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The Modified Variational Iteration Method on the Newell-Whitehead-Segel Equation Using He's polynomials

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Abstract: In this paper, we apply a modified version of the variational iteration method (MVIM) for solving Newell-Whitehead-Segel equation. The Newell-Whitehead-Segel equation models the interaction of the effect of the diffusion term with the nonlinear effect of the reaction term which appeared in the investigation of fluid dynamics. The proposed modification is made by introducing He's polynomials in the correction functional of the variational iteration method. The use of Lagrange multiplier coupled with He's polynomials are the clear advantages of this technique over the decomposition method

Keywords: Variational Iteration Method, He's Polynomials, Newell-Whitehead-Segel equation, nonlinear differential equation.

1. Introduction

In order to model behavior and effects of many phenomena in different fields of science and engineering by mathematical concepts, nonlinear differential equations are introduced. Semi-analytical method as the Variational Iteration Method (VIM) is considered as a powerful and flexible algorithm to solve and obtain exact solution of nonlinear differential equations.

The idea of the VIM was first pioneered by He [1]. Then, the VIM is applied by He [2,3] in order to solve autonomous ordinary differential equation as well as delay differential equation. Also, new development and applications of the VIM to nonlinear wave equation, nonlinear fractional differential equations, nonlinear oscillations and nonlinear problems arising in various engineering applications is presented by He and Xh [4]. Variational iteration method is strong and efficient method which can be widely used to handle linear and nonlinear models. The VIM has no specific requirements for nonlinear operators. The method gives the solution in the form of rapidly convergent successive approximations that may give the exact solution if such a solution exists.

Several techniques including decomposition, finite element, Galerkin and cubic spline are employed to solve such equations analytically and numerically. Most of these used schemes are coupled with the inbuilt deficiencies like calculation of the so-called Adomian's polynomials and non compatibility with the physical nature of the problems. In a later work

Ghorbani et al. introduced He's polynomials [5,6] which are compatible with Adomian's polynomials but are easier to calculate and are more user friendly. This very reliable modified version [7.8] (MVIM) has been proved useful in coping with the physical nature of the nonlinear problems and hence absorbs all the positive features of the coupled techniques. The basic motivation of this paper is the application of this elegant coupling of He's polynomials and correction functional of VIM for solving Newell-Whitehead-Segel equation equations. The numerical results are very encouraging.

Nourazar et al. [9] obtained the exact solution of the Newell-Whitehead-Segel equation by using the homotopy perturbation method. Also, Pue-on [10] presented application of the Laplace adomian decomposition method for solving the Newell-Whitehead-Segel Equation. Analytic solution for Newell–Whitehead–Segel equation by differential transform method is presented by Aasaraai [11]. Application of the the homotopy perturbation method to the exact solution of nonlinear differential equations is presented by Nourazar et al. [12,13]. Soori et al. [14,15] presented application of the Variational Iteration Method and the Homotopy Perturbation Method to the fisher type equation.

The Newell-Whitehead-Segel equation describes the appearance of the stripe pattern in two dimensional systems. The equation has wide range of applications in mechanical and chemical engineering, ecology, biology, Rayleigh-Benard convection, faraday instability, nonlinear optics, chemical reactions and bio-engineering.

The Newell-Whitehead-Segel equation is written as:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial^2 x} + \alpha u - \beta u^p, \tag{1}$$

Where α, β and k are real numbers with $k > 0$ and p is a positive integer.

2. Analysis of Variational Iteration Method Using He's Polynomials

To illustrate the basic concept of the MVIM, we consider the following general differential equation:

$$Lu(t)+Nu(t)= g(t), \tag{2}$$

Where *L* is a linear operator, *N* is a nonlinear operator, and *g(t)* is a known analytic function. According to the variational iteration method, we can construct the following correction functional:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)(u_n(\xi) + N\tilde{u}_n(x,t) - g(t)d\xi, \tag{3}$$

Where λ is a general Lagrange multiplier which can be identified optimally via variational theory and $\tilde{u}_n$ is considered as a restricted variation which means $\tilde{u}_n = 0$. Now, we apply He's polynomials [15, 16]

$$\sum_{n=0}^{\infty} p^{(n)} u_n = u_n(x,t) + p\int_0^t \lambda(\xi)\left[\sum_{n=0}^{\infty} p^{(n)} L(u_n) + \sum_{n=0}^{\infty} p^{(n)}\, N(\tilde{u}_n)\right] d\xi - \int_0^T \lambda(\xi)\, g(\xi) d\xi$$

which is the MVIM [20 – 25] and is formulated by the coupling of VIM and He's polynomials. The comparison of like powers of p gives solutions of various orders.

