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Bayesian Panel Data Model Based on Markov Chain Monte Carlo

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Abstract

The general aim of this paper is to deal with problems of estimation , prediction, and model building for panel data model .Bayesian approach based on Markov chain Monte Carlo (MCMC) employed to make inferences on panel data model coefficients under some conditions on the prior distribution . We investigate the posterior density and identify the analytic form of the Bayes factor for checking the model.

Keywords: Panel Data Model , Likelihood function , Bayesian approach , Markov chain Monte Carlo (MCMC), Prior distribution, Posterior distribution , Bayes factor.

1. Introduction

One of the aims of science is to describe and predict events in the world in which we live. One way this is accomplished is by finding a formula or equation that relates quantities in the world, in linear model we attempt to model the relationship between variables,[5],[10].

Linear models play a central part in modern statistical methods. On the one hand ,these models are able to approximate a large amount of metric data structures in their entire range of definition or at least piecewise. The theory of generalized models enables us, through appropriate link function , to apprehend error structures that deviate from the normal distribution , hence ensuring that a linear model is maintained in principle,[9].

Linear statistical methods are widely used as part of this learning process. In the biological, physical, and social sciences , as well as in business and engineering ,linear models are used in both the planning stages of research and analysis of the resulting data,[10],[12] .

In spite of the availability of highly innovative tools in statistics , the main tool of the applied statistician remains the linear model. The linear model involves the simplest and seemingly most restrictive statistical properties: independence, normality, constancy of variance , and linearity. However, the model and the statistical methods associated with it are surprisingly versatile and robust. More importantly, mastery of the linear model is a prerequisite to work with advanced statistical tools because most advanced tools are generalizations of the linear model. The linear model is thus central to the training of any statistician, applied or theoretical,[10].

Panel (or longitudinal) data are cross – sectional and time – series . There are multiple entities, each of which has repeated measurements at different time periods . Panel data have a cross – sectional (entity or subject) variable and a time series variable . In Stata , this arrangement is called the long form (as opposed to the wide form) . While the long form has both group (individual level) and time variables ,the wide form includes either group or time variables. Panel data usually give the researcher a large number of data points, increasing the degrees of freedom and reducing the collinearity among explanatory variables. Panel data models have becomes increasingly popular among applied researchers due to their heightened capacity for capacity for capturing the complexity of human behaviour as compared to cross-sectional or time –series data models. As a consequence , more and richer panel data sets also have become increasingly available,[1],[6],[13].

Bayesian estimation and inference has a number of advantages in statistical modelling and data analysis. The Bayesian statistical paradigm is conceptually simple and general because inference involve only probability calculation as opposed to maximization of a function like the log likelihood .On the other hand , the probability calculations usually entail complicated or even intractable integrals. In Bayesian statistics, uncertainty about the value of a parameter is expressed using the tools of the probability theory . Density functions of the parameters reflect the current credibility of possible update the uncertainty distributions for parameters, and then draw sensible conclusions using theses updated distribution,[3],[7],[8],[11],[14].

Classical statistics treats parameters as fixed unknown quantities , Bayesian statistics is based on different philosophy; parameters are treated as random variables . The probability distribution changes as new data are acquired. Bayesian statistical differs from classical statistics in two important respects: the use of prior distribution to characterize knowledge of the parameter values prior to data collection and the use of posterior distribution – that is, the conditional distribution of the parameters given the data- as the basis of inference. Some statisticians are uneasy about the use of priors, but when done with care, the use of priors is quite sensible. In some situations, we might have strong prior beliefs that will influence our analysis. In the other situations, where we feel that know little or nothing about the parameters, we can chose " noninformative " prior , one could also use an " improper prior" (i.e., one with infinite mass) such as the uniform distribution an real line. By stating a prior, we make clear how much or how little, we believe we know a prior about the parameter. In constant to the controversy about the use of priors, there is little argument that the posterior provides a powerful inferential machinery,[3],[7],[8],[11],[14].

