TOWARDS ENVIRONMENTAL SUSTAINABILITY THROUGH A FOURIER SERIES APPROACH IN FORECASTING SOLID WASTE GENERATION IN A TVET INSTITUTION: A CASE STUDY OF ACCRA TECHNICAL UNIVERSITY

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Abstract

Globally, waste volumes are increasing quickly, even faster than the rate of urbanization. According to the World Bank Report (2012), there is a direct correlation between the per capita level of income in cities and the amount of waste per capita that is generated. As the world moves towards an urban future, the amount of solid waste being generated continues to compound. As part of the Sustainable Development Goals, we are to ensure the availability and sustainable management of water and sanitation. TVET institutions in Africa must be strategically involved in the attainment of this goal. Management of solid waste in our municipalities continues to be a major challenge for local governments in urban areas across the world. Accra Polytechnic, like other TVET institutions, has a concentration of thousands of students who generate solid waste on a daily basis, which if not managed properly could have dire consequence on our environment. Due to the lack of available adequate data on solid waste generation and the waste characteristics in our institutions, integrated solid waste management is always a challenge. The prediction of students' solid waste generation will play an important role in solid waste management in our institutions. Traditional forecasting models for solid waste generation rely on demographic and socioeconomic factors on a per capita basis. Most of these models are designed based on the configuration of semi-empirical mathematical models. This paper looks at the determination of the solid waste generated per capita per day by students on Accra Polytechnic campus using a Fourier series approach.

Keywords: Solid Waste, SDG, TVET, Fourier Series, per capita.

1.0 Introduction

1.1 Ensuring Environmental Sustainability

Between the years 2000 and 2015, the United Nations (UN) championed a crusade to better the lot of citizens all over the world. These were enshrined in the eight Millennium Development Goals (MDGs), that were established following the Millennium Summit in the year 2000. Goal 7 of the MDGs was specifically to ensure environmental sustainability. The targets included integrating the principle of sustainable development into country policies and programs and reverse loss of environmental resources. A key indicator was the proportion of urban population with access to improved sanitation. (MDGs Report: 2006).

According to available statistics, about 2.1 billion people in the universe have gained access to improved sanitation. At the same time, the proportion of people practicing open defecation has reduced by nearly 50 percent since 1990. (MDG Monitor: 2015). Despite these achievements, it is incumbent on authorities within our communities to ensure the availability and sustainable management of water and sanitation.

1.2 The Sustainable Development Goals

As the MDGs were supposed to be achieved by 2015, a further process to agree and develop post-2015 development goals was needed, and this gave birth to Sustainable Development Goals (SDGs). In September 2015, the UN adopted the 2030 Development Agenda – Transforming our World – which contained 17 goals and 169 targets, most of which are a continuation and expansion of the MDGs that were never achieved.

Goal 6 of the SDGs is to ensure availability and sustainable management of water and sanitation for all. According to an SDG report (Millenium Development Goal Monitor :2015), this goal should be viewed as an ambitious aspiration that will be very challenging to achieve given that it may require major changes in water management among many nations to harmonize governance, data collection and sharing policies, and the use of models and tools. Data, science and applications of good management and governance principles will also be needed to achieve this goal.

Research and technology will be needed to better understand and manage the impacts of local decisions and at the same time there is a need to downscale global model projections to the local level for improved planning.

1.3 Strategic Involvement of TVET Institutions in SDG 6

Accra Polytechnic, like other TVET institutions, has a concentration of thousands of students who generate solid waste on a daily basis. The institution, which is located in the heart of the city of Accra, has over ten thousand students on its campus pursuing various programmes of study at different levels. The volume of human traffic on any week day on its campus is quite huge and one can imagine the amount of solid waste that is generated on a daily basis.

The waste generated is taken care of by the Sanitation and Environmental Unit of the institution; however a scientific approach is necessary to aid in the management of these wastes. As indicated in the SDG report, data collection and application of technology will be needed to better manage the situation. It is important to be able to predict the amount of solid waste generated per capita and use a scientific approach to analyze the resulting data to arrive at a mathematical model. This will aid school authorities is formulating policies and arriving at decisions aimed at attaining SDG 6. In the light of the above, this paper aims at using a Fourier series approach to forecast solid waste generated in Accra Polytechnic.

