd/PROTECT/CHAPTERS/CHAP16/HEAD01.HTM

Raluca I, Devon M., and Paul G. (2005), "Queueing Systems Theory and Simulations for Simple ATM Models", retreived 24th Oct, 2014 from euler.nmt.edu/~jstarret/430/430Project3/Devon/queue.pdf

Sharma J.K. (2007), "Operation Research: Theory and Application, 3rd Ed., Macmillan Ltd, India

Taha A.H (2003), "Operation Research: An Introduction, 7th Ed., Prentice Hall, India

Vasumathi A. and Dhanavanthan P. (2010), "Application of Simulation Technique in Queueing Model for ATM Facility", International Journal of Applied Engineering Research, Dindigul, Volume 1 No 30, pp 469-482.

Acknowledgement: This research was fully supported by the Tertiary Education Trust Fund (TETFUND) under 2013-2014 Research Projects (RP) Intervention for Rufus Giwa Polytechnic, Owo, Ondo State, Nigeria.

Appendix I

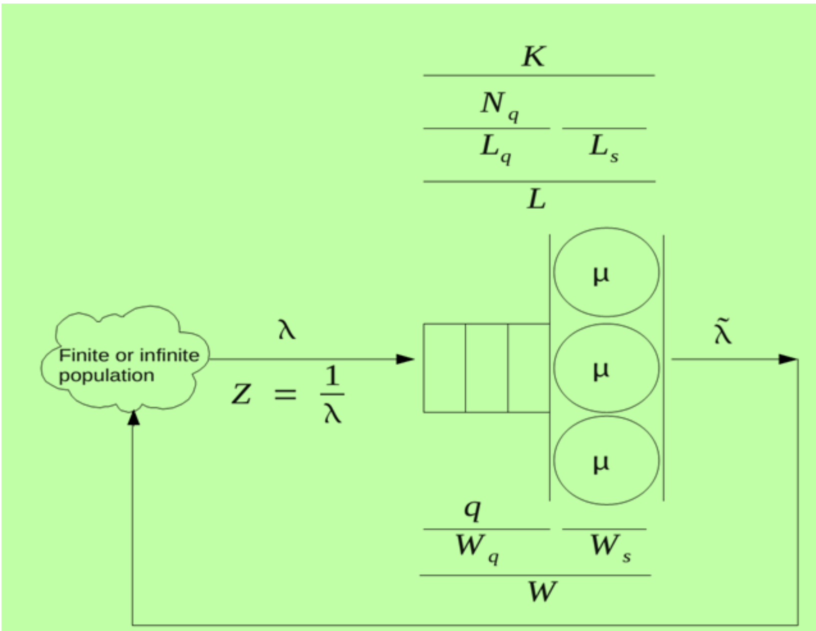

Figure 1 A typical queue model and its components

Table 1.0: The summarized queue metrics

days	*nS*	*p*	*L*	*Lq*	*Wq*	*W*	*Wiq*	*Lqq*
1	1	0.9	8.97	8.07	378.96	421.21	421.21	9.97
1	2	0.45	1.13	0.228	10.72	52.97	38.4	1.82
1	3	0.3	0.93	0.03	1.4	43.66	20.12	1.43
1	4	0.23	0.9	0.004	0.2	42.44	13.63	1.29
1	5	0.18	0.9	0.001	0.03	42.27	10.3	1.22
*2	1	1.06	n/a	n/a	n/a	n/a	n/a	n/a
2	2	0.53	1.46	0.407	23.5	84.46	64.51	2.11
2	3	0.35	1.11	0.056	3.25	64.21	31.34	1.54
2	4	0.26	1.06	0.01	0.5	61.47	20.7	1.35
2	5	0.21	1.06	0.001	0.07	61.04	15.45	1.27
3	1	0.8	3.98	3.18	159.84	200	200	4.98
3	2	0.4	0.95	0.152	7.63	47.8	33.45	1.67
3	3	0.27	0.82	0.019	0.95	41.11	18.25	1.36
3	4	0.2	0.8	0.002	0.12	40.28	12.55	1.25
3	5	0.16	0.79	0	0.01	40.18	9.56	1.19
*4	1	1.68	n/a	n/a	n/a	n/a	n/a	n/a
4	2	0.83	5.67	3.987	169.09	240.24	220.9	6.21
4	3	0.56	2.06	0.386	16.37	87.53	53.82	2.27
4	4	0.42	1.75	0.076	3.2	74.36	30.64	1.72
4	5	0.34	1.69	0.016	0.66	71.82	21.42	1.51
5	1	0.65	1.84	1.188	87.35	134.95	134.95	2.83
5	2	0.32	0.72	0.076	5.57	53.17	35.19	1.48
5	3	0.22	0.66	0.008	0.61	48.21	20.23	1.28
5	4	0.16	0.65	0.001	0.06	47.67	14.2	1.19
5	5	0.13	0.65	0	0.01	47.61	10.93	1.49
6	1	0.75	2.99	2.241	142.03	189.52	189.52	3.99
6	2	0.38	0.87	0.122	7.76	55.25	37.98	1.6
6	3	0.25	0.76	0.015	0.93	48.42	21.1	1.33
6	4	0.19	0.75	0.002	0.11	47.61	14.61	1.23
6	5	0.15	0.75	0	0.01	47.51	11.17	1.18
7	1	0.74	2.86	2.116	122.28	165.08	165.08	3.86
7	2	0.37	0.86	0.118	6.8	49.6	33.99	1.59
7	3	0.25	0.76	0.014	0.81	43.61	18.94	1.33
7	4	0.19	0.74	0.002	0.1	42.89	13.13	1.23
7	5	0.15	0.74	0	0.01	42.81	10.05	1.17
8	1	0.87	6.49	5.62	252.9	291.9	291.9	7.49
days	*nS*	*p*	*L*	*Lq*	*Wq*	*W*	*Wiq*	*Lqq*
8	2	0.43	1.07	0.2	9.01	48	34.4	1.76
8	3	0.29	0.89	0.026	1.16	40.15	18.27	1.41
8	4	0.22	0.87	0.003	0.16	39.15	12.44	1.28