3. Newell-Whitehead-Segel Equation

To illustrate the capability and reliability of the method, three cases of non linear diffusion equations are presented.

Case 1

In Eq. (1) for $\alpha = 2, \beta = 3, k = 1 \; and \; p = 2$ Newell-Whitehead-Segel equation is written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} + 2u - 3u^2, \tag{4}$$

Subject to a constant initial condition

$$u(x,0) = \lambda\,,$$

The correction functional for Eq. (4) is in the following form

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + 3{u_n}^2(x,\xi) - 2u_n(x,\xi)d\xi,\right) \tag{5}$$

where u_n is restricted variation $\delta u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary, we have

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + 3{u_n}^2(x,\xi) - 2u_n(x,\xi)d\xi,\right) \tag{6}$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi}\right) d\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1 + \lambda(\xi)\big) - \delta\int_0^t \lambda'(\xi) u_n(x,t)\, d\xi \tag{7}$$

By using the following stationary conditions

$$\delta u_n:\; 1 + \lambda(\xi) = 0$$

$$\delta u_n:\; \lambda'(\xi) = 0. \tag{8}$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$ therefore the following iteration formula becomes as

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + 3{u_n}^2(x,\xi) - 2u_n(x,\xi)d\xi, \tag{9}$$

We can select$u_0(x,y)=\lambda$ from the given condition.

Now we apply the variational iteration method using He's polynomials (MVIM)

$$u_0+pu_1+p^2u_2+\cdots=\lambda-p\int_0^t\left[\frac{\partial u_0}{\partial\xi}+p\frac{\partial u_1}{\partial\xi}+p^2\frac{\partial u_2}{\partial\xi}+\cdots\right]d\xi+p\int_0^t\left[\frac{\partial^2 u_0}{\partial^2 x}+p\frac{\partial^2 u_1}{\partial^2 x}+p^2\frac{\partial^2 u_2}{\partial^2 x}+\cdots\right]d\xi-p\int_0^t 3[u_0+pu_1+p^2u_2+\cdots]^2 d\xi+p\int_0^t 2[u_0+pu_1+p^2u_2+\cdots]\,d\xi, \quad (10)$$

By comparing the co-efficient of like powers of p, we obtain

$$p^{(0)}: u_0(x,t)=\lambda,$$

$$p^{(1)}: u_1(x,t)=\lambda-\int_0^t\left[\frac{\partial u_0}{\partial\xi}-\frac{\partial^2 u_0}{\partial^2 x}+3{u_0}^2-2u_0\right]d\xi,=\lambda+\lambda(2-3\lambda)t$$

$$p^{(2)}: u_2(x,t)=\lambda+\lambda(2-3\lambda)t-\int_0^t\left[\frac{\partial u_1}{\partial\xi}-\frac{\partial^2 u_1}{\partial^2 x}+3{u_1}^2-2u_1\right]d\xi,$$

$$=\lambda+\lambda(2-3\lambda)t+\big(\lambda(2-3\lambda)t-3\lambda^2(2-3\lambda)\big)t^2-3\lambda^2(2-3\lambda)t^3,$$

$$p^{(3)}: u_3(x,t)=\lambda+\lambda(2-3\lambda)t+\big(\lambda(2-3\lambda)t-3\lambda^2(2-3\lambda)\big)t^2-3\lambda^2(2-3\lambda)t^3-\int_0^t\left[\frac{\partial u_1}{\partial\xi}-\frac{\partial^2 u_1}{\partial^2 x}+3{u_1}^2-2u_1\right]d\xi,$$

$$=\lambda+\lambda(2-3\lambda)t+\lambda(3\lambda-1)(3\lambda-2)t^2-\lambda(3\lambda-2)(27\lambda^2-18\lambda+2)\frac{t^3}{3}+2\lambda^2(3\lambda-1)(3\lambda-2)^2t^4-3\lambda^2(3\lambda-2)^2(15\lambda^2-10\lambda+1)\frac{t^5}{5}+\lambda^3(3\lambda-1)(3\lambda-2)^3t^6-3\lambda^4(3\lambda-2)^4\frac{t^7}{7},$$

