Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.6, No.4, 2016

www.iiste.org

On Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces with Rational Inequality

P.L.Sanodia[1], A.Gupta[2], V.P.Pandey[3]

1. Department of Mathematics, IEHE Bhopal.
2. Department of Mathematics, Govt. M.V.M. Bhopal
3. Department of Mathematics, TIT Bhopal

Abstract: In this paper, we use the concepts of subcompatibility and subsequential continuity in Intuitionistic Fuzzy Metric Spaces which are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four mapstaking rational inequality.

AMS Subject Classification Codes: 47H10, 54H25

Keywords: Intuitionistic fuzzy metric space, Subcompatibility and Subsequential continuity, common fixed point theorem, implicit relation.

1. Introduction

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al.[1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [5]. Further, Alaca et al. [1] proved Intuitionistic fuzzy Banach and Intuitionisticfuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [8].Popa ([9]-[10]) introduced the idea of implicit function to prove a common fixed point theorem in metric spaces Singhand Jain [13] further extended the result of Popa ([9]-[10]) in fuzzy metric spaces. In this paper, we usethe concepts of subcompatibility and subsequential continuityin Intuitionistic Fuzzy Metric Spaces Using Implicit Relationwhich are respectively weaker than occasionally weak compatibility and reciprocal continuity. With them, we establish a common fixed point theorem for four maps.

2 Preliminary Notes

Definition 2.1.[11] A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norms if $*$ is satisfying conditions:

(i) $*$ is an commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Definition 2.2.[11] A binary operation $\Diamond$:$[0, 1] \times [0, 1] \to [0, 1]$ is a continuous t –conormif it satisfies the following conditions:

(a) $\Diamond$ is commutative and associative;

(b) $\Diamond$ is continuous;

(c) $a \Diamond 0 = a$ for all $a \in [0, 1]$

(d) $a \Diamond b = c \Diamond d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.3.[1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$;

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$

(IFM-2) $M(x, y, 0) = 0$ for all x, y in X

(IFM-3) $M(x, y, t) = 1$ for all x, y in X and $t > 0$ if and only if $x = y$

(IFM-4) $M(x, y, t) = M(y, x, t)$, for all x, y in X and $t > 0$

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(IFM-6) $M(x, y, .): [0, \infty) \to [0, 1]$ is left continuous

(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$, for all x, y in X and $t > 0$,

(IFM-8) $N(x, y, 0) = 1$ for all x, y in X
(IFM-9) $N(x, y, t) = 0$ for all x, y in X and $t > 0$ if and only if $x = y$
(IFM-10) $N(x, y, t) = N(y, x, t)$, for all x, y in X and $t > 0$

(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$

(IFM-12) $N(x, y, .): [0, \infty) \to [0, 1]$ is right continuous

(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$, for all x, y in X and $t > 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \lozenge)$ such that t- norm $*$ and t-conorm $\lozenge$ are associated that is,

$x \lozenge y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Example 2.5. Let (X,d) be a metric space. Define t -norm $a * b = \min\{a,b\}$ and t – conorm $a \lozenge b = \max\{a,b\}$ and for all $x,y \in X$ and $t > 0$.

$M_d(x,y,t) = \frac{t}{t+d(x,y)}$, $N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$

Then $(X, M, N, *, \lozenge)$ is an IFM – space and theintuitionistic fuzzy metric space (M, N)induced by the metric d isoften referred to as the standard intuitionistic fuzzy metric.

Remark 2.6.[1] In intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$, M(x, y, .) is non -decreasing and N(x, y,.) is non-increasing for all x, y in X.

Definition 2.7.[1] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$:

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$

(c) $(X, M, N, *, \lozenge)$ is said to be complete if and only if every Cauchy sequence in X is

Convergent.

Example 2.8. Let X = {1/n**:** n = 1, 2, 3 …} $\cup$ {0} and let $*$ be the continuous t-norm and $\Diamond$ be the continuous t-conorm defined by a $*$ b = a b and a $\Diamond$ b = min. {1, a+b} respectively, for all a, b $\in$ [0, 1]. For each t $\in$ (0, ∞) and x, y in X, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases} \quad \text{and} \quad N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Clearly, (X, M, N, $*$, $\Diamond$) is complete intuitionistic fuzzy metric space.