The calculation required by the Bayesian inference – in particular , the need to integrate the parameters out of joint density – have been a serious obstacle on the application of Bayesian methods. However, new Monte Carlo techniques such as Markov chain Monte Carlo and importance sampling have provided powerful methods for attacking this problem,[2],[4],[11].

The general aim of this paper is to deal with problems of estimation , prediction, and model building for panel data model .Bayesian approach based on Markov chain Monte Carlo (MCMC) employed to make inferences on panel data model coefficients under some conditions on the prior distribution . We investigate
the posterior density and identify the analytic form of the Bayes factor for checking the model

2.Panel Data Model and The Prior Distribution

Consider the model:

$$Y_{it} = \mu + \sum_{j=1}^{K} \beta_j X_{jit} + \varepsilon_{it} , i = 1, \dots, N, t = 1, \dots, T, \qquad (1)$$

where, Y_{it} the value of response variable for i^{th} unit at time t , X_{jit} the explanatory variables , $\mu , \beta_j , j = 1, \dots, K$ are fixed parameters and ε_{it} is an error term with $\varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$.

Now , if the parameter μ is specified as :

$$\mu = \beta_0 + u_i \ , \tag{2}$$

where, $u_i \sim N(0, \sigma_u^2)$, then, the model (1) is

$$Y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j x_{jit} + u_i + \varepsilon_{it} \ . \tag{3}$$

The model (3) is rewrite as follows :

$$Y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j x_{jit} + \omega_{it} \ , \tag{4}$$

where, $\omega_{it} = u_i + \varepsilon_{it}$, $\omega_{it} \sim N(0 , \sigma_\omega^2)$, $\sigma_\omega^2 = \sigma_\varepsilon^2 + \sigma_u^2$, thus by using matrix notation the model (4) is

$$Y = F\theta + \omega \ , \tag{5}$$

where, $F = [e , X]$, $e = [1,1, \dots ,1]^T$ has length NT , $Y = [Y_{11}, \dots, Y_{1T}, Y_{21}, \dots Y_{2T}, \dots, Y_{N1}, \dots, Y_{NT}]^T$ has length NT , $X = [X_1, X_2, \dots, X_N]^T$ is a $NT \times K$ design matrix of fixed effects , $\theta = [\beta_0, \beta_1, \dots, \beta_K]^T$ has length $K + 1$, and $\omega = [\omega_{11}, \dots, \omega_{1T}, \omega_{21}, \dots \omega_{2T}, \dots, \omega_{N1}, \dots, \omega_{NT}]^T$ has length NT .From model (5) , we have $Y \sim N(F\theta , \Psi)$, where $\Psi = E(\omega\omega^T) = I_N \otimes (\sigma_\varepsilon^2 I_t + \sigma_u^2 ee^T)$

$$= \sigma_\varepsilon^2 (I_N \otimes I_t) + \sigma_u^2 (I_N \otimes ee^T),$$

replace I_T by ($E_T + J_T$) and ee^T by $T J_T$, where $J_T = \frac{1}{T} ee^T$ and $E_T = I_T - J_T$, then

$$\Psi = \sigma_\varepsilon^2 [I_N \otimes (E_T + J_T)] + \sigma_u^2 (I_N \otimes T J_T)$$

$$= \sigma_\varepsilon^2 (I_N \otimes E_T) + \sigma_\varepsilon^2 (I_N \otimes J_T) + T\sigma_u^2 (I_N \otimes J_T) ,$$

by collecting terms with the same matrices, we get

$\Psi = \sigma_\varepsilon^2 (I_N \otimes E_T) + (\sigma_\varepsilon^2 + T\sigma_u^2) (I_N \otimes J_T) = \sigma_\varepsilon^2 Q + \sigma_1^2 P$, where , $\sigma_1^2 = (\sigma_\varepsilon^2 + T\sigma_u^2)$ and

$\Psi^{-1} = \frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}$, $|\Psi| =$ product of its characteristic roots, [1] $\rightarrow |\Psi| = (\sigma_\varepsilon^2)^{N(T-1)} (\sigma_1^2)^N$.