The series solution is therefore given by

$$u_n(`x,t)=\lambda+\lambda(2-3\lambda)t+\lambda(3\lambda-1)(3\lambda-2)t^2-\lambda(3\lambda-2)(27\lambda^2-18\lambda+2)\frac{t^3}{3}+2\lambda^2(3\lambda-1)(3\lambda-2)^2t^4-3\lambda^2(3\lambda-2)^2(15\lambda^2-10\lambda+1)\frac{t^5}{5}+\lambda^3(3\lambda-1)(3\lambda-2)^3t^6-3\lambda^4(3\lambda-2)^4\frac{t^7}{7}+\ldots,$$

(11)

$$`u(x,t)=\lim_{n\to\infty}u_n(`x,t)$$

We can write Eq. (11) in the closed form as

$$u(x,t)=\frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3}+\lambda-\lambda e^{2t}}. \quad (12)$$

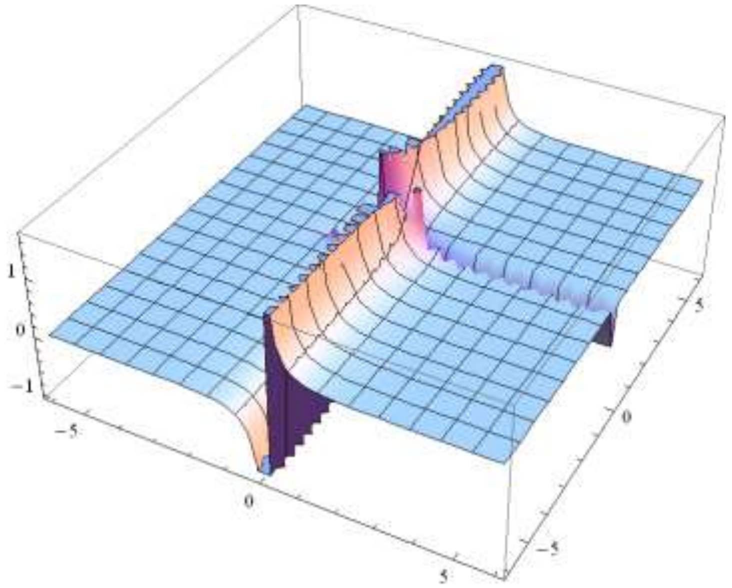

Fig1. Surface plot of the solution (12)

Case 2

In Eq. (1) for $\alpha = 1, \beta = 1, k = 1$ *and* $p = 2$ Newell-Whitehead-Segel equation is written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} - u^2 + u \qquad (13)$$

Subject to the initial condition

$$\mathrm{u}(\mathrm{x}, 0) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2},$$

The correction functional for Eq. (13) is in the following form

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial \mathrm{u_n}(\mathrm{x},\xi)}{\partial \xi} - \frac{\partial^2 \mathrm{u_n}(\mathrm{x},\xi)}{\partial^2 \mathrm{x}} + \mathrm{u_n}^2(\mathrm{x},\xi) - \mathrm{u_n}(\mathrm{x},\xi)\mathrm{d}\xi,\right) \quad (14)$$

where u_n is restricted variation $\delta u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary, we have

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi)\left(\frac{\partial \mathrm{u_n}(\mathrm{x},\xi)}{\partial \xi} - \frac{\partial^2 \mathrm{u_n}(\mathrm{x},\xi)}{\partial^2 \mathrm{x}} + \mathrm{u_n}^2(\mathrm{x},\xi) - \mathrm{u_n}(\mathrm{x},\xi)\mathrm{d}\xi,\right)$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi)\left(\frac{\partial \mathrm{u_n}(\mathrm{x},\xi)}{\partial \xi}\right)\mathrm{d}\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1 + \lambda(\xi)\big) - \delta \int_0^t \lambda'(\xi) u_n(x,t)\,\mathrm{d}\xi \qquad (15)$$

By using the following stationary conditions

$$\delta u_n:\ 1 + \lambda(\xi) = 0$$

$$\delta u_n:\ \lambda'(\xi) = 0.$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$ therefore the following iteration formula becomes as

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + u_n^2(x,\xi) - u_n(x,\xi) d\xi, \qquad (16)$$

