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Riemann-Liouville derivative and Caputo derivative for solving Extraordinary differential equation by homotopy analysis method

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Abstract:

The purpose of this paper is to study Riemann-Liouville derivative and Caputo derivative by homotopy analysis method to solve an Extraordinary differential equation. The results are obtianed by the proposed method show efficient (HAM) using Riemann-Liouville derivative and Caputo derivative .

Keywords: homotopy analysis method, Extraordinary differential equation, Riemann-Liouville derivative , Caputo derivative.

Introduction:

Extraordinary differential equation is one of types from a differential equation contianing a fractional derivative of order half along with an ordinary first –order derivative consider the following extraordinary differential equation:

$$\frac{dy(x)}{dx} + \frac{d^{\frac{1}{2}}y(x)}{dx^{\frac{1}{2}}} - 2y(x) = 0, \quad (1)$$

with an initial condition $y_0 = c$, where c constant and the solution of Extraordinary differential equation[2] is:

$$y(x) = \frac{c}{3}(2e^{4x}erfc\left(2\sqrt{x}\right) + e^{x}\mathrm{erf}(-\sqrt{x})) \quad (2)$$

In this paper we apply the homotopy analysis method for Extraordinary differential equation.To show the basic idea, let us consider the following equation:

$$N(y(x)] = 0, \quad (3)$$

where N is a nonlinear operator, x denotes the independent variable, $y(x)$ is an unknown function, respectively. Generalizing the traditional homotopy method, Liao[5],constructs the so- called Zero –order deformation equation:

$$(1-p)L[\emptyset(x;p) - y_0(x)] = phH(x)N[y(x)] \quad (4)$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(x) \neq 0$ ia an auxiliary function , L is an auxiliary linear operator, $y_0(x)$ is an initial guess of $y(x)$, $\emptyset(x;p)$ is an unknown function. When $p = 0$ and $p = 1$, then

$$\emptyset(x;0) = y_0(t), \quad \emptyset(x;1) = y(x), \tag{5}$$

respectively.Thus, as p increases from 0 to 1, the solution $\emptyset(x;p)$ varies from the initial guess $y_0(x)$ to the solution $y(x)$. Expanding $\emptyset(x;p)$ in Taylor series with respect to p, one has

$$\emptyset(x;p) = y_0(x) + \sum_{k=1}^{\infty} y_k(x)\ p^k, \tag{6}$$

where

$$y_k(x) = \frac{1}{k!}\frac{\partial^k \emptyset(x;p)}{\partial p^k}\bigg|_{p=0}, \tag{7}$$

if the auxiliary linear operater, the initial guess, the auxiliary parameter h, and the auxiliary function ara properly chosen, the series(6) converages at $p = 1$, thus

$$y(t) = y_0(t) + \sum_{k=1}^{\infty} y_k(t). \tag{8}$$

Which must be one of the solution of the origional nonlinear equation, as proved by Liao[5].

According to the equation (7), the govering equation can be deduced from the zero-order deformation equation (4). Define the vector

$$\vec{y}_m = \{y_0(x), y_1(x), \dots, y_m(x)\} \tag{9}$$

Differentiating equation (4) k times with respect to the embedding parameter p and then setting $p = 0$ and finally dividing them by $k!$, we have the so-called kth-order deformation equation:

$$L[y_k(x) - \chi_k y_{k-1}(x)] = hH(x)R_k(\vec{y}_{k-1}), \tag{10}$$

where

$$R_k(\vec{y}_{k-1}) = \frac{1}{(k-1)!}\frac{\partial^{k-1} N[\emptyset(x;p)]}{\partial p^{k-1}}\bigg|_{p=0}, \tag{11}$$

and

$$\chi_k = \begin{cases} 0 \ if\ k \leq 1 \\ 1\ \ if\ k > 1 \end{cases} \tag{12}$$

It shuould be emphasized that $y_k(x)$ for $k \geq 1$ is governed by the linear equation (7)with the linear boundary condition that comes from the original problem

In this paper, we will give definitions of fractional derivative introduced by Riemann-Liouville derivative and Caputo derivative.

Definition.1.(Riemann-Liouville derivative) let $\beta > 0$ denoted a real number and n the smallest integer exceeding β such that $n - \beta > 0$ $(n = 0\, if\ \beta < 0)$ assuming $f(x)$ to be afunction of class $C^{(n)}$ (the class of functions with continuous nthderivatives) the fractional derivative of a function $f(x)$ of order β is given by:

$$\frac{d^\beta f(x)}{dx^\beta} = \frac{1}{n-\beta}\frac{d^n}{dx^n}\int_0^x \frac{f(t)dt}{(x-t)^{1-n+\beta}}, \tag{13}$$

Definition.2. (Caputo derivative) fractional derivative of $f(x)$ in the Caputo sense [3] is defined as:

$$D_x^\beta f(x) = \frac{1}{\Gamma(n-\beta)}\int_0^x (x-\tau)^{n-\beta-1}\frac{d^n f(t)}{dt^n}dt,$$

$$n-1 < \beta \leq n, n \in \mathbb{N}, x > 0 \tag{14}$$

β is the order of the derivative. For the Caputo's derivative we have:

$1 - D^\beta C = 0,$ C is constant,

$2 - D^\beta x^\alpha = 0,\ \ \alpha \leq \beta - 1,$

$3 - D^\beta x^\alpha = \frac{\Gamma(1+\alpha)}{\Gamma(1-\beta+\alpha)}x^{\alpha-\beta},\ \ \alpha > \beta - 1,$

Solution of an Extraordinary differential equation by homotopy analysis method.