1.4 Fourier Series as a Forecasting Tool

In mathematics, a Fourier series offers up a means through which it becomes possible to represent wave-like functions by utilizing the sum of sine waves (Weisstein, 2016a). The Fourier series, then, offers an expansion of a given periodic function f (x) in terms of an unending sum of cosines and sines (Weisstein, 2016a). The use of a Fourier series in research toward the attainment of a specific answer is referred to as harmonic analysis, a process that serves as a means through which it is possible to break an arbitrary periodic function into a given set of terms that may be plugged into the equation, solved as an individual problem, and then the results recombined as a means of either obtaining a solution to the original problem or as a means of approximating the results to the accuracy or practicality desired (Weisstein, 2016a).

The Fourier series is, in essence, an expansion of the continuous function into set weighted sums of cosines and sines or the set weighted sum of complex exponentials (Glynn, 2007). The process is referred to as harmonic analysis given the similarities to the graphs of the resultant answers to a musical harmony or sound wave. While a Fourier series is not the only form of harmonic analysis that may be conducted, it is one of the most widely applied of the different forms of harmonic analysis, particularly when it comes to processing complex signaling, the practice of medical imaging and, perhaps the most complex field of all, quantum mechanics.

Due to the ease of application of this practice to the complex nature of the problem, it seems not a large leap to the application of the Fourier series to the problem of waste generation, waste removal, and waste disposal in a high density population area like a TVET campus. As research shows, Fourier series has been applied to the matter of waste generation, waste removal, and waste disposal, including the conversion of disposed waste to energy sources, making the application of this type of analysis all the more rational in this context. (Boursier, Bourgeat, & Tromeur-Dervout, 2004; Cacuci, 2010; Committee on the Waste Isolation Pilot Plant, Commission on Geosciences, Environment and Resources, Division on Earth and Life Studies, & the National Research Council, 1996; Jinno & Kawamura, 1993; Gurka, Hiatt, & Titus, 1984; Yang, Yan, Chin, Liang, Chen, & Zheng, 2004).

Additional research into this area has likewise shown the manner in which any researcher may apply a Fourier series to the realm of physical application, taking the concept from a theoretical one into the realm of practical applications (Jackson, 1941). In building on the body of past knowledge and historical research it is possible to find current solutions to the tangible problem found within the boundaries of Accra Polytechnic’s walls.

The amount of waste generated per capita constantly fluctuates based on the number of individuals within the given area and this means that a continual wave would be, and could be, produced if the data is to be graphed out over a period of years, and while future predictions could be made as to the amount of waste generated, such predictions are not necessarily likely to be accurate if going beyond the immediate future, as a great many factors work to contribute to the income generated per capita, and changes to those rates may occur at any time.

2.0 Methodology

The forecasting of solid waste generation can be modeled through the use of different approaches. For the purpose of this study, the campus of Accra Polytechnic was divided into three strata; the administration and office blocks, the classroom blocks and the students’ hostel blocks. The blocks served as the strata in this case. Within each stratum, five particular floors were identified and bins placed on each floor used in the study. These bins were attended to at a particular time in the morning for fifteen continuous week days for three weeks. The amount of waste was measured using appropriate equipment and this constituted the data for the analysis.

While the Fourier series could be applied, in this instance, using the sum of sine and cosine, given the large volumes of waste, the more practical means of applying the Fourier series to this particular task at hand is through the use of the weighted sums of complex exponential numerical values (Glynn, 2007). Average amount of waste generated per floor can be written exponentially and, taking these values, input into a Fourier series. Utilizing this particular approach allows for the resolution of the values necessary to create the graphical representation of the same while at the same time works to ensure that the numerical values stay manageable in size when writing out the valuations and working to provide a solution, both practical and mathematical, to the targeted issues at hand within Accra Polytechnic's current societal matrix. Beyond this study, the mathematical review of the solid waste management challenge in Accra Polytechnic, a TVET institution within the capital city of Ghana, will provide the Accra Metropolitan Assembly with the information it needs to be able to take the appropriate steps to work to manage and, ultimately, mitigate the high refuse problems present in Accra at this time.

3.0 Findings

3.1 Types of Solid Waste Generated

Different types of solid wastes were found in the various bins located on the selected floors for the study. These included:

(i) Rubbish, typically combustible, which consisted of materials such as paper, cardboard, plastics, textiles, rubber, leather and wood.
(ii) Non-combustible rubbish which consisted of glass, tin cans, aluminum cans, ferrous and non-ferrous metals and dirt.
(iii) Trash, which included all non-putrescible waste that would normally be in a students' hostel including brush and waste from minor hostel room repairs that is burnable.
(iv) Food wastes, raw or cooked, which were discarded. They included uneaten portions of meals and trimmings from food preparation activities.

3.2 Trend Daily Solid Waste Data

Table 3.1 shows the pattern of daily solid waste data (in kilograms) averaged for each stratum. Each stratum consisted of five floors and the average solid waste generated is found per stratum. This was for a period of fifteen working days. It is observed that amount of solid waste generated differs from one stratum to the other.