We select

$$u_o(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2},$$

Now we apply the variational iteration method using He's polynomials (MVIM)

$$u_0 + pu_1 + p^2u_2 + \cdots = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} - p\int_0^t \left[\frac{\partial u_0}{\partial \xi} + p\frac{\partial u_1}{\partial \xi} + p^2\frac{\partial u_2}{\partial \xi} + \cdots\right] d\xi + p\int_0^t \left[\frac{\partial^2 u_0}{\partial^2 x} + p\frac{\partial^2 u_1}{\partial^2 x} + p^2\frac{\partial^2 u_2}{\partial^2 x} + \cdots\right] d\xi - p\int_0^t [u_0 + pu_1 + p^2u_2 + \cdots]^2 d\xi + p\int_0^t [u_0 + pu_1 + p^2u_2 + \cdots]\, d\xi, \qquad (17)$$

By comparing the co-efficient of like powers of p, we obtain

$$p^{(0)}: u_0(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2},$$

$$p^{(1)}: u_1(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} - \int_0^t \left[\frac{\partial u_0}{\partial \xi} - \frac{\partial^2 u_0}{\partial^2 x} + u_0^2 - u_0\right] d\xi, = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} + \left(\frac{5}{3}\frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3}\right)t$$

$$p^{(2)}: u_2(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^2} + \left(\frac{5}{3}\frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3}\right)t - \int_0^t \left[\frac{\partial u_1}{\partial \xi} - \frac{\partial^2 u_1}{\partial^2 x} + u_1^2 - u_1\right] d\xi,$$

$$= \frac{1}{5}\frac{5e^{\frac{x}{\sqrt{6}}} + \left(5e^{\frac{x}{\sqrt{6}}}\right)^2 - 1}{e^{\frac{x}{\sqrt{6}}}\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} + \left(\frac{5}{3}\frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3}\right)t + \left(\frac{25}{36}\frac{e^{\frac{x}{\sqrt{6}}}\left(2e^{\frac{x}{\sqrt{6}}}-1\right)}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^4}\right)t^2 - \left(\frac{25}{27}\frac{\left(2e^{\frac{x}{\sqrt{6}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^6}\right)t^3$$

$$u_n(x,t) = \frac{1}{5}\frac{5e^{\frac{x}{\sqrt{6}}} + \left(5e^{\frac{x}{\sqrt{6}}}\right)^2 - 1}{e^{\frac{x}{\sqrt{6}}}\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3} + \left(\frac{5}{3}\frac{e^{\frac{x}{\sqrt{6}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^3}\right)t + \left(\frac{25}{36}\frac{e^{\frac{x}{\sqrt{6}}}\left(2e^{\frac{x}{\sqrt{6}}}-1\right)}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^4}\right)t^2 - \left(\frac{25}{27}\frac{\left(2e^{\frac{x}{\sqrt{6}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^6}\right)t^3 + \cdots,$$

(18)

We can write Eq. (18) in the closed form as

$$u(x,t) = \frac{1}{\left(1+e^{\frac{x}{\sqrt{6}}\frac{5}{6}t}\right)^{2}}. \tag{19}$$

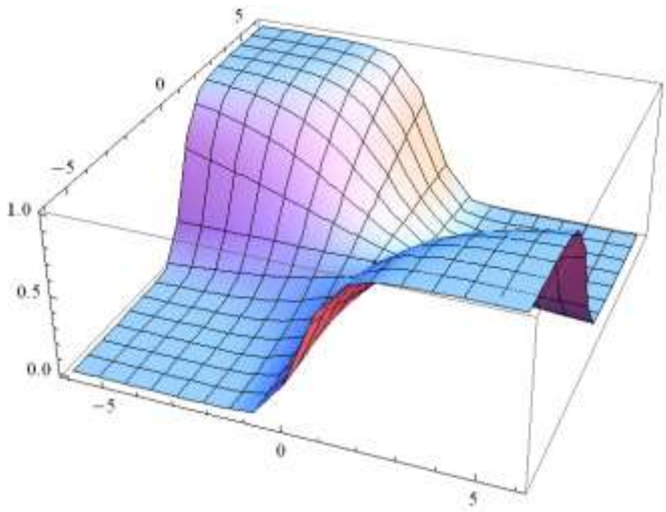

Fig2. Surface plot of the solution (19)

Case3

In Eq. (1) for $\alpha = 1, \beta = 1, k = 1$ *and* $p = 4$ Newell-Whitehead-Segel equation is written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x} - u^4 + u \tag{20}$$

Subject to the initial condition

$$u(x,0) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}}.$$

The correction functional for Eq. (20) is in the following form

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + {u_n}^4(x,\xi) - u_n(x,\xi)d\xi,\right) \tag{21}$$

where u_n is restricted variation $\delta u_n = 0$, λ is a Lagrange multiplier and u_0 is an initial approximation or trial function.