8	5	0.17	0.87	0	0.02	39	9.43	1.21
9	1	0.78	3.42	2.643	205.21	265.28	265.28	4.42
9	2	0.39	0.91	0.137	10.57	70.63	48.97	1.63
9	3	0.26	0.79	0.017	1.29	61.35	26.98	1.35
9	4	0.19	0.78	0.002	0.16	60.22	18.62	1.24
9	5	0.16	0.78	0	0.02	60.08	14.21	1.18
10	1	0.73	2.72	1.989	126.48	172.97	172.97	3.72
10	2	0.37	0.84	0.112	7.17	53.66	36.64	1.58
10	3	0.24	0.75	0.013	0.85	47.34	20.49	1.32
10	4	0.18	0.73	0.002	0.1	46.59	14.22	1.22
10	5	0.15	0.73	0	0.01	46.5	10.89	1.17
11	1	0.53	1.13	0.601	54.54	102.73	102.73	2.13
11	2	0.27	0.57	0.04	3.65	51.84	32.8	1.36
11	3	0.18	0.54	0.004	0.35	48.54	19.52	1.21
11	4	0.13	0.53	0	0.03	48.22	13.89	1.15
11	5	0.11	0.53	0	0	48.19	10.78	1.11
12	1	0.81	4.24	3.432	205.09	253.45	253.4	5.24
12	2	0.41	0.97	0.158	9.47	57.82	40.61	1.68
12	3	0.27	0.83	0.02	1.18	49.54	22.07	1.37
12	4	0.2	0.81	0.003	0.15	48.51	15.16	1.25
12	5	0.16	0.81	0	0.02	48.37	11.54	1.19
13	1	0.58	1.4	0.815	64.61	110.84	110.84	2.4
13	2	0.29	0.64	0.054	4.29	50.52	32.62	1.41
13	3	0.19	0.59	0.006	0.44	46.67	19.13	1.24
13	4	0.15	0.58	0.001	0.04	46.27	13.53	1.17
13	5	0.12	0.58	0	0	46.24	10.47	1.13