Figure 3.1 shows a time series graph for the solid waste data over the fifteen-day period. It is observed from the graph that the amount of solid waste generated in the Students' Hostel stratum exceeds that of the Lecture venues. The Office stratum generates the least among the three.

3.3 Population Prediction for Study Period

It is reported that solid waste generation is a strong function of population in an area. The amount of solid waste generated in each of these three strata depends on the population resident in these areas as well as those who migrate to the place. It is therefore necessary to predict the population in these three strata under consideration to predict the solid waste generation. To predict the population four empirical models are considered in this work. These are;

(i) Arithmetic Projection; which is assumed that population is increasing in arithmetical progression and assumption is made that this pattern persisted during the fifteen-day period. Using the past trend of population in the three strata, the population was projected mathematically as: $P_n = P_0(1 + rt)$.
(ii) Geometric Projection; with the assumption that population tends to grow at a geometric rate and it compounds like interest or money. The geometric curve portrays the idea that with each unit of time, the absolute addition to population gets bigger and bigger. The projection was done by using the formula: $P_n = P_0(1 + r)^t$.
(iii) Incremental Increase Method; which increases in population over unit time is considered for projection within the period. It was calculated from the formula: $P_n = P_0 + nx + \frac{1}{2}[(n(n + 1)]y$.
(iv) Decreasing Rate of Growth; this was assumed that decrease in growth rate would continue during the period of taking the data. It was calculated as $P_n = P_0[1 + (r + r_1)t]$.

The interpretation of these symbols can be found in table 3.2, in the appendix. The suitability of the projection method to predict the population is identified from the existing data that was collected from available records. This was used for the population prediction model. Based on these base period data, the population for the period is estimated using three methods, that is; arithmetic projection, geometric projection and incremental increase method. These estimated data for all study strata are compared with actual data. Based on comparison, appropriate method of population projection is selected for each study stratum. It must be noted that all these population prediction models are empirical.

3.4 Development of a Fourier Series Forecasting Model

From the data represented in Table 3.1, the first period gives an upper limit of 28000 and the lower limit of 4000. From the observed graph the occurrence of solid waste tends to replicate itself in an upward trend. The maximum point for the first period and minimum point are as indicated. The graph in Fig. 3.2 shows a sinusoidal function; therefore, for continuous modeling of the system. The cosine x was integrated using Fourier series method. The equation as given by Enrique, 1992 is $\int \cos x \, dx$, where; $x = (-3, -2, -1,0,1,2,3); y = (0, -1,0,1,0, -1,0)$. The upper boundary, y = 1 and the lower boundary = -1, on the y axis.

In this section, $f(x)$, denotes a function of the real variable x. this function is usually taken to be periodic, of period 2π, which is to say that $f(x + 2\pi) = f(x)$, for all real numbers x. We will attempt to write such a function as an infinite sum, or series of simpler 2π periodic functions. We will start by using an infinite sum of sine and cosine functions on the interval $[-\pi, \pi]$, as Fourier did.

For a periodic function $f(x)$ that is integral on $[-\pi, \pi]$, the numbers: $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx,\ \ n \geq 0$ and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx,\ \ n \geq 1$, are called the Fourier coefficients of f. The rest of the analysis which requires rigorous mathematics of the Fourier series is performed in the appendix. However, the final model is found as:
$Y_m = 1153.8x + 1600 - 12000\sum_{n=0}^{\infty} \sin nx$.

Therefore, the Fourier series straight line equation forecasting model is as above.

4.0 Discussions and Conclusions

4.1 Solid Waste per Capita

The amount of solid waste generated per person per day is found from the tables given in the appendices. Table 3.1, gives the average amount of solid waste (in kgs) generated within each stratum over the 15-day period. In the first stratum, an average of 36.07kg is computed as the amount of solid waste generated per day for offices located within the administration block and other identified blocks. The average number of occupants of the offices for the five floors selected for the study is also given in Table 3.3, as 188. The ratio of the amount of solid waste generated to the population within the stratum gives the solid waste per capita. For the offices, this is found to be 0.1919kg. This means each office worker on Accra Polytechnic campus, on average, generates 191.9g of solid waste per day.

The second stratum refers to Lecture rooms where, for the purpose of this study, five floors were selected from different blocks. From Table 3.1, it is seen that an average of 48.07kg of solid waste was generated per day. With an estimated population of 803, from Table 3.3, the solid waste per capita is also estimated as 0.0599kg. This is interpreted as; every student of Accra Polytechnic generates an average solid waste of 59.9g per day within the lecture halls area.