Consider the following Extraordinary differential equation :

$$\frac{dy(x)}{dx} + \frac{d^{\frac{1}{2}}y(x)}{dx^{\frac{1}{2}}} - 2y(x) = 0,$$

with an initial condition $y_0 = c,$

note that we define the nonlinear operator as, $N(y(x)) = \frac{dy(x)}{dx} + \frac{d^{\frac{1}{2}}y(x)}{dx^{\frac{1}{2}}} - 2y(x),$ and we define the linear operator as, $L(y(x) = \frac{dy(x)}{dx},$

for solving this equation by HAM, we chose $H(x) = 1$ and we construct the zeroth-order deformation Equation as:

$$(1-p)L[\frac{dy(x)}{dx} - c] = phH(t)[\frac{dy(x)}{dx} + \frac{d^{1l2}y(x)}{dx^{\frac{1}{2}}} - 2y(x)], \quad (15)$$

for $p = 0$ and $p = 1$, we can write

$y_0(x;0) = c, \quad y(x;1) = y(x),$

the solution of the kth-order deformation Equation for $k \geq 1$ becomes

$$y_k(x) = \chi_k\, y_{k-1}(x) + phH(t)L^{-1}[\frac{dy_{k-1}(x)}{dx} + \frac{d^{1l2}y_{k-1}(x)}{dx^{\frac{1}{2}}} - 2y_{k-1}(x)], \quad (16)$$

where

$$y_1(x) = h\int(\frac{dy_0(x)}{dx} + \frac{d^{1l2}y_0(x)}{dx^{\frac{1}{2}}} - 2y_0(x))dx, \quad (17)$$

and

$$y_k(x) = \; y_{k-1}(x) + h\int(\frac{dy_{k-1}(x)}{dx} + \frac{d^{1l2}y_{k-1}(x)}{dx^{\frac{1}{2}}} - 2y_{k-1}(x))dx. \quad (18)$$

Now there are two case to solve an Extraordinary differential equation. First, we will use Riemann-Liouville derivative of the fractional derivative in equation the solution shall be as follows:

$$y_0(x) = c,$$

$$y_1(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx,$$

$$y_2(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx + \frac{2h^2c}{\sqrt{\pi}}x^{--\frac{1}{2}} - h^2cx - \frac{16}{3}\frac{h^2cx^{\frac{3}{2}}}{\sqrt{\pi}} + 2ch^2x^2,$$

$$y_3(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx + \frac{4h^2cx^{\frac{1}{2}}}{\sqrt{\pi}} - 2h^2cx - \frac{32h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 4h^2cx^2 + \frac{2h^3cx^{\frac{1}{2}}}{\sqrt{\pi}}$$
$$-\frac{28h^3cx^{\frac{3}{2}}}{3\sqrt{\pi}} + h^3cx^2 + \frac{32h^3cx^{\frac{5}{2}}}{5\sqrt{\pi}} - \frac{4^3hcx^3}{3}, \; ---$$

$$y_4(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx + \frac{6h^2cx^{\frac{1}{2}}}{\sqrt{\pi}} - 3h^2cx - \frac{16h^2cx^{\frac{3}{2}}}{\sqrt{\pi}} + 6h^2cx^2 + \frac{6h^3cx^{\frac{1}{2}}}{\sqrt{\pi}}$$
$$-\frac{28h^3cx^{\frac{3}{2}}}{\sqrt{\pi}} + 3h^3cx^2 + \frac{96h^3cx^{\frac{5}{2}}}{5\sqrt{\pi}} - 4h^3cx^3 + \frac{2h^4cx^4}{3} + \frac{224h^4cx^{\frac{5}{2}}}{15\sqrt{\pi}} - \frac{512h^4cx^{\frac{7}{2}}}{105\sqrt{\pi}} + h^4cx$$
$$-\frac{5h^4cx^2}{2} + \frac{2h^4cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{12h^4cx^{\frac{3}{2}}}{\sqrt{\pi}},$$

$$y_5(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx + \frac{8h^2cx^{\frac{1}{2}}}{\sqrt{\pi}} - 4h^2cx - \frac{64h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 8h^2cx^2 + \frac{12h^3cx^{\frac{1}{2}}}{\sqrt{\pi}}$$
$$-\frac{56h^3cx^{\frac{3}{2}}}{\sqrt{\pi}} + 6h^3cx^2 + \frac{192h^3cx^{\frac{5}{2}}}{5\sqrt{\pi}} - 8h^3cx^3 + \frac{8h^4cx^4}{3} + \frac{896h^4cx^{\frac{5}{2}}}{15\sqrt{\pi}} - \frac{2048h^4cx^{\frac{7}{2}}}{105\sqrt{\pi}} + 4h^4cx$$
$$-10h^4cx^2 + \frac{8h^4cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{48h^4cx^{\frac{3}{2}}}{\sqrt{\pi}} + 2h^5cx + \frac{19h^5cx^3}{3} + \frac{2h^5cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{40h^5cx^{\frac{3}{2}}}{3\sqrt{\pi}} - \frac{1408h^5cx^{\frac{7}{2}}}{105\sqrt{\pi}}$$
$$-\frac{4h^5cx^5}{15} - \frac{2h^5cx^4}{3} - 8h^5cx^2 + \frac{512h^5cx^{\frac{9}{2}}}{189\sqrt{\pi}} - \frac{328h^5cx^{\frac{5}{2}}}{15\sqrt{\pi}},$$

$$y_6(x) = \frac{2hc}{\sqrt{\pi}}x^{\frac{1}{2}} - 2hcx + \frac{10h^2cx^{\frac{1}{2}}}{\sqrt{\pi}} - 5h^2cx - \frac{80h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 10h^2cx^2 + \frac{h^3cx^{\frac{1}{2}}}{\sqrt{\pi}} -$$
$$\frac{280h^3cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 10h^3cx^2 + \frac{64h^3cx^{\frac{5}{2}}}{\sqrt{\pi}} - \frac{40h^3cx^3}{3} + \frac{20h^4cx^4}{3} + \frac{448h^4cx^{\frac{5}{2}}}{3\sqrt{\pi}} - \frac{1024h^4cx^{\frac{7}{2}}}{21\sqrt{\pi}} + 10h^4cx$$
$$-25h^4cx^2 + \frac{20h^4cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{120h^4cx^{\frac{3}{2}}}{\sqrt{\pi}} + 10\hbar^5cx + \frac{95\hbar^5cx^3}{3} + \frac{10h^5cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{200h^5cx^{\frac{3}{2}}}{3\sqrt{\pi}} - \frac{1408h^5cx^{\frac{7}{2}}}{21\sqrt{\pi}}$$
$$-\frac{4h^5cx^5}{3} - \frac{10h^5cx^4}{3} - 40h^5cx^2 + \frac{2560h^5cx^{\frac{9}{2}}}{189\sqrt{\pi}} - \frac{328h^5cx^{\frac{5}{2}}}{3\sqrt{\pi}} + 3h^6cx - 15h^6cx^2 + \frac{37h^6cx^3}{2}$$
$$-\frac{15h^6cx^4}{2} + \frac{2h^6cx^5}{3} + \frac{4h^6cx^6}{45} + \frac{2h^6cx^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{40h^6cx^{\frac{3}{2}}}{3\sqrt{\pi}} + \frac{24h^6cx^{\frac{5}{2}}}{\sqrt{\pi}} - \frac{704h^6cx^{\frac{7}{2}}}{35\sqrt{\pi}} - \frac{4096h^6cx^{\frac{11}{2}}}{3465\sqrt{\pi}},$$

..