The likelihood function is the joint density of the $Y's$ that is

$$L(Y ; \theta, \Psi) = (2\pi)^{\frac{-NT}{2}} |\Psi|^{\frac{-1}{2}} \exp\left\{ \frac{-1}{2} (Y - F\theta)^T \Psi^{-1} (Y - F\theta) \right\}$$

$$= (2\pi)^{\frac{-NT}{2}} (\sigma_\varepsilon^2)^{\frac{-N(T-1)}{2}} (\sigma_1^2)^{\frac{-N}{2}} \exp\{\frac{-1}{2} (Y - F\theta)^T \left[\frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}\right] (Y - F\theta)\}.$$

Then, the likelihood estimators of parameters θ , σ_ε^2 , σ_1^2 are

$$\hat{\theta} = (F^T \Psi^{-1} F)^{-1} (F^T \Psi^{-1} Y) , \hat{\sigma}_\varepsilon^2 = \frac{1}{N(T-1)} (Y - F\hat{\theta})^T Q (Y - F\hat{\theta})$$ and

$$\hat{\sigma}_1^2 = \frac{1}{N} (Y - F\hat{\theta})^T P ((Y - F\hat{\theta}).$$

To specify a complete Bayesian model , we need a prior distribution on $(\theta, \sigma_\varepsilon^2, \sigma_1^2)$ .we will use the uniform distribution $U(0,1)$ of the vector parameters θ , as well as we will assume that the prior distribution on σ_ε^2 and σ_1^2 are invers gamma with of parameters $\alpha_\varepsilon, \beta_\varepsilon$,$\alpha_1$and β_1 respectively .i.e.

$\pi_0(\sigma_\varepsilon^2) = \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} (\alpha_\varepsilon)^{-(\alpha_\varepsilon+1)} \exp\left(-\frac{\beta_\varepsilon}{\sigma_\varepsilon^2}\right)$ and $\pi_0(\sigma_1^2) = \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} (\alpha_1)^{-(\alpha_1+1)} \exp\left(-\frac{\beta_1}{\sigma_1^2}\right)$, where $\alpha_\varepsilon, \beta_\varepsilon$,α_1 and β_1 are hyperparameters that determine the priors and must be chosen by the statistician.

3 . The Posterior Distribution

From the model (5) we have $Y|\theta, \sigma_\varepsilon^2, \sigma_1^2 \sim N_{NT}(F\theta, \Psi)$. Then ,the likelihood function is

$$L(Y|\theta, \sigma_\varepsilon^2, \sigma_1^2) = \prod_{\substack{i=1 \\ t=1}}^{NT} (2\pi)^{\frac{-1}{2}} \exp\{-\frac{1}{2}(Y - F\theta)^T \Psi^{-1} (Y - F\theta)\}.$$

In the exponent, we add and subtract $F\hat{\theta}$ to obtain

$$[(Y - F\theta)^T \Psi^{-1} (Y - F\theta)] = [(Y - F\hat{\theta} + F\hat{\theta} - F\theta)^T \Psi^{-1} (Y - F\hat{\theta} + F\hat{\theta} - F\theta)]$$

$$= [(Y - F\hat{\theta}) - F(\theta - \hat{\theta})]^T \Psi^{-1} [(Y - F\hat{\theta}) - F(\theta - \hat{\theta})]$$

$$= [(Y - F\hat{\theta})^T \Psi^{-1} (Y - F\hat{\theta}) - (Y - F\hat{\theta})^T \Psi^{-1} F(\theta - \hat{\theta}) -$$

$$(\theta - \hat{\theta})^T F^T \Psi^{-1}(Y - F\hat{\theta}) + (\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})],$$

since, $(F^T \Psi^{-1} F)\,\hat{\theta} = F^T \Psi^{-1} Y$, then

$$[(Y - F\theta)^T \Psi^{-1}(Y - F\theta)] = (Y - F\hat{\theta})^T \Psi^{-1}(Y - F\hat{\theta}) + (\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta}) \qquad (6)$$