The third stratum is the students' hostel. Five floors were selected from the two hostel blocks and the average solid waste generated was estimated as 60.67kg, from Table 3.1. The population estimate for these floors is 643, from Table 3.3; and the solid waste generated per capita per day is also estimated as 0.0944kg. Each student admitted into any of the school hostels generates an average of 94.4g of solid waste per day.

It may look a bit strange that office workers generate the highest amount of solid waste per day. This can be explained with their population compared with the others. This is followed by the students in residence, which can be considered to be normal. These students spend much time at the hostel. The solid waste generated per capita for the lecture halls area was the least. This is also explained by the huge volume of human traffic around such areas per day.

4.2 Forecasting Solid Waste

The primary objective of this research is to develop a forecasting model that will be able to predict the amount of solid waste generated per person per day. Based on the primary data collected and the interaction with the officers in the Sanitation Unit, it was observed that accurate forecasting has not been practiced. The mathematical model has been developed to forecast the amounts of solid waste generated, while the solid waste per capita has also been found. It is recommended that model validation will have to be thoroughly done to confirm the veracity of the developed model. When validation is done, the model will help its users to make a reliable decision as to actualize the amount of solid waste generated at various locations in the city and the country at large.

4.3 Reducing Solid Waste Per Capita and Improving Solid Waste Disposal

Waste reduction is said to be a logical starting point for sustainable solid waste management, by reducing the amounts of waste that must be managed, by collection and disposal (UNEP, 1996). Source waste which is mostly organic is essential to note. The sanitation officers in Accra Polytechnic can practice the following to help reduce solid waste per capita as well as improving its disposal.

(i) Composting of the putrescible part of the solid wastes to reduce the waste being disposed off. The compost becomes fertilizer to be used in backyard gardens or serve as raw material in organic fertilizer blending factories or plants.

(ii) The institution must develop and enforce an efficient system of collecting the garbage as scheduled, which is daily for now.

(iii) Attempt must be made to segregate wastes and embark on a recycle program.

(iv) Increasing human resources at the sanitation unit and exposing them to state-of-the-art waste management practice.

(v) Organising a waste management training session for all personnel in the sanitation unit and extending to cover the entire polytechnic community.

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APPENDIX

Table 3.1: Average amount of solid waste (in kgs) generated within each stratum

Stratum	Day														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Office	30	30	31	37	32	30	30	34	41	40	31	38	28	37	35
Lecture Rooms	50	41	41	43	51	42	55	53	42	43	43	50	50	55	45
Students' Hostel	59	61	60	54	55	53	66	69	59	65	54	71	66	63	65