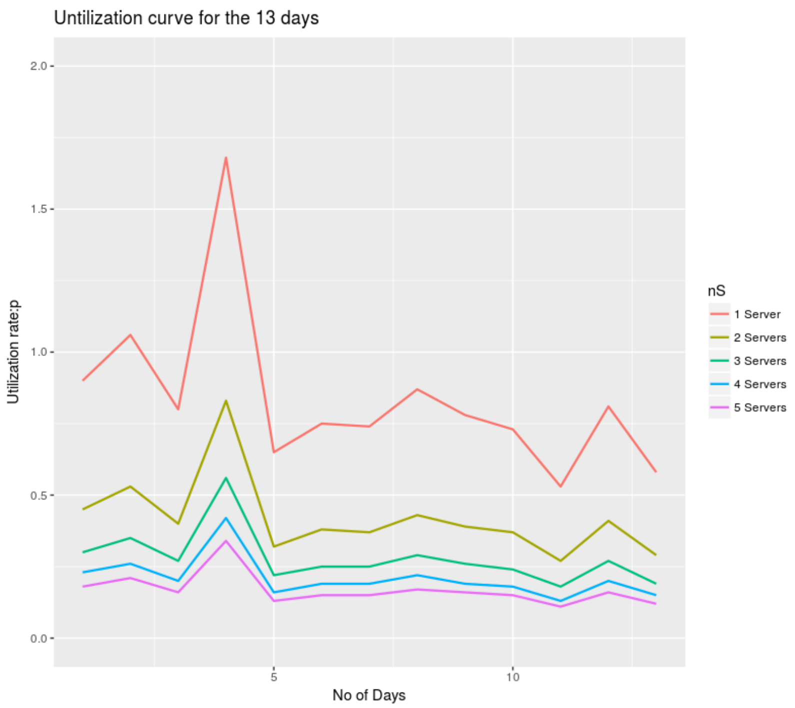

Figure 2: Utilization Rate

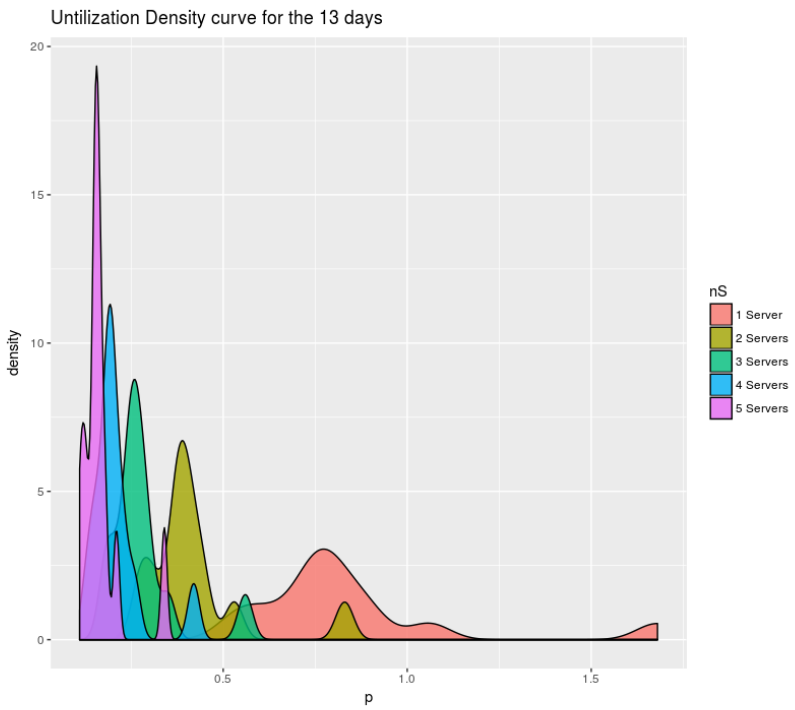

Figure 3: Density curve analysis of Utilization Rate

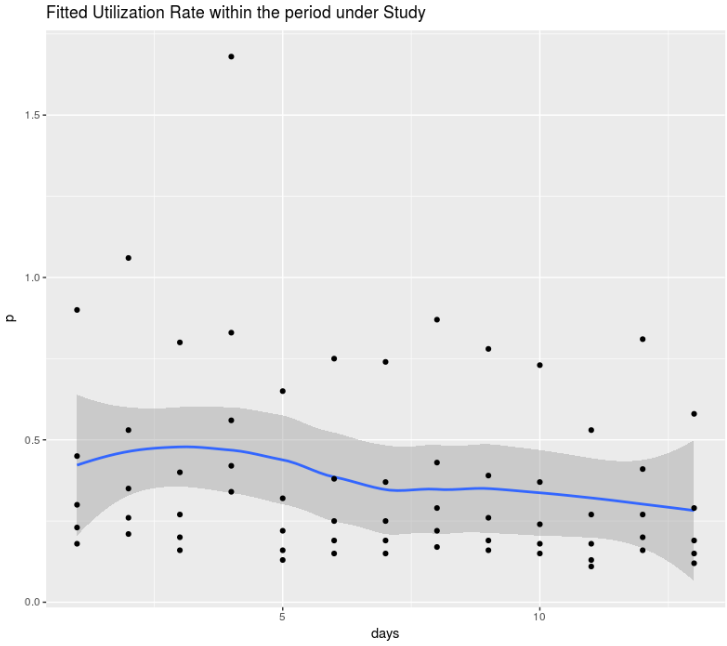