The solution when $h = -1$, we get

$$y_0(x) = c,$$

$$y_1(x) = 2cx - 2c\frac{x^{\frac{1}{2}}}{\sqrt{\pi}},$$

$$y_2(x) = 2cx^2 + cx - \frac{16c}{3}\frac{x^{\frac{3}{2}}}{\sqrt{\pi}},$$

$$y_3(x) = \frac{4c}{3}x^3 + 3cx^2 - \frac{32c}{5}\frac{x^{\frac{5}{2}}}{\sqrt{\pi}} - \frac{4c}{3}\frac{x^{\frac{3}{2}}}{\sqrt{\pi}},$$

$$y_4(x) = \frac{2c}{3}x^4 + 4cx^3 + \frac{1}{2}x^2 - \frac{512}{105\sqrt{\pi}}x^{\frac{7}{2}} - \frac{64}{15\sqrt{\pi}}x^{\frac{5}{2}},$$

$$y_5(x) = \frac{4c}{15}x^5 + \frac{10c}{3}x^4 + \frac{5c}{3}x^3 - \frac{512c}{189}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{128c}{21}\frac{x^{\frac{7}{2}}}{\sqrt{\pi}} - \frac{8c}{15}\frac{x^{\frac{5}{2}}}{\sqrt{\pi}},$$

$$y_6(x) = \frac{4c}{45}x^6 + 2cx^5 + \frac{5c}{2}x^4 + \frac{c}{6}x^3 - \frac{4096cx^{\frac{11}{2}}}{3465\sqrt{\pi}} - \frac{1024c}{189}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{64}{35}\frac{x^{\frac{7}{2}}}{\sqrt{\pi}},$$

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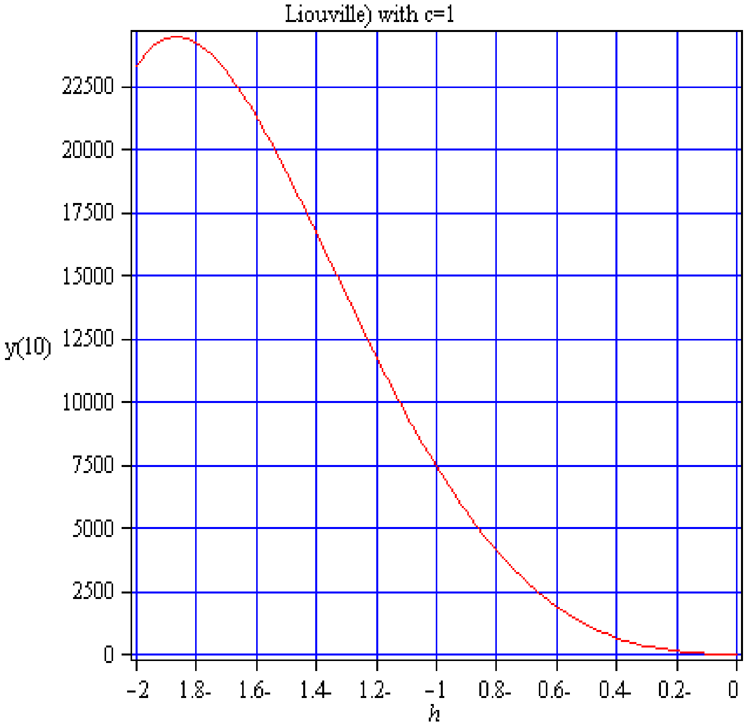
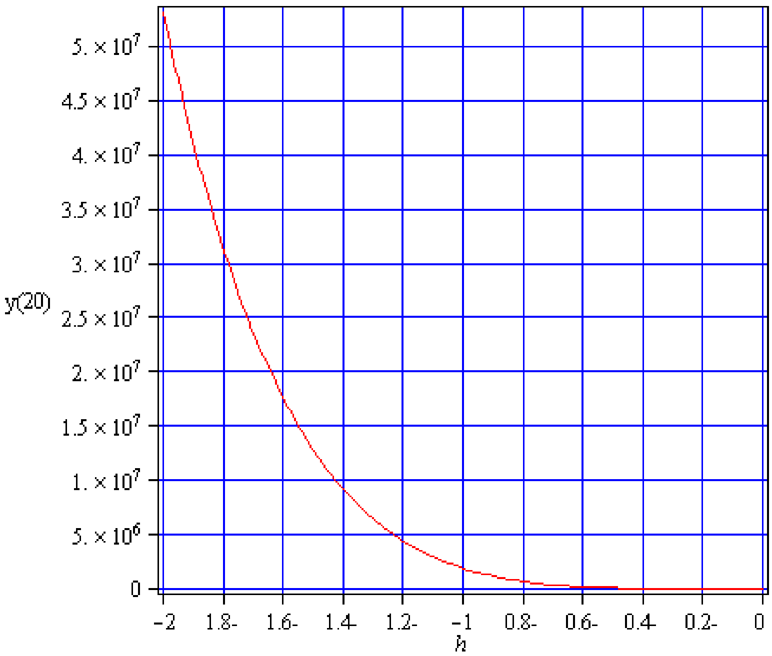
The approximate solution for Equation (1), will be as followed

$$y(x) = c(1 + 3x + \frac{11}{2}x^2 + \frac{43}{6}x^3 + \frac{13}{2}x^4 + \frac{34}{15}x^5 + \frac{4}{45}x^6 - 2\frac{x^{\frac{1}{2}}}{\sqrt{\pi}} - \frac{20}{3}\frac{x^{\frac{3}{2}}}{\sqrt{\pi}}$$
$$-\frac{65}{5\sqrt{\pi}}x^{\frac{5}{2}} - \frac{64}{5\sqrt{\pi}}x^{\frac{7}{2}} - \frac{1536}{189}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{4096cx^{\frac{11}{2}}}{3465\sqrt{\pi}} + \cdots),$$