The joint posterior density of the coefficients θ and the variances σ_ε^2 and σ_1^2 given by the expression

$$\pi_1(\theta|\sigma_\varepsilon^2, \sigma_1^2) \propto L(Y|\theta, \sigma_\varepsilon^2, \sigma_1^2)\pi_0(\theta, \sigma_\varepsilon^2, \sigma_1^2)$$

$$\propto (2\pi)^{\frac{-NT}{2}} (\sigma_\varepsilon^2)^{(\frac{-N(T-1)}{2}} (\sigma_1^2)^{\frac{-N}{2}} \exp\{-\tfrac{1}{2}(Y - F\hat{\theta})^T \left(\frac{Q}{\sigma_\varepsilon^2} + \frac{P}{\sigma_1^2}\right)(Y - F\hat{\theta})\}$$

$$\exp\ \{-\tfrac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})\} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)}(\sigma_\varepsilon^2\ )^{-(\alpha_\varepsilon+1)} \exp\left\{\frac{-\beta_\varepsilon}{\sigma_\varepsilon^2}\right\} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}(\sigma_1^2\ )^{-(\alpha_1+1)} \exp\ \left\{\frac{-\beta_1}{\sigma_1^2}\right\}$$

$$\propto (\sigma_\varepsilon^2\ )^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \exp - \left\{\frac{\frac{1}{2}(Y - F\hat{\theta})^T Q (Y - F\hat{\theta}) + \beta_\varepsilon}{\sigma_\varepsilon^2}\right\} (\sigma_1^2)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}$$

$$\exp - \left\{\frac{\frac{1}{2}(Y - F\hat{\theta})^T P\ (Y - F\hat{\theta}) + \beta_1}{\sigma_1^2}\right\} \exp\ \{-\tfrac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})\}.$$

From this expression , we can deduce the following conditional and marginal posterior distributions

$$\pi_1(\theta|\sigma_\varepsilon^2, \sigma_1^2, Y) \propto \exp\ \{-\frac{1}{2}(\theta - \hat{\theta})^T F^T \Psi^{-1} F(\theta - \hat{\theta})\}, \qquad (7)$$

and

$$\pi_1(\sigma_\varepsilon^2|\theta, \sigma_1^2, Y) \propto\ (\sigma_\varepsilon^2\ )^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \exp\left\{-\frac{\frac{1}{2}(Y - F\hat{\theta})^T Q (Y - F\hat{\theta}) + \beta_\varepsilon}{\sigma_\varepsilon^2}\right\}, \qquad (8)$$

$$\pi_1(\sigma_1^2|\theta, \sigma_\varepsilon^2, Y) \propto\ (\sigma_1^2)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)} \exp\left\{-\frac{\frac{1}{2}(Y - F\hat{\theta})^T P\ (Y - F\hat{\theta}) + \beta_1}{\sigma_1^2}\right\}. \qquad (9)$$

Therefore , it follows that

$$(\theta|\sigma_\varepsilon^2, \sigma_1^2, Y) \sim N(\hat{\theta}, F^T \Psi^{-1} F), \qquad (10)$$

$$\sigma_\varepsilon^2\ |\ \theta, \sigma_1^2, Y \sim IG\left(\alpha_\varepsilon + \frac{N(T-1)}{2}, \beta_\varepsilon + \frac{1}{2}\ (Y - F\hat{\theta})^T Q\ (Y - F\hat{\theta}\ )\ \right), \qquad (11)$$

$$\sigma_1^2\ |\theta, \sigma_\varepsilon^2, Y \sim IG\left(\alpha_1 + \frac{N}{2}, \beta_1 + \frac{1}{2}\ (Y - F\hat{\theta})^T P\ \ (Y - F\hat{\theta})\ \right). \qquad (12)$$