The following definition of weakly commuting mappings in intuitionistic fuzzy metric

space is given on the lines of Sessa [12].

Definition 2.9[12]. Let A and S be maps from an intuitionistic fuzzy metric space

(X, M, N, $*$, $\Diamond$)into itself. The maps A and S are said to be weakly commuting if

M (ASz, SAz, t) ≥ M (Az, Sz, t), and N (ASz, SAz, t) ≤ N (Az, Sz, t) for all z $\in$ X and t > 0

Definition 2.10[14]. Let A and S be maps from an IFM-space (X, M, N, $*$, $\Diamond$) into itself.

The maps A and S are said to be compatible if for all t >0,

$\lim_{n\to\infty}$ M (ASx_n,SAx_n, t) = 1 and $\lim_{n\to\infty}$ N (ASx_n,SAx_n, t) = 0,whenever { x_n } is asequence in X such that $\lim_{n\to\infty}$ Ax_n= $\lim_{n\to\infty}$ Sx_n= zfor some z $\in$ X.

Definition 2.11[6]: Two mappings A and S of a IFM space (X, M, N, $*$, $\Diamond$) will be called reciprocally continuous if $ASu_n \to Az$ and $SAu_n \to Sz$,whenever {u_n } is a sequence such that Au_n , $Su_n \to$ z for some z in X .

If A and S are both continuous, then they are obviously reciprocally continuous butconverse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings A and S implies their reciprocal continuity but not conversely.

Definition 2.12. Let (X, M, N, *, ◊) be a intuitionistic fuzzy metric space. A and S be selfmaps on X. A point x in X is called a coincidence point of A and S iff Ax = Sx. In thiscase, w = Ax = Sx is called a point of coincidence of A and S.

Definition 2.13. A pair of self mappings (A, S) of aIFM space(X, M, N, *, ◊)is said to be weakly compatible if they commute at the coincidence pointsi.e., if Au = Su for some u in X, then ASu = SAu.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.14[2]. Two self mappings A and S of a IFM space(X, M, N, *, ◊) are said to be occasionally weakly compatible (owc) iff there is a point xin X which is coincidence point of A and S at which A and S commute. In this paper, we weaken the above notion by introducing a new concept calledsubcompatibility just as defined by H. Bouhadjera[4] in metric space, as follows:

Definition 2.15. Let (X, M, N,*,◊) be a intuitionistic fuzzy metric space. Self maps Aand S on X are said to be subcompatibleiff there exists a sequence {x_n} in X such that

$\lim_{n\to\infty}$ Ax_n= $\lim_{n\to\infty}$ Sx_n=zfor somez $\in$ X and satisfy

$\lim_{n\to\infty}$ M (ASx_n,SAx_n, t) = 1 and $\lim_{n\to\infty}$ N (ASx_n,SAx_n, t) = 0.

Obviously, two owc maps are subcompatible, however the converse is not true in general.
The example below shows that there exist subcompatible maps which are not owc

Example 2.16. Let X $\in$ [0, ∞). For each t $\in$ (0, ∞) and x, y $\in$ X, define (M, N) by

M(x, y, t) = $\begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases}$ and N(x, y, t) = $\begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$

Define A and S as follows:

A(x) = x^2, S(x) = $\begin{cases} x + 2\ if x \in [0, 4] \cup (9, \infty) \\ x + 12\ if x \in (4, 9] \end{cases}$

Let $\{x_n\}$ be a sequence in X defined by $x_n = 2 + \frac{1}{n}$ for n = 1, 2, 3….

Then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 4$, $4 \in X$ and

$ASx_n \to 16$, $SAx_n \to 16$

When $n\to\infty$. Thus, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$.

i.e. A and S are subcompatible. On the other hand, we have

Ax = Sxiff x = 2 and AS (2) ≠ SA (2), hence A and S are not owc.