With the above correction functional stationary we have

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta\int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + {u_n}^4(x,\xi) - u_n(x,\xi)d\xi,\right)$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi)\left(\frac{\partial u_n(x,\xi)}{\partial \xi}\right) d\xi,$$

$$\delta u_{n+1}(x,t) = \delta u_n(x,t)\big(1+\lambda(\xi)\big) - \delta \int_0^t \lambda'(\xi) u_n(x,t)\, d\xi \qquad (22)$$

By using the following stationary conditions

$$\delta u_n:\ 1+\lambda(\xi) = 0$$

$$\delta u_n:\ \lambda'(\xi) = 0$$

This gives the Lagrange multiplier $\lambda(\xi) = -1$ therefore the following iteration formula becomes as

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \frac{\partial u_n(x,\xi)}{\partial \xi} - \frac{\partial^2 u_n(x,\xi)}{\partial^2 x} + u_n{}^4(x,\xi) - u_n(x,\xi) d\xi, \qquad (23)$$

We choose

$$u_o(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}},$$

Now we apply the variational iteration method using He's polynomials (MVIM):

$$u_0 + pu_1 + p^2u_2 + \cdots = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} - p\int_0^t \left[\frac{\partial u_0}{\partial \xi} + p\frac{\partial u_1}{\partial \xi} + p^2\frac{\partial u_2}{\partial \xi} + \cdots\right] d\xi + p\int_0^t \left[\frac{\partial^2 u_0}{\partial^2 x} + p\frac{\partial^2 u_1}{\partial^2 x} + p^2\frac{\partial^2 u_2}{\partial^2 x} + \cdots\right] d\xi - p\int_0^t [u_0 + pu_1 + p^2u_2 + \cdots]^4 d\xi + p\int_0^t [u_0 + pu_1 + p^2u_2 + \cdots] d\xi, \qquad (24)$$

By comparing the co-efficient of like powers of *p*, we obtain

$$p^{(0)}: u_0(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}},$$

$$p^{(1)}: u_1(x,t) =$$

$$\frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} - \int_0^t \left[\frac{\partial u_0}{\partial \xi} - \frac{\partial^2 u_0}{\partial^2 x} + u_0{}^4 - u_0\right] d\xi, = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \left(\frac{7}{5}\frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^{\frac{5}{3}}}\right)t,$$

$$p^{(2)}: u_2(x,t) = \frac{1}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{2}{3}}} + \left(\frac{7}{5}\frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1+e^{\frac{x}{\sqrt{6}}}\right)^{\frac{5}{3}}}\right)t - \int_0^t \left[\frac{\partial u_1}{\partial \xi} - \frac{\partial^2 u_1}{\partial^2 x} + u_1{}^4 - u_1\right] d\xi,$$

$$=\frac{7e^{\frac{3x}{\sqrt{10}}}+7\left(e^{\frac{3x}{\sqrt{10}}}\right)^2-1}{7e^{\frac{3x}{\sqrt{10}}}\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}+\left(\frac{7}{5}\frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t+\left(\frac{49}{100}\frac{e^{\frac{3x}{\sqrt{10}}}\left(2e^{\frac{3x}{\sqrt{10}}}-3\right)}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}}\right)t^2$$

$$-\left(\frac{98}{25}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^2}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{14}{3}}}\right)t^3-\left(\frac{243}{125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^3}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{17}{3}}}\right)t^4,$$

$u_n(x,t)=$

$$\frac{7e^{\frac{3x}{\sqrt{10}}}+7\left(e^{\frac{3x}{\sqrt{10}}}\right)^2-1}{7e^{\frac{3x}{\sqrt{10}}}\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}+\left(\frac{7}{5}\frac{e^{\frac{3x}{\sqrt{10}}}}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{5}{3}}}\right)t+\left(\frac{49}{100}\frac{e^{\frac{3x}{\sqrt{10}}}\left(2e^{\frac{3x}{\sqrt{10}}}-3\right)}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{8}{3}}}\right)t^2-\left(\frac{98}{25}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^2}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{14}{3}}}\right)t^3-$$

$$\left(\frac{243}{125}\frac{\left(e^{\frac{3x}{\sqrt{10}}}\right)^3}{\left(1+e^{\frac{3x}{\sqrt{10}}}\right)^{\frac{17}{3}}}\right)t^4\ldots,$$