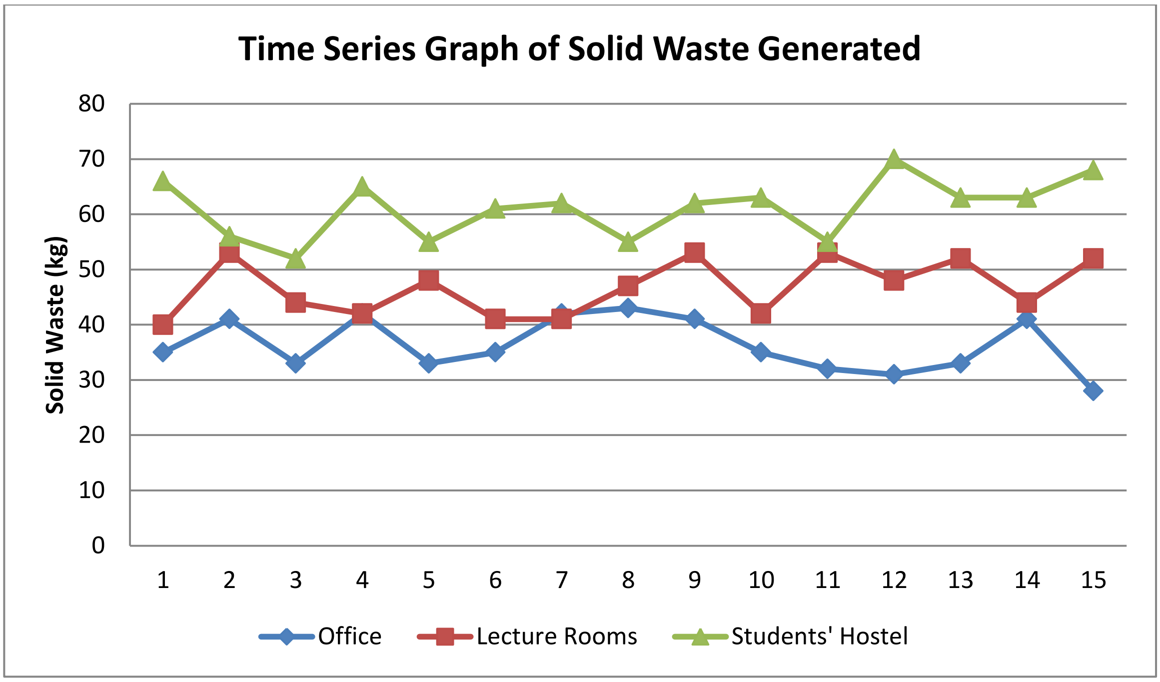

Fig. 3.1 Graph of Solid Waste Generated for the 15-day Period

Table 3.2: Population Prediction Models for Study Strata

Projection Method	Formula Used	Interpretation of Variables
Arithmetic	$P_n = P_0(1 + rt)$	P_n= Population in the nth period from the base year. P_0= Population in base year. r = Rate of Increase in Population. r_1 = Average decrease in the percentage increase. t = Number of periods from the base year. x = Average increase in population. y = Incremental increase in population.
Geometric	$P_n = P_0(1 + r)^t$	
Incremental Increase	$P_n = P_0 + nx + \frac{1}{2}[(n(n+1)]y$	
Decreasing Rate	$P_n = P_0[1 + (r + r_1)t]$	

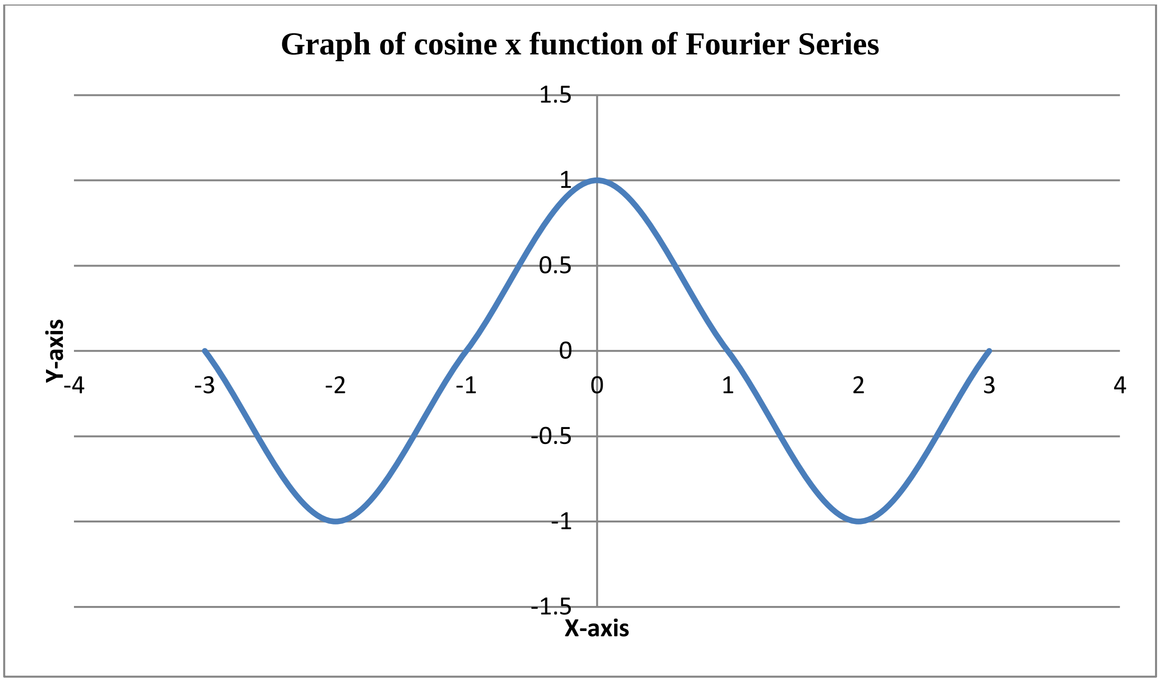

Fig.3.2: Graph Of Cosine *X* Function of Fourier Series

Table 3.3: Estimated Population within each stratum

Stratum	Week			
	1	2	3	Average
Office	181	194	190	188
Lecture Rooms	792	812	804	803
Students' Hostel	645	652	633	643

Table 3.4: Solid Waste Generated Per Capita

Stratum	Estimated Solid Waste Generated Per Capita over the 15-Day Period	
	Kilogrammes	Grammes
Offices	0.1919	191.9
Lecture Rooms	0.0599	59.9
Students' Hostels	0.0944	94.4