Now, our second objective is to introduce subsequential continuity in intuitionistic fuzzymetric space which weakens the concept of reciprocal continuity which was introduced byPant [8] just as introduced by H. Bouhadjera[4] in metric space, as follows:

Definition 2.17. Let (X, M, N,*, ◊) be a intuitionistic fuzzy metric space. Self maps A andS on X are said to be subsequentially continuous iff there exist a sequence $\{x_n\}$ in X suchthat

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$ and satisfy $\lim_{n\to\infty} ASx_n = At$, $\lim_{n\to\infty} SAx_n = St$.

Clearly, if A and S are continuous or reciprocally continuous then they are obviouslysubsequentially continuous. The next example shows that there exist subsequentialcontinuous pairs of maps which are neither continuous nor reciprocally continuous.

Example 2.18. Let $X \in [0, \infty)$. For each $t \in (0, \infty)$ and $x, y \in X$, define (M, N) by

M(x, y, t) = $\begin{cases} \frac{t}{t+|x-y|}, & t > 0 \\ 0, & t = 0 \end{cases}$ and N(x, y, t) = $\begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$

Define A and S as follows:

A(x) = $\begin{cases} 1 + x if x \in [0,1] \\ 2x - 1\ if x \in (1, \infty) \end{cases}$, S(x) = $\begin{cases} 1 - x if x \in [0, 1) \\ 3x - 2\ if x \in [1, \infty). \end{cases}$

Clearly A and S are discontinuous at x = 1.

Let $\{x_n\}$ be a sequence in X defined by $x_n = \frac{1}{n}$ for n = 1, 2, 3…

Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, $1 \in X$ and

$ASx_n \to 2 = A(1)$, $SAx_n \to 1 = S(1)$ when $n\to\infty$, therefore,A and S are subsequentialcontinuous.
Now, let $\{x_n\}$ be a sequence in X defined by $x_n = 1 + \frac{1}{n}$ for n = 1, 2, 3…

Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, $1 \in X$ and

$ASx_n \to 1 \neq 2 = A(1)$ when $n\to\infty$, so A and S are not reciprocally continuous.

Lemma 2.19. Let $\{u_n\}$ be a sequence in an intuitionistic fuzzy metric space(X, M, N,*, ◊).
If there exists a constant $k \in (0, 1)$ such that
$M(u_n, u_{n+1}, kt) \geq M(u_{n-1},u_n,t)$ and $N(u_n, u_{n+1}, kt) \leq N(u_{n-1},u_n,t)$ for all t > 0 and n = 1, 2, 3…..
Then $\{u_n\}$ is a Cauchy sequence in X.

3. Implicit Relation

Let M_4 be the setof all real continuous functions $\emptyset, \Psi$:$[0, 1]^3 \to R$, non-decreasing in the first argument and satisfyingthe following conditions:

A. $\emptyset(u, u, 1) \geq 0 \Rightarrow u \geq 1$
B. $\Psi(u, u, 0) \leq 0 \Rightarrow u \leq 0$ $for\ all\ u.$

4. Main Result

Now, we prove ours main theorem using definition of subcompatible and subsequential continuous maps as follows:

Theorem 4.1. Let A, B, S and T be four self maps of a Intuitionistic fuzzy metric space (X, M, N,*, ◊) with continuous t-norm * and continuous t-conorm ◊ defined by t*t ≥ tand (1-t)◊ (1-t) ≤ (1-t) for all t$\in [0, 1]$. If the pairs (A, S) and (B, T) are subcompatible and subsequentially continuous, then

(a) A and S have a coincidence point.

(b) B and T have a coincidence point.