4. Bayes factor

We would like to choose between a fully Bayesian panel data model with (K+1) of parameters against a Bayesian panel data model with $(q+1)$ of parameters ,where $q < K$, by using Bayes factor for two hypotheses

$$\left.\begin{array}{l} H_0: Y_{it} = \beta_0 + \sum_{j=1}^{q} \beta_j x_{jit} + \omega_{it}, \text{ or } H_0: F^0\theta^0 + \omega \\ \text{against} \\ H_1: Y_{it} = \beta_0 + \sum_{j=1}^{K} \beta_j x_{jit} + \omega_{it}, \text{ or } H_1: F\theta + \omega \end{array}\right\}, \qquad (13)$$

where ,θ^0 is $(q+1)$ vectors of parameters , F^0 is an $NT \times (q+1)$ design matrix and $q < K$. We compute the Bayes factor, B_{01} of H_0 relative to H_1 for testing problem(13) as follows

$$B_{01} = \frac{m(Y|H_0)}{m(Y|H_1)}, \qquad (14)$$

where $m(Y|H_i)$ is the marginal density of Y under model H_i, $i = 0,1$.

We have:

$$m(Y|\,H_0) = \iint \left(\int f(Y|\theta^0, \sigma_\varepsilon^2, \sigma_1^2) \pi_1(\theta^0|\sigma_\varepsilon^2, \sigma_1^2) \pi_0(\sigma_\varepsilon^2, \sigma_1^2) d\theta^0 \right) d\sigma_\varepsilon^2 d\sigma_1^2).$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \iint (\sigma_1^2)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)} \exp\left\{ -\frac{\frac{1}{2}(Y - F^0\theta^0)^T P\, (Y - F^0\theta^0) + \beta_1}{\sigma_1^2} \right\}$$

$$(\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \exp\left\{ -\frac{\frac{1}{2}(Y - F^0\theta^0)^T Q (Y - F^0\theta^0) + \beta_\varepsilon}{\sigma_\varepsilon^2} \right\}$$

$$\left[\int \frac{-1}{2} (\theta - \theta^0)^T F^T \Psi^{-1} F (\theta - \theta^0) d\theta^0 \right] d\sigma_\varepsilon^2 \; d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \int (\sigma_1^2)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)} \exp\left\{ -\frac{\frac{1}{2}(Y - F^0\theta^0)^T P\, (Y - F^0\theta^0) + \beta_1}{\sigma_1^2} \right\}$$

$$\int \left[(\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \exp\left\{ -\frac{\frac{1}{2}(Y - F^0\theta^0)^T Q (Y - F^0\theta^0) + \beta_\varepsilon}{\sigma_\varepsilon^2} \right\} d\sigma_\varepsilon^2 \right] d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T p\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int (\sigma_\varepsilon^2\ )^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-\ F^0\theta^0)+\beta_\varepsilon\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\ \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2\ ]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T p\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \frac{\left(\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}}{(\sigma_\varepsilon^2\ )^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T p\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{(\sigma_\varepsilon^2\ )}\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T p\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{(\sigma_\varepsilon^2\ )}\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+2\right)-1} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \left(\frac{N(T-1)}{2}+\alpha_\varepsilon+2\right) \left(\frac{1}{2}\,(Y-F^0\theta^0)^T Q(Y-F^0\theta^0)+\ \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-)F^0\theta^{0T}P\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\} d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \left(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2\right)$$

$$\left(\frac{1}{2}(Y-F^0\theta^0)^TQ\,(Y-F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\} \times \quad \left(\frac{1}{2}(Y-F^0\theta^0)^TP(Y-F^0\theta^0) + \beta_1\right)^{\left(\alpha_1+\frac{N}{2}+1\right)} \times$$

$$\left(\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)} d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma\left(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2\right) \left(\frac{1}{2}(Y-F^0\theta^0)^TQ(Y-F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int \frac{\left(\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0)+\beta_1\right)^{\left(\alpha_1+\frac{N}{2}+1\right)}}{(\sigma_1^2)^{\left(\alpha_1+\frac{N}{2}+1\right)}} \times \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times\left[\left(\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}\right]$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2)\left(\frac{1}{2}(Y-F^0\theta^0)^TQ(Y-F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right)^{\left(\alpha_1+\frac{N}{2}+2\right)-1} \times \exp\left\{-\frac{\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0)+\beta_1}{\sigma_1^2}\right\}$$