Figure 4: Fitted Utilization Rate for the period under investigation

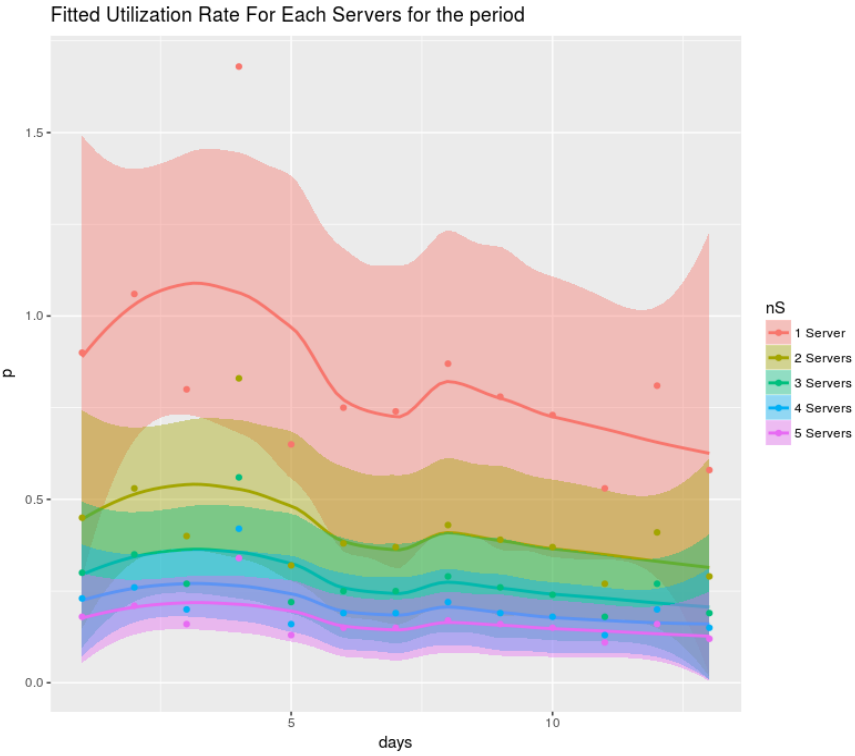

Figure 5: Fitted Utilization Rate of Each Server for the period under investigation