(25)

Eq. (25) in the closed form is as

$$u(x,t)=\left(\frac{1}{2}tanh\left(-\frac{3}{2\sqrt{10}}\left(x-\frac{7}{\sqrt{10}}t\right)\right)+\frac{1}{2}\right)^{\frac{2}{3}}. \qquad (26)$$

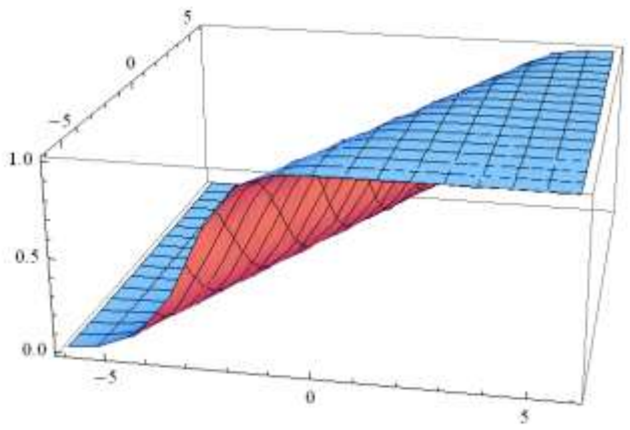

Fig3. Surface plot of the solution (26)

4. Conclusion

In this paper, we applied the variational iteration method using He's polynomials (MVIM) for finding the solutions of Newell-Whitehead-Segel equation. The use of Lagrange multiplier coupled with He's polynomials are the clear advantages of this technique over the decomposition method.

References

[1] J.H. He, Variational iteration method - a kind of non-linear analytical technique: Some examples. Int. J. NonLinear Mech. 34 (1999), 699-708.

[2] J.H. He, Variational iteration method for autonomous ordinary differential systems. Applied Math.andComput. 114 (2000), 115–123.

[3] J.H. He, Variational iteration method for delay differential equations.Commun. Nonlinear Sci. and Numer.Simul. 2 (1997), 235-236.

[4] J.H. He and X.h.Wu, Variational iteration method: new development and applications. Comput.and Math. Appl. 54 (2007), 881–894.

[5] A. Ghorbani , J. S. Nadjfi. He's homotopy perturbation method for calculating Adomian's polynomials, Int. J. Nonlin. Sci. Num. Simul. 8 (2):229-332 (2007).

[6] Ghorbani, A. Beyond Adomian Polynomials: He Polynomials. Chaos, Solitons & Fractals, 39, 1486-1492 (2009).

[7] M.A. Noor and S. T. Mohyud-Din, Modified Variational Iteration Method for a Boundary Layer Problem in Unbounded Domain, Int. J. Nonlinear Sci. Numer. Simul. 9, 141-157 (2008).

[8] M.A. Noor and S. T. Mohyud-Din, Modified variational iteration method for solving fourth-order boundary value problems, J. Appl. Math. Comput. 29, 81-94 (2009)

[9] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On The Exact Solution of Newell-Whitehead-Segel Equation Using the Homotopy Perturbation Method. Aust. J. Basic and Appl. Sci. 5 (2011), 1400-1411.

[10] P. Pue-on, Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation. Appl. Math. Sci. 7 (2013), 6593-6600.

[11] A. Aasaraai, Analytic solution for Newell–Whitehead–Segel equation by differential transform method. Middle-East J. Sci. Res. 10 (2011), 270-273.

[12] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On the Exact Solution of Burgers-Huxley Equation Using the Homotopy Perturbation Method. J. Appl. Math. and Phy. 3 (2015), 285-294.

[13] S.S. Nourazar, M. Soori and A. Nazari-Golshan, On the Homotopy Perturbation Method for the Exact Solution of Fitzhugh–Nagumo Equation. Int. J. Math. Comput. 27 (2015), 32-43.

[14] M. Soori, S.S. Nourazar, and A. Nazari-Golshan, Application of the Variational Iteration Method and the Homotopy Perturbation Method to the Fisher Type Equation, Int. J. Math. Comput. 27 (2015), 1-9.

[15] M. Soori, The Homotopy Perturbation Method and the Variational Iteration Method to Nonlinear Differential Equations (2011).