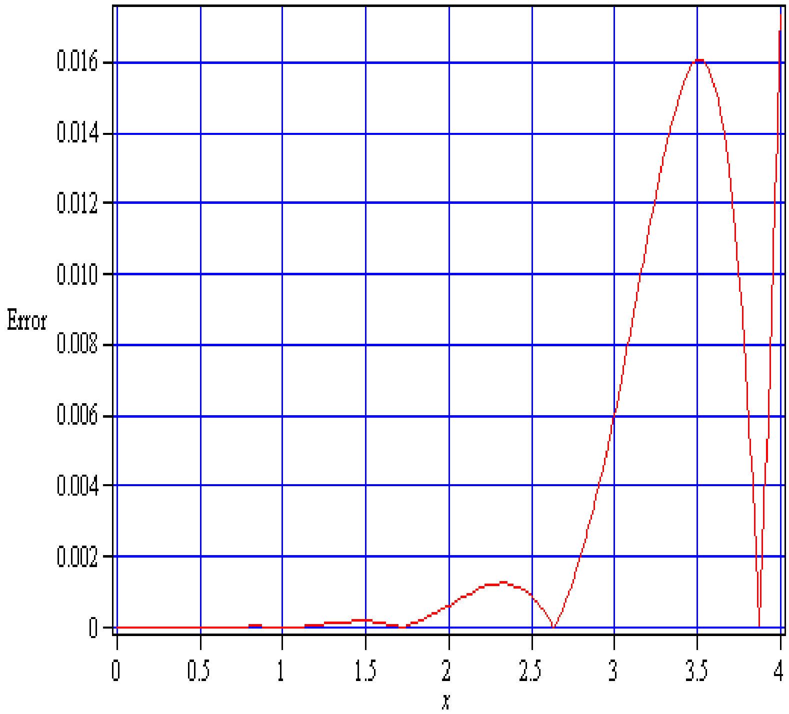
Figure(1).The h-curve of y(10) based on the six terms of HAM(Riemann-Liouville) with c=1



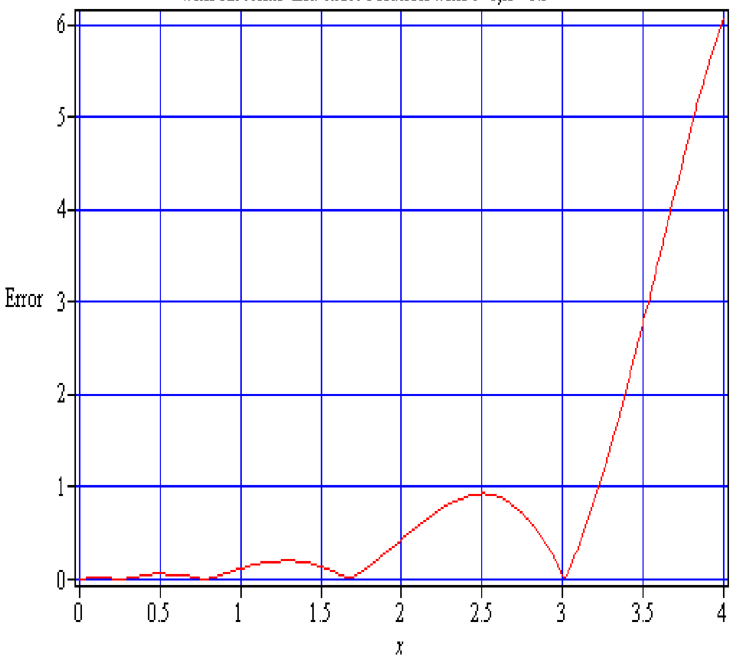
Figure(2).The h-curve of y(20) based on the six terms of HAM(Riemann-Liouville) with c=2



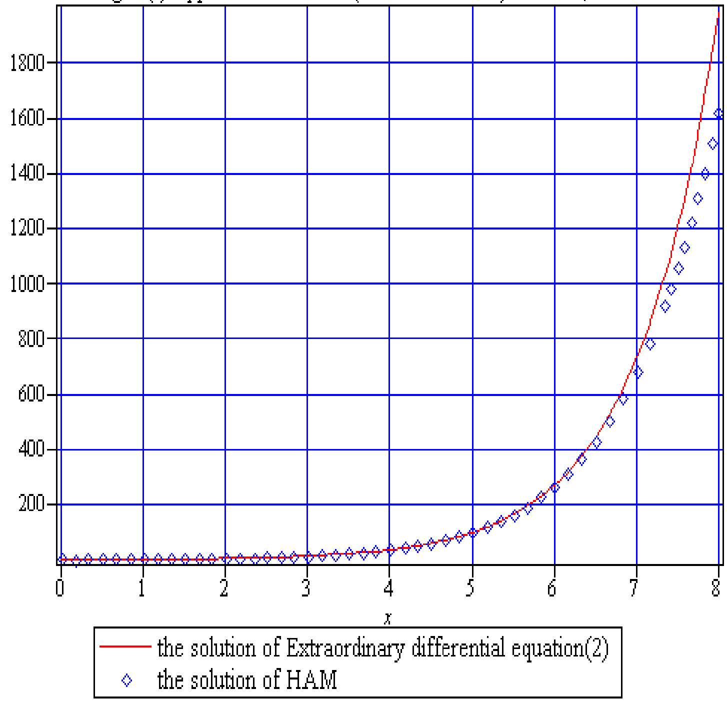
Figure(3),The absolute errors between the solutions obtained using HPM(Riemann-Liouville) with six terms and exact solution with c=1,h=-1



Figure(4),The absolute errors between the solutions obtained using HPM(Riemann-Liouville) with six terms and exact solution with c=1,h=-1.5