$$\times\left(\frac{1}{2}(Y-F^0\theta^0)^TP\,(Y-F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma\left(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2\right) \left(\frac{1}{2}(Y-F^0\theta^0)^TQ(Y-F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\Gamma\left(\frac{N}{2} + \alpha_1 + 2\right)\left(\frac{1}{2}(Y-F^0\theta^0)^TP(Y-F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}.$$

$$m(Y|H_1) = \iint\left(\int f\,(Y,\theta,\sigma_\varepsilon^2\,,\sigma_1^2)\,\pi_1(\theta|\sigma_\varepsilon^2\,,\sigma_1^2)\pi_0(\sigma_\varepsilon^2\,,\sigma_1^2)d\theta\right)d\sigma_\varepsilon^2\,d\sigma_1^2\,.$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \iint \; (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^TP\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times (\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\} \times \left[\int \frac{-1}{2}\,(\theta-\hat{\theta})^T\ F^T\Psi^{-1}F\ (\theta-\hat{\theta})\,d\theta\right] d\sigma_\varepsilon^2\ d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \left[(\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\} d\sigma_\varepsilon^2\right] d\sigma_1^2\ .$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int (\sigma_\varepsilon^2)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\} \times \left(\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times \left(\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\beta_1\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2\ ]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \frac{\left(\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}}{(\sigma_\varepsilon^2)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} [\int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{(\sigma_\varepsilon^2)}\right)^{\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon}{\sigma_\varepsilon^2}\right\}$$

$$\times \left(\frac{1}{2}\,(Y-F\theta)^T Q(Y-F\theta)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} d\sigma_\varepsilon^2]d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \int (\sigma_1^2)^{-\left(\alpha_1+\frac{N}{2}+1\right)} \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P\,(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}}{(\sigma_{\varepsilon}^{2}\ )}\right)^{\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+2\right)-1} \times \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}}{\sigma_{\varepsilon}^{2}}\right\}$$

$$\times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)} d\sigma_{\varepsilon}^{2}]d\sigma_{1}^{2}$$

$$= (2\pi)^{\frac{-NT}{2}}\frac{\beta_{1}^{\alpha_{1}}}{\Gamma(\alpha_{1})}\ \frac{\beta_{\varepsilon}^{\alpha_{\varepsilon}}}{\Gamma(\alpha_{\varepsilon})}\ \Gamma\left(\frac{N(T-1)}{2}+\alpha_{\varepsilon}+2\right)\left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int\ (\sigma_{1}^{2})^{-\left(\alpha_{1}+\frac{N}{2}+1\right)}\exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^{T}P\ (Y-F\theta)+\beta_{1}}{\sigma_{1}^{2}}\right\}d\sigma_{1}^{2}$$

$$= (2\pi)^{\frac{-NT}{2}}\frac{\beta_{1}^{\alpha_{1}}}{\Gamma(\alpha_{1})}\ \frac{\beta_{\varepsilon}^{\alpha_{\varepsilon}}}{\Gamma(\alpha_{\varepsilon})}\Gamma\left(\frac{N(T-1)}{2}+\alpha_{\varepsilon}+2\right)\left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int\ (\sigma_{1}^{2})^{-\left(\alpha_{1}+\frac{N}{2}+1\right)}\exp\left\{-\frac{\frac{1}{2}\left((Y-F\theta)^{T}P(Y-F\theta)+\beta_{1}\right)^{-\left(\alpha_{1}+\frac{N}{2}+1\right)}+\beta_{1}}{\sigma_{1}^{2}}\right\}$$

$$\times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}P(Y-F\theta)+\beta_{1}\right)^{\left(\alpha_{1}+\frac{N}{2}+1\right)}\ \left(\tfrac{1}{2}\ (Y-F\theta)^{T}P(Y-F\theta)+\beta_{1}\right)^{-\left(\alpha_{1}+\frac{N}{2}+1\right)}\ d\sigma_{1}^{2}$$