(c) For some $\emptyset, \Psi \in M_4$ and for all x, y ∈ X and every t > 0.

$$\emptyset\left\{\begin{matrix}\dfrac{aM(Ax, By, t) + bM(Sx, By, t)}{a + b}, \dfrac{cM(Sx, Ty, t) + dM(Ty, Ax, t)}{c + d}, \\ \dfrac{M(Sx, Ax, t) + M(Ty, By, t)}{2}\end{matrix}\right\} \geq 0$$

$$\Psi\left\{\begin{matrix}\dfrac{aN(Ax, By, t) + bN(Sx, By, t)}{a + b}, \dfrac{cN(Sx, Ty, t) + dN(Ty, Ax, t)}{c + d}, \\ \dfrac{N(Sx, Ax, t) + N(Ty, By, t)}{2}\end{matrix}\right\} \leq 0$$

Then A, B, S, T have a unique common fixed point.

Where a and b or ($c$and d) can not be simultaneously zero.

Proof: Since the pair (A, S) and (B, T) are aresubcompatible and subsequentially continuous, then, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, $z \in X$ and satisfy

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = N(Az, Sz, t) = 0$.

$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z'$, $z' \in X$ and satisfy

$\lim_{n\to\infty} M(BTy_n, TBy_n, t) = M(Bz', Tz', t) = 1$ and $\lim_{n\to\infty} N(BTy_n, TBy_n, t) = N(Bz', Tz', t) = 0$.

Therefore, Az = Sz and $Bz' = Tz'$; that is, z is a coincidence point of A and S and z' is a coincidence point of B and T.

Now, we prove z = z'.

Put x = x_n and y = y_n in inequality (c), we get

$$\emptyset\left\{\begin{matrix}\frac{aM(Ax_n, By_n, t) + bM(Sx_n, By_n, t)}{a+b}, \frac{cM(Sx_n, Ty_n, t) + dM(Ty_n, Ax_n, t)}{c+d}, \\ \frac{M(Sx_n, Ax_n, t) + M(Ty_n, By_n, t)}{2}\end{matrix}\right\} \geq 0$$

taking the limit as $n \to \infty$ yields

$$\emptyset\left\{\begin{matrix}\frac{aM(z,z',t) + bM(z,z',t)}{a+b}, \frac{cM(z,z',t) + dM(z',z,t)}{c+d}, \\ \frac{M(z,z,t) + M(z',z',t)}{2}\end{matrix}\right\} \geq 0$$

$\emptyset\{ M(z,z',t), M(z,z',t), 1\} \geq 0$

$$\Psi\left\{\begin{matrix}\frac{aN(Ax_n, By_n, t) + bN(Sx_n, By_n, t)}{a+b}, \frac{cN(Sx_n, Ty_n, t) + dN(Ty_n, Ax_n, t)}{c+d}, \\ \frac{N(Sx_n, Ax_n, t) + N(Ty_n, By_n, t)}{2}\end{matrix}\right\} \leq 0$$

$$\Psi\left\{\begin{matrix}\frac{aN(z,z',t) + bN(z,z',t)}{a+b}, \frac{cN(z,z',t) + dN(z',z,t)}{c+d}, \\ \frac{N(z,z,t) + N(z',z',t)}{2}\end{matrix}\right\} \leq 0$$

$\Psi\{ N(z,z',t), N(z,z',t), 0\} \leq 0$

In view of $\emptyset, \Psi$ we get z = z'

Again, we claim that Az = z.

Put x = z and y = y_n in inequality (c), we get

$$\emptyset\left\{\begin{matrix}\frac{aM(Az, By_n, t) + bM(Sz, By_n, t)}{a+b}, \frac{cM(Sz, Ty_n, t) + dM(Ty_n, Az, t)}{c+d}, \\ \frac{M(Sz, Az, t) + M(Ty_n, By_n, t)}{2}\end{matrix}\right\} \geq 0$$

$$\emptyset\left\{\begin{matrix}\frac{aM(Az, z', t) + bM(Az, z', t)}{a+b}, \frac{cM(Az, z', t) + dM(z', Az, t)}{c+d}, \\ \frac{M(Az, Az, t) + M(z', z', t)}{2}\end{matrix}\right\} \geq 0$$