Figure(5).Approximate solutions(Riemann-Liouville) with c=1,h=-1



Figure(6).Approximate solutions(Riemann-Liouville) with c=1,h=-1.5

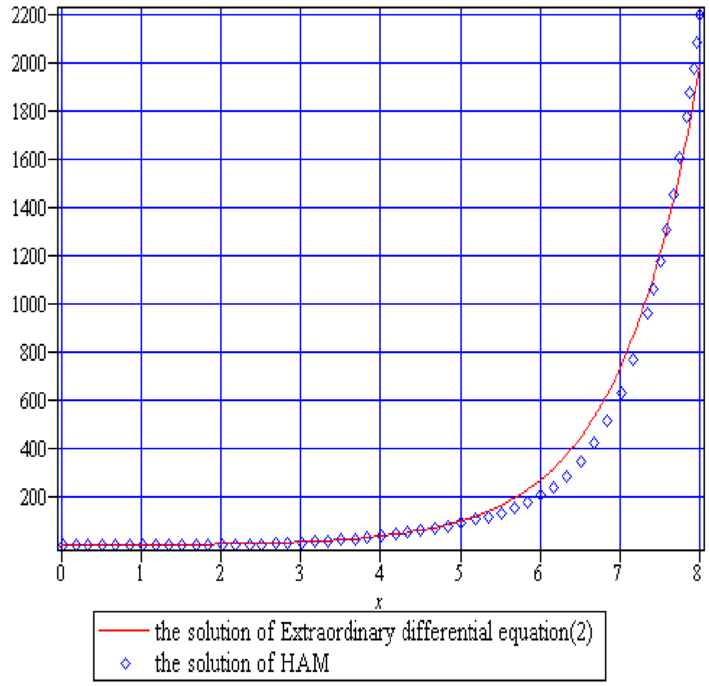


Table.1. Approximate solutions (Riemann-Liouville derivative)with $c = 3$.

x	HAM($h = -1$)	HAM($h = -1.3$)	The solution of (2)
0.1	2.593980055	2.600468107	2.593975906
0.2	2.715511959	2.717798689	2.715509289
0.3	2.913380332	2.905433262	2.913393641
0.4	3.159482909	3.142395434	3.159508620
0.5	3.446678260	3.426561220	3.446693963
0.6	3.773666560	3.758762150	3.773646433
0.7	4.141577460	4.139663430	4.141510100
0.8	4.552870220	4.569310290	4.552768087
0.9	5.010911580	5.047459530	5.010813280
1	5.519811130	5.57411626	5.519771433

Second, we will use Caputo derivative of the fractional derivative in equation the solution shall be as follows:

$y_0(x) = c$

$y_1(x) = -2hcx,$

$y_2(x) = -2chx - 2h^2cx - \frac{8}{3}\frac{h^2cx^{\frac{3}{2}}}{\sqrt{\pi}} + 2ch^2x^2,$

$y_3(x) = -2hcx - 4h^2cx - \frac{16h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 4h^2cx^2 - 2h^3cx - \frac{16h^3cx^{\frac{3}{2}}}{3\sqrt{\pi}}$
$+3h^3cx^2 + \frac{64h^3cx^{\frac{5}{2}}}{15\sqrt{\pi}} - \frac{4h^3cx^3}{3},$

$y_4(x) = -2hcx - 6h^2cx - \frac{8h^2cx^{\frac{3}{2}}}{\sqrt{\pi}} + 6h^2cx^2 - 6h^3cx - \frac{16h^3cx^{\frac{3}{2}}}{\sqrt{\pi}} + 9h^3cx^2 + \frac{64h^3cx^{\frac{5}{2}}}{5\sqrt{\pi}}$
$-4h^3cx^3 - 2h^4cx - \frac{8h^4cx^{\frac{3}{2}}}{\sqrt{\pi}} + 3h^4cx^2 + \frac{176h^4cx^{\frac{5}{2}}}{15\sqrt{\pi}} - 2h^4cx^3 - \frac{128h^4cx^{\frac{7}{2}}}{35\sqrt{\pi}} + \frac{2h^4cx^4}{3},$
$y_5(x) == -2hcx - 8h^2cx - \frac{32h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 8h^2cx^2 - 12h^3cx - \frac{32h^3cx^{\frac{3}{2}}}{\sqrt{\pi}} + 18h^3cx^2 + \frac{128h^3cx^{\frac{5}{2}}}{5\sqrt{\pi}}$
$-8h^3cx^3 - 8h^4cx - \frac{32h^4cx^{\frac{3}{2}}}{\sqrt{\pi}} + 12h^4cx^2 + \frac{704h^4cx^{\frac{5}{2}}}{15\sqrt{\pi}} - 8h^4cx^3 - \frac{512h^4cx^{\frac{7}{2}}}{35\sqrt{\pi}} +$

$\frac{8h^4cx^4}{3} + \frac{2048h^5cx^{\frac{9}{2}}}{945\sqrt{\pi}} - \frac{256h^5cx^{\frac{7}{2}}}{21\sqrt{\pi}} + \frac{64h^5cx^{\frac{5}{2}}}{3\sqrt{\pi}} - \frac{32h^5cx^{\frac{3}{2}}}{3\sqrt{\pi}} - \frac{4h^5cx^5}{15} + \frac{2h^5cx^4}{3} - \frac{h^5cx^3}{3} +$

$2h^5cx^2 - 2h^5cx,$

$y_6(x) = -2hcx - 10h^2cx - \frac{40h^2cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 10h^2cx^2 - 20h^3cx - \frac{160h^3cx^{\frac{3}{2}}}{3\sqrt{\pi}} + 30h^3cx^2 +$
$\frac{128h^3cx^{\frac{5}{2}}}{3\sqrt{\pi}} - \frac{40h^3cx^3}{3} - 2 - 0h^4cx - \frac{80h^4cx^{\frac{3}{2}}}{\sqrt{\pi}} + 30h^4cx^2 + \frac{352h^4cx^{\frac{5}{2}}}{3\sqrt{\pi}} --$