$$= (2\pi)^{\frac{-NT}{2}}\frac{\beta_{1}^{\alpha_{1}}}{\Gamma(\alpha_{1})}\ \frac{\beta_{\varepsilon}^{\alpha_{\varepsilon}}}{\Gamma(\alpha_{\varepsilon})}\Gamma\left(\frac{N(T-1)}{2}+\alpha_{\varepsilon}+2\right)\ \left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\ \beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int \frac{\left(\frac{1}{2}(Y-F\theta)^{T}P\ (Y-F\theta)+\beta_{1}\right)^{\left(\alpha_{1}+\frac{N}{2}+1\right)}}{(\sigma_{1}^{2})^{\left(\alpha_{1}+\frac{N}{2}+1\right)}}\times \exp\ \left\{-\frac{\frac{1}{2}(Y-F\theta)^{T}P\ (Y-F\theta)+\beta_{1}}{\sigma_{1}^{2}}\right\}$$

$$\times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}P\ \ (Y-F\theta)+\ \beta_{1}\right)^{-\left(\alpha_{1}+\frac{N}{2}+1\right)}d\sigma_{1}^{2}$$

$$= (2\pi)^{\frac{-NT}{2}}\frac{\beta_{1}^{\alpha_{1}}}{\Gamma(\alpha_{1})}\ \frac{\beta_{\varepsilon}^{\alpha_{\varepsilon}}}{\Gamma(\alpha_{\varepsilon})}\ \Gamma\left(\frac{N(T-1)}{2}+\alpha_{\varepsilon}+2\right)\ \times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F\theta)^{T}P(Y-F\theta)+\beta_{1}}{(\sigma_{1}^{2}\ )}\right)^{\left(\alpha_{1}+\frac{N}{2}+1\right)}\ \times\ \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^{T}P(Y-F\theta)+\beta_{1}}{\sigma_{1}^{2}}\right\}$$

$$\times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}P(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N}{2}+1\right)}d\sigma_{1}^{2}$$

$$= (2\pi)^{\frac{-NT}{2}}\frac{\beta_{1}^{\alpha_{1}}}{\Gamma(\alpha_{1})}\ \frac{\beta_{\varepsilon}^{\alpha_{\varepsilon}}}{\Gamma(\alpha_{\varepsilon})}\ \Gamma\left(\frac{N(T-1)}{2}+\alpha_{\varepsilon}+2\right)\ \times \left(\tfrac{1}{2}\ (Y-F\theta)^{T}Q(Y-F\theta)+\beta_{\varepsilon}\right)^{-\left(\alpha_{\varepsilon}+\frac{N(T-1)}{2}+1\right)}$$

$$\times \int \left(\frac{\frac{1}{2}(Y-F\theta)^T P(Y-F\theta)+\beta_1}{(\sigma_1^2\ )}\right)^{\left(\alpha_1+\frac{N}{2}+2\right)-1} \times \exp\left\{-\frac{\frac{1}{2}(Y-F\theta)^T P(Y-F\theta)+\beta_1}{\sigma_1^2}\right\}$$

$$\times \left(\frac{1}{2}\ (Y-F\theta)^T P(Y-F\theta)+\beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)} d\sigma_1^2$$

$$= (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma\left(\frac{N(T-1)}{2}+\alpha_\varepsilon+2\right)\left[\frac{1}{2}\ (Y-F\theta)^T Q\ (Y-F\theta)+\ \beta_\varepsilon\right]^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)}$$

$$\times \Gamma\left(\frac{N}{2}+\alpha_1+2\right)\ \left(\frac{1}{2}\ (Y-F\theta)^T P\ \ (Y-F\theta)+\ \beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}.$$

$$\therefore B_{01} = \frac{\left(\frac{1}{2}(Y-F^0\theta^0)^T Q\ (Y-F^0\theta^0)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \left(\frac{1}{2}(Y-F^0\theta^0)^T P\ \ (Y-F^0\theta^0)+\beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}}{\left(\frac{1}{2}(Y-F\theta)^T Q\ (Y-F\theta)+\beta_\varepsilon\right)^{-\left(\alpha_\varepsilon+\frac{N(T-1)}{2}+1\right)} \left(\frac{1}{2}(Y-F\theta)^T P\ \ (Y-F\theta)+\beta_1\right)^{-\left(\alpha_1+\frac{N}{2}+1\right)}}$$