$\emptyset\{M(Az, z', t), M(Az, z', t), 1 \} \geq 0$

$$\Psi\left\{\begin{matrix}\frac{aN(Az, By_n, t) + bN(Sz, By_n, t)}{a+b}, \frac{cN(Sz, Ty_n, t) + dN(Ty_n, Az, t)}{c+d}, \\ \frac{N(Sz, Az, t) + N(Ty_n, By_n, t)}{2}\end{matrix}\right\} \leq 0$$

$$\Psi\left\{\begin{matrix}\frac{aN(Az,z',t)+bN(Az,z',t)}{a+b},\frac{cN(Az,z',t)+dN(z',Az,t)}{c+d},\\ \frac{N(Az,Az,t)+N(z',z',t)}{2}\end{matrix}\right\}\le 0$$

$\Psi\{N(Az,z',t),N(Az,z',t),0\}\le 0$

In view of $\emptyset,\Psi$ we get Az = z'= z

Again, we claim that Bz = z.

Put x = z and y = z in inequality (c), we get

$$\emptyset\left\{\begin{matrix}\frac{aM(Az,Bz,t)+bM(Sz,Bz,t)}{a+b},\frac{cM(Sz,Tz,t)+dM(Tz,Az,t)}{c+d},\\ \frac{M(Sz,Az,t)+\ M(Tz,Bz,t)}{2}\end{matrix}\right\}\ge 0$$

$$\emptyset\left\{\begin{matrix}\frac{aM(z,Bz,t)+bM(z,Bz,t)}{a+b},\frac{cM(z,Bz,t)+dM(Bz,z,t)}{c+d},\\ \frac{M(z,z,t)+\ M(Bz,Bz,t)}{2}\end{matrix}\right\}\ge 0$$

$\emptyset\{\ M(z,Bz,t),M(z,Bz,t),1\}\ge 0$

$$\Psi\left\{\begin{matrix}\frac{aN(Az,Bz,t)+bN(Sz,Bz,t)}{a+b},\frac{cN(Sz,Tz,t)+dN(Tz,Az,t)}{c+d},\\ \frac{N(Sz,Az,t)+\ N(Tz,Bz,t)}{2}\end{matrix}\right\}\le 0$$

$$\Psi\left\{\begin{matrix}\frac{aN(z,Bz,t)+bN(z,Bz,t)}{a+b},\frac{cN(z,Bz,t)+dN(Bz,z,t)}{c+d},\\ \frac{N(z,z,t)+\ N(Bz,Bz,t)}{2}\end{matrix}\right\}\le 0$$

$\Psi\{\ N(z,Bz,t),N(z,Bz,t),0\}\le 0$

In view of $\emptyset,\Psi$ we get z = Bz= Tz

Therefore, z = Az = Bz = Sz = Tz, that is z is common fixed point of A, B, S and T.

For Uniqueness: Suppose that there exist another fixed point w of A, B, S and T.

By condition (c), take x = z, y = w, we have

$$\emptyset\left\{\begin{matrix}\frac{aM(Az,Bw,t)+bM(Sz,Bw,t)}{a+b},\frac{cM(Sz,Tw,t)+dM(Tw,Az,t)}{c+d},\\ \frac{M(Sz,Az,t)+\ M(Tw,Bw,t)}{2}\end{matrix}\right\}\ge 0$$

$$\emptyset\left\{\begin{matrix}\frac{aM(z,w,t)+bM(z,w,t)}{a+b},\frac{cM(z,w,t)+dM(w,z,t)}{c+d},\\ \frac{M(z,z,t)+\ M(w,w,t)}{2}\end{matrix}\right\}\ge 0$$

$\emptyset\{\ M(z,w,t),M(z,w,t),1\}\ge 0$

$$\Psi\left\{\begin{matrix}\frac{aN(Az,Bw,t)+bN(Sz,Bw,t)}{a+b},\frac{cN(Sz,Tw,t)+dN(Tw,Az,t)}{c+d},\\ \frac{N(Sz,Az,t)+\ N(Tw,Bw,t)}{2}\end{matrix}\right\}\leq 0$$

$$\Psi\left\{\begin{matrix}\frac{aN(z,w,t)+bN(z,w,t)}{a+b},\frac{cN(z,w,t)+dN(w,z,t)}{c+d},\\ \frac{N(z,z,t)+\ N(w,w,t)}{2}\end{matrix}\right\}\leq 0$$

$\Psi\{\ N(z,w,t),N(z,w,t),0\}\leq 0$

In view of $\emptyset,\Psi$ we get z = w.Therefore, uniqueness follows.