$20h^4cx^3 - \frac{256h^4cx^{\frac{7}{2}}}{7\sqrt{\pi}} + \frac{20h^4cx^4}{3} + \frac{2048h^5cx^{\frac{9}{2}}}{189\sqrt{\pi}} - \frac{1280h^5cx^{\frac{7}{2}}}{21\sqrt{\pi}} + \frac{320h^5cx^{\frac{5}{2}}}{3\sqrt{\pi}} - \frac{32h^5cx^{\frac{3}{2}}}{3\sqrt{\pi}}$

$-\frac{4h^5cx^5}{3} + \frac{10h^5cx^4}{3} - \frac{5h^5cx^3}{3} + 10h^5cx^2 - 10h^5cx + \frac{4h^6cx^6}{45} - \frac{5h^6cx^4}{2} + 5h^6cx^3$

$-2h^6cx - \frac{2048h^6cx^{\frac{11}{2}}}{2079\sqrt{\pi}} + \frac{512h^6cx^{\frac{9}{2}}}{63\sqrt{\pi}} - \frac{864h^6cx^{\frac{7}{2}}}{35\sqrt{\pi}} + \frac{320h^6cx^{\frac{5}{2}}}{\sqrt{\pi}} - \frac{40h^6cx^{\frac{3}{2}}}{3\sqrt{\pi}},$

.

The solution when $h = -1,$ we obtain:

$y_0(x) = c,$

$y_1(x) = 2cx\ ,$

$y_2(x) = 2cx^2 - \frac{8c}{3}\frac{x^{\frac{3}{2}}}{\sqrt{\pi}},$

$y_3(x) = \frac{4c}{3}x^3 + 2cx^2 - \frac{64c}{15}\frac{x^{\frac{5}{2}}}{\sqrt{\pi}},$

$y_4(x) = \frac{2c}{3}x^4 + 2x^3 - \frac{128}{35\sqrt{\pi}}x^{\frac{7}{2}} - \frac{16}{15\sqrt{\pi}}x^{\frac{5}{2}},$

$y_5(x) = \frac{4c}{15}x^5 + 2cx^4 + \frac{c}{3}x^3 - \frac{2048c}{945}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{256c}{105}\frac{x^{\frac{7}{2}}}{\sqrt{\pi}} - \frac{8c}{15}\frac{x^{\frac{5}{2}}}{\sqrt{\pi}},$

$y_6(x) = \frac{4c}{45}x^6 + \frac{4c}{3}x^5 + \frac{5c}{6}x^4 - \frac{2048cx^{\frac{11}{2}}}{2079\sqrt{\pi}} - \frac{512c}{189}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{32}{105}\frac{x^{\frac{7}{2}}}{\sqrt{\pi}},$

.

The approximate solution for Equation (1), will be as followed

$$y(x) = c\left(1 + 2x + 3x^2 + \frac{11}{3}x^3 + \frac{63}{18}x^4 + \frac{24}{15}x^5 + \frac{4}{45}x^6 - \frac{8}{3}\frac{x^{\frac{3}{2}}}{\sqrt{\pi}} - \frac{88}{15}\frac{x^{\frac{5}{2}}}{\sqrt{\pi}} - \frac{32}{5}\frac{x^{\frac{7}{2}}}{\sqrt{\pi}} - \frac{512}{105}\frac{x^{\frac{9}{2}}}{\sqrt{\pi}} - \frac{2048}{2079}\frac{x^{\frac{11}{2}}}{\sqrt{\pi}} + \cdots\right),$$

Figure (7): The h-curve of y(10) based on the six terms of HAM(Caputo)with c=1.

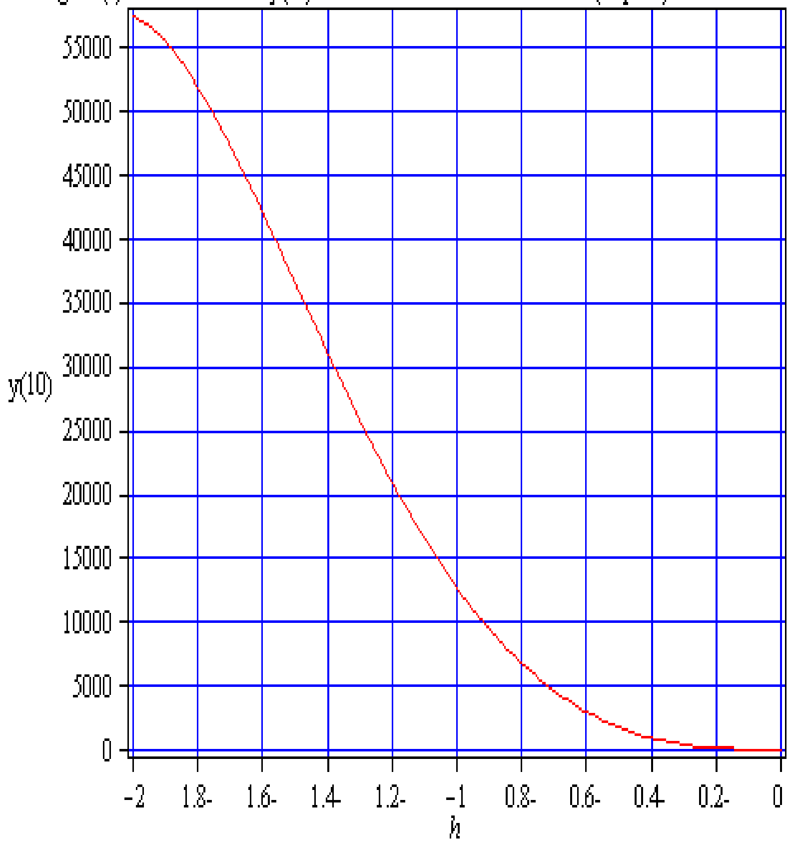


Figure (8): The h-curve of y(20) based on the six terms of HAM(Caputo)with c=2.