5. Conclusion

The conclusions which are obtained throughout this paper are given as follows :

1. The likelihood function of the panel data model is

$$L(Y|\theta\ ,\sigma_\varepsilon^2\ ,\sigma_1^2) = (2\pi)^{\frac{-NT}{2}} (\sigma_\varepsilon^2)^{\left(\frac{-N(T-1)}{2}\right.} (\sigma_1^2)^{\frac{-N}{2}} \exp\{-\frac{1}{2}\left(Y-F\hat{\theta}\right)^T \left(\frac{Q}{\sigma_\varepsilon^2}+\frac{P}{\sigma_1^2}\right)$$

$$\left(Y-F\hat{\theta}\right)\}\ \exp\ \{-\frac{1}{2}\left(\theta-\hat{\theta}\right)^T F^T\Psi^{-1}F\left(\theta-\hat{\theta}\right)$$

2. The posterior of θ , σ_ε^2 and σ_1^2 are respectively :

$$(\theta|\sigma_\varepsilon^2, \sigma_1^2, Y) \sim N(\hat{\theta}, F^T\Psi^{-1}F),$$

$$\sigma_\varepsilon^2\ |\theta, \sigma_1^2, Y \sim IG\left(\alpha_\varepsilon+\frac{N(T-1)}{2}, \beta_\varepsilon+\frac{1}{2}\ (Y-F\hat{\theta})^T Q\ (Y-F\hat{\theta}\ )\ \right) \text{ and}$$

$$\sigma_1^2\ |\theta, \sigma_\varepsilon^2, Y \sim IG\left(\alpha_1+\frac{N}{2}, \beta_1+\frac{1}{2}\ (Y-F\hat{\theta})^T P\ \ (Y-F\hat{\theta})\right)$$

3. The marginal density of Y under model $H_i, i = 0,1$ are

$$m(Y|H_0) = (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma\left(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2\right) \left(\frac{1}{2}(Y - F^0\theta^0)^T Q (Y - F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)}$$

$$\Gamma\left(\frac{N}{2} + \alpha_1 + 2\right)\left(\frac{1}{2}(Y - F^0\theta^0)^T P (Y - F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}$$

and

$$m(Y|H_1) = (2\pi)^{\frac{-NT}{2}} \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\Gamma(\alpha_\varepsilon)} \Gamma\left(\frac{N(T-1)}{2} + \alpha_\varepsilon + 2\right) \left[\frac{1}{2}(Y - F\theta)^T Q (Y - F\theta) + \beta_\varepsilon\right]^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)}$$

$$\Gamma\left(\frac{N}{2} + \alpha_1 + 2\right) \left(\frac{1}{2}(Y - F\theta)^T P (Y - F\theta) + \beta_1\right)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}$$

4. The Bayes factor for testing $H_0: Y_{it} = \beta_0 + \sum_{j=1}^{q} \beta_j x_{jit} + \omega_{it}$ against

$H_1: Y_{it} = \beta_0 + \sum_{j=1}^{K} \beta_j x_{jit} + \omega_{it}$ is given by the following form:

$$B_{01} = \frac{\left(\frac{1}{2}(Y-F^0\theta^0)^T Q (Y-F^0\theta^0) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \left(\frac{1}{2}(Y-F^0\theta^0)^T P (Y-F^0\theta^0) + \beta_1\right)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}}{\left(\frac{1}{2}(Y-F\theta)^T Q (Y-F\theta) + \beta_\varepsilon\right)^{-\left(\alpha_\varepsilon + \frac{N(T-1)}{2} + 1\right)} \left(\frac{1}{2}(Y-F\theta)^T P (Y-F\theta) + \beta_1\right)^{-\left(\alpha_1 + \frac{N}{2} + 1\right)}}$$

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