If we put S = T, in Theorem 4.1, we get the following result:

Corollary 4.2. Let A, B and S be threeself maps of a Intuitionistic fuzzy metric space

(X, M, N,*, ◊) with continuous t-norm * and continuous t-conorm ◊ defined by t*t ≥ tand

(1-t)◊ (1-t) ≤ (1-t) for all t ∈ $[0,1]$. If the pairs (A, S) and (B, S) are subcompatible and

subsequentially continuous, then

(a) A and S have a coincidence point.

(b) B and S have a coincidence point.

(c) For some $\emptyset,\Psi\in\ M_4$ and for all x, y ∈ X and every t > 0.

$$\emptyset\left\{\begin{matrix}\frac{aM(Ax,By,t)+bM(Sx,By,t)}{a+b},\frac{cM(Sx,Sy,t)+dM(Sy,Ax,t)}{c+d},\\ \frac{M(Sx,Ax,t)+\ M(Sy,By,t)}{2}\end{matrix}\right\}\geq 0$$

$$\Psi\left\{\begin{matrix}\frac{aN(Ax,By,t)+bN(Sx,By,t)}{a+b},\frac{cN(Sx,Sy,t)+dN(Sy,Ax,t)}{c+d},\\ \frac{N(Sx,Ax,t)+\ N(Sy,By,t)}{2}\end{matrix}\right\}\leq 0$$

Then A, B, S have a unique common fixed point.

Where a and b or ($c$and d) can not be simultaneously zero.

References

[1] C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, Chaos, Solitons& Fractals, 29 (2006), 1073-1078.

[2] M.A. Al-Thagafi, N. Shahzad, Generalised I-nonexpansiveself maps and invariants approximations. Acta Math. Sin, 24 (5) (2008), 867-876.

[3] K. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and system, 20 (1986) 87-96.

[4].H. Bouhadjera and C. Godet-Thobie, Common fixed point theorems for pairs of subcompatible maps,arXiv: 0906.3159v1 [math.FA] 17 June 2009.

[5].I.Kramosil and J. Michalek,Fuzzy metric and statistical metric spaces,Kybernetica 11 (1975), 336-344.

[6] S. Muralisankar and G. Kalpana, Common fixed point theorems in intuitionistic fuzzy metric spaces using reneral contractive condition of integral type, Int. J. Contemp. Math. Sciences, 4 (11) (2009), 505-518.

[7].R. P. Pant and K. Jha, A remark on common fixed points of four mappings in a fuzzy metric space, J. Fuzzy Math. 12 (2), (2004), 433-437.

[8] J.H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons& Fractals, 22(2004) 1039-1046.

[9]. Popa, V. 1999. Some fixed point theorems for compatible mappings satisfying on implicit relation, Demonsratio Math., 32, 157 – 163

[10].Popa, V. 2000. A general coincidence theorem for compatible multivalued mappings satisfying an implicit relation, Demonsratio Math., 33, 159-164.

[11].Schweizer, B. , and Sklar, A. 1960. Statistical metric spaces, Pacific J. Math., 10, 313-334.

[12].S. Sessa, On a weak commutativity condition of mappings in fixed point considerations, Publ. Inst. Math.(Beogard) (N.S.), 32(46) (1982), 149-153 .

[13].B. Singh, A. Jain and A.A. Masoodi, Semi-compatibility, weak compatibility and fixed point theorem in fuzzy metric space, International Mathematical Forum, 5(61),(2010), 3041-3051.

[14] D. Turkoglu, C. Alaca and C. Yildiz, Compatible maps and Compatible maps of type (α) and (β) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39 (3) (2006), 671-684.

[15]. L. A. Zadeh, Fuzzy sets, Inform. and control 89 (1965), 338-353.