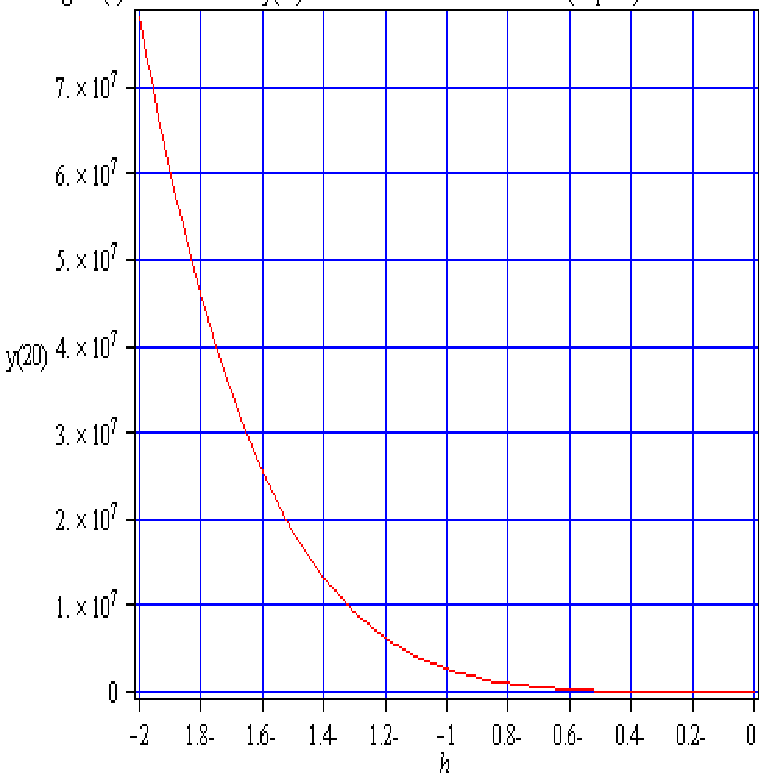
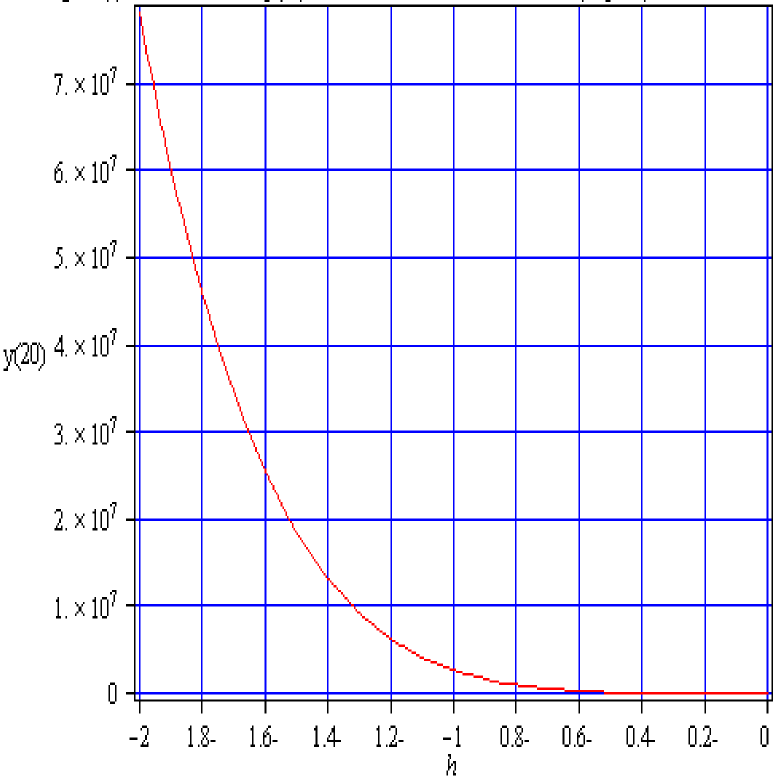
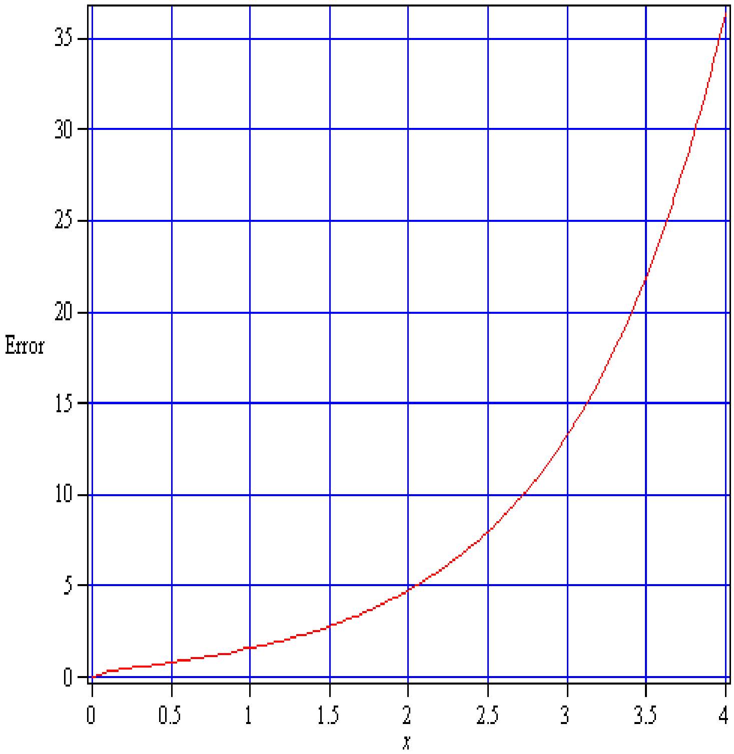