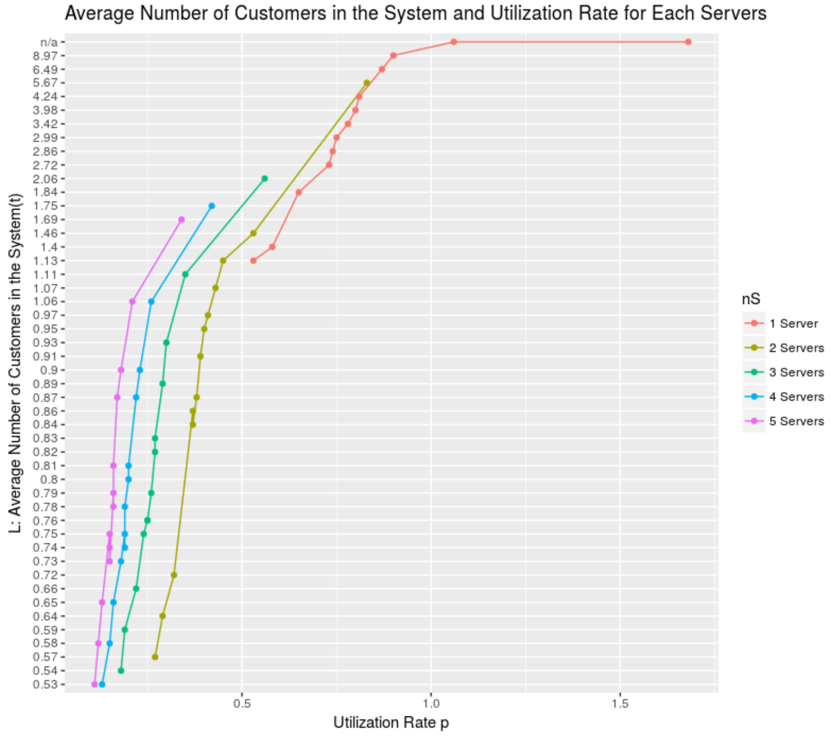

Figure 6: Average Number of Customers in the System

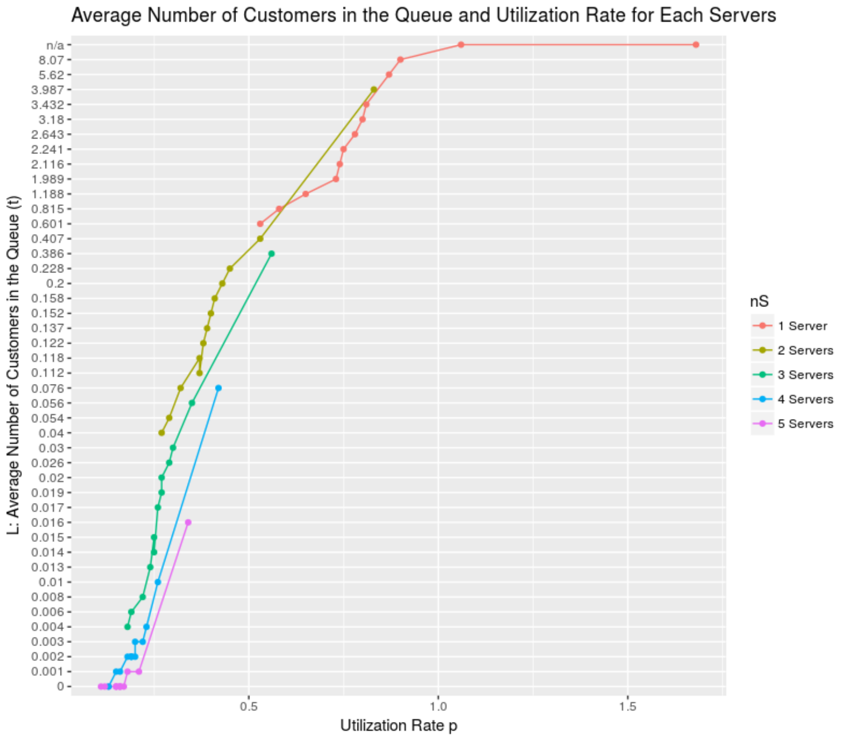

Figure 7: Average Number of Customers in the Queue

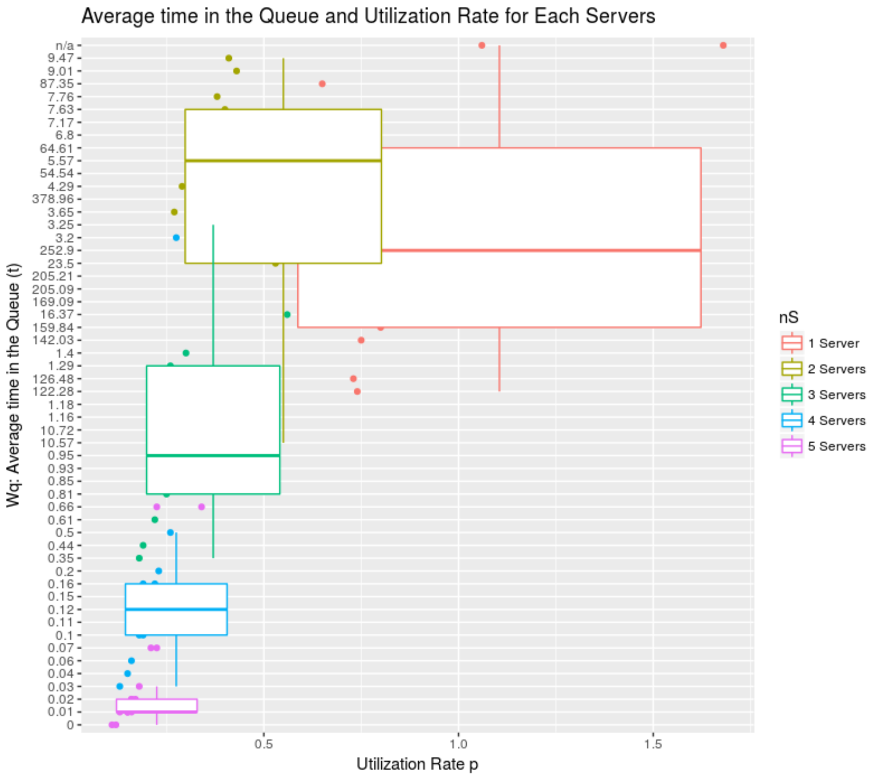

Figure 8: Average waiting time in the Queue