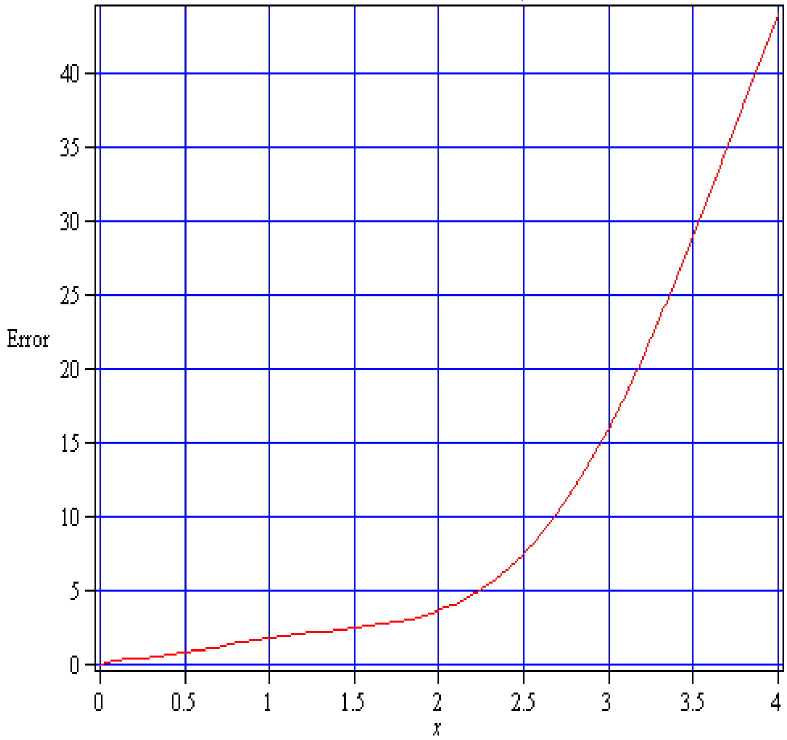
Figure (8): The h-curve of y(20) based on the six terms of HAM(Caputo)with c=2.



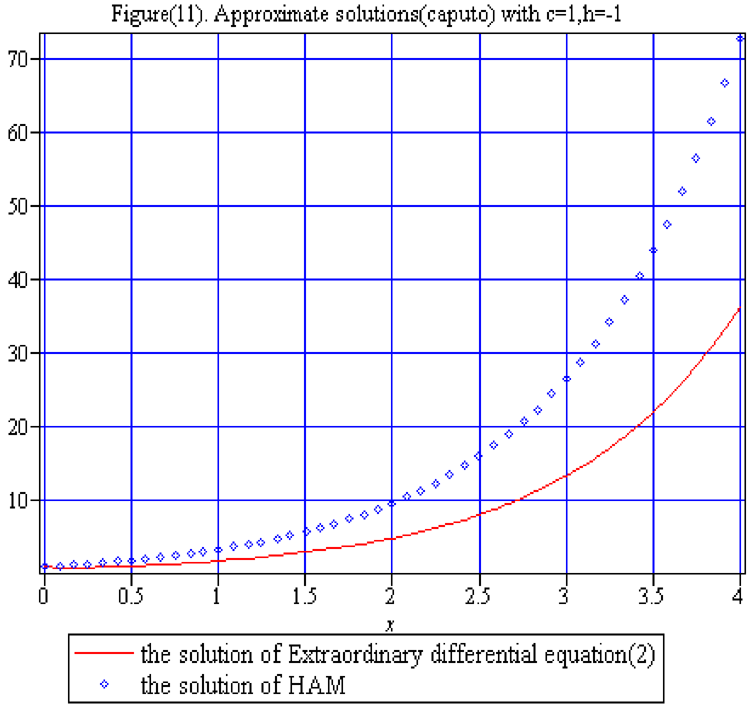
Figure(9). The absolute errors between the solutions obtained using HPM(caputo) with six terms and exact solution with c=1,h=-1



Figure(10). The absolute errors between the solutions obtained using HPM(caputo) with six terms and exact solution with c=1,h=-1.5



Figure(11). Approximate solutions(caputo) with c=1,h=-1



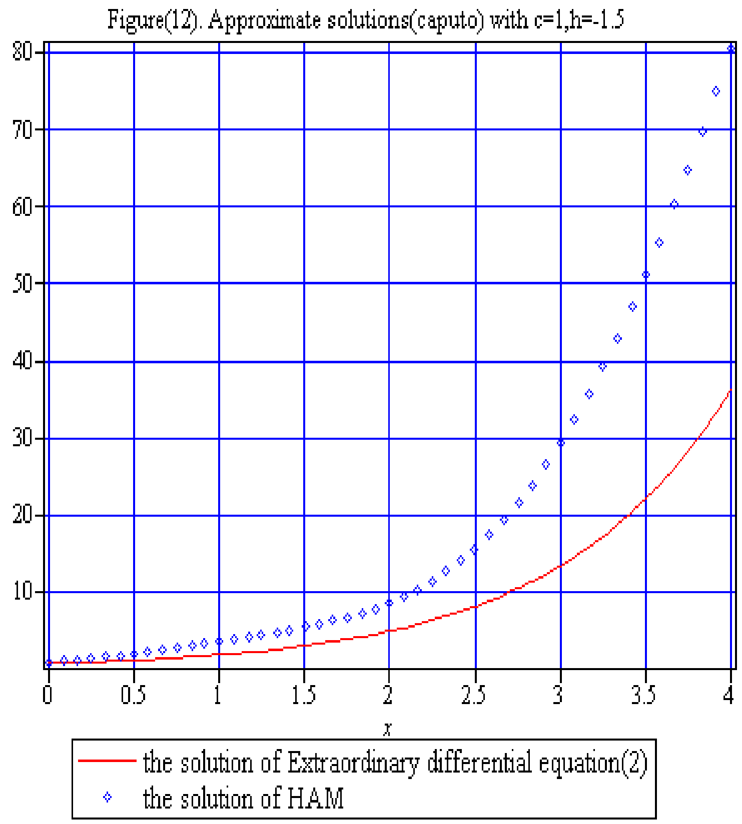


Table.2. Approximate solutions (Caputo derivative) with $c = 3$.

x	HAM($h = -1$)	HAM($h = -1.3$)	The solution of (2)
0.1	3.527130571	3.521008337	2.593975906
0.2	4.056271889	4.043310096	2.715509289
0.3	4.618243115	4.606009373	2.913393641
0.4	5.224840887	5.223084349	3.159508620
0.5	5.884826608	5.901209538	3.446693963
0.6	6.606201710	6.643784136	3.773646433
0.7	7.396984190	7.453196810	4.141510100
0.8	8.265579660	8.332372580	4.552768087
0.9	9.221006730	9.285863280	5.010813280
1	10.27305821	10.32060297	5.519771433

Conclusion:

In this work, we apply homotopy analysis method for solving Extraordinary differential equation. The presented work Riemann-Liouville derivative and Caputo derivative we get the solution using Riemann-Liouville derivative approach to the exact solution i-e

Riemann-Liouville derivative is more approiate. Computations in this paper are performed using maple 13.

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